Constructing Wideband Mosaics with DSA

- Algorithm options in the image domain
- How to choose between options

U.Rau, NRAO

DSA Algorithm Group Meeting

9 Sept 2025

Wideband Mosaic : Measurement and Model

$$I^{obs}_{fp} = I^{psf}_{fp} \times (P_{fp} . I^{sky}_{f}) \qquad \text{where } I^{sky}_{f} = I^{sky}_{f0} (f / f0)^{\alpha}$$

$$\Rightarrow \quad V^{obs}_{fp} = S_{fp} . (A_{fp} \times V^{sky}_{f}) \qquad \text{where } V^{sky}_{f} = F [I^{sky}_{f}]$$

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Basic	Reconstruct I^{sky}	For each frequency and pointing
Mosaic	Reconstruct I^{sky} across multiple P_p	For each frequency
Wideband	Reconstruct I^{sky}_{f0} , α in the presence of P_f	For each pointing
Wideband Mosaic	Reconstruct I^{sky}_{f0} , α with both P_p and P_f	–

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Observed wb-mosaic : $$I^{obs}_{wbmos} = \sum_{fp} I^{psf}_{fp} \times (P_{fp} . I^{sky}_{f})$$

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Basic Imaging (single pointing, single frequency)

Gridding algorithm/kernel	When & How ?
Prolate spheroidal func and/or w-term kernel $I^{obs} = I^{psf} \times (P \cdot I^{sky})$ $\Rightarrow I^{model} = P \cdot I^{sky}$	PB does not vary during the imaging time range or across baselines => P is multiplicative on the sky => Deconvolve. Then PB-correct
AW-Projection kernel : Aperture illumination w/wo w-term kernel and PS $I^{obs} = \frac{P^* \cdot [I^{psf} \times (P \cdot I^{sky})]}{sqrt\{ \lvert P^*P \rvert \}}$ $\Rightarrow I^{model} \approx P \cdot I^{sky}$	PB varies per visibility : pointing offsets, beam squint (azimuthal asymm.) + rotation. => Gridding : pseudo-inverse of P per vis → Phase corrections are applied. => Image domain convolution eqn is not exact → Need to iterate

Normalization choices : 1 , flatnoise = sqrt{ |P*P| } , flatsky = |P*P|

Mosaic , Wideband and Both

Mosaic Imaging

PSF changes with pointing

The UV-coverage changes across time, as a mosaic is constructed…

- PSF rotation with time
- Projection effects with elevation

Resulting angular resolution can change too.

Mosaic Imaging

PSF changes with pointing

PB changes with pointing

Sky emission extends across multiple pointings

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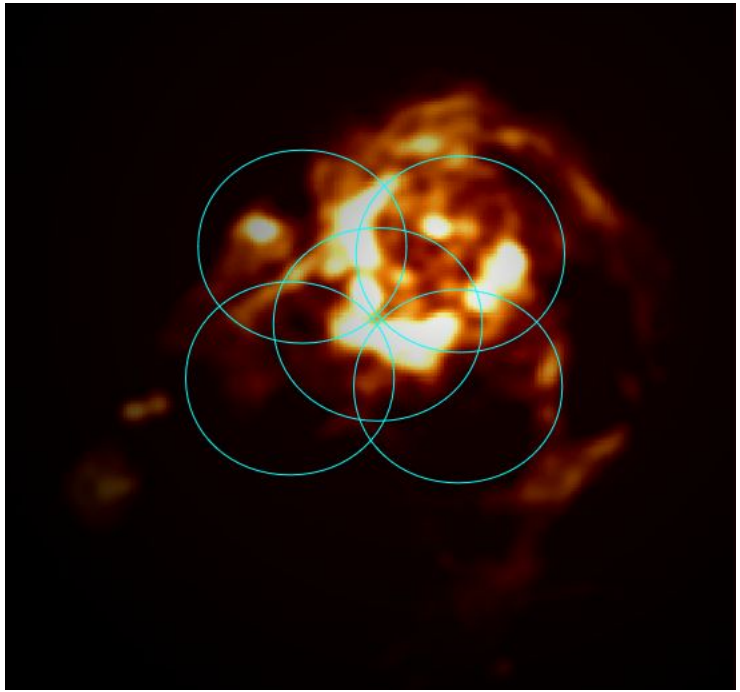
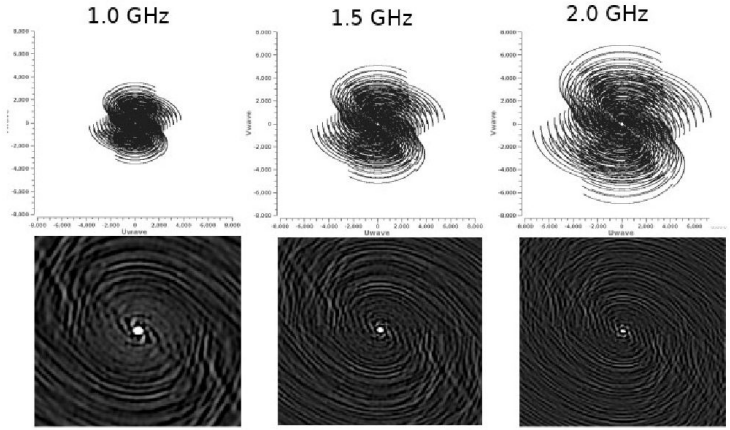


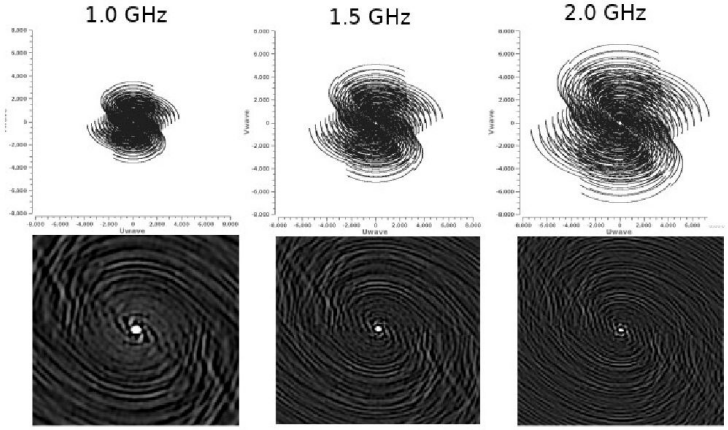
Wideband Imaging

PSF changes with frequency

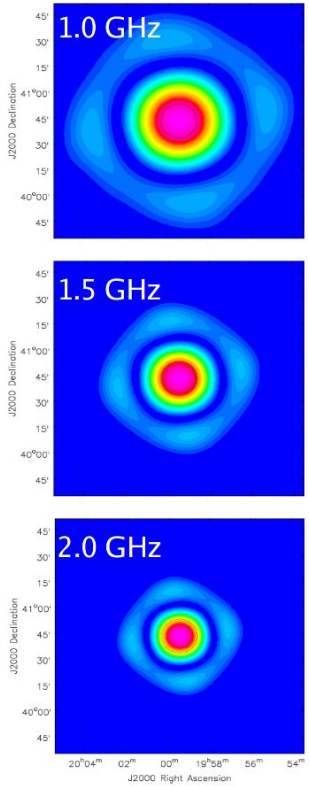


Wideband Imaging

PSF changes with frequency

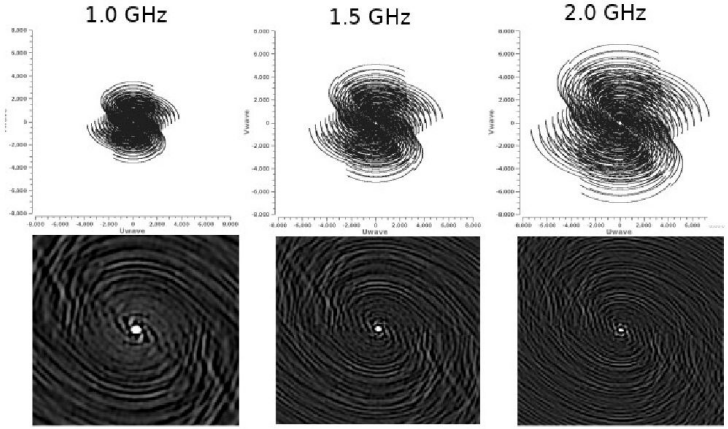


PB changes with frequency

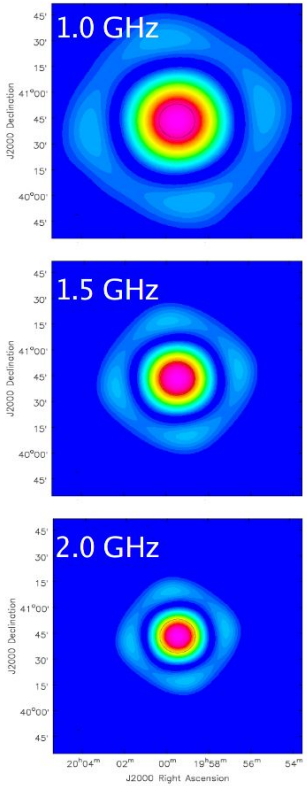


Wideband Imaging

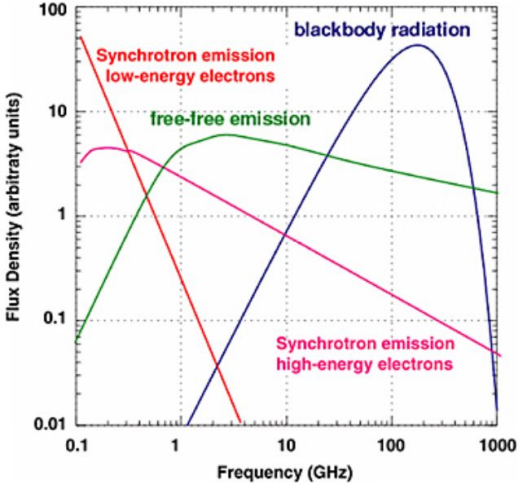
PSF changes with frequency



PB changes with frequency



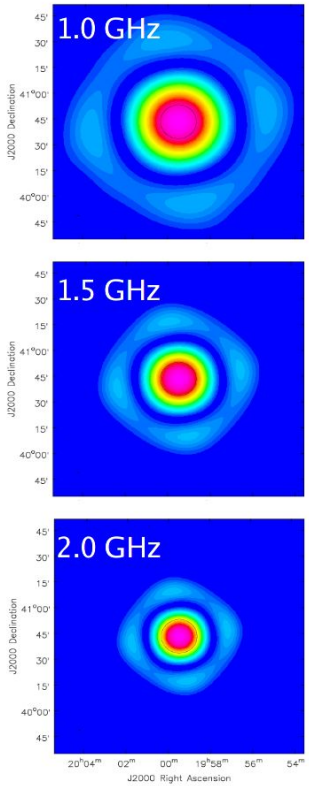
Sky changes with frequency

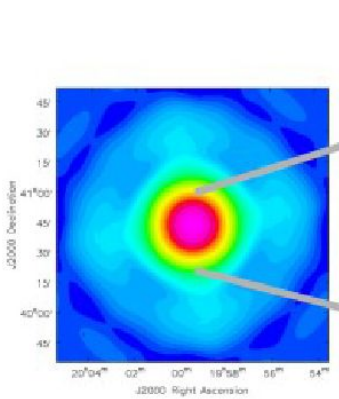


Wideband Imaging with full PB fov

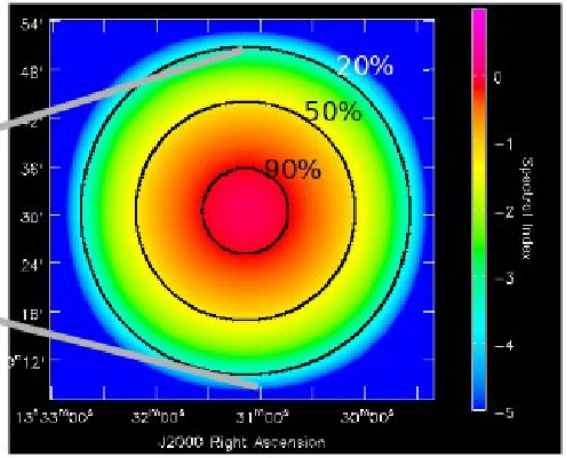
Primary Beam introduces an artificial “ spectral index “

Single Pointing ⇒ Can ignore the PB spectral index in the middle of the FOV





Average PBs



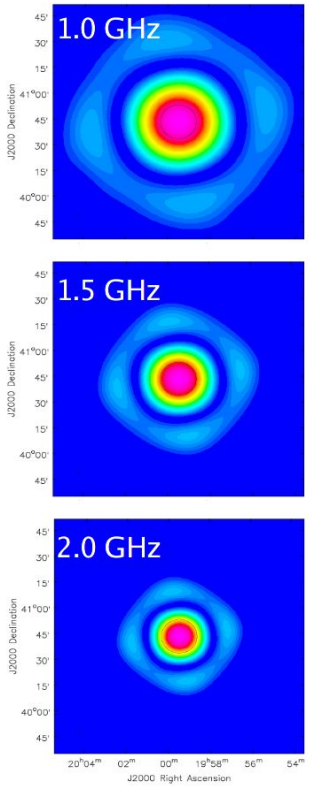
Spectral Index of PB

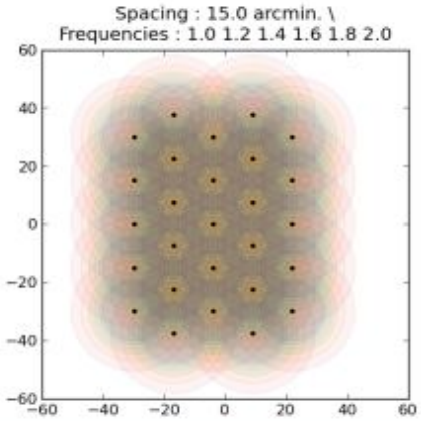
(About -1.4 at the HPBW)

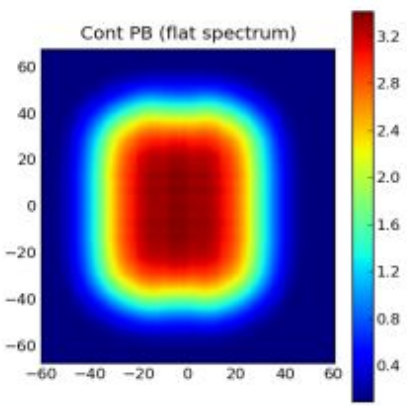
Wideband Mosaic

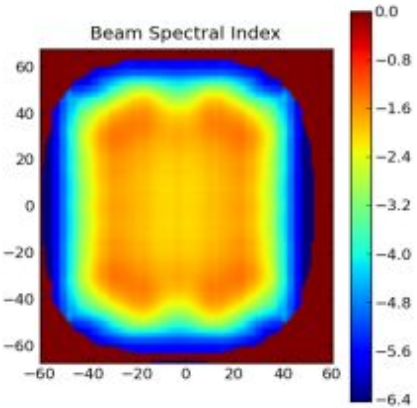
Primary Beam introduces an artificial " spectral index "

Mosaic ⇒ Cannot ignore the PB spectral index anywhere in the mosaic









⇒ P_p and P_f indicate spatial and spectral weighting → wbmos 'sensitivity'

⇒ Need to exclude P_p and P_f when reconstructing the sky model

WB-Mos Options

Separate Deconvolution

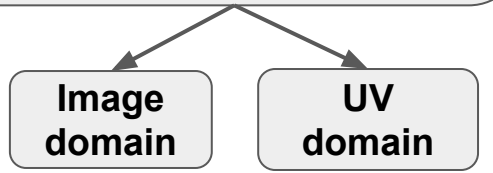
Accumulate $I^{model} \approx P \cdot I^{sky}$

If UV coverage and SNR allow accurate recon per channel and pointing

Joint Deconvolution

Accumulate $I^{obs} \approx I^{psf} \times (P \cdot I^{sky})$

If joint UV coverage or SNR is needed for accurate recon.



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UV domain

Mosaic (PB-weighted sum)	**Continuum (Wideband model)**
E.g. No bright outliers High SNR extended emission	E.g. Simple spatial structure. High SNR Accurate super-resolution.
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PB correction is always approximate -> Need to iterate

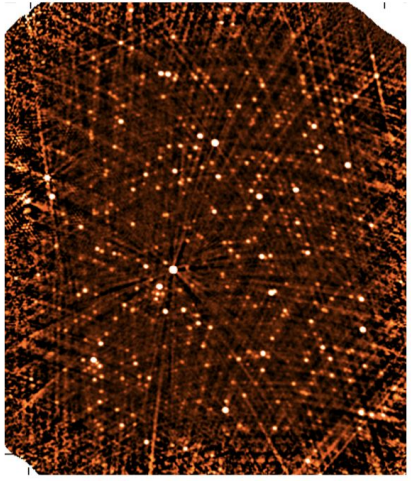
Wide-band Mosaic Options

	Separate deconvolution per pointing	**Joint deconvolution across pointings**
Separate deconvolution per channel	Image and deconvolve each chan and pointing separately Construct mosaic per chan Divide out $P_{f,mos}$ Construct (low rank) wideband model	Joint mosaic deconvolution per channel Divide out $P_{f,mos}$ Construct (low rank) wideband image model
Joint wideband deconvolution	Wideband deconvolution and low-rank model per pointing (Before deconvolution, remove P_{fp} and apply $P_{ref,p}$) Divide out P_{ref} Construct mosaic image model	Combine chans and pointings before joint deconvolution (Before deconvolution, remove P_{fp} and apply $P_{ref,p}$) Divide out $\sum_p P_{ref,p}$

For VLA psfs…

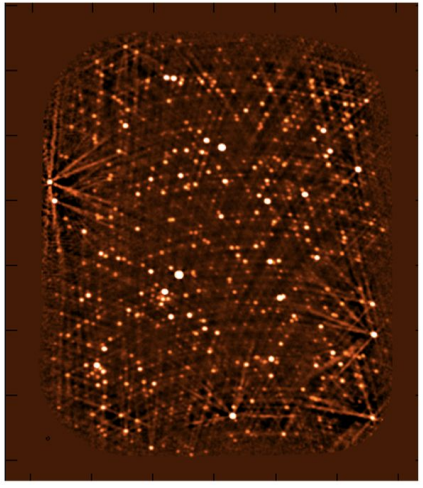
Cube + Joint Mosaic
(with static Primary Beams)

Dyn.Range = 5000:1



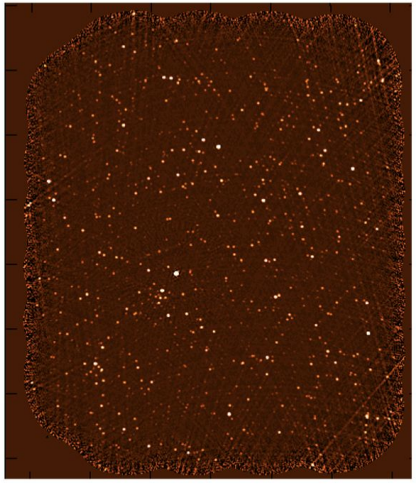
Cube + A-Projection + Joint Mosaic

Dyn.Range = 10000:1



Wideband A-Proj + Joint Mosaic + Multi-term MFS

Dyn.Range = 40000:1



Method Intensity Range	I/I_{true} $> 20\mu Jy$	I/I_{true} $5 - 20\mu Jy$	I/I_{true} $< 5\mu Jy$	$\alpha - \alpha_{true}$ $> 50\mu Jy$	$\alpha - \alpha_{true}$ $10 - 50\mu Jy$
Cube	0.9 ± 0.1	0.9 ± 0.3	0.9 ± 0.5	-0.5 ± 0.2	-0.6 ± 0.5
Cube + AWP	1.0 ± 0.05	1.0 ± 0.2	1.0 ± 0.3	-0.15 ± 0.1	-0.1 ± 0.25
MTMFS + WB-AWP	1.0 ± 0.02	1.0 ± 0.04	1.0 ± 0.15	-0.05 ± 0.05	-0.1 ± 0.2

Example for VLA PSFs :

Joint mosaics and wideband recon were better than other options

Ref : Deep wideband single pointings and mosaics in radio interferometry: How accurately do we reconstruct intensities and spectral indices of faint sources? (U.Rau *et al* 2016 *AJ* 152 124)

For DSA ?

Evaluate assumptions for RC images (per frequency/pointing)

- RC : For each RC image, is $I^{obs}_{fp} = I^{psf}_{fp} \times (P_{fp} . I^{sky}_{f})$ true ?
  - A basic prerequisite for DAT
- DAT : How accurately can image reconstruction be done per RC image ?
  - When is a joint mosaic or wideband recon needed ?

Simulate observed RC images per channel and pointing. Test combination options.

- Simplest : Perfect image recon per chan/pointing + one-step wbmos combination.
- Iterative : Joint reconstruction (for mosaic or wideband or both)
  - Approximate reverse transform
  - Accurate forward transform in the image domain
    - Calculate residuals with RC images I^{obs}_{fp}. (similar to ‘Clark CLEAN’)

Questions to address with this info (From meeting participants - 9 Sept 2025)

- Constraints on co-addition to form mosaics.
  - Storing PSFs ?
  - What granularity at which to save RC images….. Pointings, frequency, timeranges…
- Limitations of knowledge of freq-dep of sources
- Goal - wideband : ~ 3:1 BWR
- Outputs
  - Multiple freq resolutions and bws.
  - 1.3GHz continuum with in-band spectral model. (at continuum sensitivity)
    - And/Or 130MHz subband images.
  - Don’t forget Polarization – sky and instrument.