# Parameterized Deconvolution for Wide-Band Radio Synthesis Imaging

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Ph.D. Thesis Defense

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# Goals of this thesis

(1) Develop an image reconstruction algorithm for broad-band radio interferometry

- Model and reconstruct the amplitude and spectrum of the sky brightness
  ( solve for the parameters of a multi-scale multi-frequency image model )

- Account for the frequency dependence of the telescope
  ( spatial-frequency coverage and the antenna primary beam )

(2) Study the broad-band synchrotron spectrum of the M87 radio galaxy

- Measure the broad-band spectrum across the M87 radio halo
  ( new data between 1.1 and 1.8 GHz, plus existing images at 74 MHz, 327 MHz, 1.4 GHz )

- Calculate synchrotron ages and compare with dynamical ages
  ( compare measured spectra with synchrotron ageing models )

# Outline

- Introduction : Imaging with a broad-band radio interferometer

- Algorithms : Multi-scale multi-frequency wide-field imaging

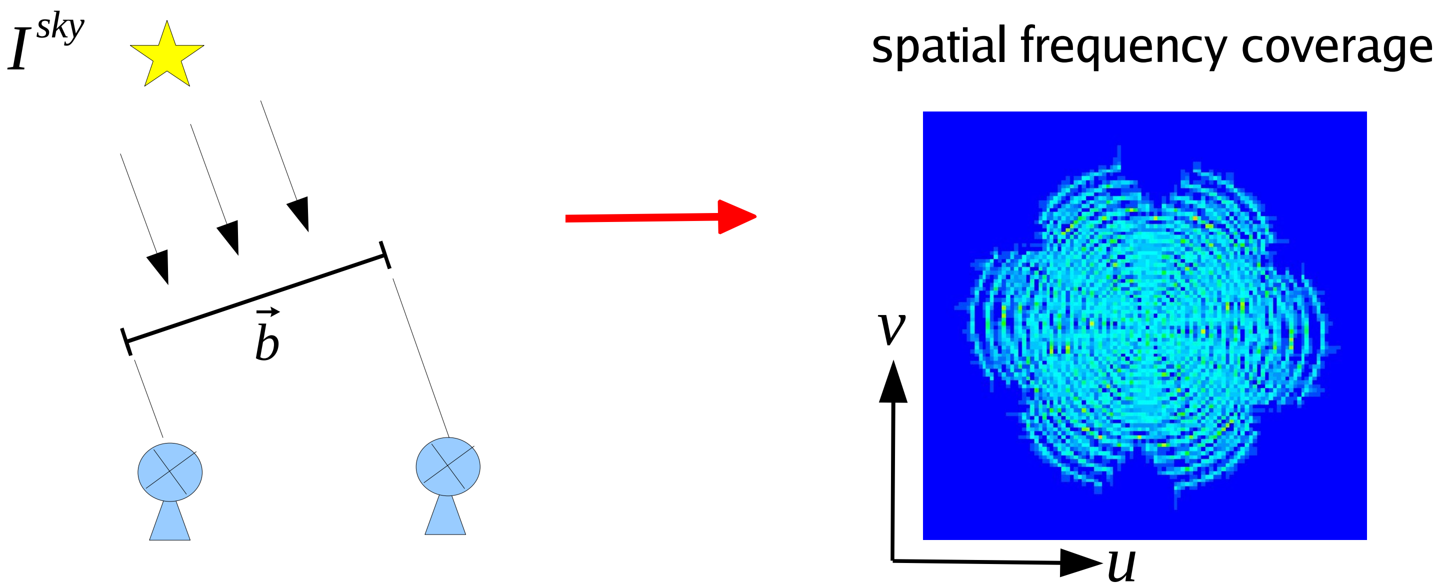
- Examples : Imaging results, feasibility study and errors

- Astrophysics : A study of the broad-band spectrum across the M87 radio halo

- Conclusion : Dissertation summary and next steps

# Imaging with a radio interferometer - 1

An interferometer measures the spatial Fourier transform of the "sky brightness"




$$[S] = \frac{\vec{b}}{\lambda}$$

Measurement Equation :

$$V = [S][F]\, I^{sky}$$

Data

Spatial frequency coverage

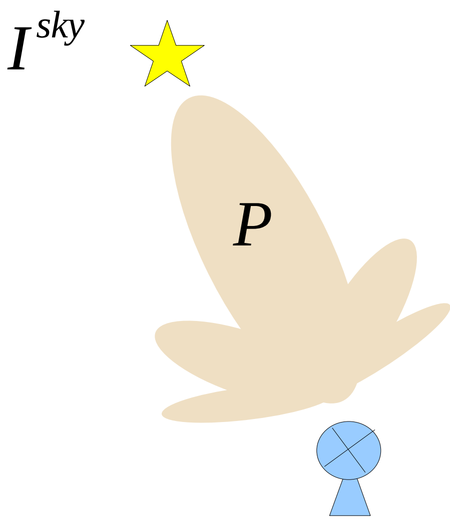
Fourier transform

Sky Brightness

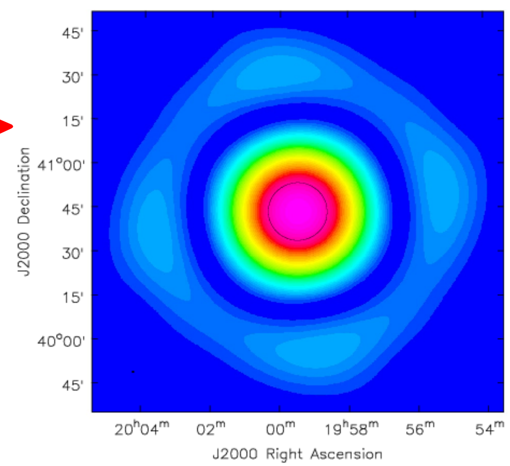
Solve this equation to reconstruct $I^{sky}$ from $V$

# Imaging with a radio interferometer - 2

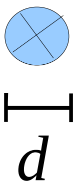
## An interferometer measures the spatial Fourier transform of the "observed sky"




Antenna primary beam




$P$ : Intensity of the diffraction pattern of the antenna aperture




$$\text{HPBW} = \frac{\lambda}{d}$$

Measurement Equation :

$$V = [S][F](P \cdot I^{sky})$$

Data

Spatial frequency coverage

Fourier transform

Primary Beam

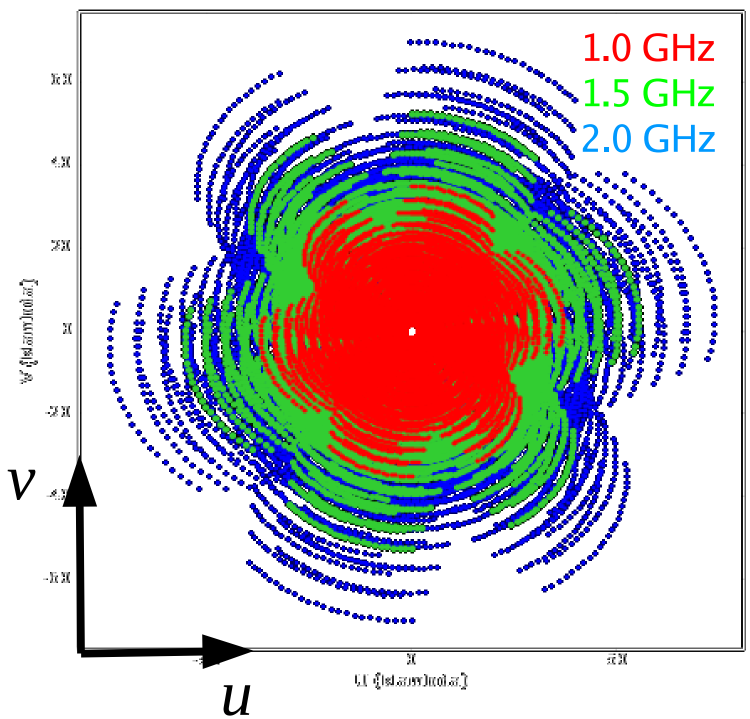
Sky Brightness

Solve this equation to reconstruct $P \cdot I^{sky}$ from $V$

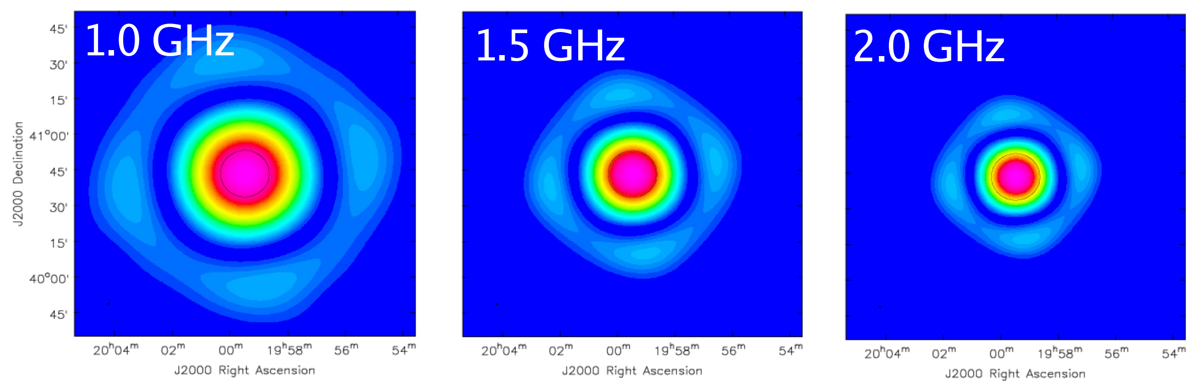
# Imaging with a broad-band radio interferometer

## Spatial-frequency coverage and antenna primary beam vary with frequency

EVLA multi-frequency uv-coverage




EVLA multi-frequency primary beams




Multi-Frequency Synthesis (MFS)

“combine data from all channels and make a single image of total intensity”

MFS increases imaging sensitivity and fidelity

..... but, need to model both spatial and spectral structure...

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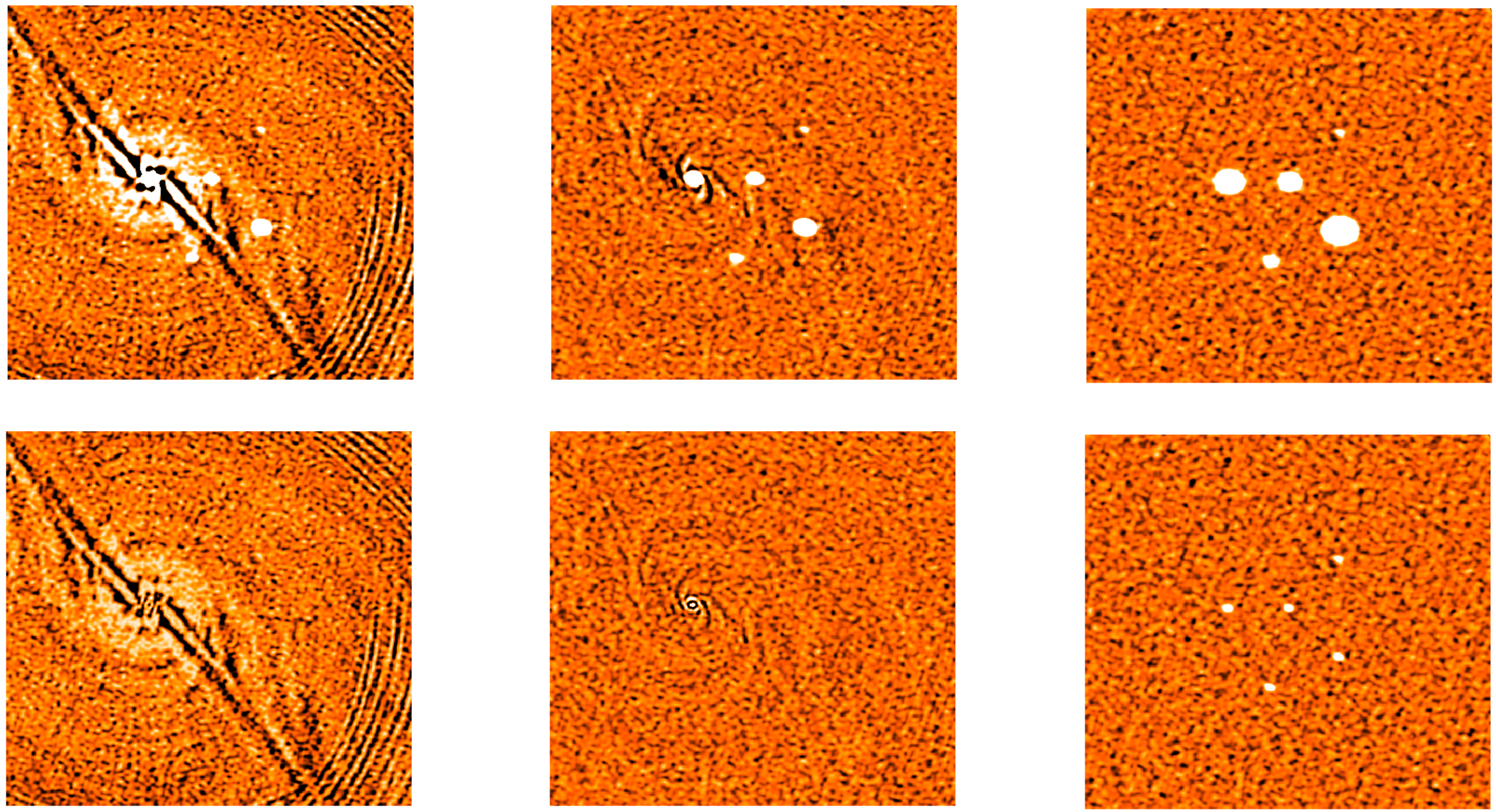
# Comparison of Existing Wide-band Imaging Methods

*EVLA Memo 101, Rau & Cornwell, 2006*

CLEAN with MFS | Multi-Frequency MF-CLEAN (Miriad) | Single-channel CLEAN

Restored Image

Residual Image



dynamic range ~ $10^3$ | dynamic range ~ $10^5$ | dynamic range ~ $10^5$

- more errors with extended emission
- no wide-field (primary-beam) corrections

# Evolution of relevant imaging algorithms

| | CLEAN *(Hogbom,1974, Clark,1980, Schwab & Cotton, 1983)* | Multi-Scale (MS) CLEAN *(Cornwell, 2008)* | Multi-Frequency (MF) CLEAN *(Sault & Wieringa, 1994)* | Wide-Field A-Projection *(Bhatnagar, Cornwell, Golap, Uson, 2008)* | This thesis project |
|---|---|---|---|---|---|
| Point source model | Yes | Yes | Yes | Yes | Yes |
| Multi-scale source model | ~ No | Yes | ~ No | Yes | Yes |
| Spectral flux model | No | No | Yes | ~ No | Yes |
| Primary-beam correction | No | No | No | Yes | Yes |

# MS-MFS : Multi-Scale Multi-Frequency Synthesis

## Multi-Frequency Spectral Model

Taylor polynomial with multi-scale coefficient images

$$I_\nu^{sky} = \sum_t I_t \left( \frac{\nu - \nu_0}{\nu_0} \right)^t \quad \text{where} \quad I_t = \sum_s \left[ I_s^{shp} * I_{s,t} \right]$$

Model :

A collection of "blobs" whose amplitudes follow a polynomial in frequency

↓

Solve for $I_{s,t}$ (linear least squares) and calculate Taylor coefficients $I_t$

↓

Interpret the Taylor coefficients :
Power Law with varying index

$$I_\nu^{sky} = I_{\nu_0}^{sky} \left( \frac{\nu}{\nu_0} \right)^{\alpha + \beta \log(\nu/\nu_0)} \longrightarrow$$

$$I_0 = I_{\nu_0}^{sky}$$

$$I_1 = I_{\nu_0}^{sky} \alpha$$

$$I_2 = I_{\nu_0}^{sky} \left( \frac{\alpha(\alpha - 1)}{2} + \beta \right)$$

Output :

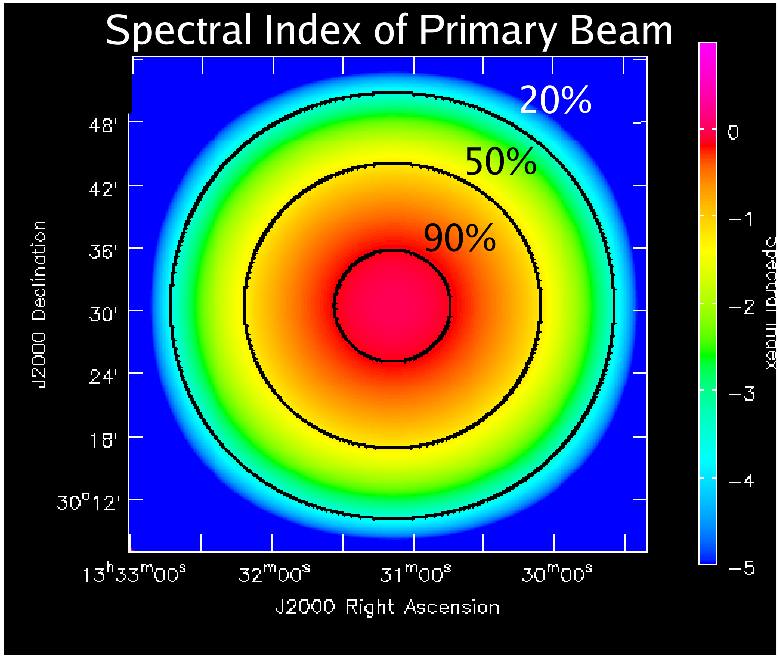
(1) List of coefficient images

(2) Intensity image
Spectral index image
Spectral curvature image

# MS-MFS with primary-beam correction

Frequency dependence of the Primary Beam

$$P_\nu = \sum_t P_t \left( \frac{\nu - \nu_0}{\nu_0} \right)^t \rightarrow P_\nu = P_{\nu_0} \left( \frac{\nu}{\nu_0} \right)^{\alpha_P + \beta_P \log(\nu/\nu_0)} \rightarrow$$




Include the primary beam in the wide-band flux model

$$I_\nu^{model} = I_{\nu_0}^{sky} P_{\nu_0} \left( \frac{\nu}{\nu_0} \right)^{[\alpha + \alpha_P] + [\beta + \beta_P] \log(\nu/\nu_0)}$$

Run MS-MFS

Remove the primary beam from Taylor coefficients

Two ways to remove the antenna primary beam

(1) After MS-MFS imaging
( average primary beam )

(2) During MS-MFS imaging
( time-varying primary beams)
(A-Projection)

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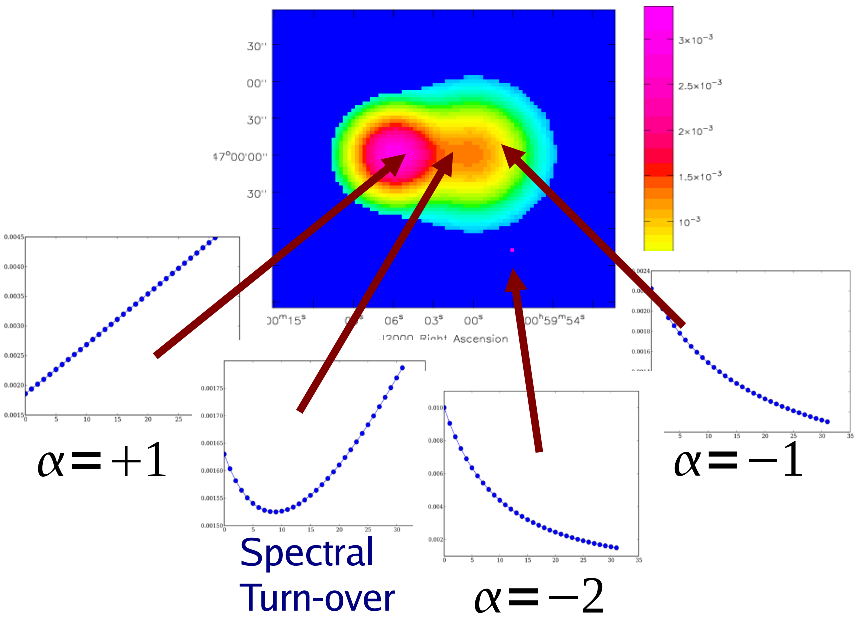
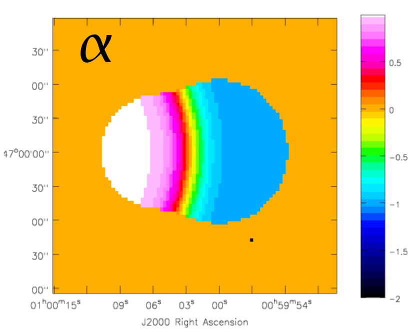
# Example : MS-MFS on simulated EVLA data (1-2 GHz)

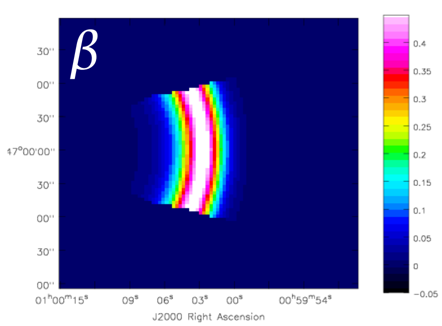
## Intensity Image




## Average Spectral Index

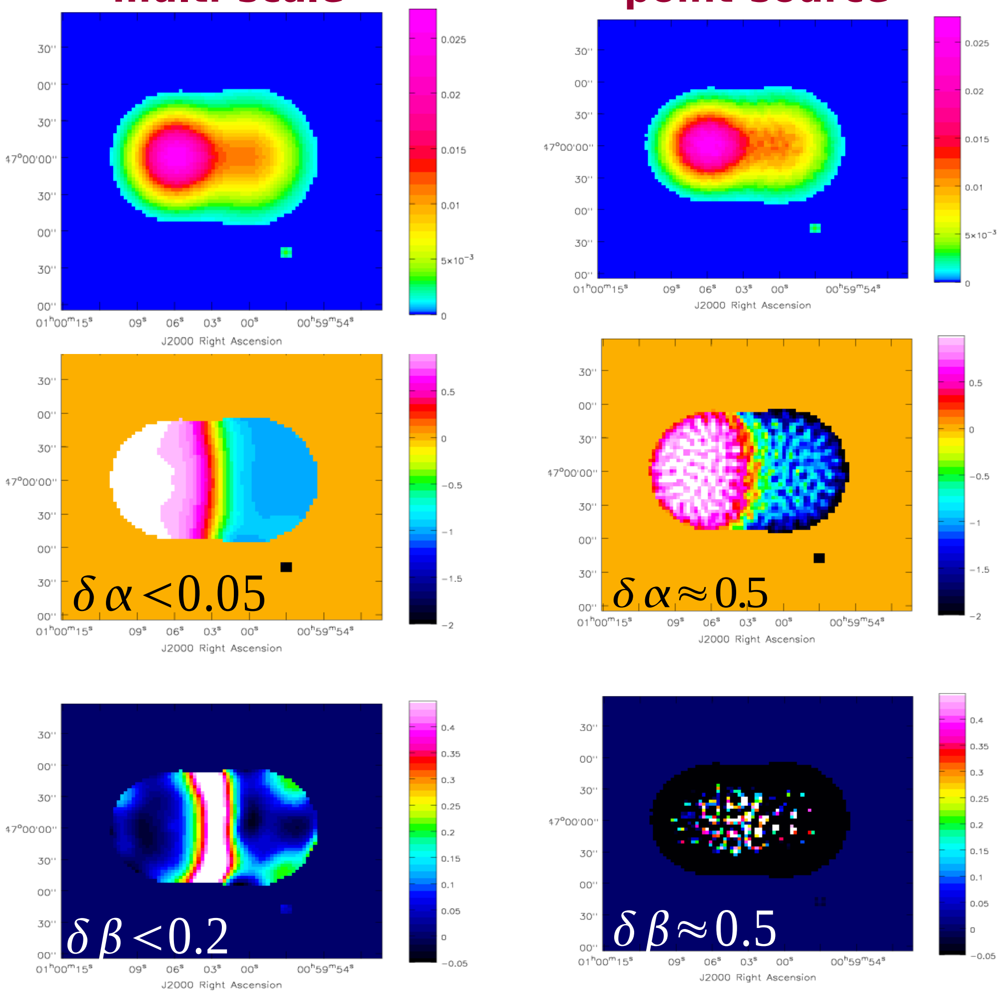



## Gradient in Spectral Index



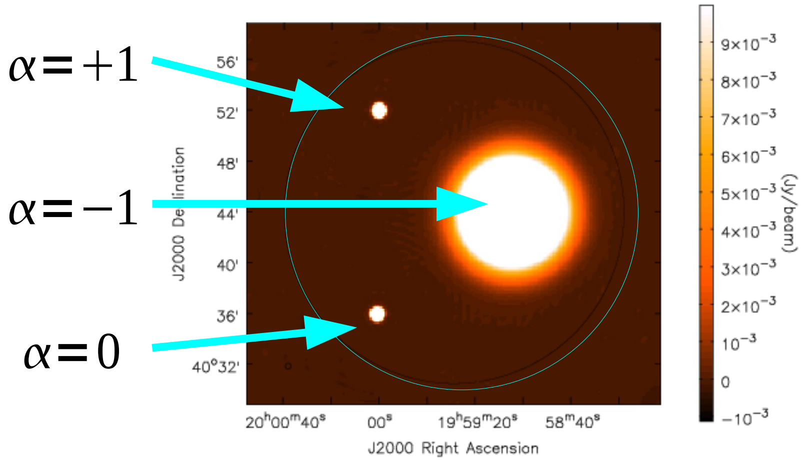

**MFS**

**multi-scale** **point-source**

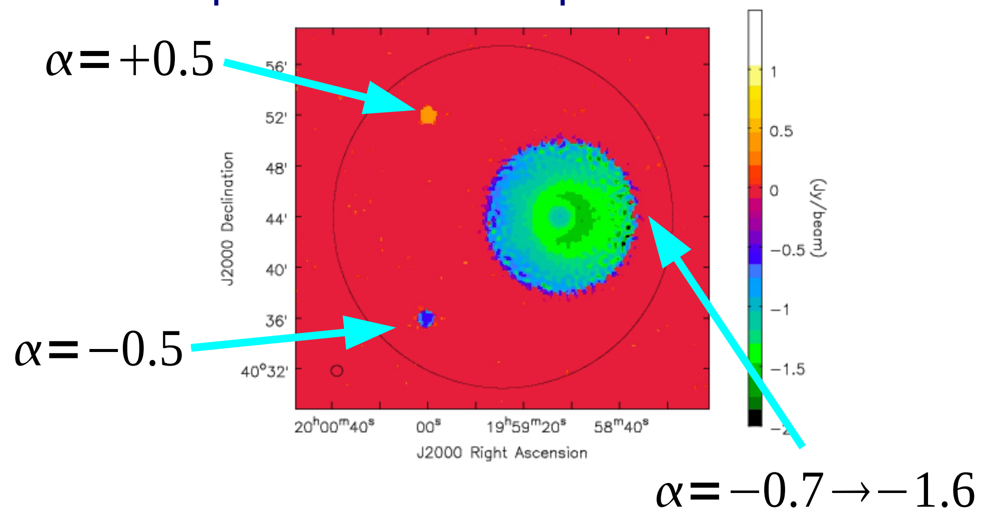


# Example : MS-MFS with Primary-Beam correction

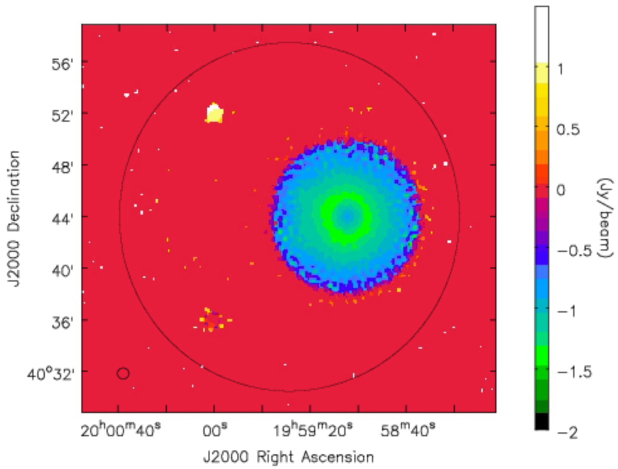
Deconvolved Stokes I image at Ref-Freq.



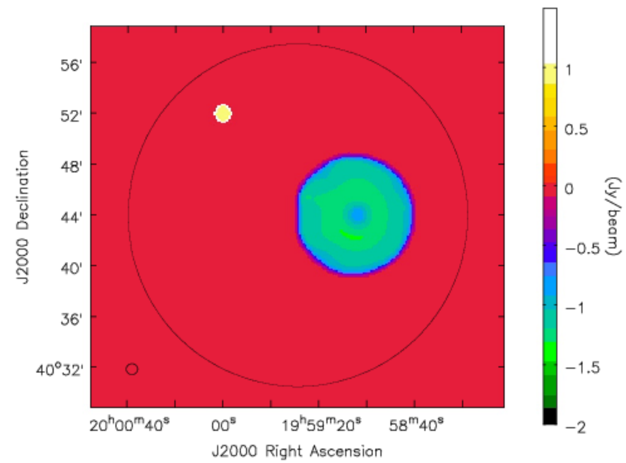

Spectral Index –NO pb-correction




Spectral Index WITH primary beam correction




(1) After MS-MFS Imaging



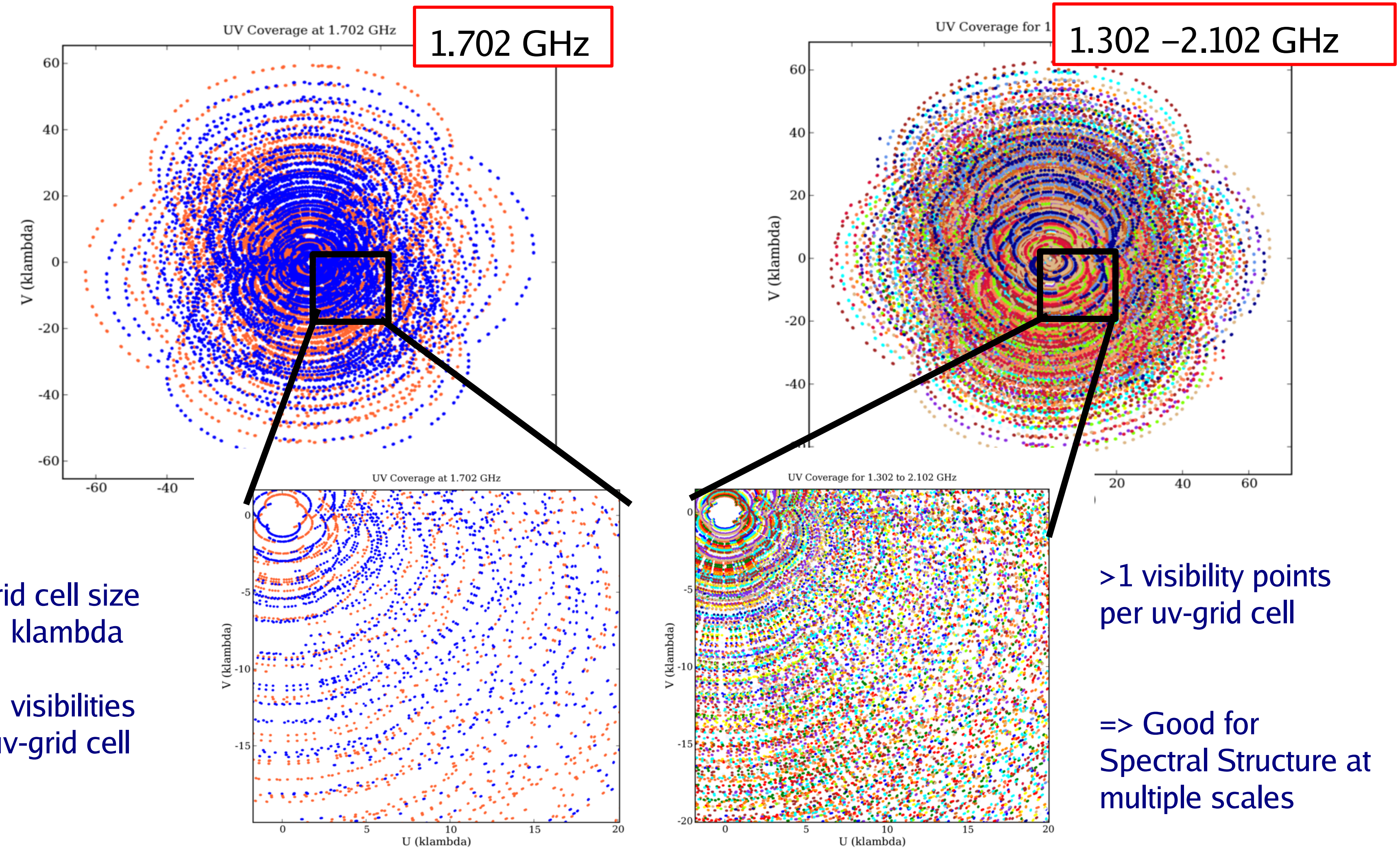

(2) During MS-MFS Imaging

“ remove an average primary beam and the average frequency dependence “

“ remove a time-varying primary beam and its frequency dependence “

# VLA spatial-frequency coverage (observation of Cygnus A)

( Cycle through 9 frequency bands, 20 one-minute snapshots per band, spread over 8 hours)




uv-grid cell size
~ 0.2 klambda

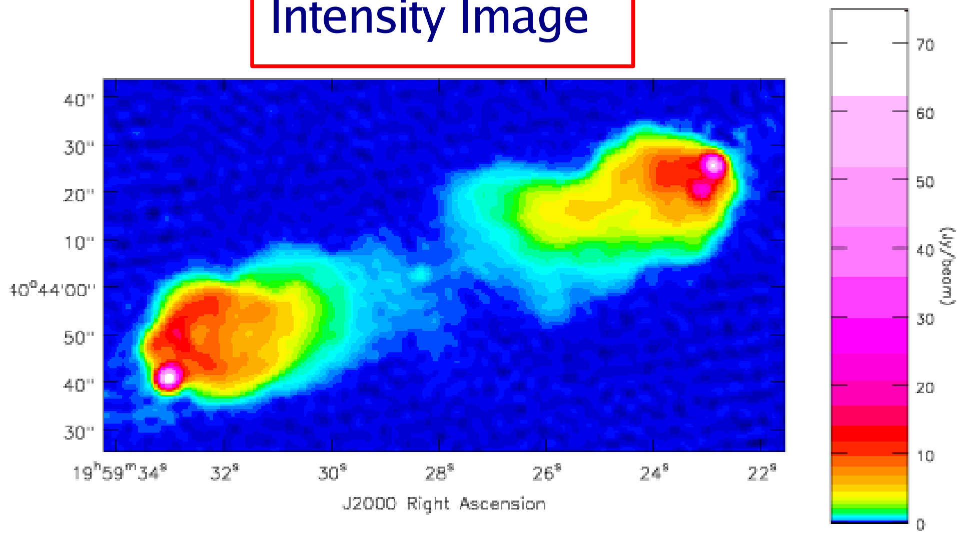
< 0.5 visibilities
per uv-grid cell

>1 visibility points
per uv-grid cell

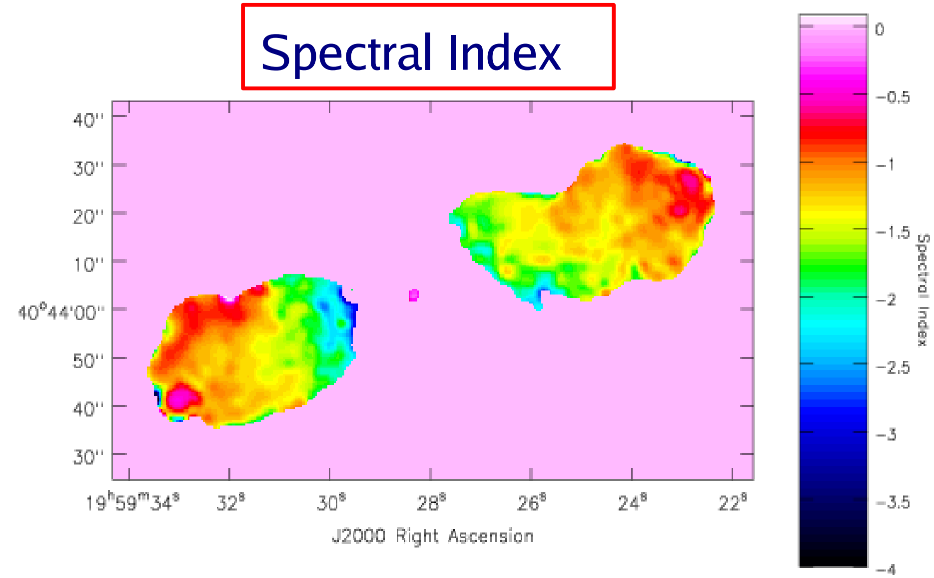
=> Good for
Spectral Structure at
multiple scales

# Example : Cygnus A (Stokes I, Spectral Index)

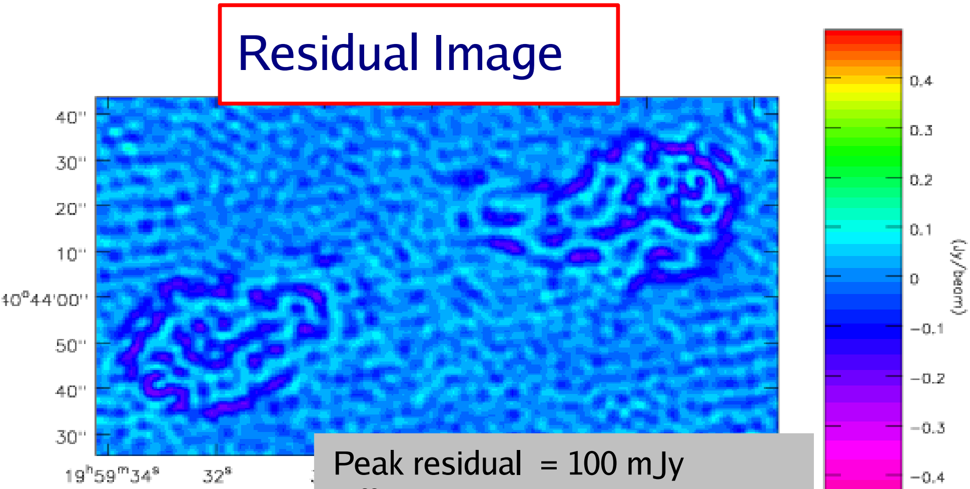
Intensity Image




Spectral Index




Residual Image



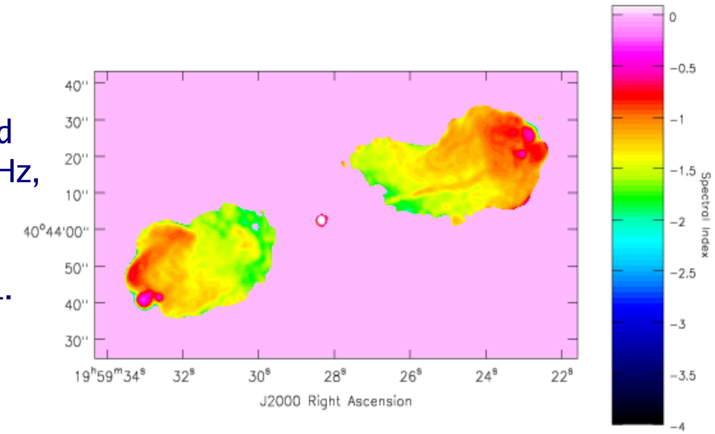

Peak residual = 100 mJy
Off-source rms = 30 mJy

- Has detail and fidelity of Multi-Scale deconvolution
- Error on estimated spectral index <= 0.2

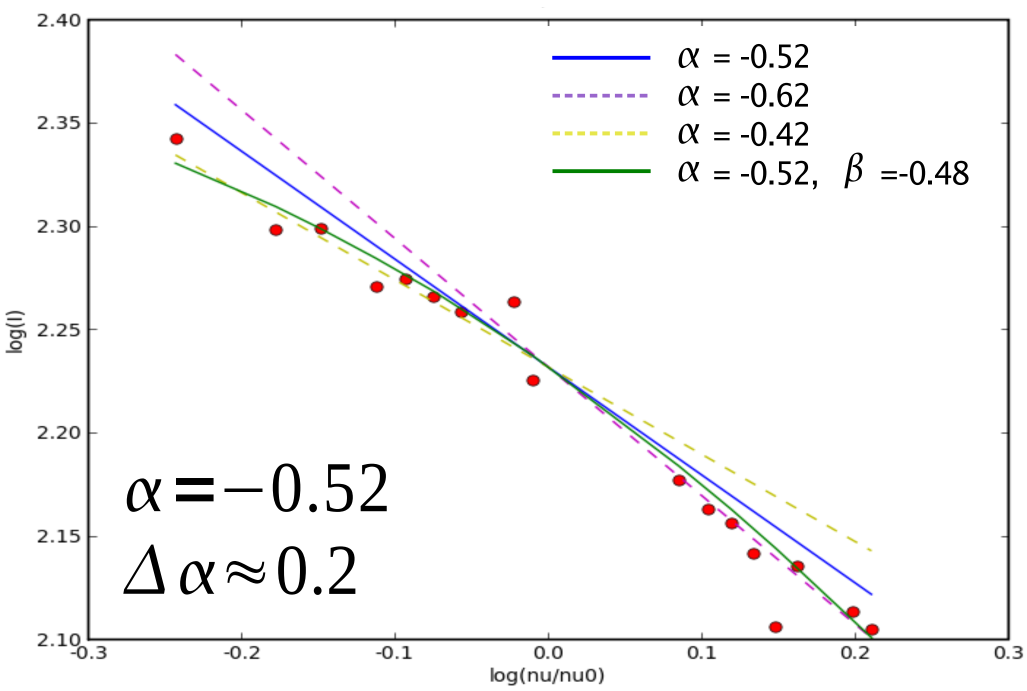
For comparison :

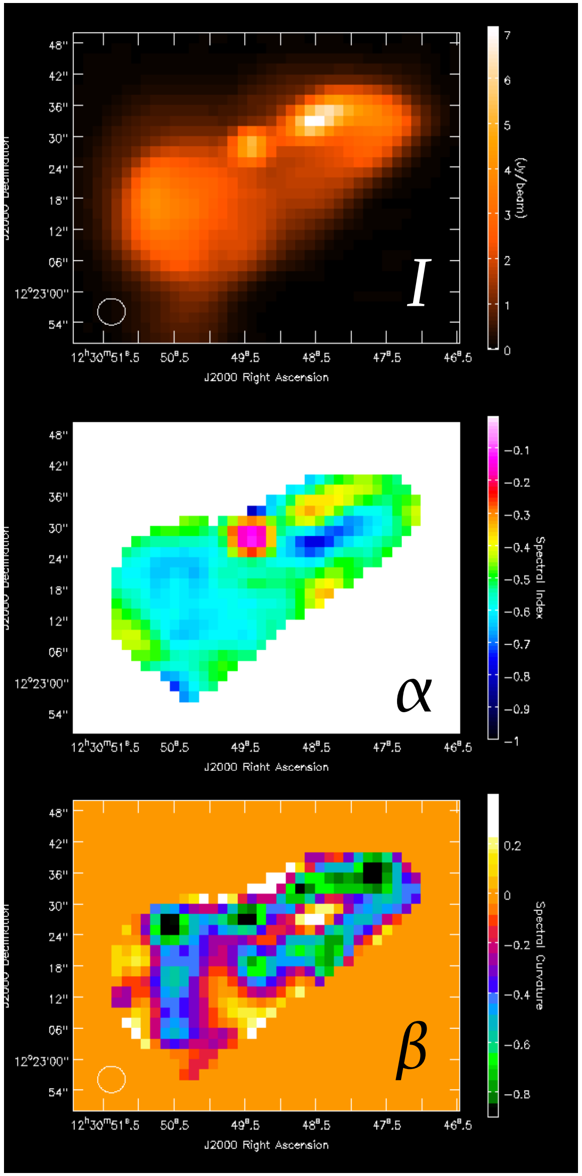
Sp.Ndx. map constructed from 1.4GHz and 4.8GHz,

Images obtained from C.Carilli et al, Ap.J. 1991. (VLA A,B,C,D Array at L and C band)



# Example : M87 spectral curvature (1.1 –1.8 GHz)



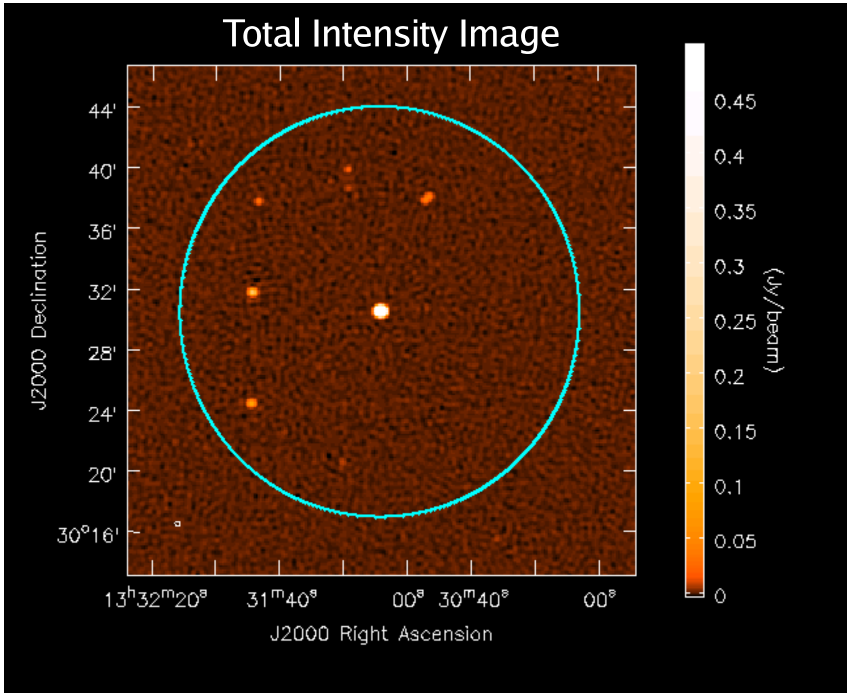

From existing P-band (327 MHz), L-band(1.42 GHz) and C-band (5.0 GHz) images of the core/jet

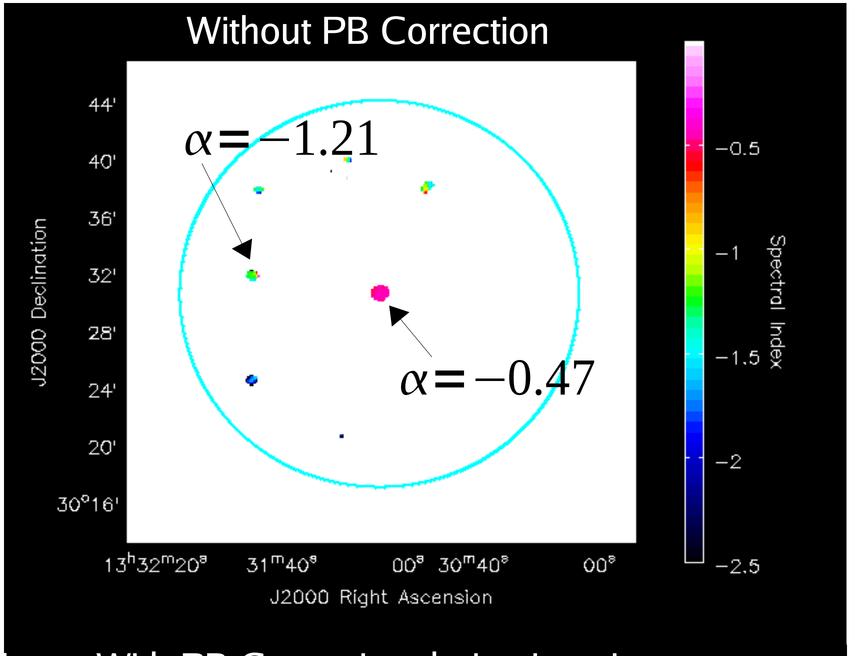
P-L spectral index : -0.36 ~ -0.45

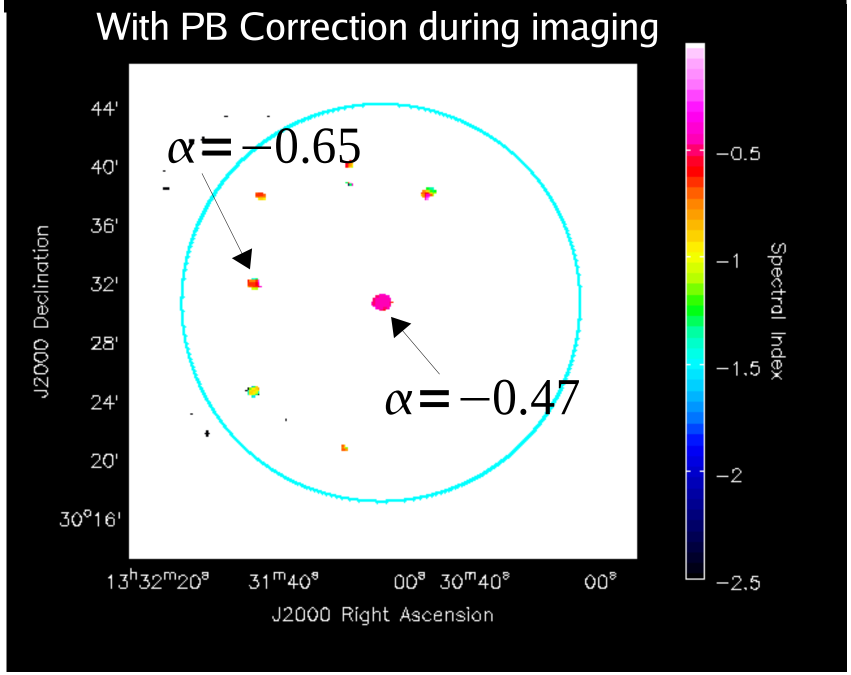
L-C spectral index : -0.5 ~ -0.7

Need SNR > 100 to fit spectral index variation ~ 0.2

# Example : 3C286 field + freq/time-varying PB correction




Verified spectral-indices by pointing directly at one background source.

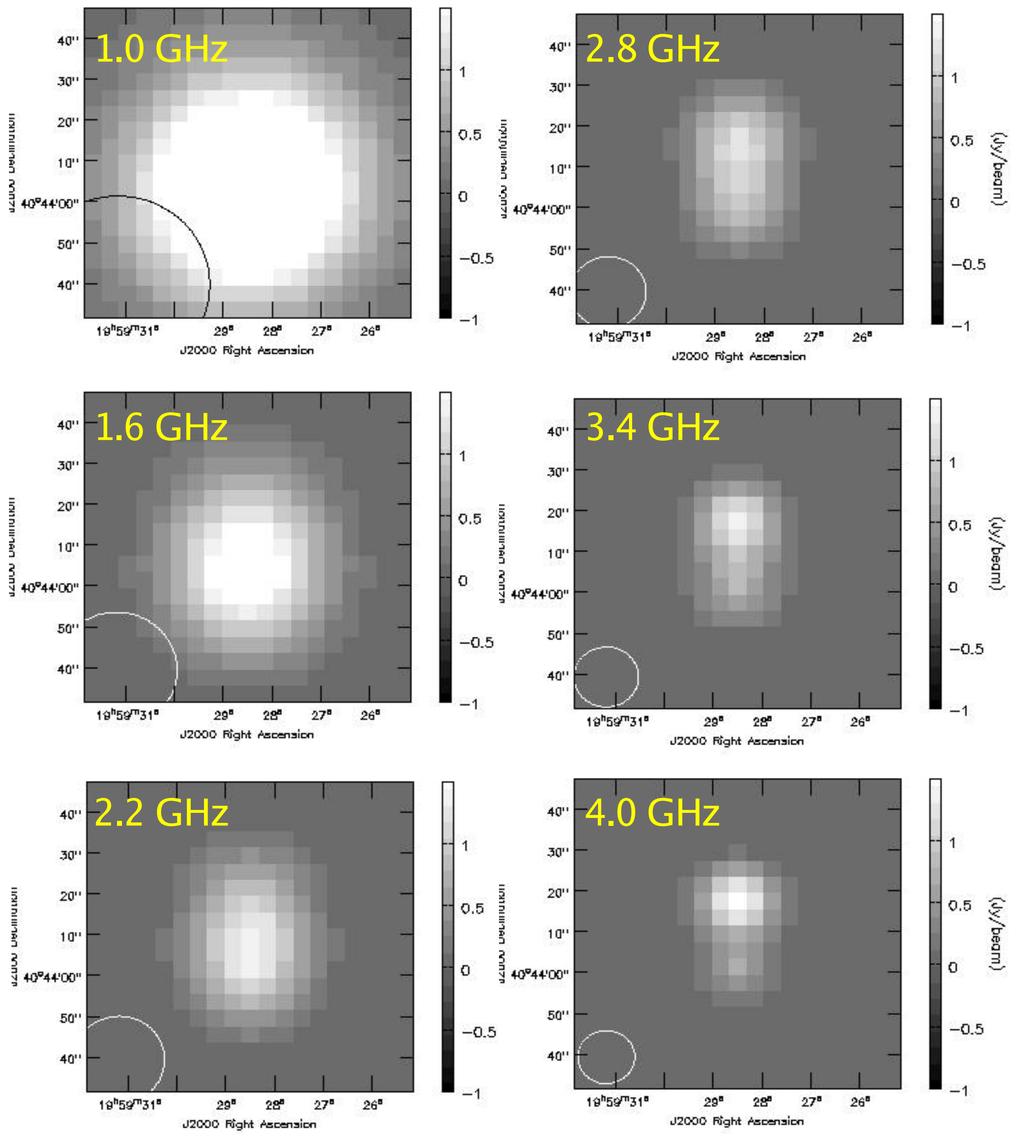
→ compared $\alpha_{center}$ with 'corrected' $\alpha_{off.center}$

Obtained $\delta\alpha$ = 0.05 to 0.1 for SNR or 1000 to 20

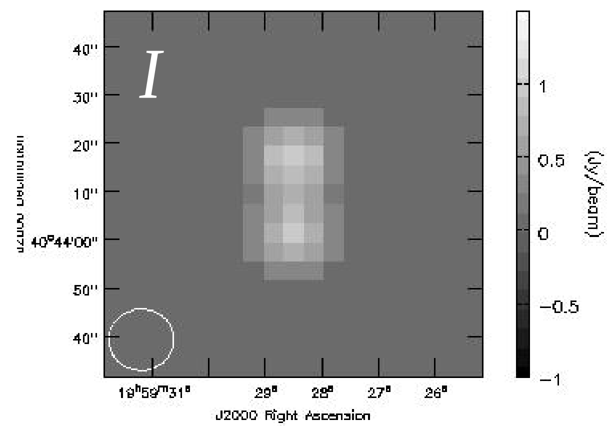
Also verified via holography observations at two frequencies

# Moderately Resolved Sources

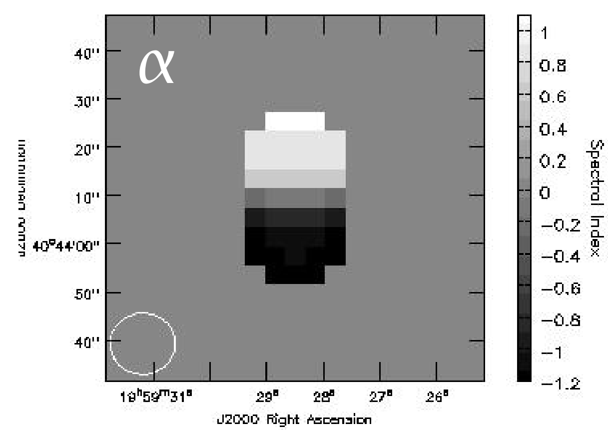
The spectrum can be reconstructed at the angular resolution of the highest frequency




Restored Intensity image




Spectral Index map

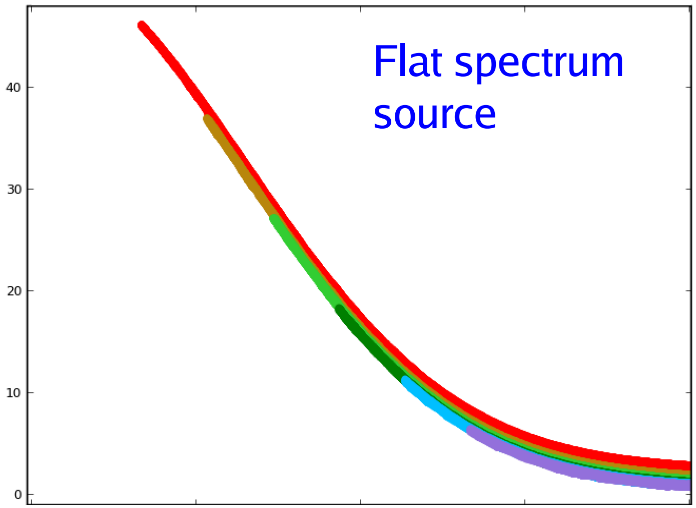


# Very large spatial scales –without short-spacing data

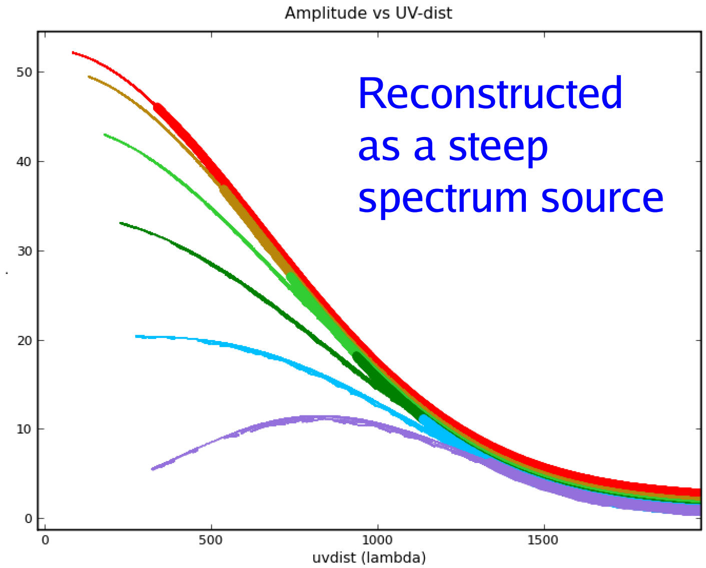
The multi-frequency data do not constrain the spectrum at large scales

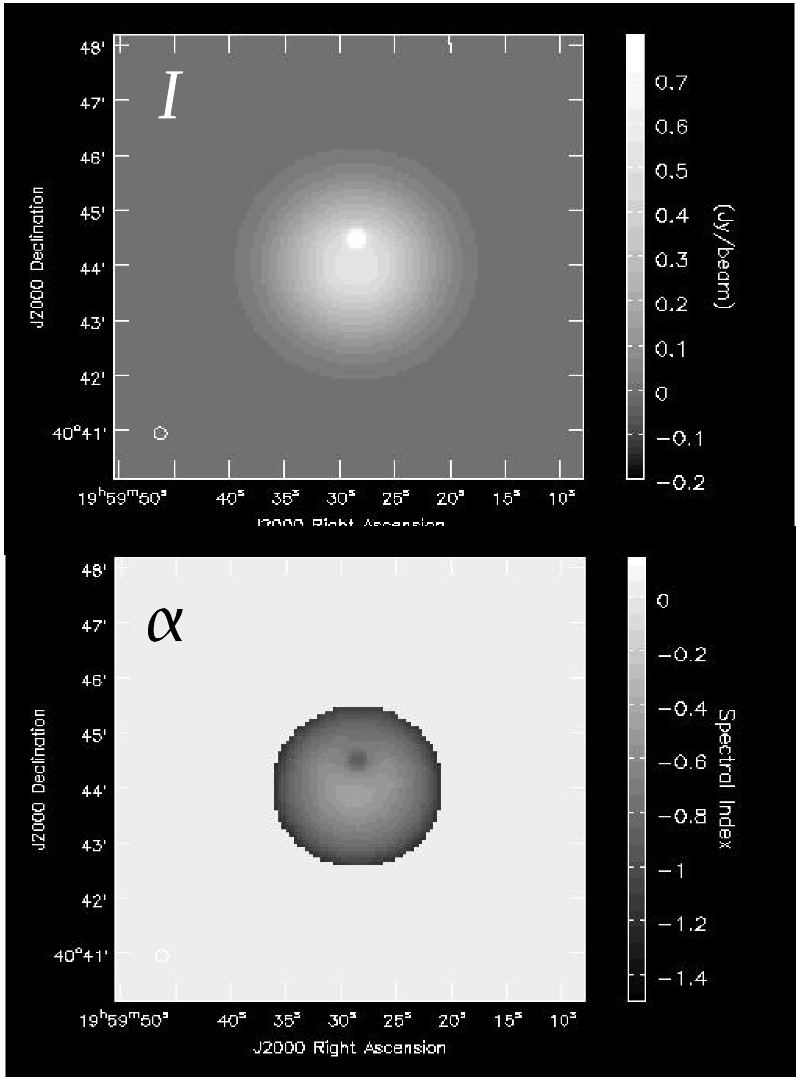
Amplitude (vs) Spatial Frequency

Data




Data
+
Model



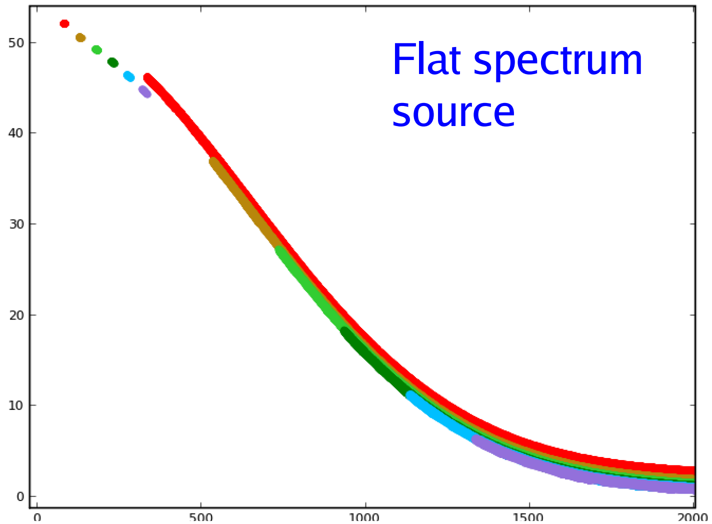



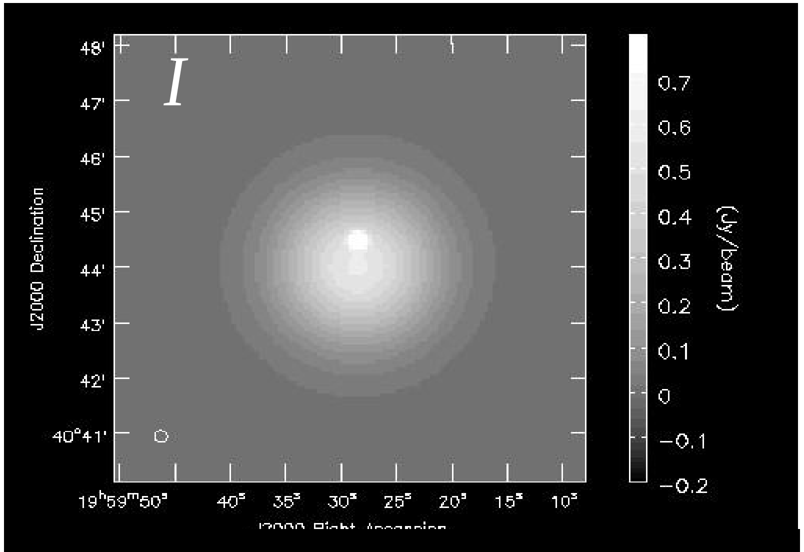
# Very large spatial scales –with short-spacing data

Extra short-spacing information can help constrain the spectrum

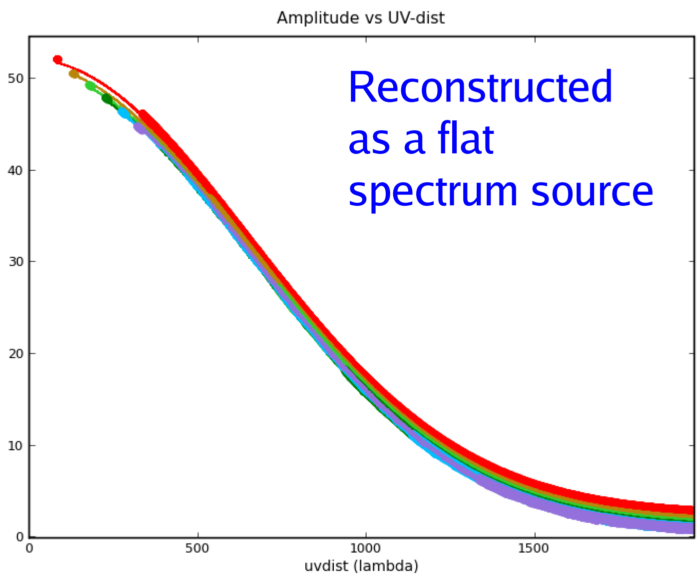
Amplitude (vs) Spatial Frequency

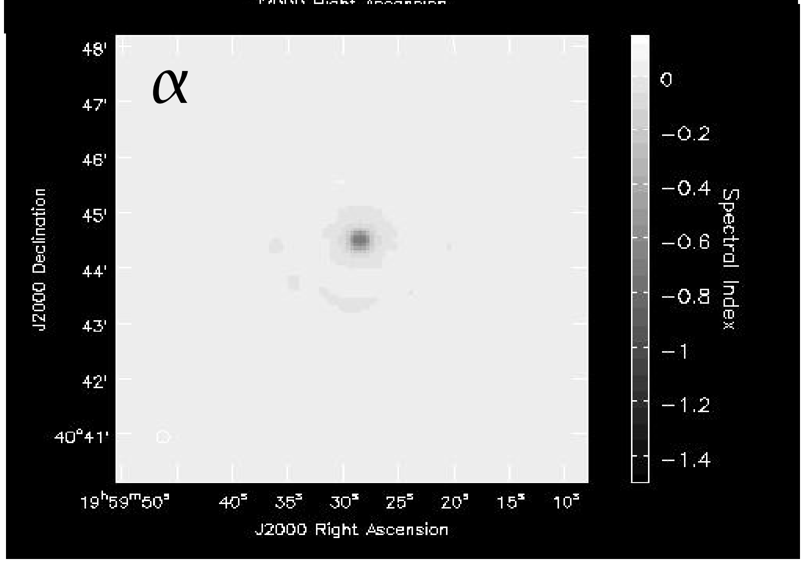
Data



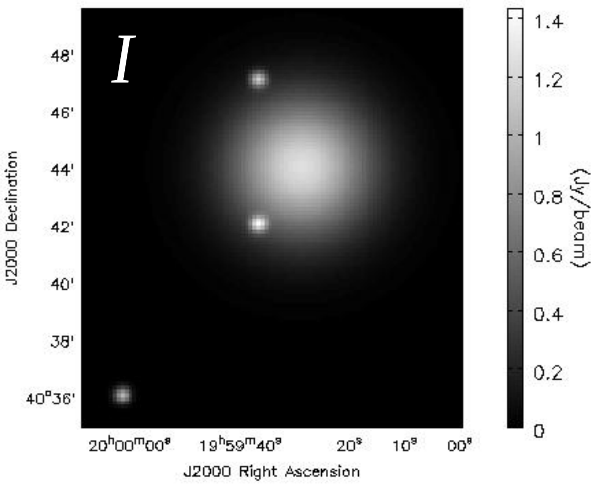



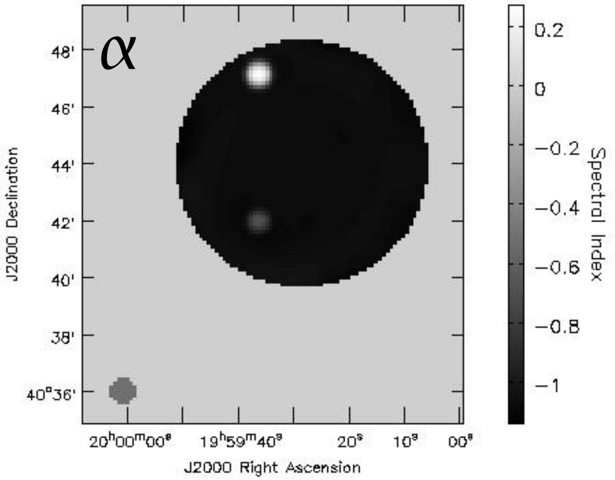

Data
+
Model



# Overlapping Sources with different spectra




MS-MFS image model naturally separates sources with different spatial scale and spectrum.

### Multi-Frequency background subtraction

$$I^{front} = I^{total} - I^{back}$$

$$\alpha^{front} = \frac{I_1^{total} - I_1^{back}}{I_0^{total} - I_1^{total}}$$

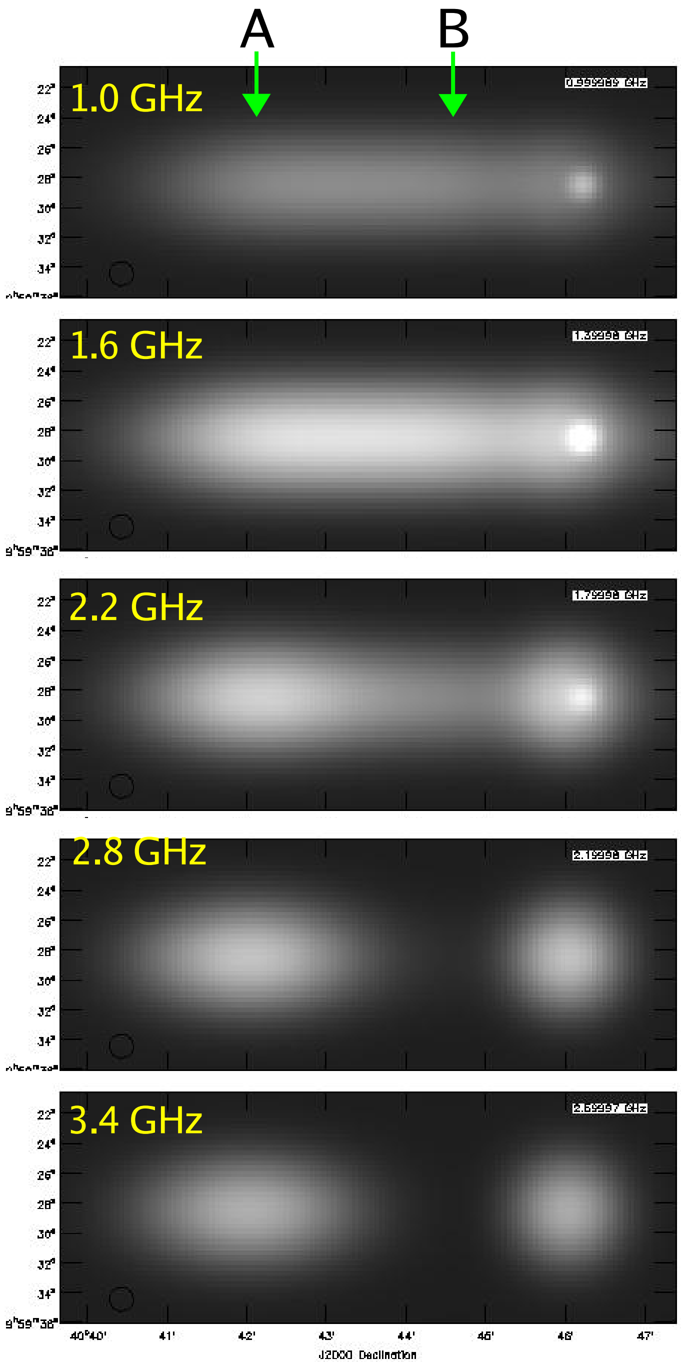
Example : Foreground source : I=1.0, $\alpha$ = -0.5

Measured : I = 1.434, $\alpha$ = -0.68

Background : I = 0.429, $\alpha$ = -1.08

Corrected foreground : I = 1.005, $\alpha$ = -0.51

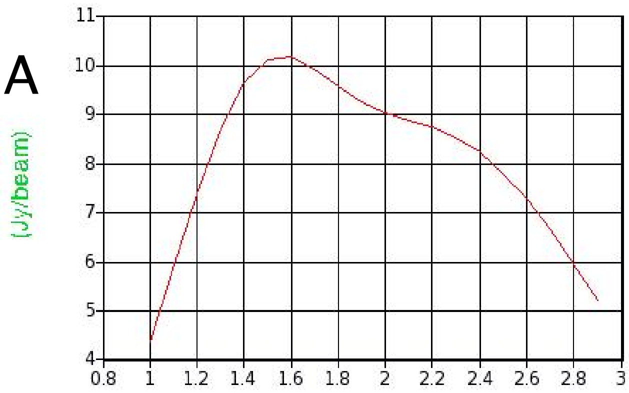
# Non power-law spectra and band-limited signals

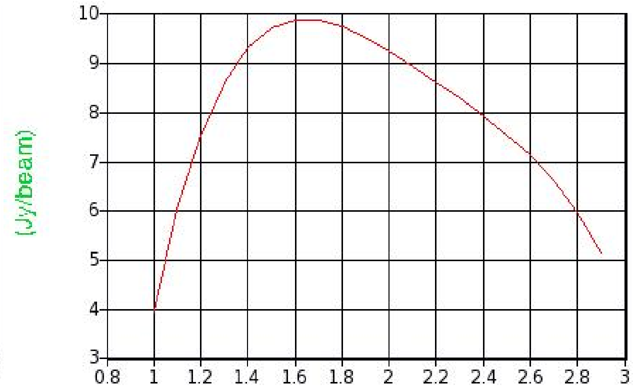



True Spectrum

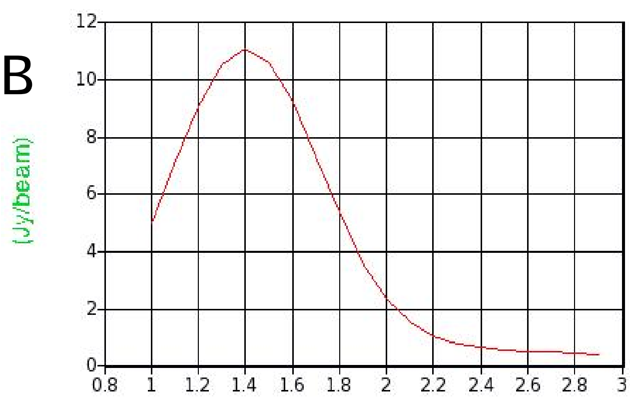
Reconstructed Spectrum

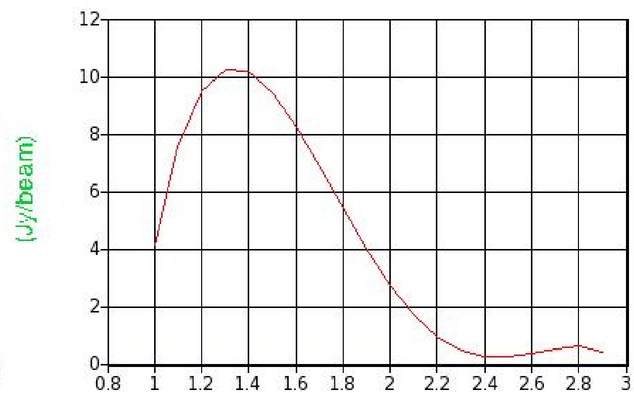
A




B

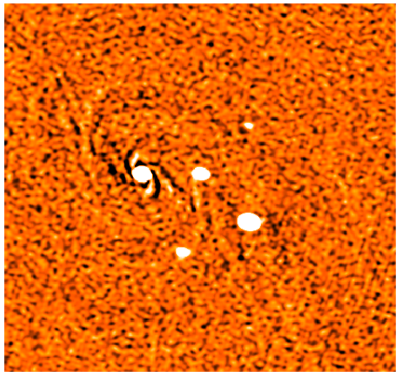


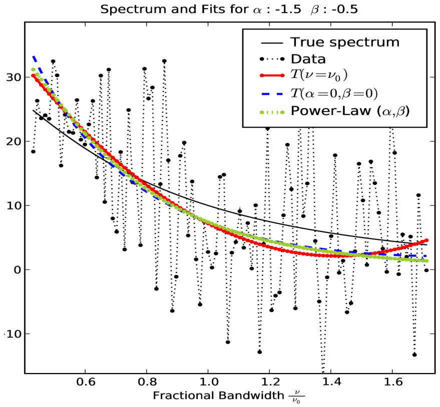

Angular resolution depends on the highest sampled frequency at which the emission exists.

# Three types of error

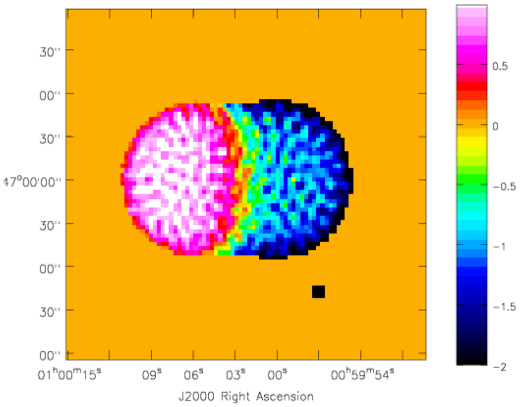
(1) Artifacts in the continuum image due to using too few terms in the Taylor-series expansion of the spectrum.



(2) Error in spectral index and curvature due to too few or too many Taylor terms, or unconstrained spectra.




(3) Error propagation during the division of one noisy image by another



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# M87 –the radio galaxy at the core of the Virgo cluster

Hot and dense cluster cores : $t_{cool} \propto \frac{1}{density}$

=> 'Cooling Flow"

For Virgo, expected $t_{cool}$ = 1 Gyr

But... 1 Gyr << Hubble-time
No evidence (X-rays) of cooling below ~ $10^7$ K

=> Current picture : Heating via feedback from an active galactic nucleus (AGN)

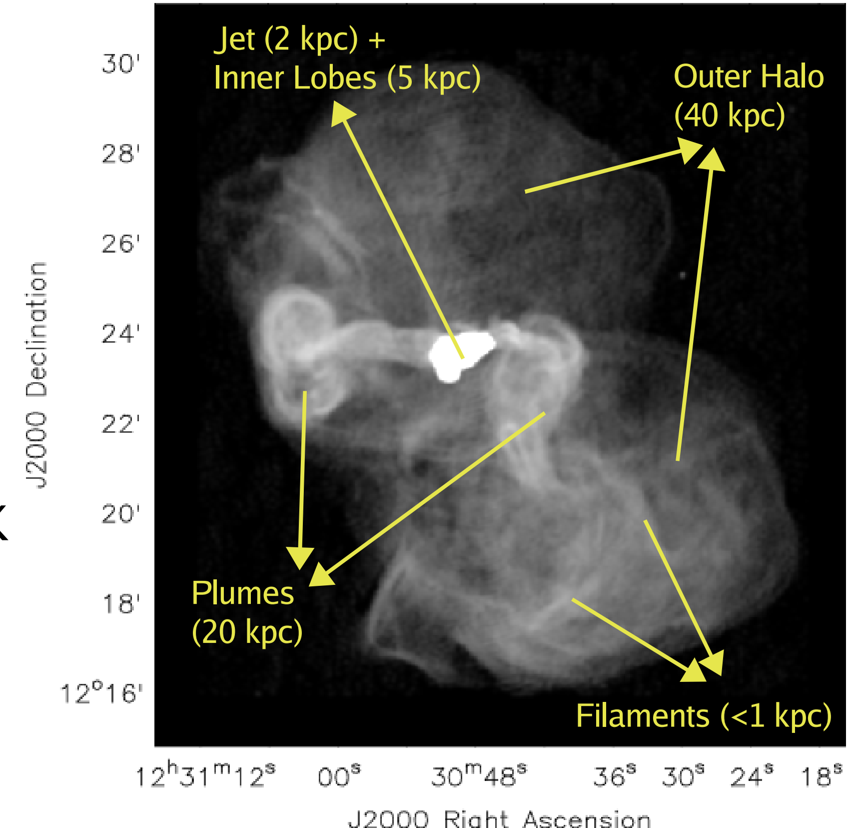



Image Credits : F. *Owen*

Question : How is energy transported from the AGN to the intra-cluster medium (ICM) ?

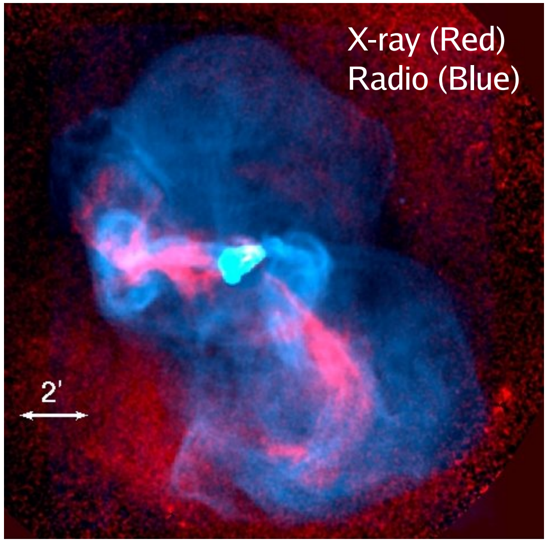
Current picture : Physical transport via buoyant bubbles rising through the ICM.
Dynamical age of outer halo : ~ 60-100 Myr *(Churazov, 2001)*

.... but, this may not be the complete picture....
.... what are the particles in the halo doing ?

# M87 –Study the spectral history of the radio halo

GOAL

To understand the physics of the energetic particles as they travel outward from the core.




*(Forman et. al. 2005)*

WHY ?

To investigate if there is more than just dynamical expansion and passive ageing of particles.

Clues :
+ The inner radio lobes coincide with an X-ray cavity, but the plumes and halo do not.
+ An apparent correlation between the X-ray and radio emission in the plumes.

METHOD

- Measure the broad-band spectrum of the halo and compare with spectral evolution models
- Derive synchrotron ages and compare with dynamical ages.

Existing information :
+ Low-resolution halo measurements show a drop-off between 1 and 10 GHz *(Rottmann et al, 1996)*
+ Very high-resolution jet measurements show a single power law $\alpha = -0.5$ from radio to optical to x-rays

*(Bicknell & Begelman, 1996, Perlman &Wilson, 2005)*

# Synchrotron radiation and spectral ageing

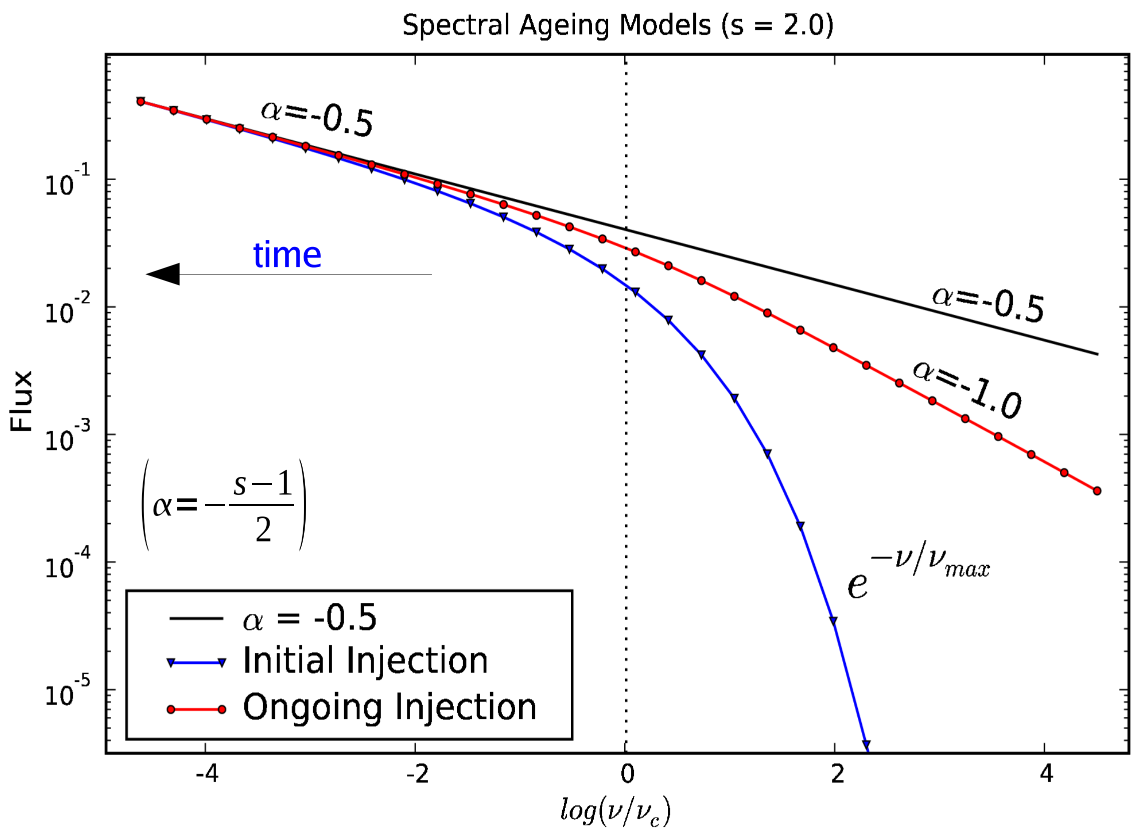
Particle energy distribution $\propto E^{-s}$

Synchrotron spectrum $\propto \nu^{\alpha}$

Energy loss rate : $\dot{E} \propto B E^2$

=> Particles with high energy and in high B-fields radiate faster.

=> Beyond a "critical frequency" $\nu_c$ the spectrum steepens.




Synchrotron age :

As the particles age, $\nu_c$ moves to lower frequencies.

$$t_{syn} \propto B^{-\frac{3}{2}} \nu_c^{-\frac{1}{2}}$$

Two ageing models :

(1) Initial injection
- exponential drop-off above $\nu_c$

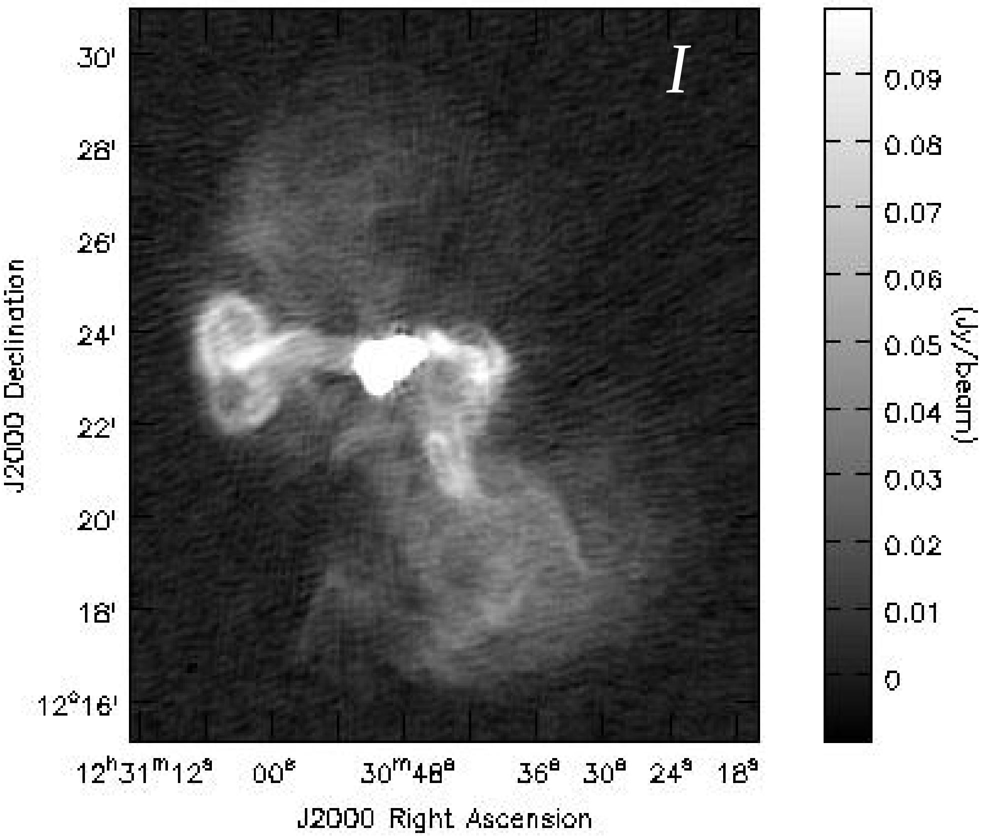
(2) Ongoing injection
- power-law steepens by 0.5 above $\nu_c$

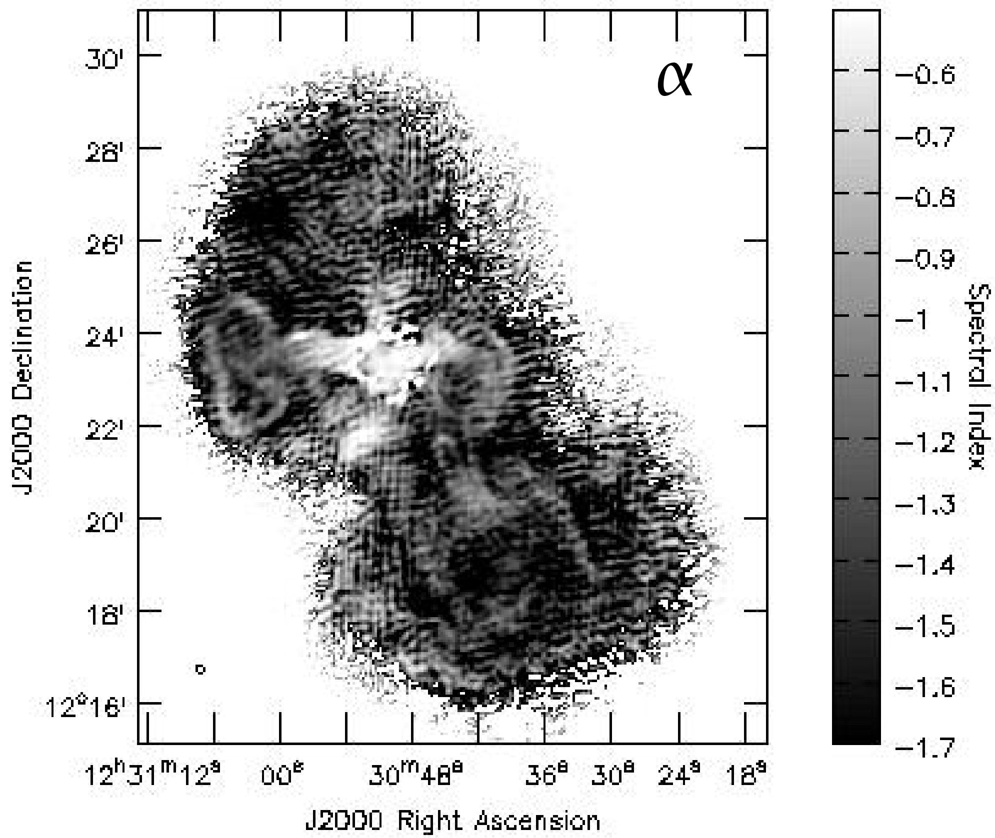
# M87 –L-band observations and imaging results

Need : High angular resolution images of the halo from 74 MHz to 10 GHz

Have : Images at 74 MHz, 327 MHz, 1.4 GHz.

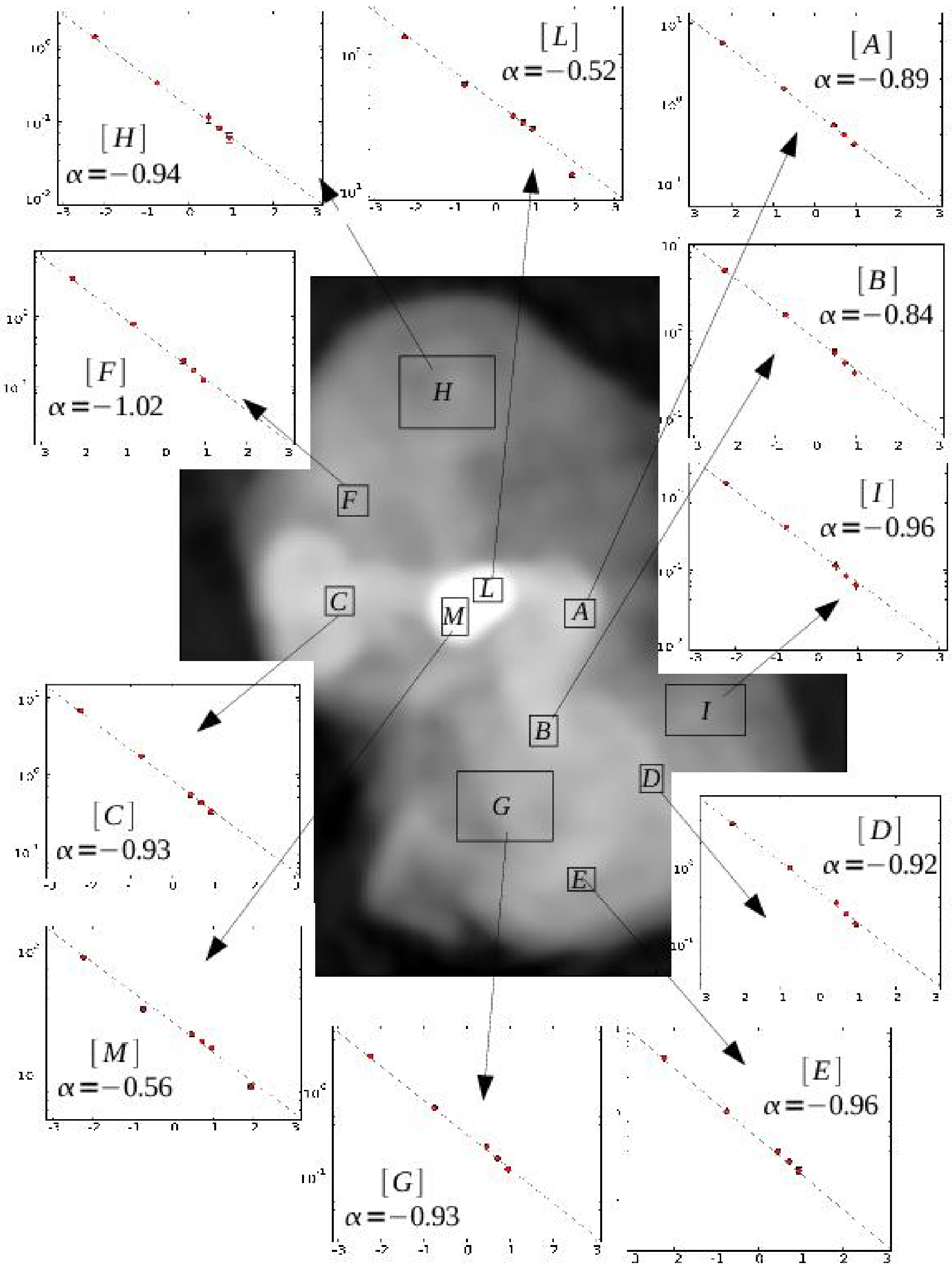
First step : Get spectral index between 1 and 2 GHz to constrain the slope at 1.4 GHz




10 VLA snapshots at 16 frequencies between 1.1 and 1.8 GHz, spread across 10 hrs

# M87 : spectral index across the halo




(1) Straight lines => pure power-laws
(75 MHz –1.8 GHz)

- inner radio lobes : $\alpha$ = -0.55
- halo regions : $\alpha$ = -0.85 to -1.0

(2) Slight steepening at 1 GHz

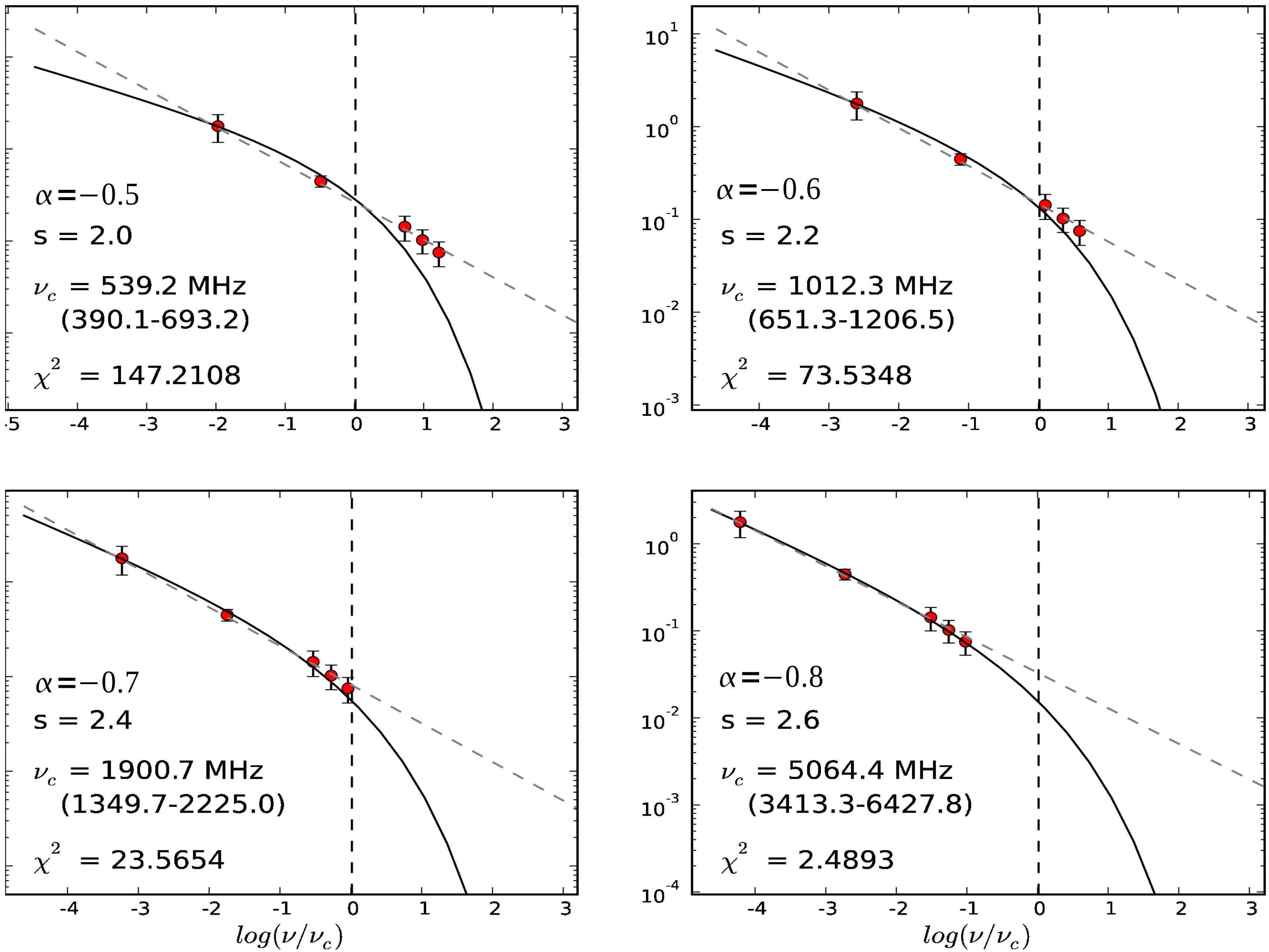
- consistent with low-resolution data
- however, steepening is within error bars

Two questions can be answered :

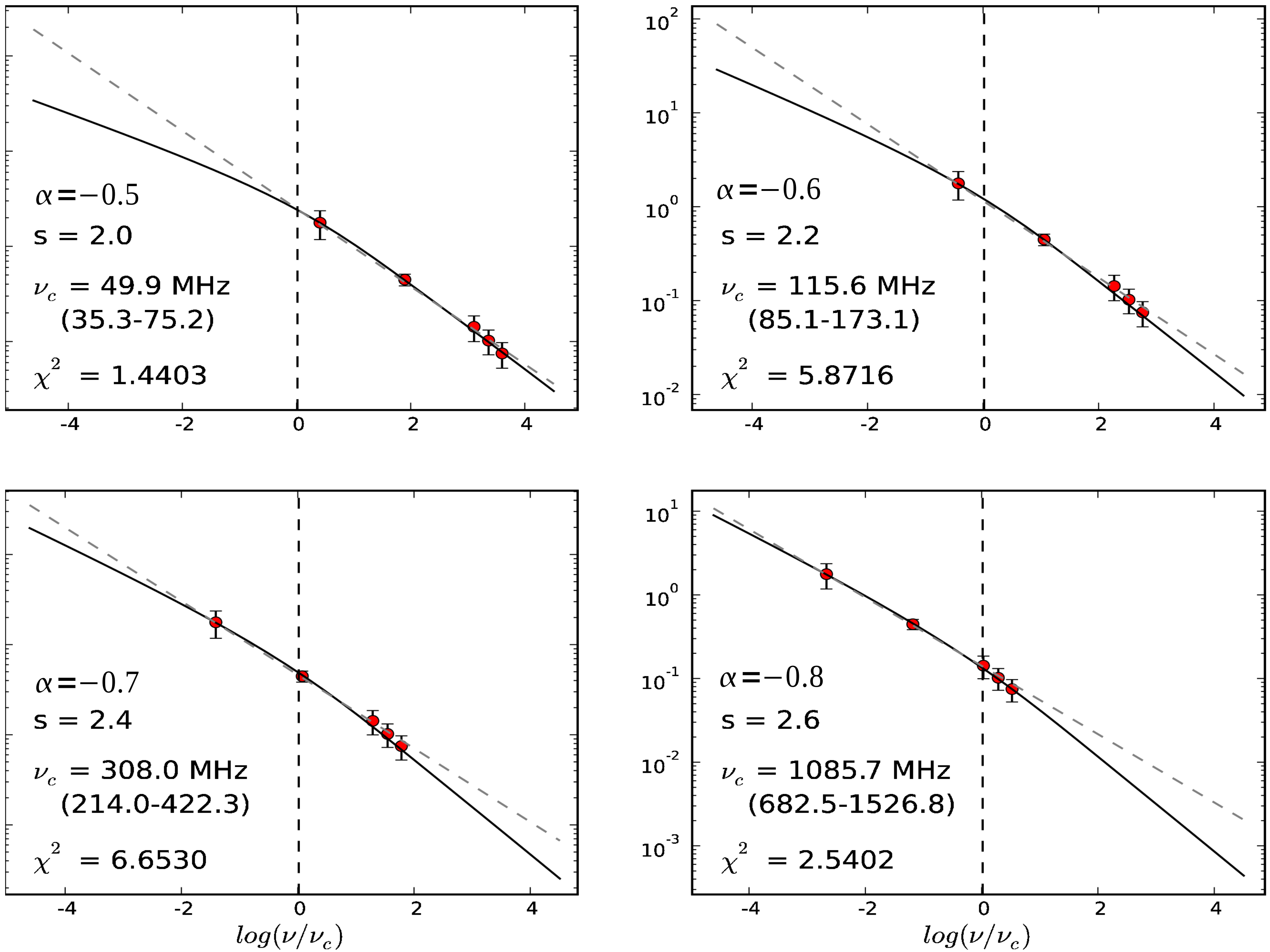
- Does a pure power-law between 75 MHz and 1.8 GHz rule out some spectral models ?

- Do models that fit the slight steepening give plausible synchrotron ages ?

# M87 –Initial injection models : can rule some out



# M87 –Ongoing injection models : cannot rule any out



# M87 results : Inner lobes, plumes and halo

## Jet/Inner Lobes

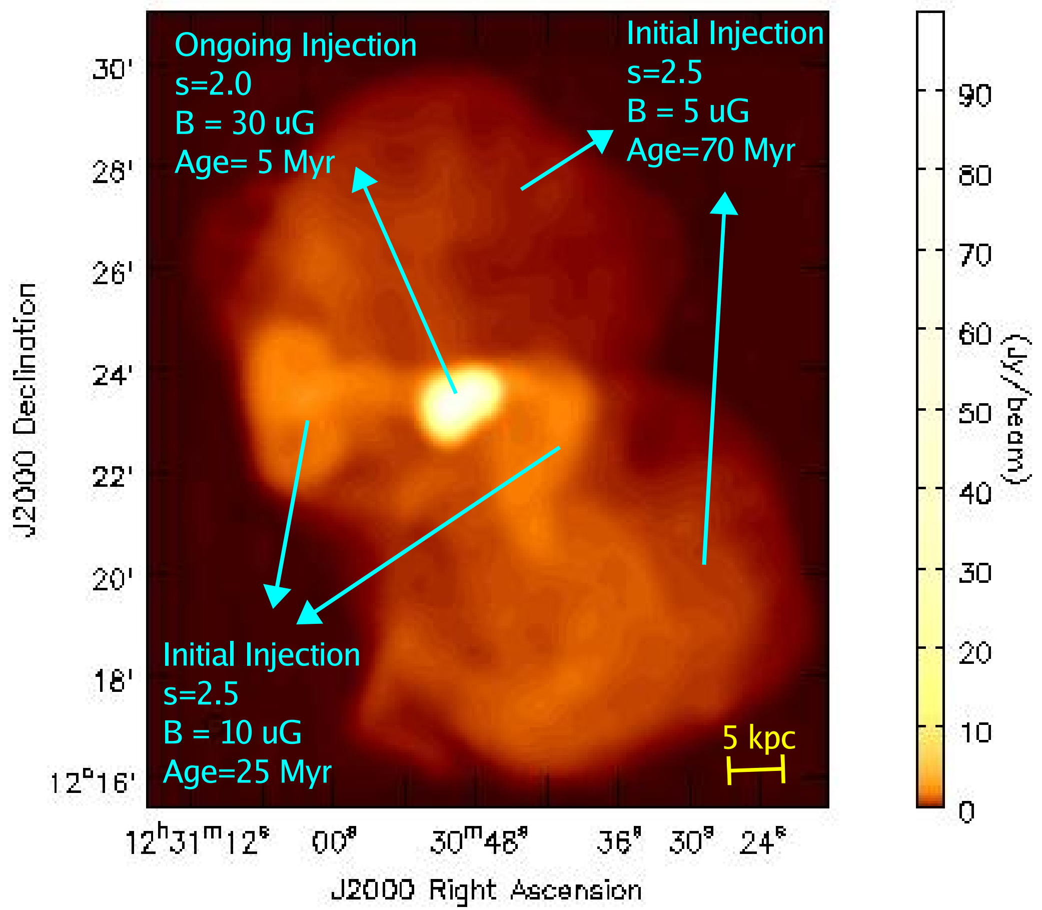
- consistent with ongoing injection with s=2.0
- no valid fits for s > 2.0
- no valid fits for initial-injection

=> The jet is continuously injecting particles with an index of s=2.0, reaching 5kpc in ~ 5 MYr.

## Plumes/Halo

- consistent with initial injection with s=2.5
- no valid fits for s < 2.4

=> Particles with an initial injection index of s=2.5 are passively ageing as they move outward (20 kpc in 25 Myr, 40 kpc in 70 Myr).



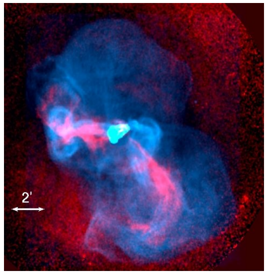

=> Is the halo the result of a previous cycle of AGN activity ?

Note : Halo spectra are also consistent with all ongoing injection models

=> cannot rule out ongoing activity

# M87 : Filaments –sites of local activity ?

The correlation between X-ray and radio emission suggests sites of local activity

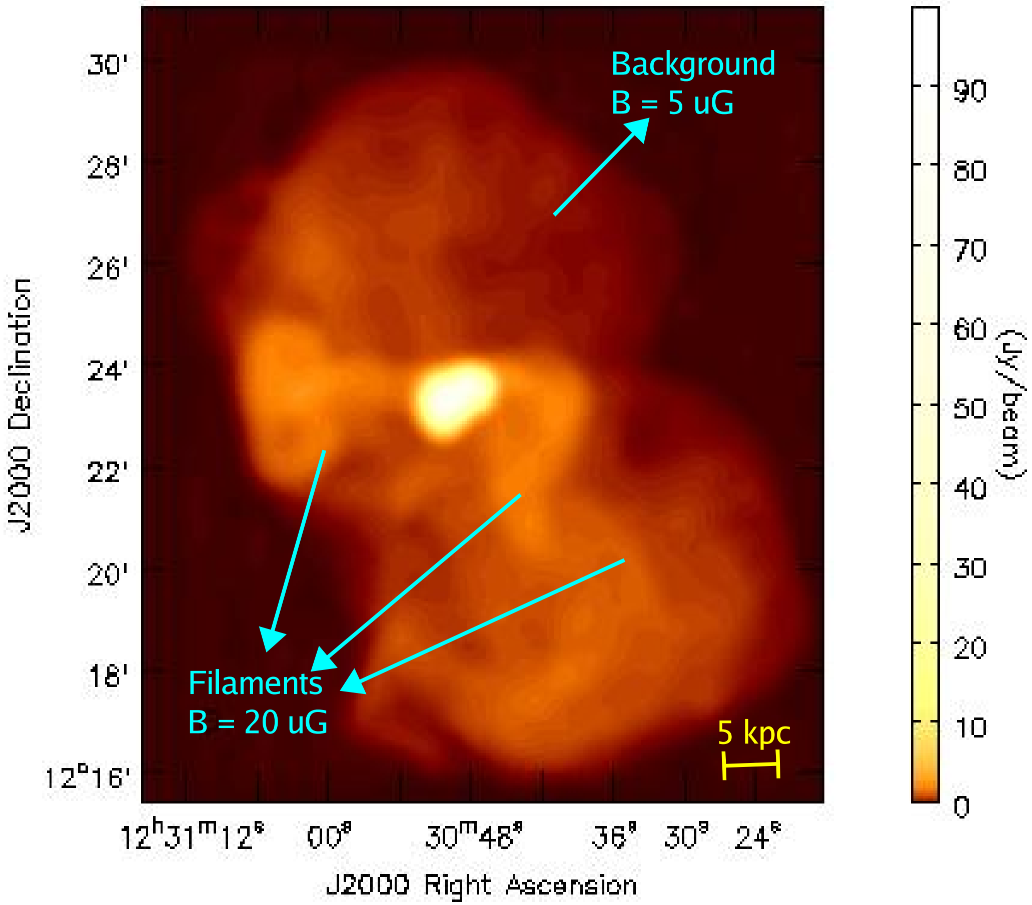



Narrow bright filaments suggest high B-fields

=> Isolate filaments from background.

- High B-fields ~ 20 uG
- spectra consistent with both ageing models with ages ranging from 100 Myr to 800 Myr

=> These results are inconclusive.




Next Steps : Map the halo between 2 and 10 GHz

- Halo : Confirm or reject the model of passive ageing (with s=2.5)
- Filaments : Is there any significant difference between filaments and background ?
- Ageing models : Consider local particle re-acceleration.

# Outline

- Introduction : Imaging with a broad-band radio interferometer

- Algorithms : Multi-scale multi-frequency wide-field imaging

- Examples : Imaging results, feasibility study and errors

- Astrophysics : A study of the broad-band spectrum across the M87 radio halo

- Conclusion : Dissertation summary and next steps

# Dissertation Summary

(1) Worked out the math for standard imaging algorithms, to understand them well enough to write code for them [ Chapters 3 and 4 ]
[Advances in Calibration and Imaging Techniques in Radio Interferometry, U.Rau, S.Bhatnagar, M.A.Voronkov, T.J.Cornwell, Proceedings of the IEEE, Vol. 97, Issue 8, p1472-1481]

(2) Evaluated existing wide-band imaging methods and identified areas requiring improvement [ Chapter 5 ]
[ Multi Frequency Synthesis Imaging for the EVLA : An initial investigation, U.Rau, T.J.Cornwell, S.T.Myers, EVLA Memo 101, 2006]

(3) Worked out the math for a reconstruction algorithm in which images are modeled as series expansions, and applied this to multi-scale and multi-frequency imaging [ Chapter 6 ]

(4) Combined these methods into MS-MFS and added a polynomial Primary-Beam model to be used with the A-Projection algorithm [ Chapter 7 ]
[ Multi-Scale Multi-Frequency Synthesis Imaging in Radio Interferometry, U.Rau, T.J.Cornwell, S.Bhatnagar, 2010 (IN PREP) ]

(5) Applied MS-MFS to simulated and real data, for validation and tests [ Chapter 8 ]
+ points to keep in mind while using MS-MFS.

(6) Applied to M87 - one step towards constraining the spectral evolution of various features in the 40 kpc radio halo [ Chapter 9 ]

Software : CASA and ASKAPSOFT (via CASACore libraries)
( MS-MFS without wide-field corrections : released in casapy v2.4 )

# Future Work

- Test MS-MFS with real wide-band EVLA data !
  - understand errors and establish a data-reduction path.

- Test MS-MFS with primary-beam correction, w-projection and mosaicing together
  - make software available

- M87 : confirm the existence (or not) of an exponential drop-off between 1 and 10 GHz.
  - Obtain real wide-band EVLA data between 1 and 2 GHz.
  - Make a wide-band mosaic image of M87 and its spectral index at 5 GHz.

- Wide-band full-polarization imaging
  - Test if the spatial and spectral models apply to Stokes Q,U,V
  - MFS with rotation-measure synthesis

- A wide-band extension of the ASP-CLEAN algorithm
  - Reduce current multi-scale related dynamic-range limits

- Wide-band snapshot imaging of time-variable sources and non power-law spectra
  - Imaging of solar flares, VLBI imaging

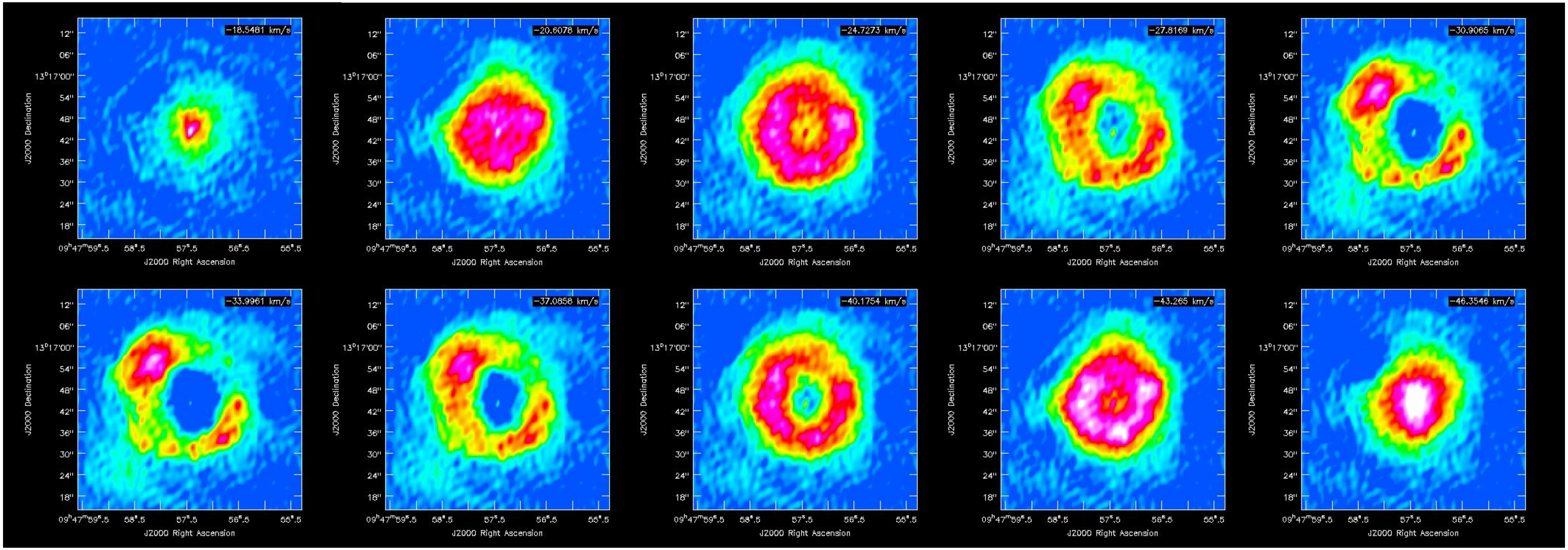
- MFS in the presence of spectral lines
  - continuum subtraction

# MS-MFS on narrow-band EVLA data !

EVLA Spectral-Line data for IRC10216 at 36 GHz : the H3CN line traces a 3D shell.

Spectral-Line width ~ 3.5 MHz,  Channel width ~ 100 kHz ( ~ 35 channels across the line ! )

MS-MFS with a $5^{th}$ order Taylor polynomial to model the spectrum.




Software : CASAPY     + calibration scripts compiled by C.Brogan for the synthesis imaging summer school

# MS-MFS on LOFAR data !

LOFAR : 115 to 160 MHz (10 subbands)

- Self-calibration using a 327 MHz image of CygA.
- Imaging : MS-MFS with two rounds of amplitude and phase self-calibration.

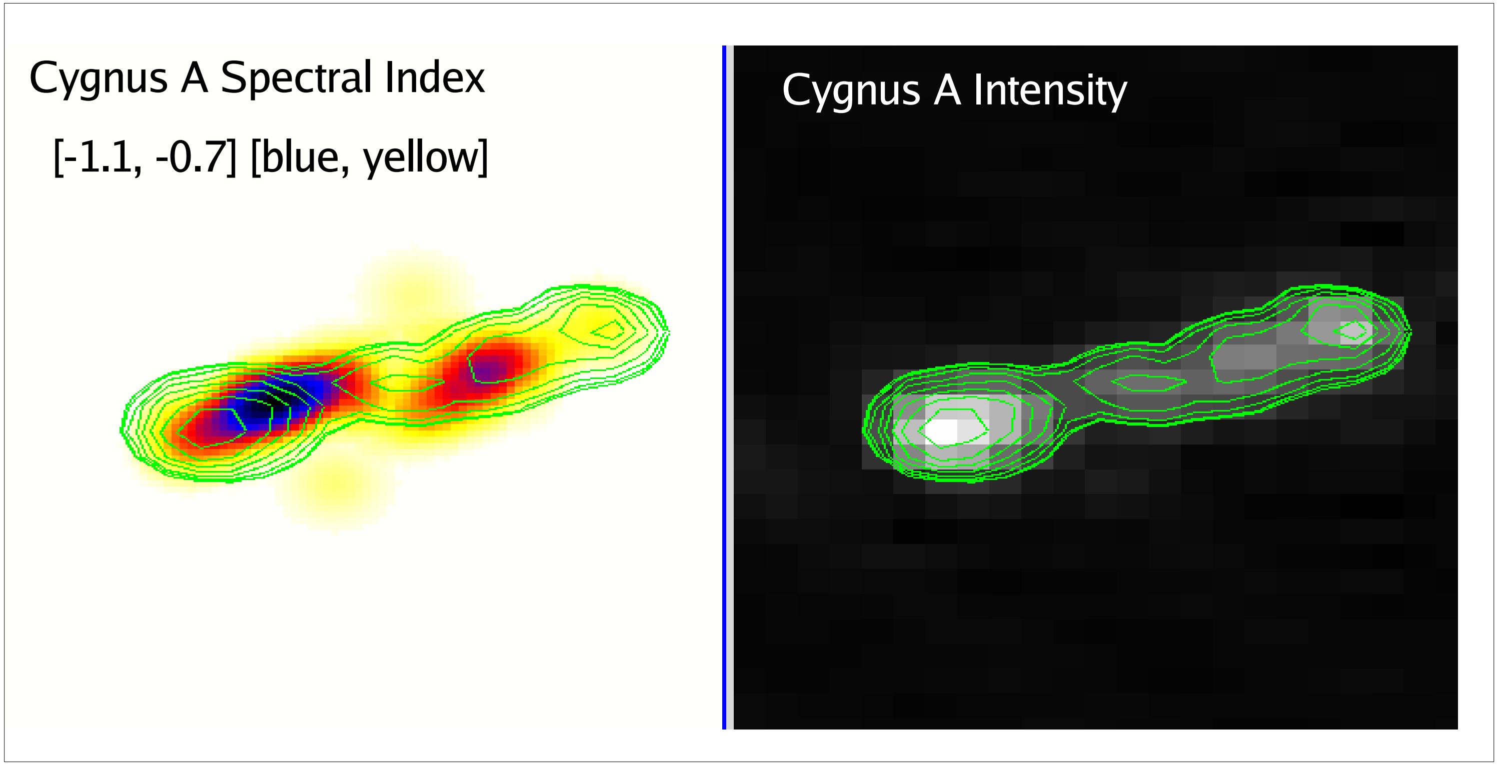



Image Credits :
Reinout van Weeren,
(Leiden Univ.)
and
Ronald Nijboer
(ASTRON)

Software : CASAPY

# Thank You !

Tim J. Cornwell

Jean A. Eilek

Frazer N. Owen

David J. Raymond, David J. Westpfahl

CASA team → Kumar Golap, Sanjay Bhatnagar

Rick Perley –Cygnus A data

Everyone at NRAO !!

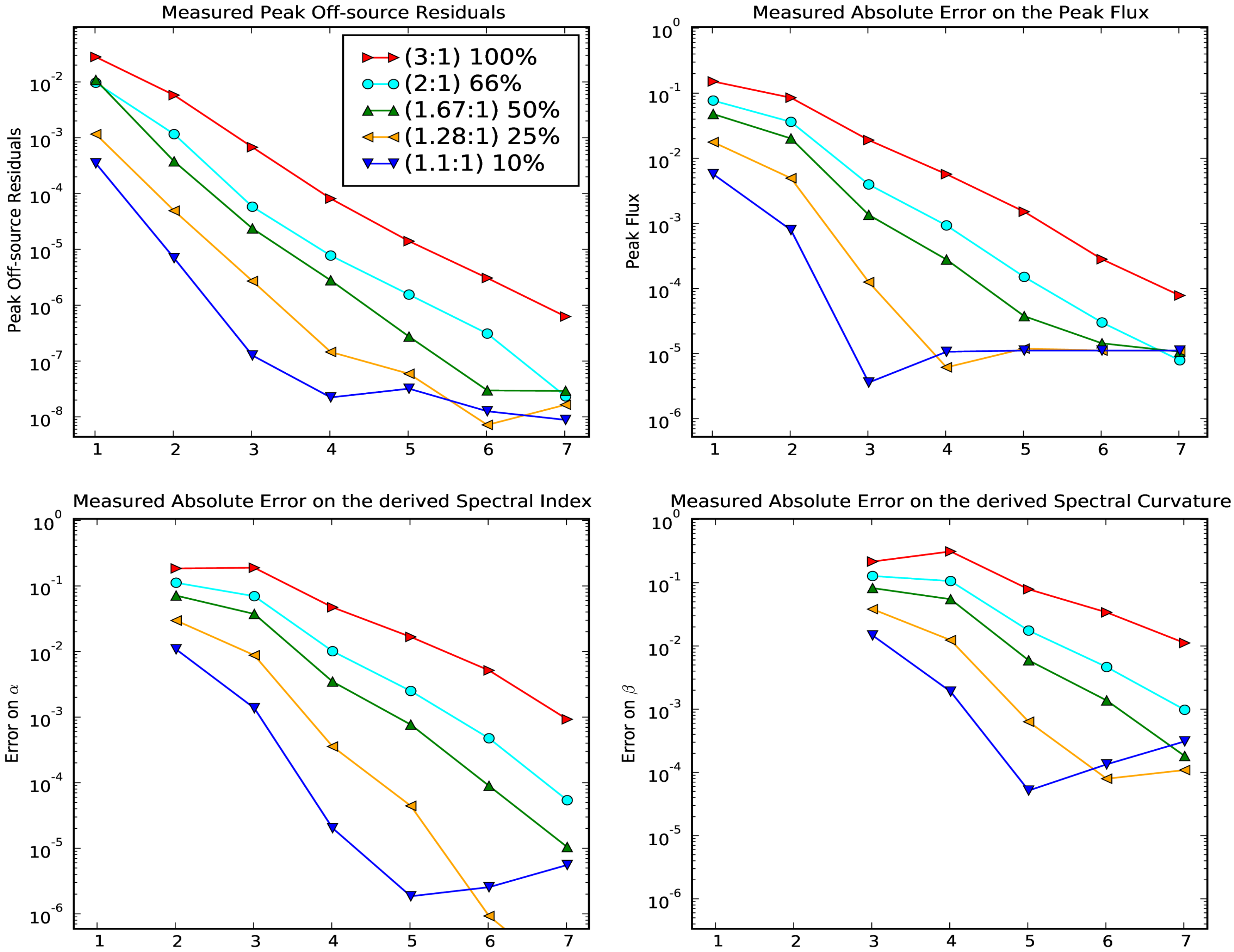
NMT : Physics Department
NRAO : Funding (CASA/Pre-doctoral fellowship)
ATNF : Funding (visits as ATNF affiliated student) + ASKAPsoft team

# MS-MFS : Computation/Performance

| | Single-Channel Imaging | MS-MFS |
|---|---|---|
| Number of deconvolution runs | $N_{chan}$ | 1 |
| Data I/O per solver Major Cycle | $N_{vis}$ / $N_{chan}$ | $N_{vis}$ |
| Memory Use per deconvolution run (multi-scale) | Image Size x $N_{scales}^{2}$ | Image Size x $(N_{taylor}$ x $N_{scales})^{2}$ |
| Runtime ( for few GB of EVLA data on CygA, M87) | ~ 30 hours parallelized : ~ 3 hours (theoretical) | ~ 12 hours parallelized : ~ 4 hours (measured) |

Trade-Off between source complexity, available uv-coverage, desired angular resolution of spectral index map, and algorithm simplicity/stability.

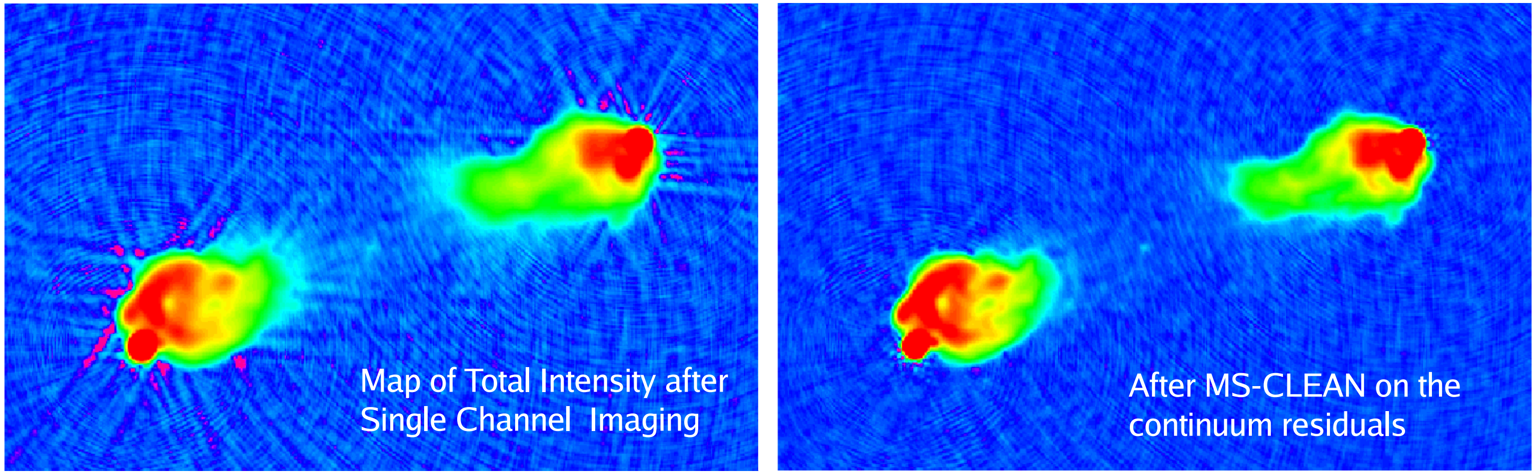
# MS-MFS : Errors due to incorrect polynomial order



# Hybrid of Spectral-Line and MFS

*EVLA Memo 101, Rau & Cornwell, 2006*

## Single-channel Multi-Scale (MS)-CLEAN followed by MFS+CLEAN on residuals



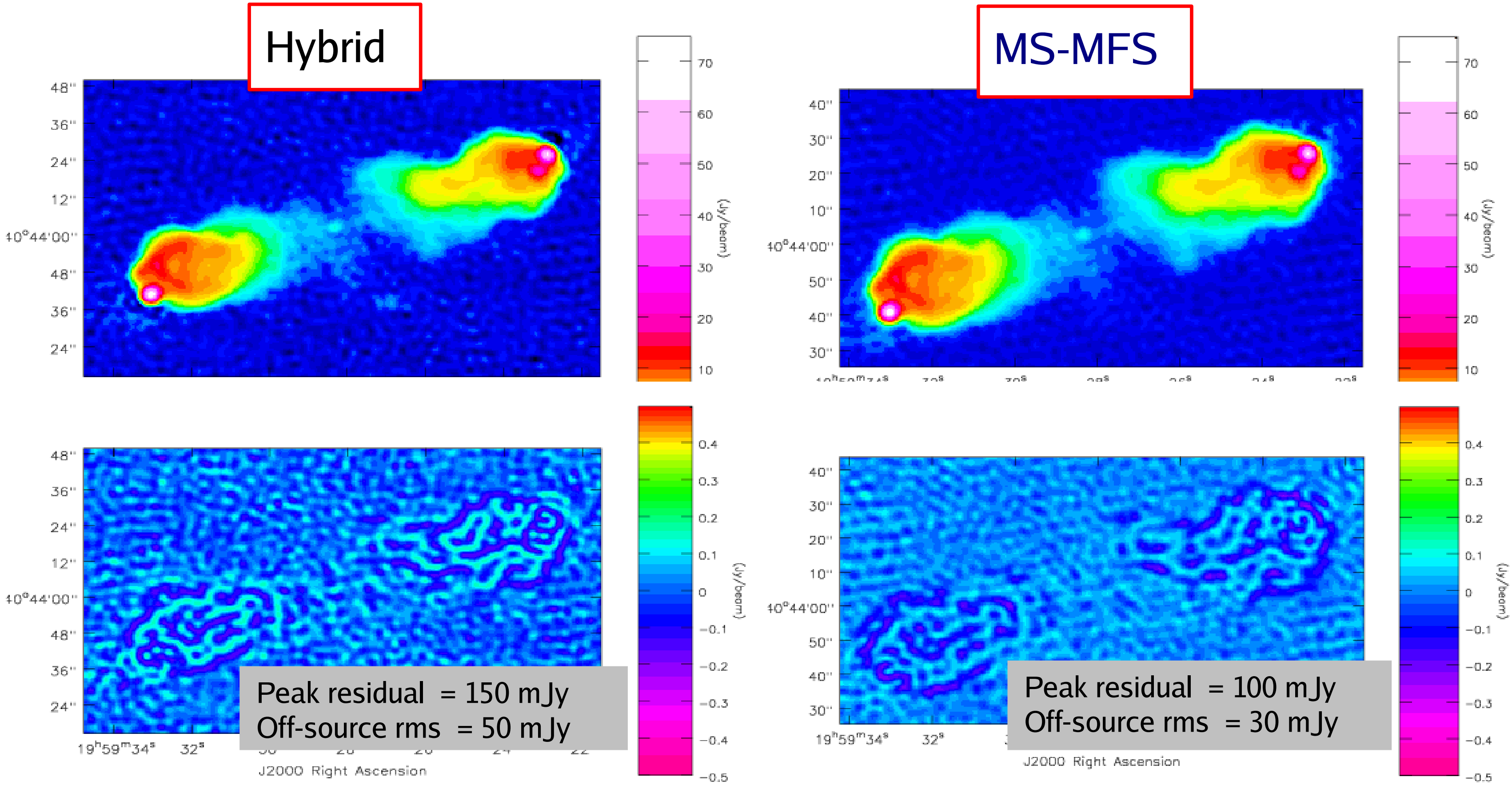

Cygnus-A+ simulation ( 40 channels, L-Band to C-Band, 4 hours ) => Ideal data

This algorithm depends on channels having sufficient uv-coverage

- limited by single-channel deconvolution errors.
- spectral information is at the resolution of the lowest frequency.
  (needs to be tested on real EVLA wide-band data)

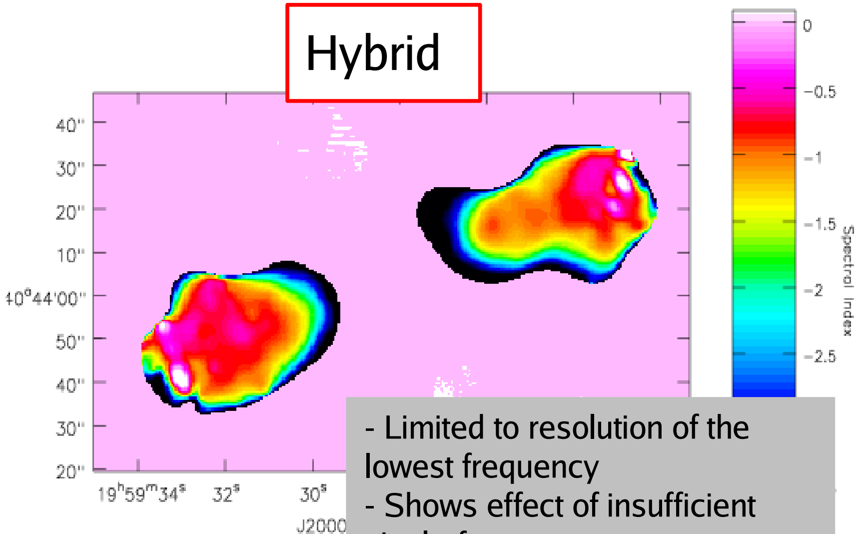
→ reconstruct spectral and spatial structure simultaneously.

# Hybrid (vs) MS-MFS –Cygnus A (Intensity)

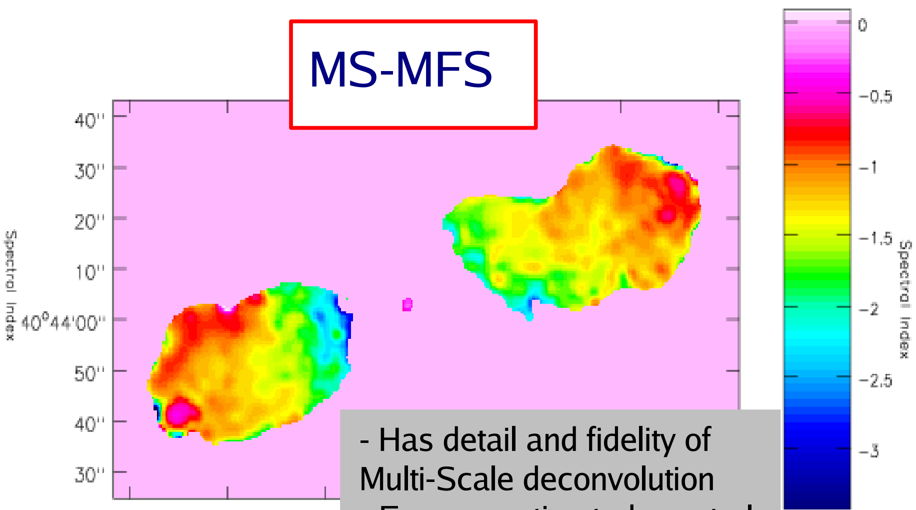



- both algorithms work well
- both have similar residual errors due to deconvolution.

# Hybrid (vs) MS-MFS –Cygnus A (Spectral Index)




- Limited to resolution of the lowest frequency
- Shows effect of insufficient single-frequency uv-coverage




- Has detail and fidelity of Multi-Scale deconvolution
- Error on estimated spectral index <= 0.2

For comparison,

Spectral Index Map constructed from images at 1.4GHz and 4.8GHz, obtained from C.Carilli et al, Ap.J. 1991. (VLA A,B,C,D Array at L and C band)

Map has been smoothed to 1 arcsec.