Polarization w/ radio interferometers

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Electromagnetic Waves Refresher

Astrophysics Motivation

Review of Definitions

Monochromatic vs Quasi-monochromatic

Circular vs Linear Bases

Analytic Signal Representation

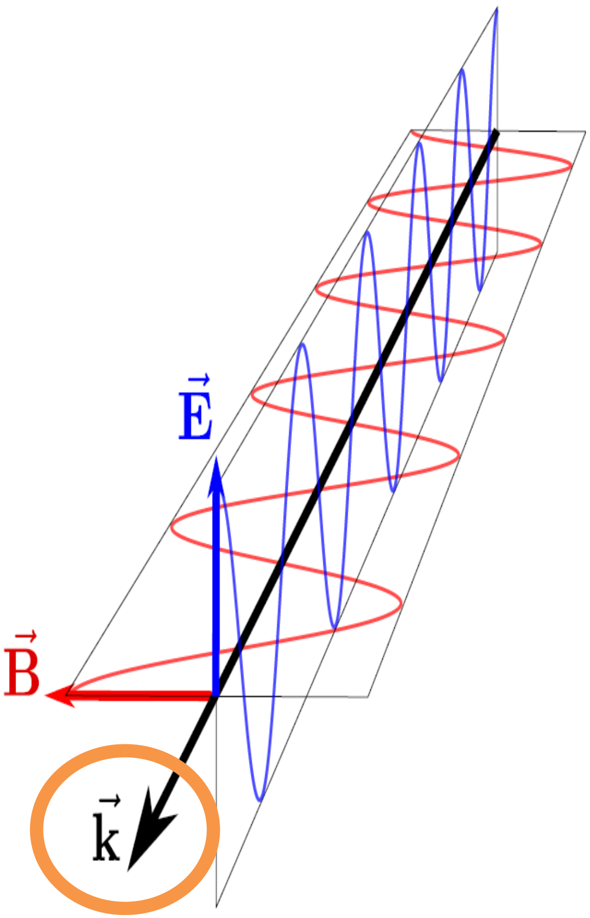
Polarimetry of Interferometers

Stokes Visibilities

Interferometer Response to Polarized Emission

Theory meets real-world

Plane Electromagnetic (EM) Wave



Vector fields describe EM waves

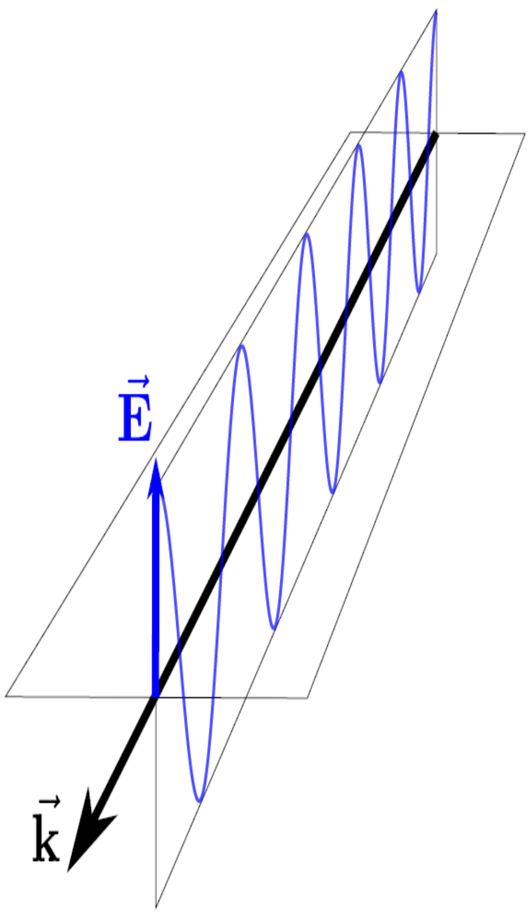
Applying Maxwell's equations for plane monochromatic waves (far field):

wave vector: $\vec{k} = \vec{E} \times \vec{B}$

By convention $\vec{k}$ points at us

Can measure $\vec{E}$ and $\vec{B}$; typically one or the other

Plane Electromagnetic (EM) Wave



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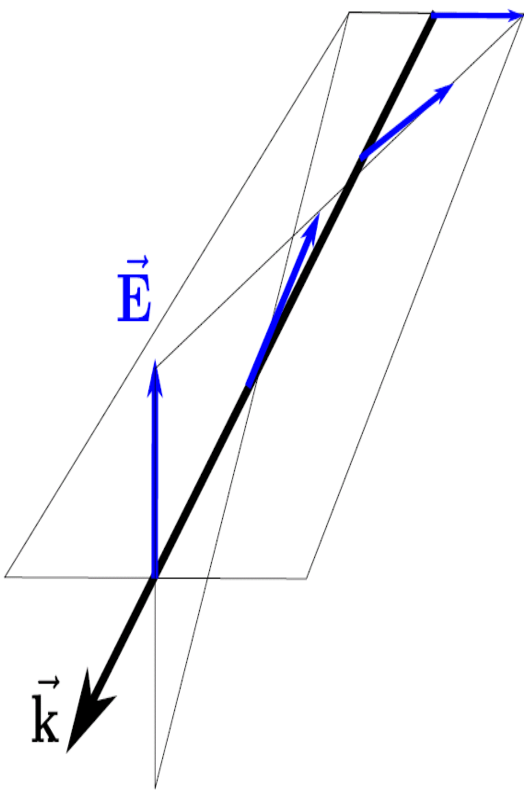
Can measure $\vec{E}$ and $\vec{B}$;
typically one or the other

Typically easiest to measure the field intensity of $\vec{E}$

Single $\vec{E}$ field vector breaks down into an x/y component for mono-chromatic waves:



$$E_x = A_x \cos(kz - \omega t + \delta_1)$$
$$E_y = A_y \cos(kz - \omega t + \delta_2)$$

$$k = \frac{2\pi}{\lambda};\ \omega = 2\pi\upsilon;\ \text{Phase: } \delta_{1/2}$$

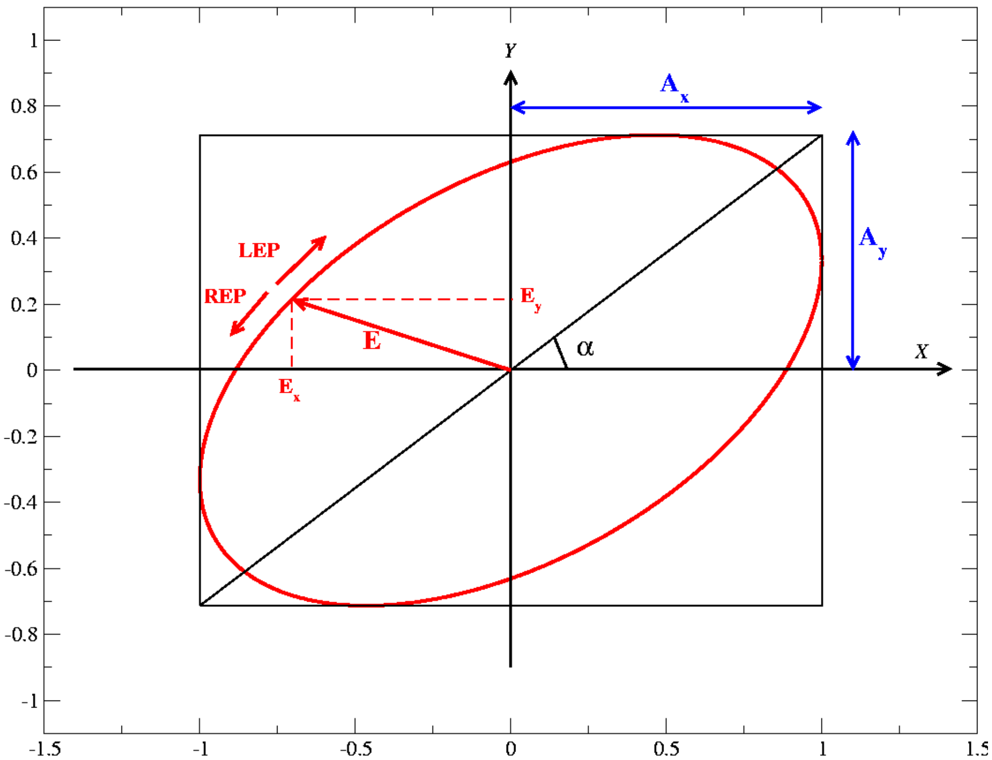
Note: $E_x \neq E_y$

$\vec{E}$ may rotate

($\vec{B}$ obeys the same wave equations)

Plane Polarized EM Wave

$$\left(\frac{E_x}{A_x}\right)^2+\left(\frac{E_y}{A_y}\right)^2-2\frac{E_xE_y}{A_xA_y}\cos\delta=\sin^2\delta$$



Essentially three parameters to describe ellipse:

$$A_x, A_y, \alpha = \tan\left(\frac{A_y}{A_x}\right)$$

Measure of ellipticity:

$\delta = \delta_1 - \delta_2$

$\delta > 0$: CW rotation

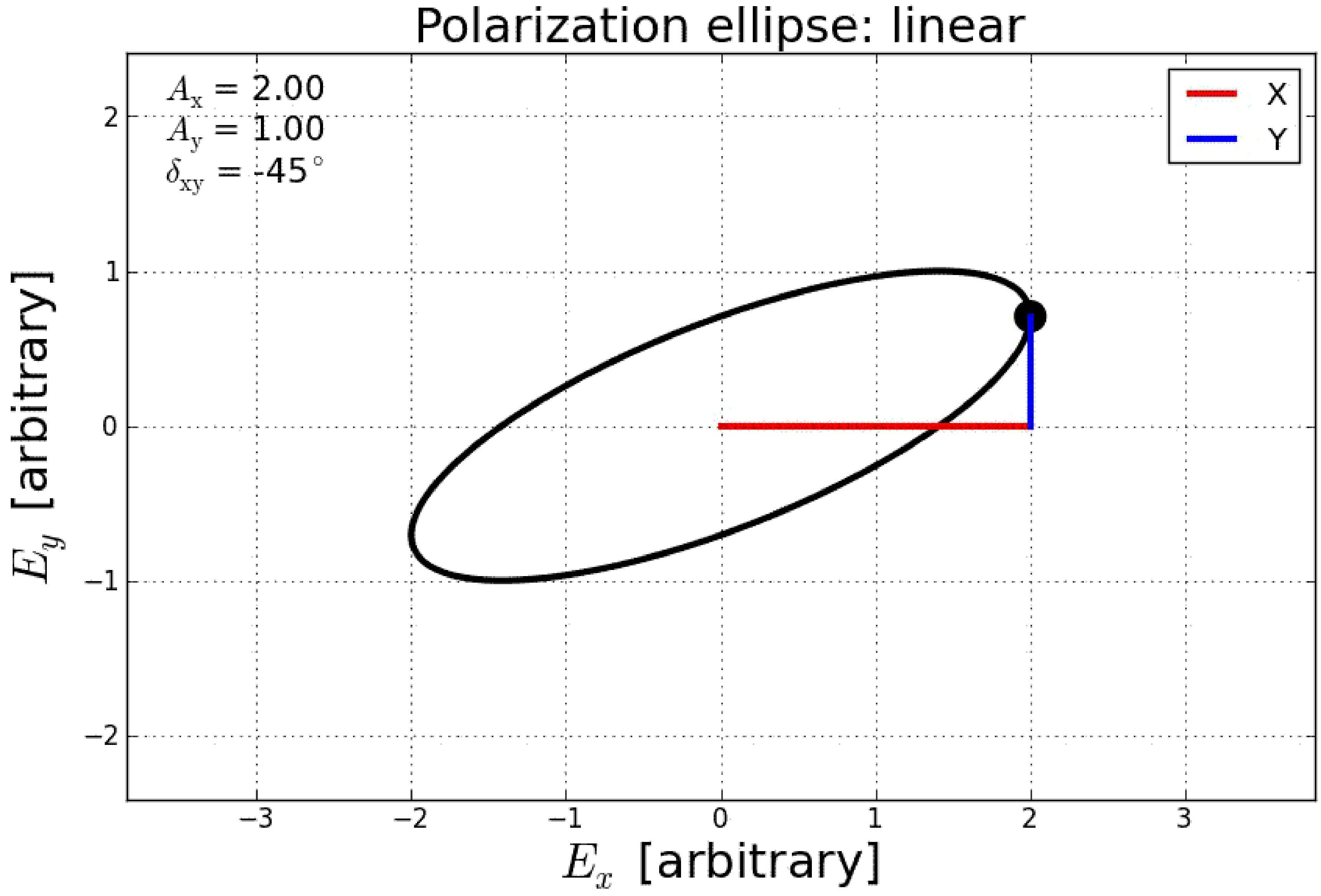
$\delta = 0$: linear polarization

$\delta < 0$: CCW rotation

If $\vec{E}$ is rotating, it is typically interpreted as:

Clockwise, the wave is Left Elliptically Polarized.

Anti-clockwise, the wave is Right Elliptically Polarized.


Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_x [arbitrary]
E_y [arbitrary]

Polarized Radio Emission

Synchrotron Emission

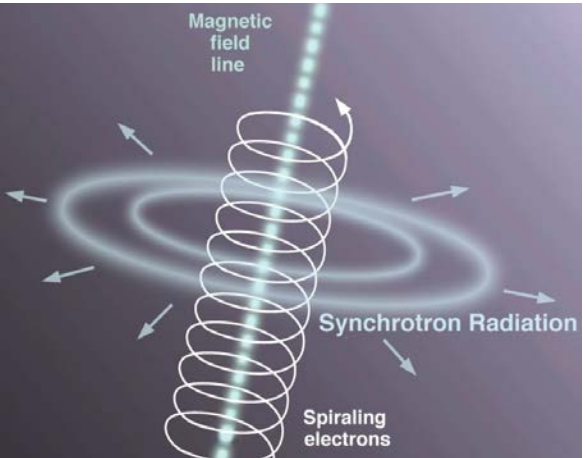


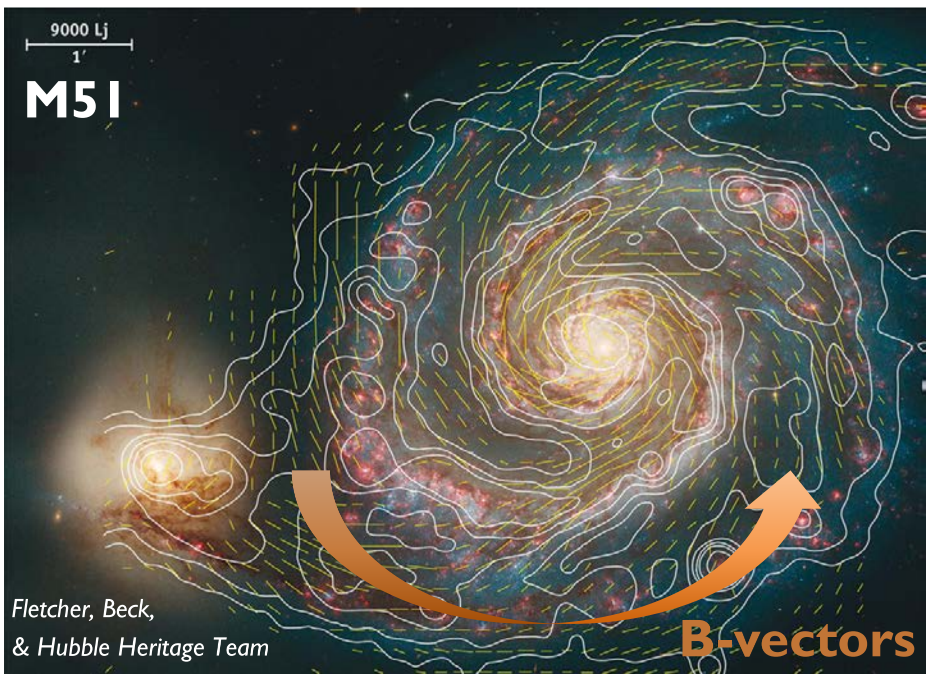
Image Credit: Gemini Observatory

Generates polarized emission

Main emission mechanism at cm-m wavelength

Up to 80% linearly polarized (no circular pol.)

$$< \vec{E}_{\text{source}} > \perp \vec{B}_{\text{source}}$$



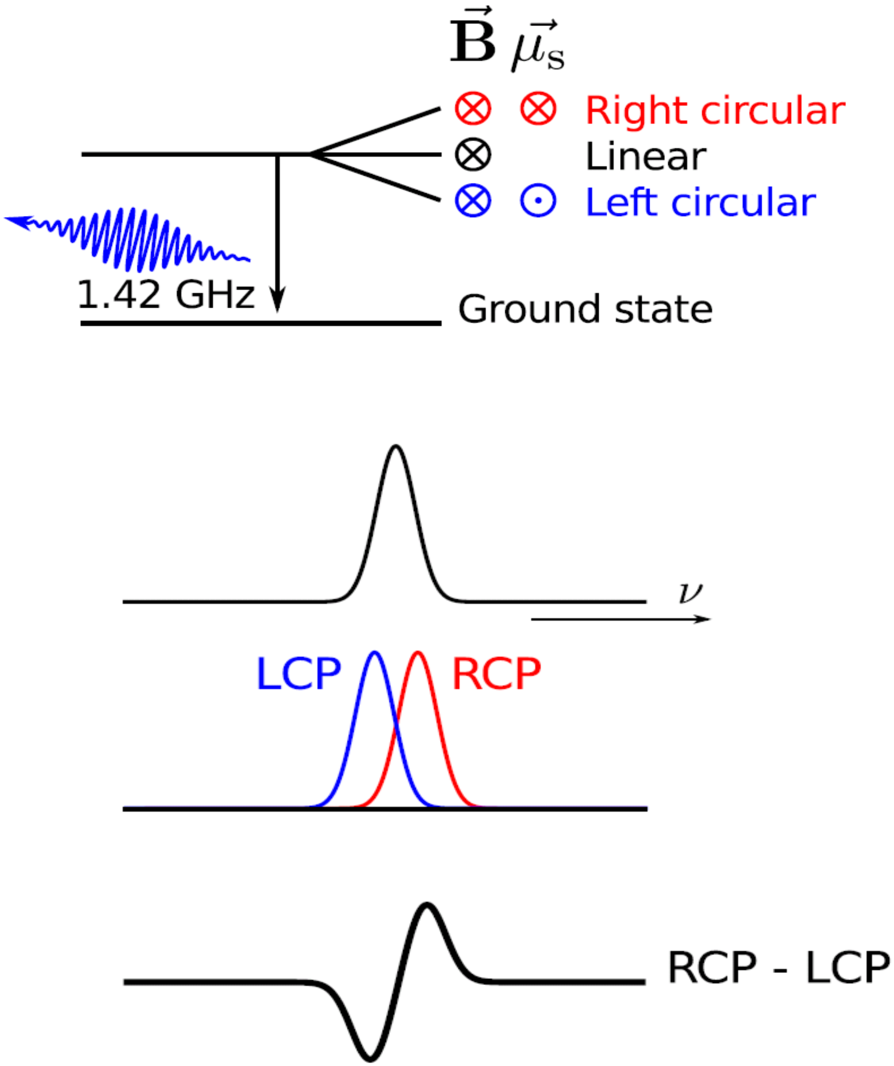
Fletcher, Beck, & Hubble Heritage Team

Polarimetry provides

- B-field direction
- Turbulence
- Indirectly: B-field strength

Polarized Radio Emission

Zeeman splitting



Process

Generates polarized emission

Only in spectral lines

If magnetic moment:

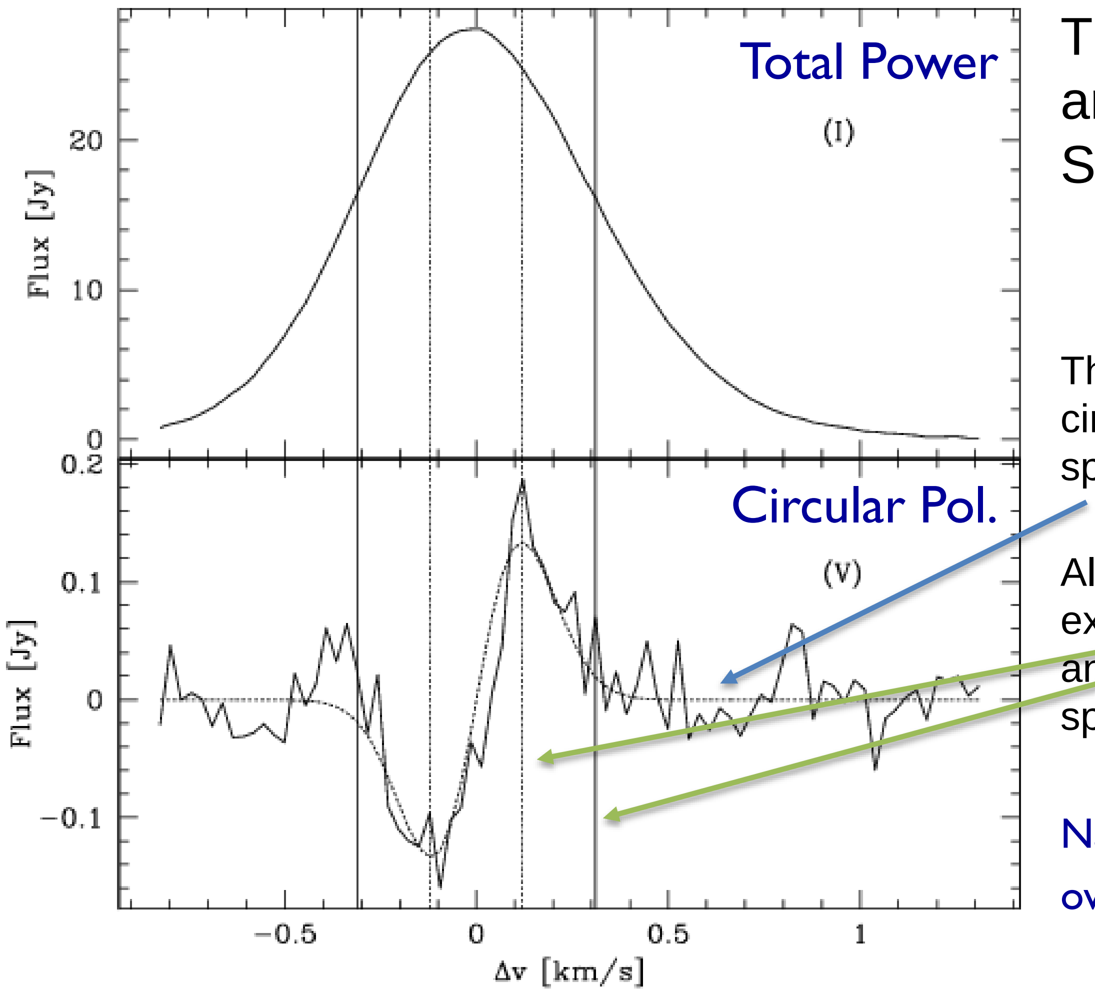
e.g. HI, OH, CN, H2O

B-field splits RCP and LCP

Separation: 2.8 Hz/mG

Polarimetry provides (if detectable) B-field strength at source

Zeeman splitting example
Vlemmings, Diamond, & van Lengevelde (2001)



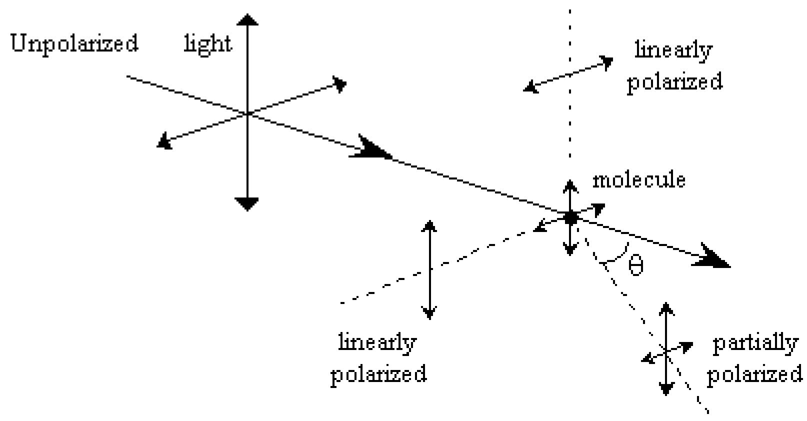
The brightest H_2O maser feature around the late type star S Per (VLBA Observation).

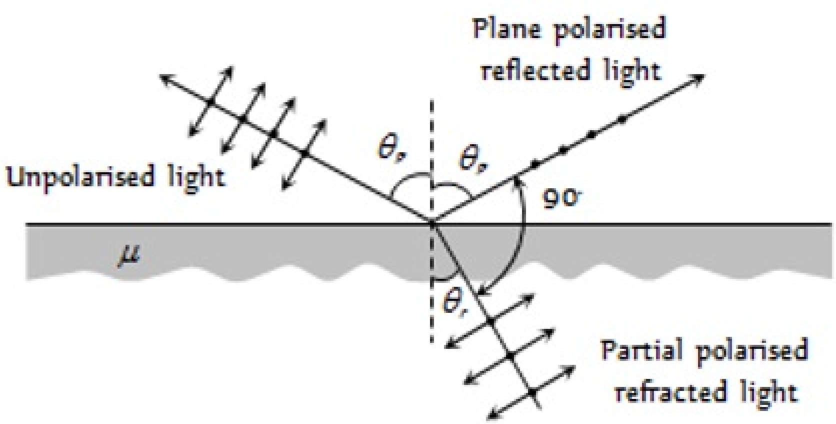
The dashed line is the fit of the synthetic circularly polarized spectrum to the observed spectrum.

Also shown are the observed (dashed) and expected (solid) positions of the minimum and maximum of the circular polarization spectrum.

Narrowing of V-spectrum attributed to overlap of multiple hyperfine components.

Polarized Radio Emission
Scattering/reflection





Unpolarized EM wave scattered by particles; the scattered wave is partially or completely polarized.

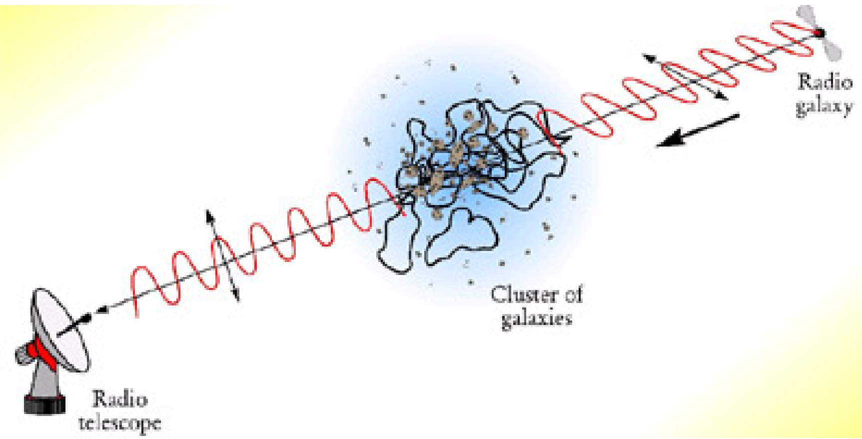
- Modifies polarization state
- Thomson scattering:
  no T dependence

Planets / Moon: dielectric transition

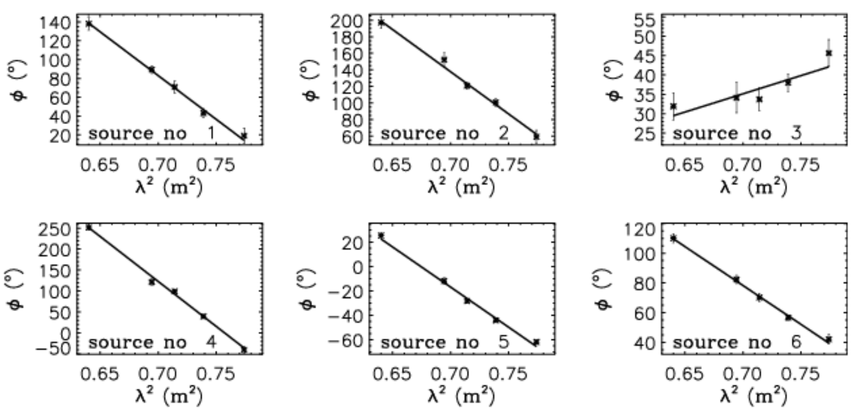
Polarimetry provides:

- Electron densities in cool gas
- Dust properties
- Lunar dielectric constant

Faraday rotation



λ^2 law



Graphs of polarization angle against wavelength squared for polarized extragalactic sources in the field (Haverkorn, Katgert, & de Bruyn 2003).

Process

- Modifies polarization state
- Delay between LCP and RCP
- Rotates linear pol. angle
- $\Delta\chi = \chi_0 + \phi\lambda^2$

$$\phi = 0.812 \int_{\text{there}}^{\text{here}} n_e \vec{B} \cdot d\vec{l}$$

Polarimetry provides

- Source plasma properties
- Intervening plasma properties
- Rare cases: 3D tomography

Stokes Parameters – Monochromatic Case

We utilize in radio astronomy the parameters defined by George Stokes (1852; $ABCD$), and introduced to astronomy by Chandrasekhar (1946; $I_l I_r UV$):

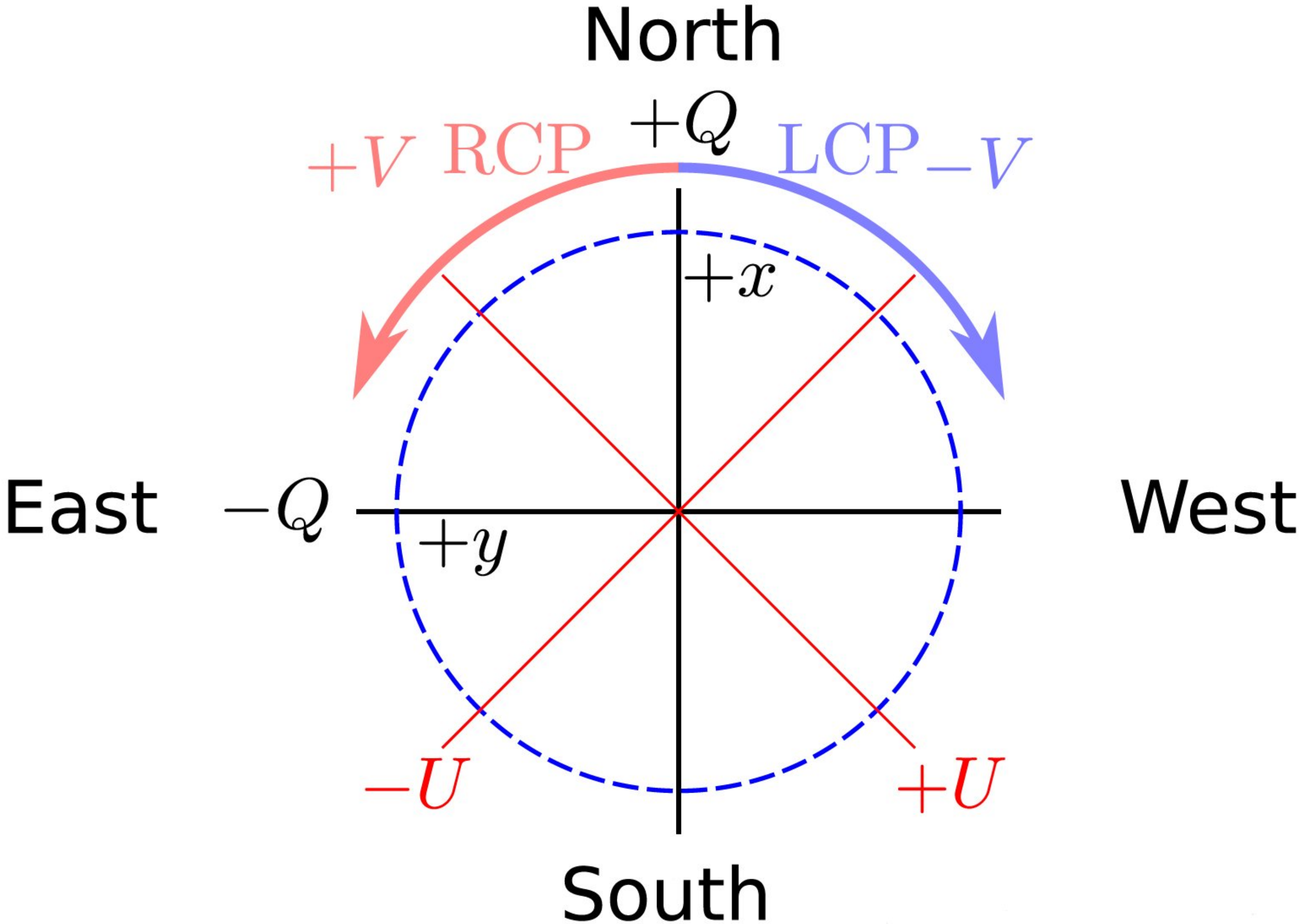
$$
\left.
\begin{aligned}
I &= A_X^2 + A_Y^2 &&= A_R^2 + A_L^2 \\
Q &= A_X^2 - A_Y^2 &&= 2A_R A_L \cos\delta_{RL} \\
U &= 2A_X A_Y \cos\delta_{XY} &&= 2A_R A_L \sin\delta_{RL} \\
V &= 2A_X A_Y \sin\delta_{XY} &&= A_R^2 - A_L^2
\end{aligned}
\right\}
$$

Units of power:
Jy, or Jy/beam

where A_X and A_Y are the Cartesian amplitude components of the E-field, and δ_{XY} is the phase lag between them, and A_R and A_L are the opposite circular amplitude components of the E-field, and δ_{RL} the phase lag between them.

Monochromatic radiation is 100% polarized: $I^2 = Q^2 + U^2 + V^2$

IAU convention (since 1973)



Real Fields, Real Physics

- The monochromatic case is useful for visualizing the definitions, but is not realistic in astronomy.
- Real wideband signals comprise emission from an uncountable number of distant radiators, and are statistical in nature
- 100% polarization is not possible with such systems
- For analysis, we employ the 'quasi-monochromatic' representation:
  - Restrict attention to a very narrow slice of frequency of width $\Delta\upsilon$, for which the fields are described by a single amplitude and phase for a period t~1/$\Delta\upsilon$
  - Since the integration time $T \gg t$, average the short-duration statistical measures to derive the Stokes parameters for timescales of interest.

Stokes Parameters – quasi-monochromatic

- Monochromatic radiation does not exist
- Narrow slices of frequency width $\Delta\upsilon$, for which fields are described by a single amplitude and phase for a period of t$\sim\frac{1}{\Delta\upsilon}$.
- Averaging time $\tau \gg \Delta\upsilon^{-1}$

$$
\begin{aligned}
I &= \left\langle A_X^2 \right\rangle + \left\langle A_Y^2 \right\rangle &&= \left\langle A_R^2 \right\rangle + \left\langle A_L^2 \right\rangle \\
Q &= \left\langle A_X^2 \right\rangle - \left\langle A_Y^2 \right\rangle &&= \left\langle 2 A_R A_L \cos\delta_{RL} \right\rangle \\
U &= \left\langle 2 A_X A_Y \cos\delta_{XY} \right\rangle &&= \left\langle 2 A_R A_L \sin\delta_{RL} \right\rangle \\
V &= \left\langle 2 A_X A_Y \sin\delta_{XY} \right\rangle &&= \left\langle A_R^2 \right\rangle - \left\langle A_L^2 \right\rangle
\end{aligned}
$$

- Note in this case: $I^2 > Q^2 + U^2 + V^2$ *(100% polarization is not possible)*
- Fractional pol.: linear $p = \sqrt{Q^2 + U^2}/I \leq 1$; circular: $v = \left||V|\right|/I \leq 1$

Stokes Parameters for Analytic Signal Representation

- An analytic signal is in signal theory a complex function of time, which imaginary part is the Hilbert transform of the real part.
- We denote the analytic Electric field with script letter $\mathcal{E}$
- The (real) Stokes parameters are thus:

$$
\begin{aligned}
I &= \langle \mathcal{E}_X \mathcal{E}_X^* \rangle + \langle \mathcal{E}_Y \mathcal{E}_Y^* \rangle &&= \langle \mathcal{E}_R \mathcal{E}_R^* \rangle + \langle \mathcal{E}_L \mathcal{E}_L^* \rangle \\
Q &= \langle \mathcal{E}_X \mathcal{E}_X^* \rangle - \langle \mathcal{E}_Y \mathcal{E}_Y^* \rangle &&= \langle \mathcal{E}_R \mathcal{E}_L^* \rangle + \langle \mathcal{E}_L \mathcal{E}_R^* \rangle \\
U &= \langle \mathcal{E}_X \mathcal{E}_Y^* \rangle + \langle \mathcal{E}_Y \mathcal{E}_X^* \rangle &&= i\left(\langle \mathcal{E}_R \mathcal{E}_L^* \rangle - \langle \mathcal{E}_L \mathcal{E}_R^* \rangle\right) \\
V &= i\left(\langle \mathcal{E}_X \mathcal{E}_Y^* \rangle - \langle \mathcal{E}_Y \mathcal{E}_X^* \rangle\right) &&= \langle \mathcal{E}_R \mathcal{E}_R^* \rangle - \langle \mathcal{E}_L \mathcal{E}_L^* \rangle
\end{aligned}
$$

- The relations are valid for a single antenna. All derived values are real.
- How about interferometry?

Stokes Visibilities

Combining the complex fields at antennas1 and 2, the Stokes visibilities can be written as

$$
\begin{aligned}
\mathcal{I} &= \left(\mathcal{E}_{X1}\mathcal{E}_{X2}^{*} + \mathcal{E}_{Y1}\mathcal{E}_{Y2}^{*}\right) &&= \left(\mathcal{E}_{R1}\mathcal{E}_{R2}^{*} + \mathcal{E}_{L1}\mathcal{E}_{L2}^{*}\right)\\
\mathcal{Q} &= \left(\mathcal{E}_{X1}\mathcal{E}_{X2}^{*} - \mathcal{E}_{Y1}\mathcal{E}_{Y2}^{*}\right) &&= \left(\mathcal{E}_{R1}\mathcal{E}_{L2}^{*} + \mathcal{E}_{L1}\mathcal{E}_{R2}^{*}\right)\\
\mathcal{U} &= \left(\mathcal{E}_{X1}\mathcal{E}_{Y2}^{*} + \mathcal{E}_{Y1}\mathcal{E}_{X2}^{*}\right) &&= i\left(\mathcal{E}_{R1}\mathcal{E}_{L2}^{*} - \mathcal{E}_{L1}\mathcal{E}_{R2}^{*}\right)\\
\mathcal{V} &= i\left(\mathcal{E}_{X1}\mathcal{E}_{Y2}^{*} - \mathcal{E}_{Y1}\mathcal{E}_{X2}^{*}\right) &&= \left(\mathcal{E}_{R1}\mathcal{E}_{R2}^{*} - \mathcal{E}_{L1}\mathcal{E}_{L2}^{*}\right)
\end{aligned}
$$

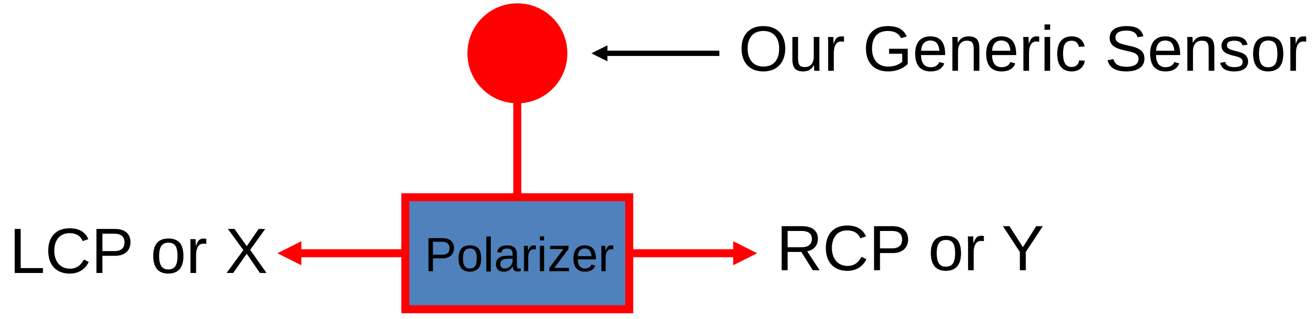
The angle brackets <> were dropped for better readability

The script symbols $\mathcal{I}, \mathcal{Q}, \mathcal{U}, \mathcal{V}$ remind us that these Stokes Visibilities are complex numbers, related to the (real) source brightness through Fourier transform, e.g.: $\mathcal{I}(u, v) = \int_{lm} I e^{+2\pi i v(ul+vm)/c} \mathrm{d}l\, \mathrm{d}m$.

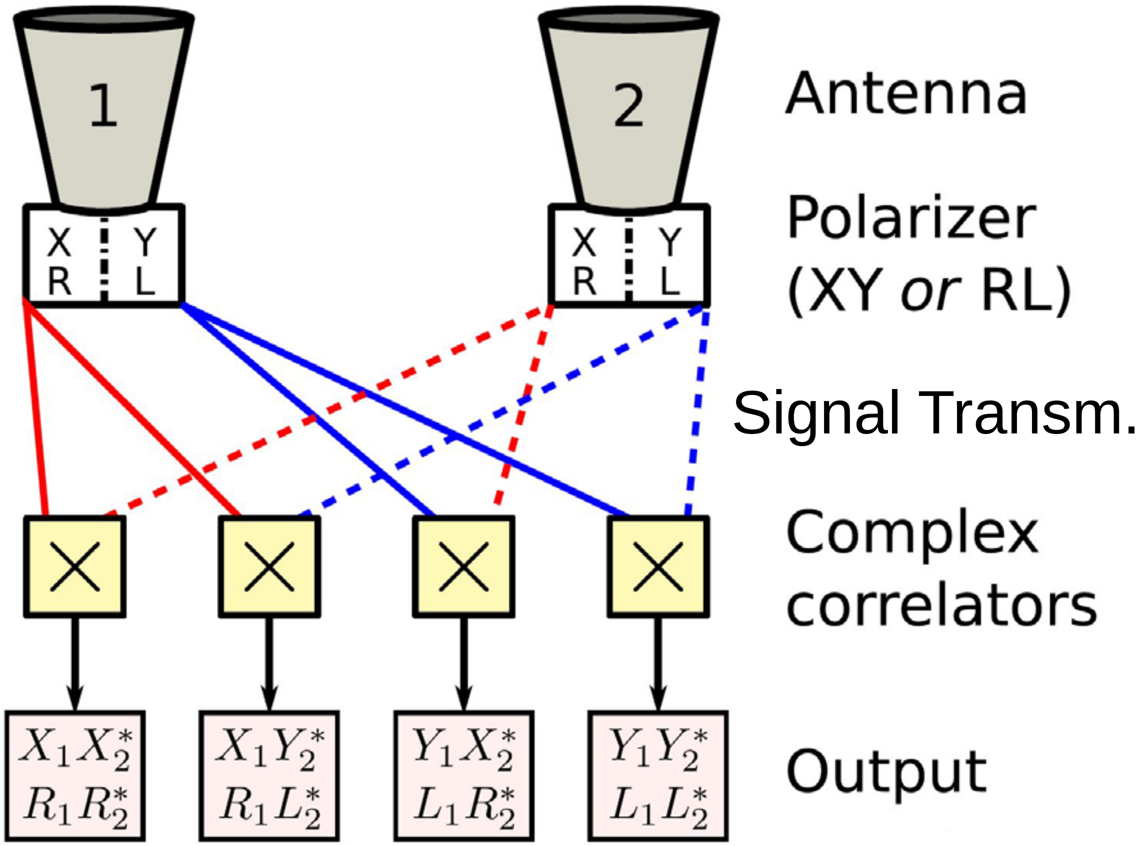
$$\mathcal{I} \Leftrightarrow \mathrm{I}, \quad \mathcal{Q} \Leftrightarrow \mathrm{Q}, \quad \mathcal{U} \Leftrightarrow \mathrm{U}, \quad \mathcal{V} \Leftrightarrow \mathrm{V}$$

Relation to Sensors

- The discussed Stokes representation was formulated in terms of electric fields measured at two locations.
- What is the relation to real sensors (antennas)?
- Antennas are polarized – they provide two simultaneous voltage signals whose values are (ideally) representations of the two electric field components – either in circular or linear basis.



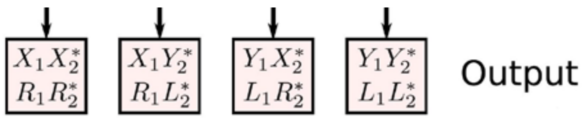
Relation to Sensors



Two antennas, each with two differently polarized outputs, produce four complex correlations.

From these four outputs, we want to generate the four complex Stokes' visibilities $\mathcal{I}, \mathcal{Q}, \mathcal{U}, \mathcal{V}$.

Relating the Products to Stokes' Visibilities



Let R_1, L_1, R_2, L_2 be the complex representation (analytic signal) of the RCP and LCP voltages emerging from our (perfect) antennas.

We can then utilize the definitions given earlier to show that the four complex correlations between these fields are related to the desired visibilities by (ignoring gain factors):

$$V_{R1R2} = \left\langle R_1 R_2^* \right\rangle = (\mathcal{I} + \mathcal{V})/2$$

$$V_{L1L2} = \left\langle L_1 L_2^* \right\rangle = (\mathcal{I} - \mathcal{V})/2$$

$$V_{R1L2} = \left\langle L_1 R_2^* \right\rangle = (\mathcal{Q} + i\mathcal{U})/2$$

$$V_{L1R2} = \left\langle R_1 L_2^* \right\rangle = (\mathcal{Q} - i\mathcal{U})/2$$

Solving for Stokes Visibilities

The solutions are straightforward:

Circular	Cartesian
$\mathcal{I} = V_{R1R2} + V_{L1L2}$	$\mathcal{I} = V_{X1X2} + V_{Y1Y2}$
$\mathcal{V} = V_{R1R2} - V_{L1L2}$	$\mathcal{Q} = V_{X1X2} - V_{Y1Y2}$
$\mathcal{Q} = V_{R1L2} + V_{L1R2}$	$\mathcal{U} = V_{X1Y2} + V_{Y1X2}$
$\mathcal{U} = i(V_{R1L2} - V_{L1R2})$	$\mathcal{V} = i(V_{X1Y2} - V_{Y1X2})$

If calibration errors dominate (and they often do), the circular basis favors measurements of linear polarization and the linear basis favors measurements of circular polarization.

Although it is true that Q, U, and V are <<1, it does not necessarily follow that Q, U, and V are much smaller than I (notable for extended objects).

Real Sensors

Sadly real sensors are:

1. Imperfectly polarized.
   Typically, the cross-polarization for circularly polarized systems is ~5% (better with linear).
2. Misaligned with the sky frame.

   Alt-Az antennas rotate w.r.t. the sky frame as they track a celestial source. The angle describing the misalignment is called the 'parallactic angle'.

   Equatorial antennas are fixed w.r.t the sky, but there will be a (small) misalignment of the feed system with the sky.

How do these imperfections affect the polarimetry?

Start with Antenna rotation (it's easier)

Antenna Rotation - Circular

For perfectly circularly polarized antennas, when both antennas are rotated by an angle ψ_P.

$$V_{R1R2} = (\mathcal{I} + \mathcal{V})/2 \qquad \mathcal{I} = V_{R1R2} + V_{L1L2}$$

$$V_{L1L2} = (\mathcal{I} - \mathcal{V})/2 \qquad \mathcal{V} = V_{R1R2} - V_{L1L2}$$

$$V_{R1L2} = (\mathcal{Q} + i\mathcal{U})e^{i2\psi_P}/2 \qquad \mathcal{Q} = V_{R1L2}e^{i2\Psi_P} + V_{L1R2}e^{-i2\Psi_P}$$

$$V_{L1R2} = (\mathcal{Q} - i\mathcal{U})e^{-i2\psi_P}/2 \qquad \mathcal{U} = -i(V_{R1L2}e^{i2\Psi_P} - V_{L1R2}e^{-i2\Psi_P})$$

The effect of antenna rotation is to simply rotate the RL and LR visibilities. Parallel hand visibilities are unaffected.

Q and U require only the cross-hand correlations. I and V require only the parallel hand correlations.

Circular vs Linear

One of the ongoing debates is the advantages and disadvantages of Linear and Circular systems, e.g.VLA mostly circular vs. ALMA linear; ngVLA linear?

Point of principle: For full polarization imaging, both systems must provide the same results. Advantages/disadvantages of each are based on points of practicalities.

Circular System	Linear System
$\mathcal{I} = V_{R1R2} + V_{L1L2}$	$\mathcal{I} = V_{X1X2} + V_{Y1Y2}$
$\mathcal{V} = V_{R1R2} - V_{L1L2}$	$\mathcal{V} = i\left(V_{X1Y2} - V_{Y1X2}\right)$
$\mathcal{Q} = e^{i2\Psi_P} V_{R1L2} + e^{-i2\Psi_P} V_{L1R2}$	$\mathcal{Q} = \left(V_{X1X2} - V_{Y1Y2}\right)\cos 2\Psi_P - \left(V_{Y1X2} + V_{X1Y2}\right)\sin 2\Psi_P$
$\mathcal{U} = i\left(e^{-i2\Psi_P} V_{L1R2} - e^{i2\Psi_P} V_{R1L2}\right)$	$\mathcal{U} = \left(V_{X1X2} - V_{Y1Y2}\right)\sin 2\Psi_P + \left(V_{Y1X2} + R_{X1Y2}\right)\cos 2\Psi_P$

For both, **Stokes 'I'** is the sum of the parallel-hands.

Stokes 'V' is the difference of the crossed hand responses for linear (good) and is the difference of the parallel-hand responses for circular (bad)

Stokes 'Q' and 'U' are differences of cross-hand responses for circular (good), and differences of parallel hands for linear (bad)

Circular vs. Linear

- Both systems provide straightforward derivation of the Stokes' visibilities from the four correlations.
- Deriving useable information from differences of large values requires both good stability and good calibration. Hence:
  - To do good circular polarization using circular system, or good linear polarization with a linear system, we need special care and special methods to ensure good calibration.
- There are practical reasons to use linear:
  - Antenna polarizers are natively linear – extra components are needed to produce circular. This hurts performance.
  - These extra components are also generally of narrower bandwidth – it's harder to build circular systems with really wide bandwidth.
  - At mm wavelengths, the needed phase shifters are not available.
- One important practical reason favoring circular:
  - Calibrator sources are often significantly linearly polarized, but have imperceptible circular polarization.
  - Gain calibration is much simpler with circular feeds, especially for 'snapshot' style observations.

Calibration Troubles …

- To understand this last point, note that for the linear system:

$$V_{Y1Y2} = G_{Y1}G^{*}_{Y2}(\mathcal{I} + \mathcal{Q}\cos 2\Psi_P + \mathcal{U}\sin 2\Psi_P)/2$$
$$V_{X1X2} = G_{X1}G^{*}_{X2}(\mathcal{I} - \mathcal{Q}\cos 2\Psi_P - \mathcal{U}\sin 2\Psi_P)/2$$

- To calibrate means to solve for G_Y and G_X terms.
- To do so requires knowledge of both Q and U.
- Virtually all calibrators have notable, and variable, linear pol.
- Meanwhile, for circular:

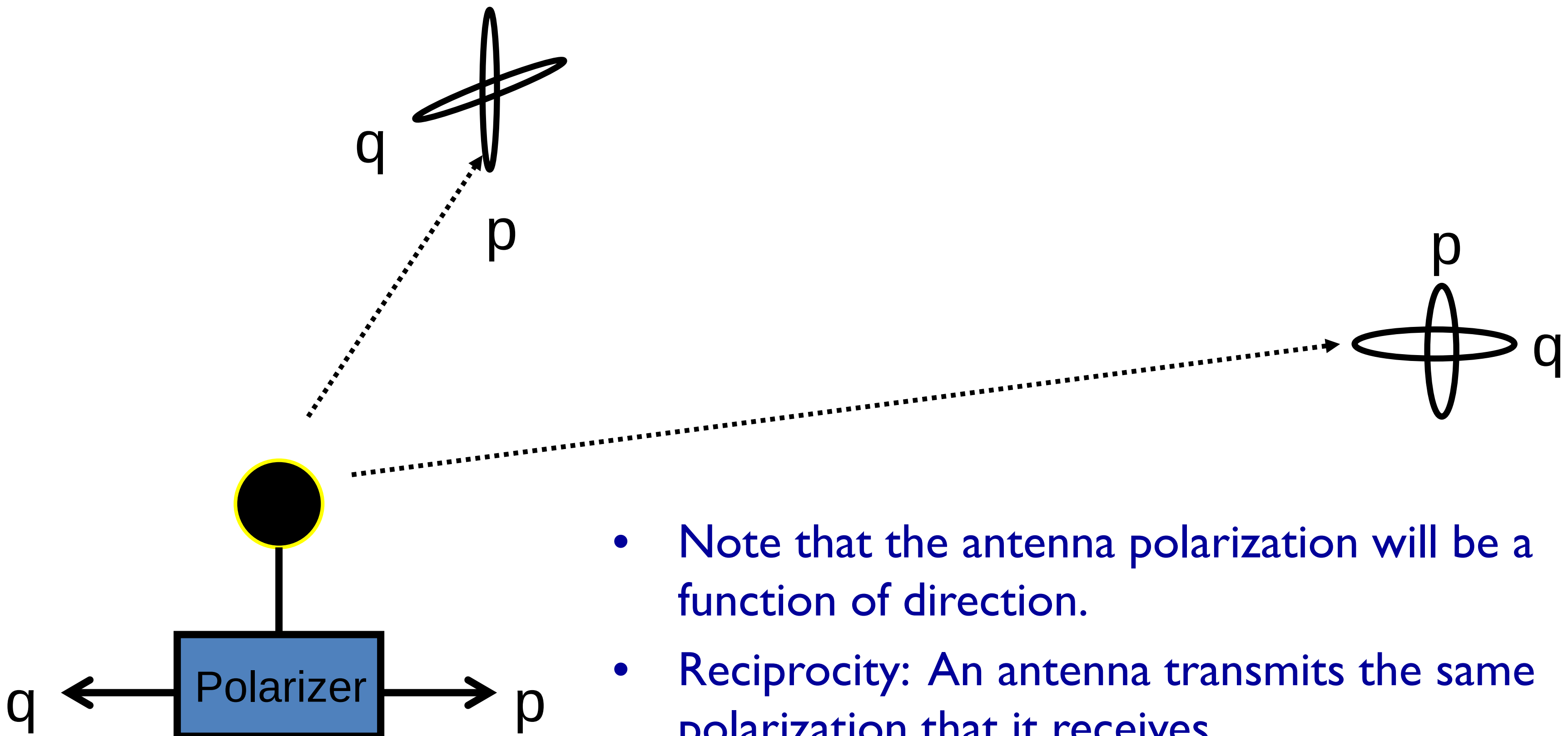
$$V_{R1R2} = G_{R1}G^{*}_{R2}(\mathcal{I} + \mathcal{V})/2$$
$$V_{L1L2} = G_{L1}G^{*}_{L2}(\mathcal{I} - \mathcal{V})/2$$

  – In this case we have **no** sensitivity to Q or U (good!). Instead, we have a sensitivity to V.
  – But as it turns out – V is nearly always negligible for the 1000-odd sources that we use as standard calibrators.

Polarization of Real Antennas

Unfortunately, antennas never provide perfectly orthogonal outputs.

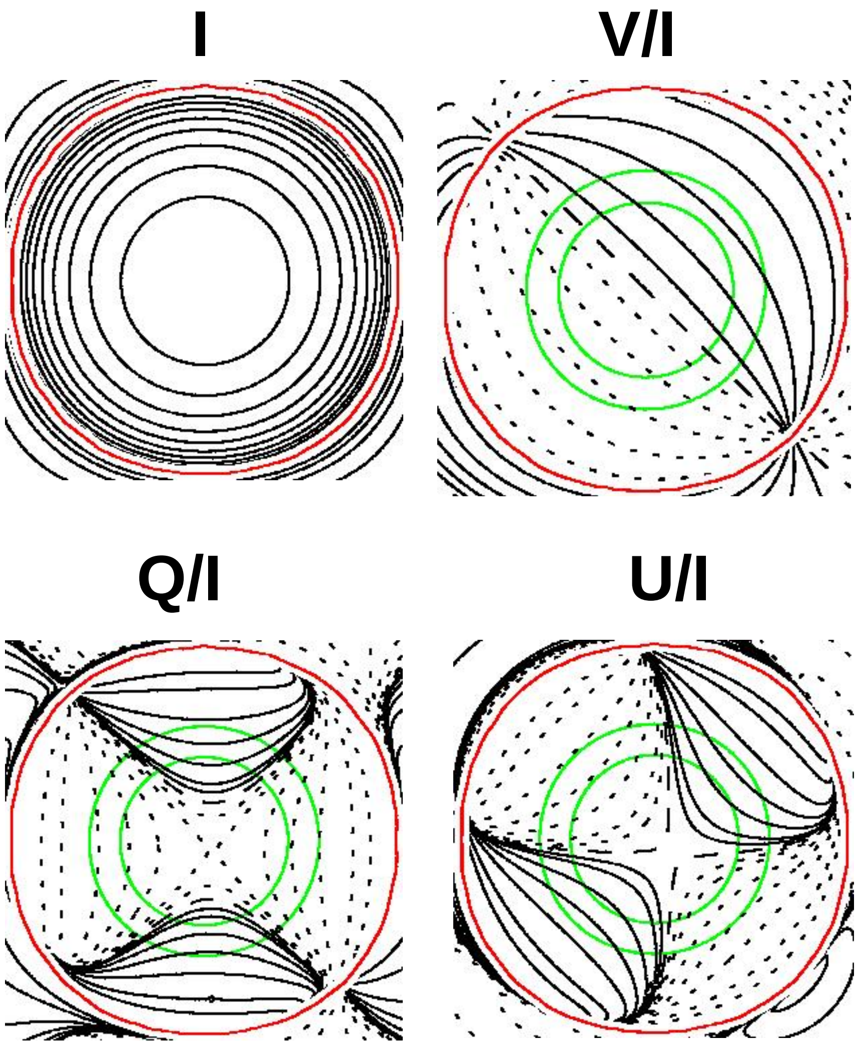
In general, the two outputs from an antenna are elliptically polarized.



- Note that the antenna polarization will be a function of direction.
- Reciprocity: An antenna transmits the same polarization that it receives.

Beam Polarization for VLA

- The beam polarization is due to the antenna and feed geometry.
- Grasp8 calculation by Walter Brisken. (EVLA Memo #58, 2003)
- Contour intervals: V/I = 4%, Q/I, U/I= 0.2%
- Very large V/I polarization is due to the VLA's offset feeds.
- The more modest linear polarization is due to the parabolic antenna.
- The beam polarizations can be removed in software – if antenna patterns are known – at considerable computational cost.



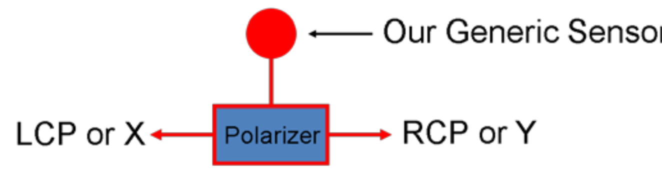
Coherency matrix

Relating Output Voltages from Real Systems to Input Electric Fields

The Stokes visibilities we want are defined in terms of the complex cross-correlations (coherencies) of electric fields ($\boldsymbol{E}_{ij}$).

Voltage vector from polarizers: $\boldsymbol{e}_i = \begin{pmatrix} p_i \\ q_i \end{pmatrix}$

p/q designate either x/y or r/l



Correlator multiplies ($\boldsymbol{E}_{ij}$ is the coherency matrix):

$$\boldsymbol{E}_{ij} = \boldsymbol{e}_i \boldsymbol{e}_j^{\dagger} = \begin{pmatrix} p_i \\ q_i \end{pmatrix} \begin{pmatrix} p_j^* & q_j^* \end{pmatrix}$$

$$\boldsymbol{E}_{ij} = \begin{pmatrix} p_i p_j^* & p_i q_j^* \\ q_i p_j^* & q_i q_j^* \end{pmatrix}$$

In a real system, $\boldsymbol{E}_{ij} \neq \boldsymbol{e}_i \boldsymbol{e}_j^{\dagger}$, it's a function of both polarizations **and** some gain factors, $\boldsymbol{E}_{ij} = \boldsymbol{g}_i \boldsymbol{e}_i \boldsymbol{g}_j^{\dagger} \boldsymbol{e}_j^{\dagger}$.

Jones Matrix Algebra

From now assume all systems linear:

$$\boldsymbol{e}'_i = \boldsymbol{J}_i \boldsymbol{e}_i$$

$\boldsymbol{J}_i$ (2x2) is called **Jones matrix**

Cross correlation:

$$\begin{aligned} \boldsymbol{E}'_{ij} &= \boldsymbol{e}'_i \boldsymbol{e}'^{\dagger}_j \\ &= \boldsymbol{J}_i \boldsymbol{e}_i \left(\boldsymbol{J}_j \boldsymbol{e}_j\right)^{\dagger} \\ &= \boldsymbol{J}_i \boldsymbol{e}_i \boldsymbol{e}^{\dagger}_j \boldsymbol{J}^{\dagger}_j \\ &= \boldsymbol{J}_i \boldsymbol{E}_{ij} \boldsymbol{J}^{\dagger}_j \end{aligned}$$

This is the measurement equation.

Invertible!

$$\boldsymbol{E}'_{ij} = \boldsymbol{J}^{-1}_i \boldsymbol{E}'_{ij} \boldsymbol{J}^{\dagger -1}_j$$

Example Jones Matrices

Perfect instrument:

$$\boldsymbol{J} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Polarization leakage:

$$\boldsymbol{J} = \begin{pmatrix} g_p & d_{q\to p} \\ d_{p\to q} & g_q \end{pmatrix}$$

Time delay:

$$\boldsymbol{J} = \begin{pmatrix} e^{2\pi i \upsilon \tau_p} & 0 \\ 0 & e^{2\pi i \upsilon \tau_q} \end{pmatrix}$$

Parallactic angle or feed rotation XY:

$$\boldsymbol{J} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Receiver gain:

$$\boldsymbol{J} = \begin{pmatrix} g_p & 0 \\ 0 & g_q \end{pmatrix}$$

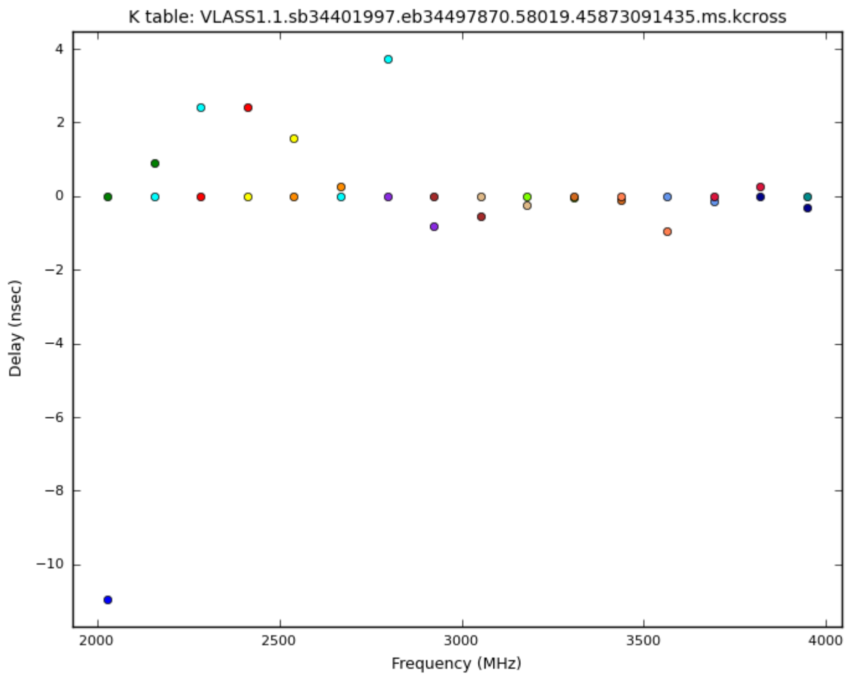
Parallactic angle or feed rotation RL:

$$\boldsymbol{J} = \begin{pmatrix} e^{+i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix}$$

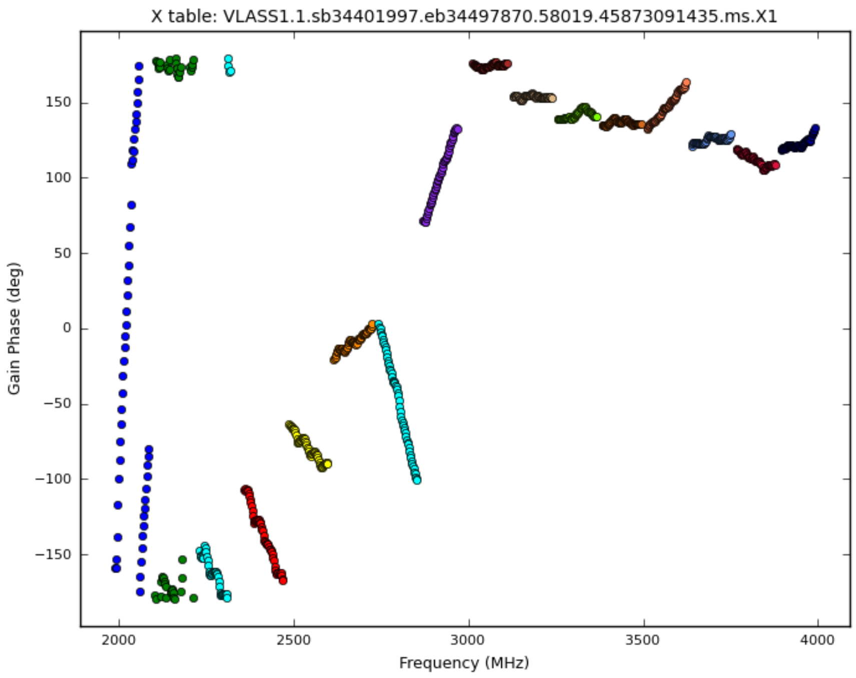
Each component of the overall system, including propagation effects, can be represented by a Jones matrix and multiplied to obtain a 'system Jones' matrix.

RL Delays/Phase – real examples

Instrumental delay between polarizations due to e.g. differences in signal path lengths.



Derived delays of parallel hands with respect to each other (one held at zero).



Derived RL phase corrections

Measuring Cross-Polarization Terms

Correction of the X/Y or R/L response for the 'leakage' is important, since the D-term amplitude is comparable to the fractional polarization.

There are two standard ways to proceed (circular base):

1. Observe a calibrator source of known polarization (preferably zero!)
2. Observe a calibrator of unknown polarization over an extended period.

Case 1: Calibrator source known to have zero polarization

$$V_{R1R2} = \mathcal{I}/2$$
$$V_{L1L2} = \mathcal{I}/2$$
$$V_{R1L2} = \mathcal{I}\left(D_{R1} + D_{L2}^*\right)/2$$
$$V_{L1R2} = \mathcal{I}\left(D_{L1} + D_{R2}^*\right)/2$$

Single observation should suffice to measure leakage terms.

Note: In this approximation, only 2Nant-1 terms can be determined. One must be assumed (usually = 0). All the others are referred to this, thus 'relative' D terms.

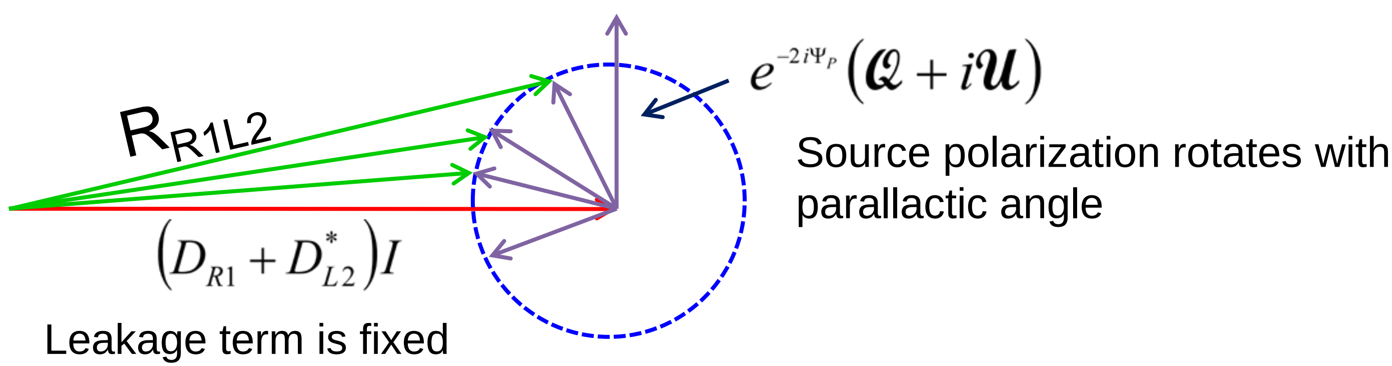
Measuring Cross-Polarization Terms

Case 2: Calibrator with significant (or unknown) polarization.

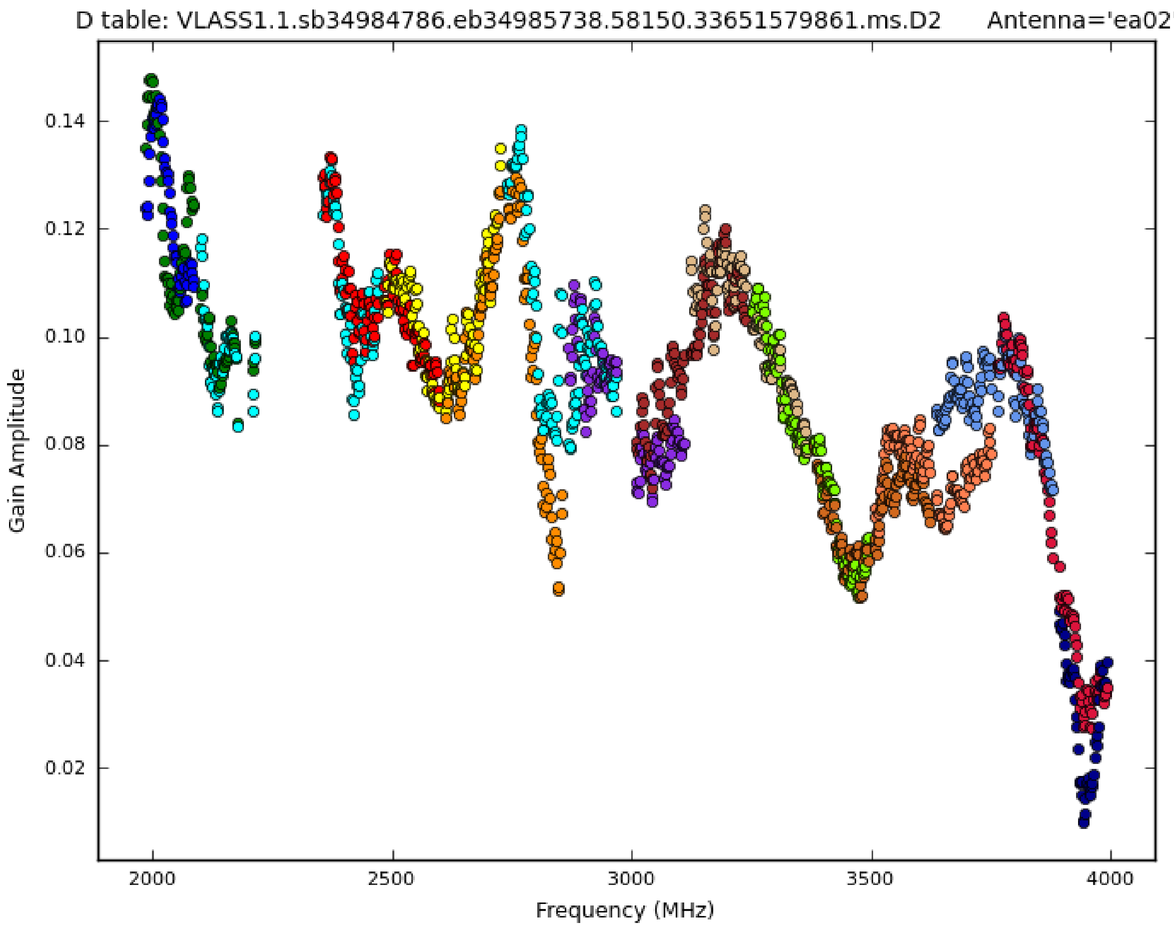
- You can determine both the (relative) D terms and the calibrator polarizations for an alt-az antenna by observing over a wide range of parallactic angle. (Conway and Kronberg first used this method.)

$$V_{L1R2} = \left[\left(D_{L1} + D^*_{R2}\right)\mathcal{I} + e^{2i\Psi_P}\left(\mathcal{Q} - i\mathcal{U}\right)\right]/2$$
$$V_{R1L2} = \left[\left(D_{R1} + D^*_{L2}\right)\mathcal{I} + e^{-2i\Psi_P}\left(\mathcal{Q} + i\mathcal{U}\right)\right]/2$$

- As time passes ψ_p changes in a known way.
- The source polarization term then rotates w.r.t. the antenna leakage term, allowing a separation.



Examples VLA D-terms



Real VLA S-band D-term amplitudes.

Significant frequency structure (2-4 MHz scale).

Antenna polarization ~8-10% for this particular VLA antenna w.r.t. the reference antenna.

I and Q Visibilities for Mars at 23 GHz

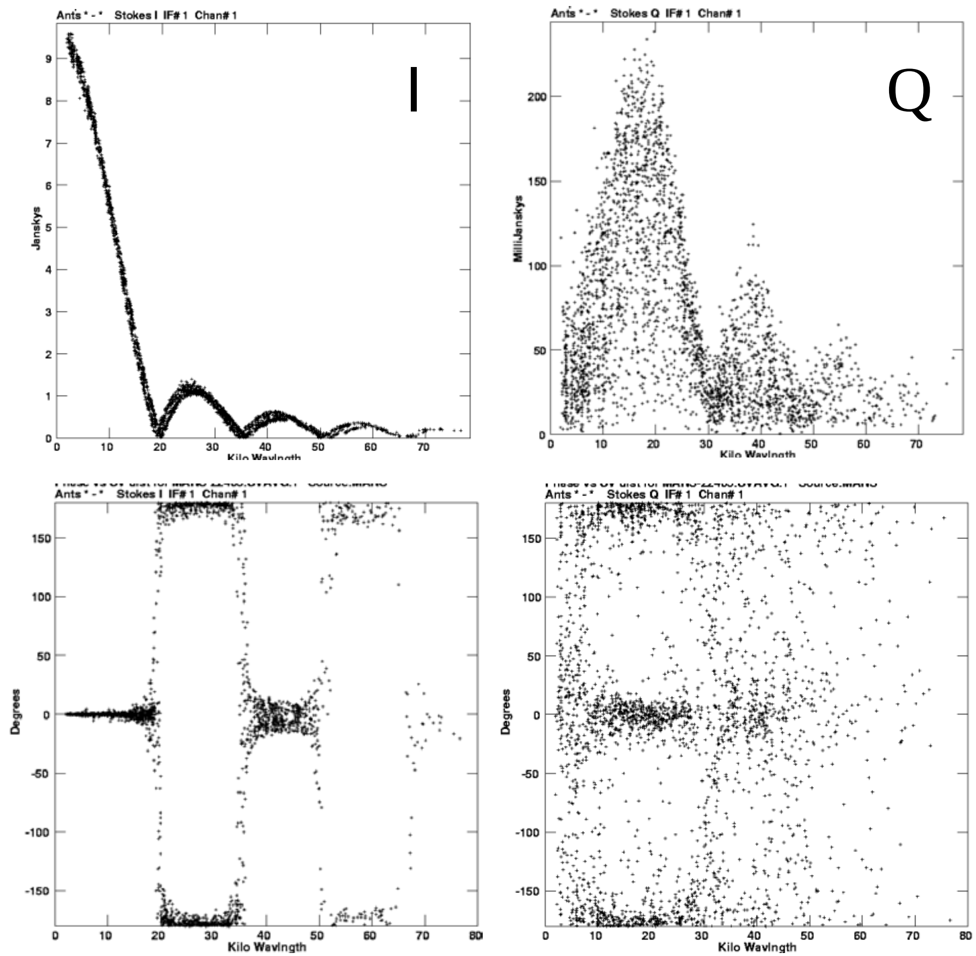
VLA, 23 GHz, 'D' config. (Jan. 2006)

Amplitude

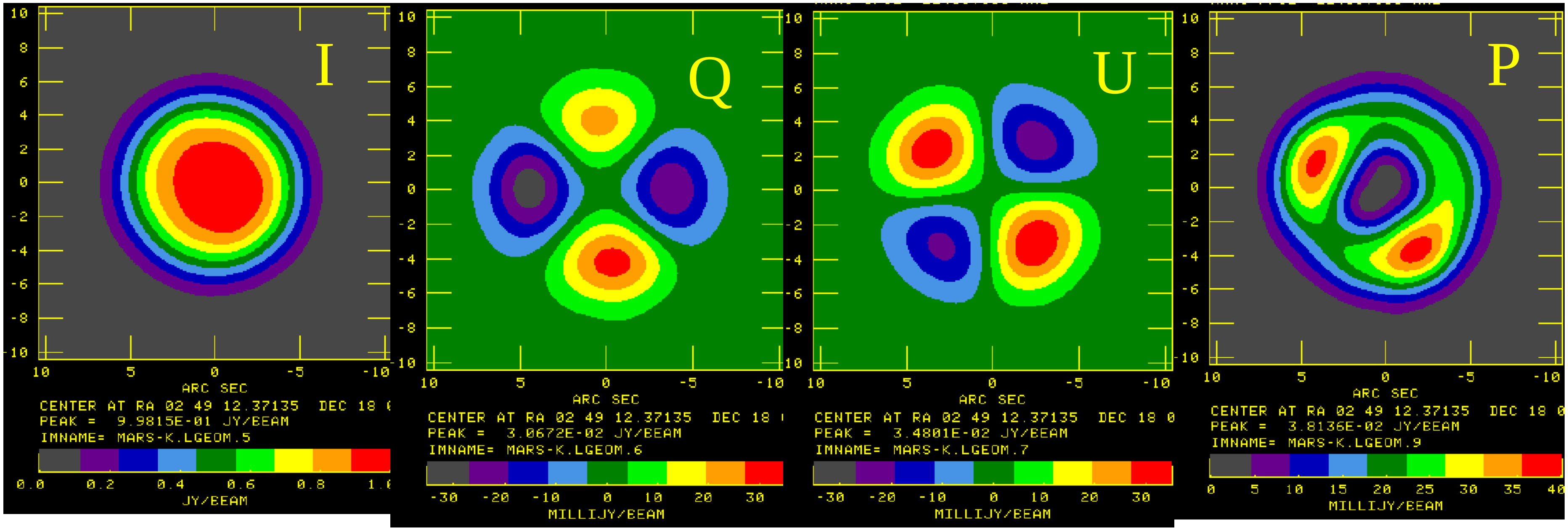
- ||I|| is close to a J_0 Bessel function.
- Zero crossing at 20 kλ
  tells that Mars is diameter ~10".
- |Q| amplitude ~0 at zero baseline
- |Q| zero at 30 kλ means
  pol. Structures ~8" scale.

Phase

- I phase alternates between 0 & π
- Q phase = both 0 and π in the 'main lobe' – this tells us there are both positive and negative structures, at different PA.



Mars I,Q,U (P)



$$P = \frac{\sqrt{Q^2 + U^2}}{I}$$

- Mars emits in the radio as a black body.
- Shown are false-color coded I,Q,U,P images.
- V is not shown – all noise – no circular polarization.
- Resolution is 3.5”, Mars’ diameter is ~10”
- From the Q and U images alone, we can deduce the polarization is radial, around the limb.

Mars – A Traditional Representation

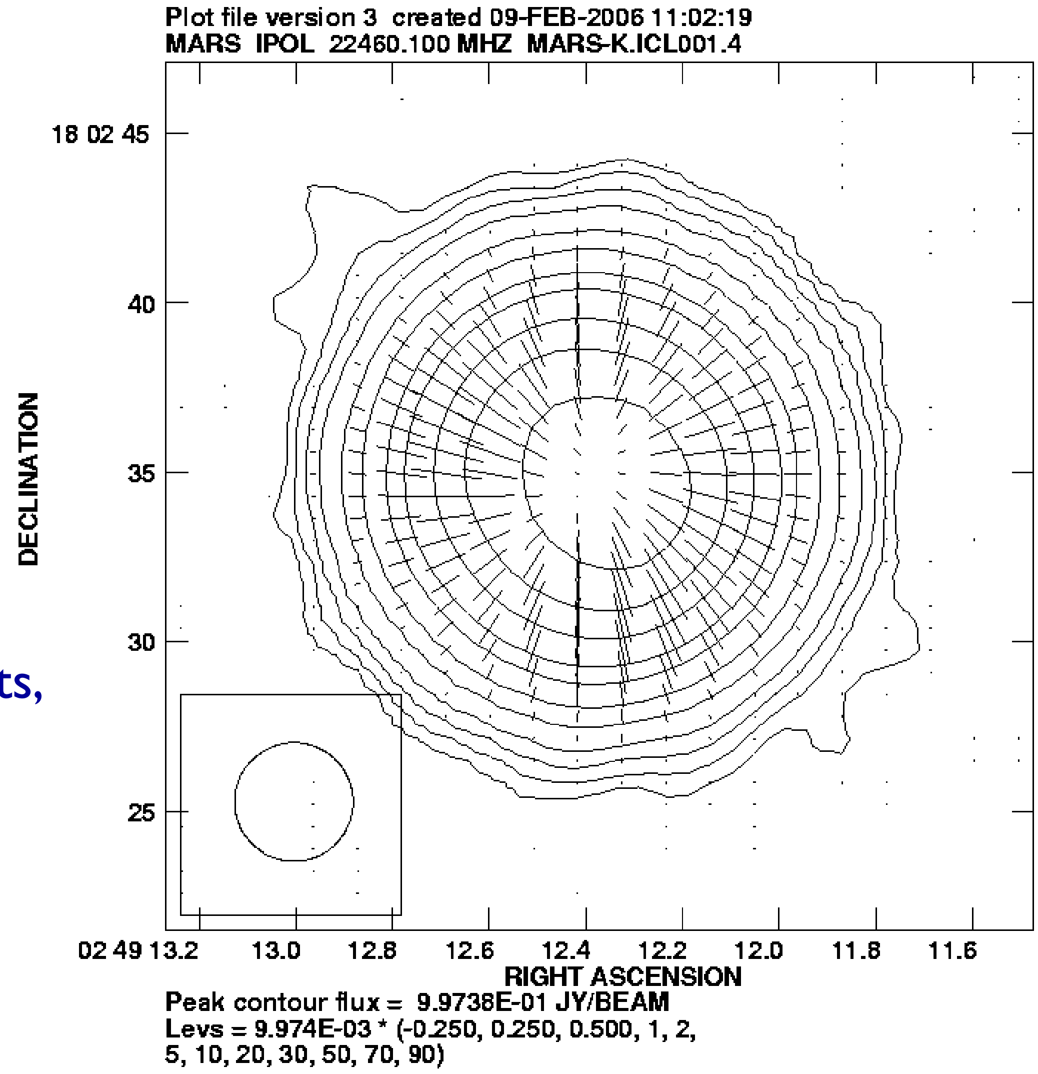
Here I, Q, and U are combined to make a more realizable map of the total and linearly polarized emission from Mars.

The dashes show the direction of the E-field.

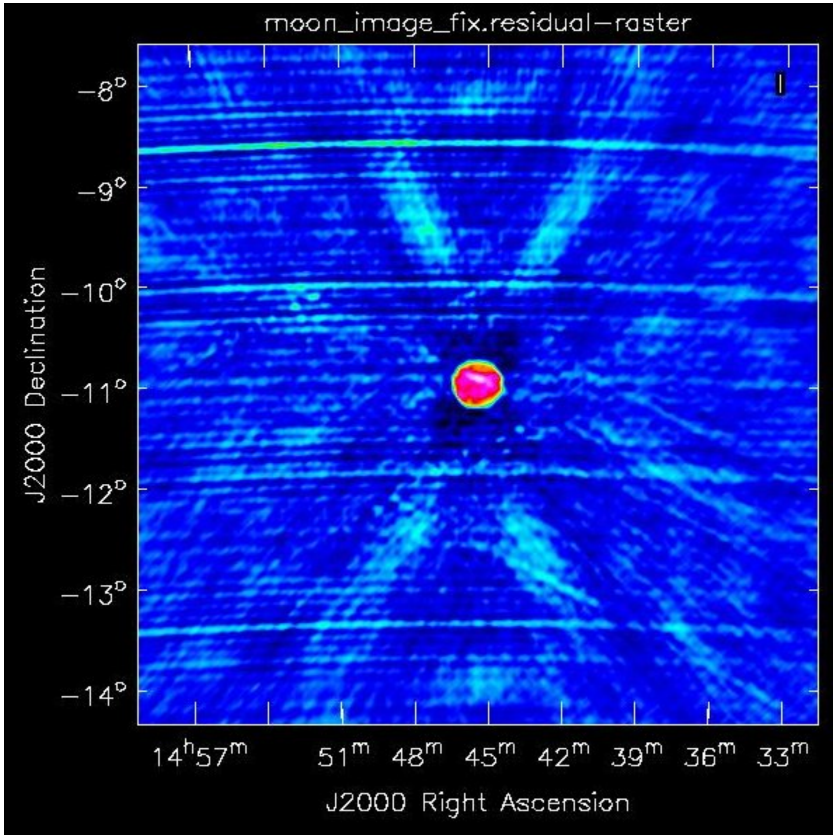
The dash length is proportional to the polarized intensity.

One could add the vector components, to show little ellipses to represent the polarization at every point.

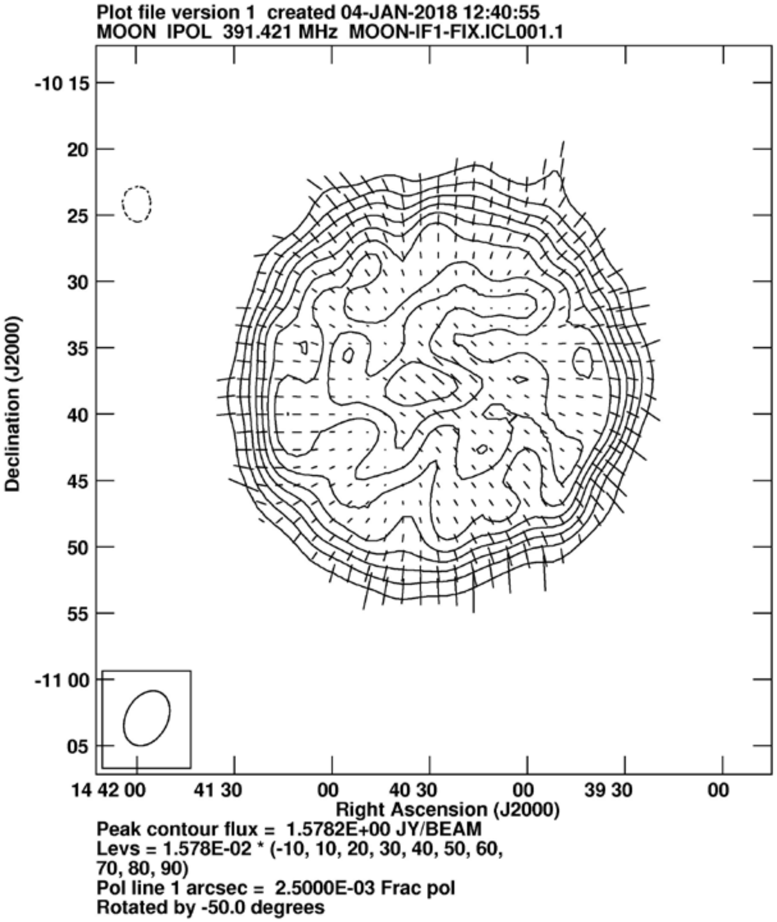
$$\chi = 0.5\ \tan^{-1}\left(\frac{U}{Q}\right)$$



VLA Moon – linear system

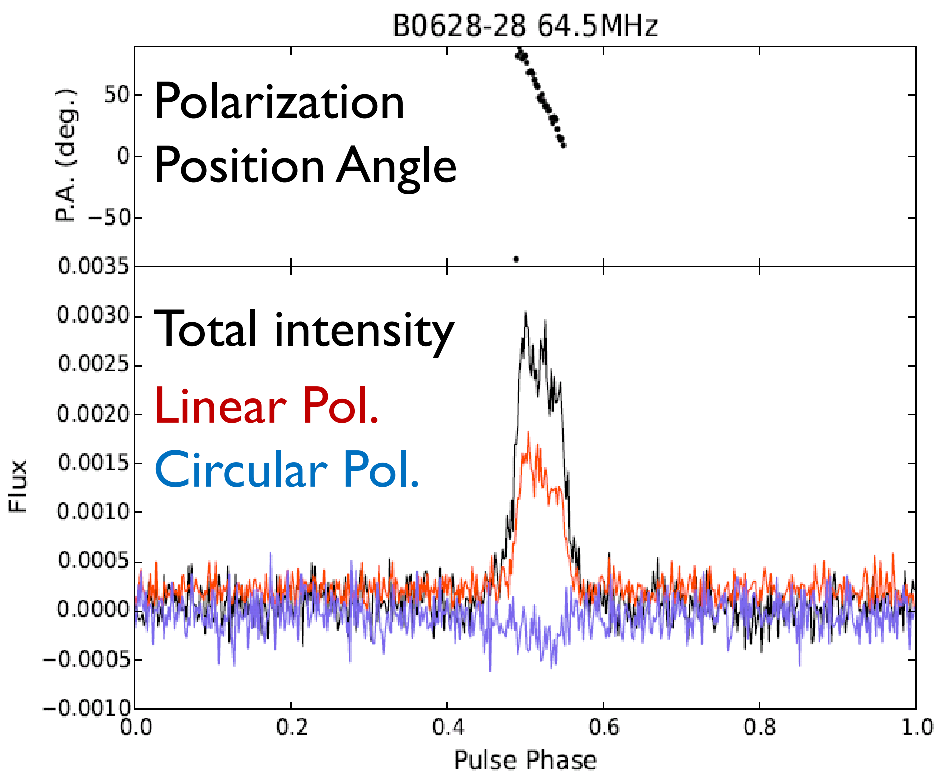


Dirty Stokes I Image of the Moon at ~350 MHz

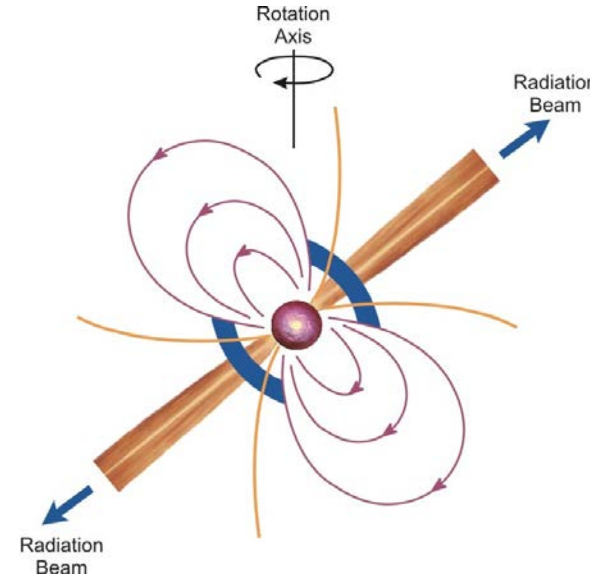


Polarization of the Moon

Pulsars



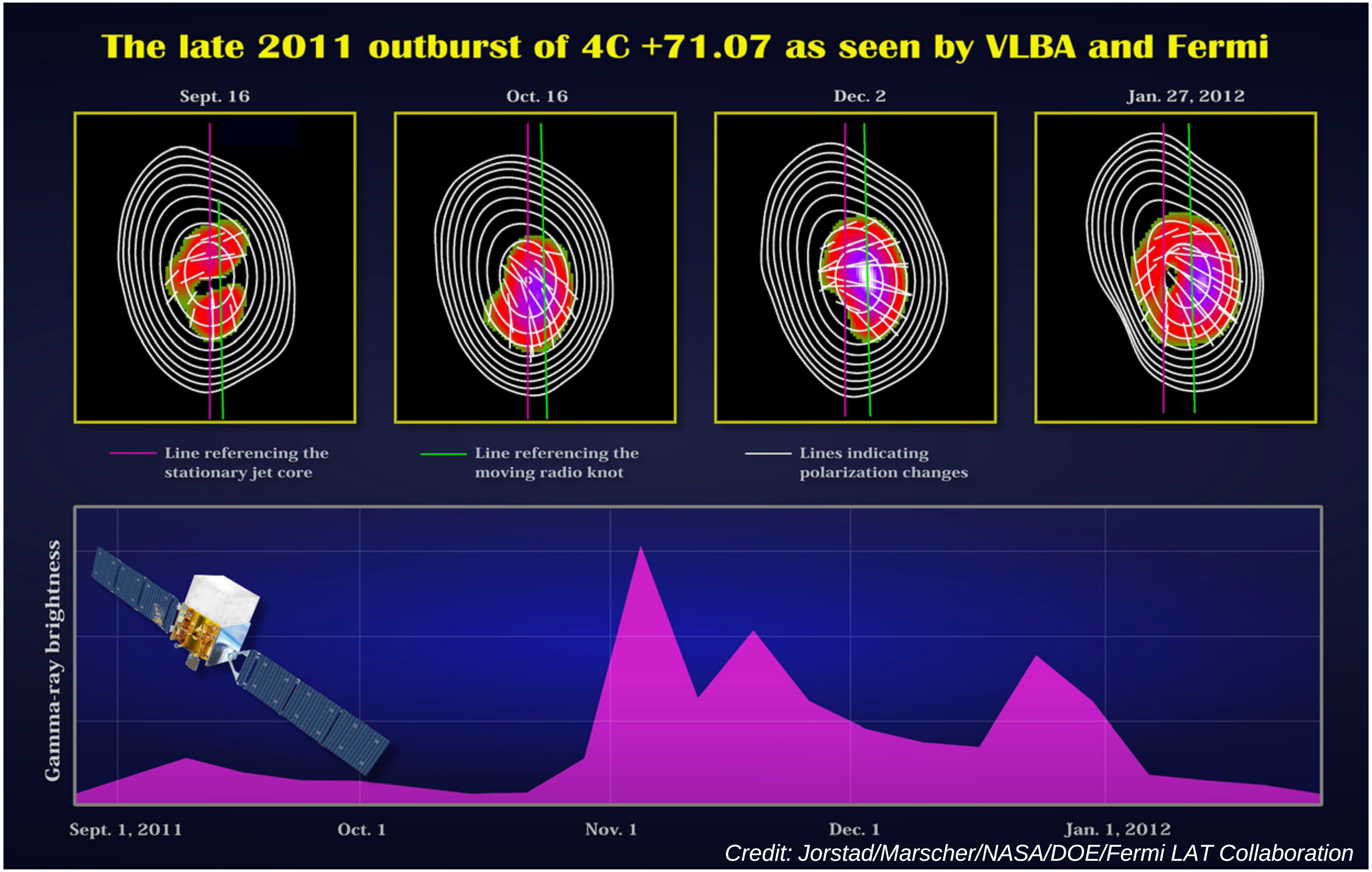
LWA1; Dike, Taylor, Stovall (2017)



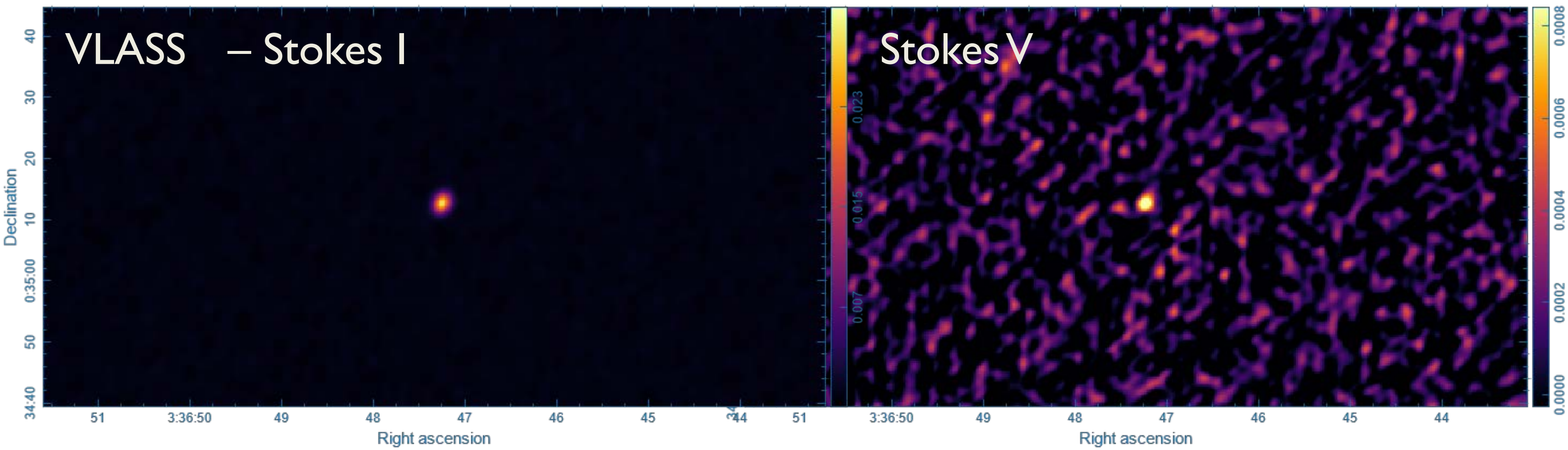
Pulsars are highly magnetized
=> strong polarized emission,
including circular

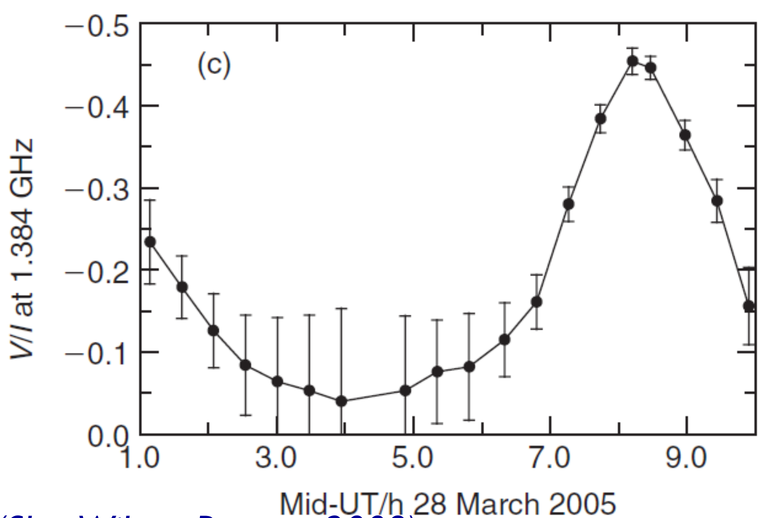
Study of which provides insights into the poorly understood emission mechanism of Pulsars.

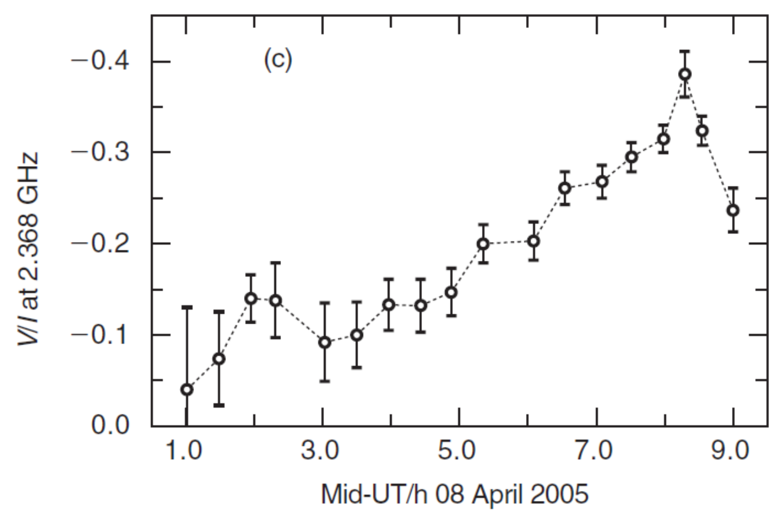
Polarization from Quasar jet



Coherent emission from variable star
Example: HR 1099







ATCA (Slee, Wilson, Ramsay 2008)

Further Reading

- K. Rohlfs & T.L. Wilson: *Tools of Radio Astronomy (Chapters 2 & 3)*
- Thompson, Moran & Swenson: *Inteferometry and Synthesis in Radio Astronomy*
- Taylor, Carilli, & Perley: *Snythesis Imaging in Radio Astronomy II*
- Bracewell: *The Fourier Transform & Its Applications*
- Hamaker/Bregman/Sault: *Understanding radio polarimetry: papers I - V (1996-2006)*
- Brentjens & de Bruyn: *Faraday rotation measure synthesis (2005)*
- *EVLA Memos by Perley & Sault (#131, #134, #135, #141, #151, #170, #178)*
- *Guide to Observing with the VLA - Polarimetry (https://science.nrao.edu/facilities/vla/docs/manuals/obsguide/modes/pol)*
- *Hales EVLA Memo #201; Schinzel EVLA Memo #205*
- *Perley EVLA Memos #207, #210*
- *Polarization Calibration (8th VLA Data Reduction Workshop) https://science.nrao.edu/science/meetings/2021/vla-data-reduction/presentations/Schinzel_Polarization.pdf*

Take Home Message

- Polarimetry is a little complicated, but do not be afraid!
- The polarized state of EM radiation gives valuable insights into magnetic fields and the physics of the emission.
- Well designed systems are stable, and have low cross-polarization, making correction relatively straightforward.
- Such systems easily allow estimation of polarization to an accuracy of the order 1 part in 10000.
- Beam-induced polarization can be corrected in software.
- Understanding polarization improves calibration and imaging even in the unpolarized case.

Thanks to Rick Perley & Michiel Brentjens from whom I extensively borrowed presented materials.