High Fidelity Imaging

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1 What is High Fidelity Imaging?

- Getting the ‘Correct Image’ – limited only by noise.
  - The best ‘dynamic ranges’ (brightness contrast) exceed 10^6 for some images.
  - (But is the recovered brightness correct?)

- Errors in your image can be caused by many different problems, including (but not limited to):
  - Errors in your data – many origins!
  - Errors in the imaging/deconvolution algorithms used
  - Errors in your methodology
  - Insufficient information

- But before discussing these, and what you can do about them, I show the effect of errors of different types on your image.

2 The Effects of Visibility Errors on Image Fidelity

- The most common, and simplest source of error is an error in the measures of the visibility (spatial coherence function).
- Consider a point source of unit flux density (S = 1) at the phase center, observed by a telescope array of N antennas.
- Formally, the sky intensity is:

$$I(l,m) = \delta(l,m)$$

- The correct visibility, for any baseline is:

$$V(u, v) = 1$$

- This are analytic expressions – they presume infinite coverage.
- In fact, we have N antennas, from which we get, at any one time

$$N_V = \frac{N(N-1)}{2} \quad \text{visibilities}$$

- Each of these N_V visibilities is a complex number, and is a function of the baseline coordinates (u_k, v_k).

2 The Effects of Visibility Errors on Image Fidelity

- The simplest image is made by direction summation over all the visibilities -- (a Direct Transform):

$$I(l,m) = \frac{1}{2N_V}\sum_{k=1}^{N_V}\left[V_k(u,v)e^{2\pi i(u_k l + v_k m)} + V_k^*(u,v)e^{-2\pi i(u_k l + v_k m)}\right]$$

- For our unit source at the image center, we get

$$I(l,m) = \frac{1}{N_V}\sum_{k=1}^{N_V}\cos\left[2\pi(u_k l + v_k m)\right]$$

- But let us suppose that for one baseline, at one time, there is an error in the amplitude and the phase, so the measured visibility is:

$$V(u,v) = (1+\varepsilon)\delta(u-u_0, v-v_0)e^{-i\phi}$$

where ε = the error in the visibility amplitude
ϕ = the error (in radians) in the visibility phase.

2 The Effects of Visibility Errors on Image Fidelity

- The map we get from this becomes

$$I(l,m) = \frac{1}{N_V}\left\{\sum_{k=2}^{N_V} \cos\left[2\pi(u_k l + v_k m)\right] + (1+\varepsilon)\cos\left[2\pi(u_1 l + v_1 m) - \phi\right]\right\}$$

- The ‘error map’ associated with this visibility error is the difference between the image and the ‘beam’:

$$\Delta I(l,m) = \frac{1}{N_V}\left((1+\varepsilon)\cos\left[2\pi(u_1 l + v_1 m) - \phi\right] - \cos\left[2\pi(u_1 l + v_1 m)\right]\right)$$

- This is a single-(spatial) frequency fringe pattern across the entire map, with a small amplitude and phase offset.
- Let us simplify by considering amplitude and phase errors separately.

1) Amplitude error only: $\phi = 0$. Then,

$$\Delta I = \frac{\varepsilon}{N_V}\cos\left[2\pi(u_1 l + v_1 m)\right]$$

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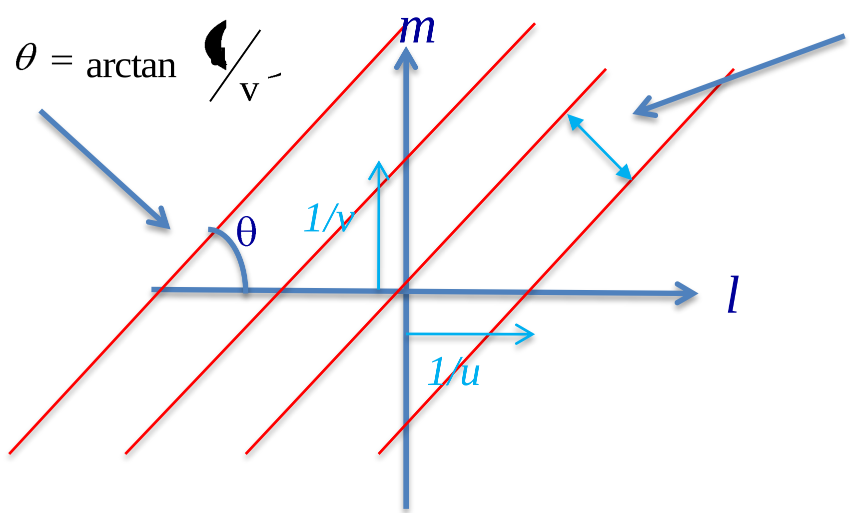
2 The Effect of an Amplitude Error on Image Fidelity

$$\Delta I = \frac{\varepsilon}{N_V}\cos\left[2\pi(u_1 l + v_1 m)\right]$$

- This is a sinusoidal wave of amplitude ε/N_V, with period $1/\sqrt{u_1^2 + v_1^2}$

tilted at an angle $\theta = \arctan\left(u/v\right)$



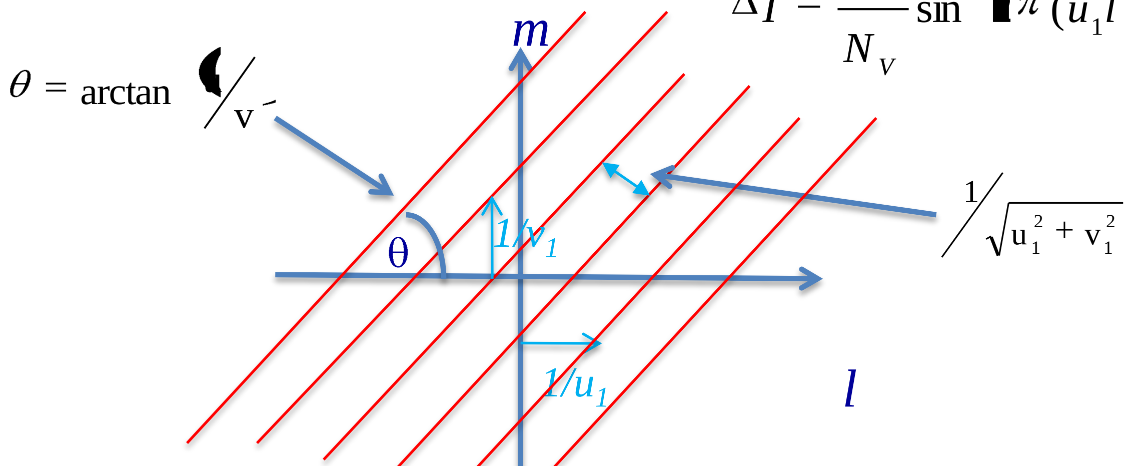
- As an example, if the amplitude error is 10% ($\varepsilon = 0.1$), and $N_V = 10^6$, the $\Delta I = 10^{-7}$ – a very small value!
- Note: The error pattern is even about the location of the source.

2 The Effect of a Phase Error on Image Fidelity

In this case: $$\Delta I = \frac{1}{N_V}\left\{\cos\left[2\pi(u_1 l + v_1 m) - \phi\right] - \cos\left[2\pi(u_1 l + v_1 m)\right]\right\}$$

- For small phase error, $\phi \ll 1$, $$\Delta I = \frac{\phi}{N_V}\sin\left[2\pi(u_1 l + v_1 m)\right]$$
- This gives the same error pattern, but with the amplitude ε replaced by ϕ, and the phase shifted by 90 degrees.

$$\Delta I = \frac{\phi}{N_V}\sin\left[2\pi(u_1 l + v_1 m)\right]$$



- From this, we derive an Important Rule:

A phase error of x radians has the same effect as an amplitude error of 100 x %

- For example, a phase error of 1/10 rad ~ 6 degrees has the same effect as an amplitude error of 10%.

Amplitude vs Phase Errors.

- This little rule explains why phase errors are deemed to be so much more important than amplitude errors.
- Modern interferometers, and cm-wave atmospheric transmission, are so good that fluctuations in the amplitudes of more than a few percent are very rare.
- But phase errors – primarily due to the atmosphere, but also from the electronics, are always worse than 10 degrees – often worse than 1 radian!
- Phase errors – because they are large – are nearly always the initial limiting cause of poor imaging.

Errors and Dynamic Range (or Fidelity):

- I now define the dynamic range as the ratio: F = Peak/RMS.
- For our examples, the peak is always 1.0, so the fidelity F is:

$$F = \frac{\sqrt{2}\,N_V}{\varepsilon}$$

- For amplitude error of 100ε %

$$F = \frac{\sqrt{2}\,N_V}{\phi}$$

- For phase error of ϕ radians

- So, taking our canonical example of 0.1 rad error on one baseline for one single visibility, (or 10% amplitude error):
- $F = 3 \times 10^6$ for $N_V = 250{,}000$ (typical for an entire day)
- $F = 5000$ for $N_V = 351$ (a single snapshot).

- Errors rarely come on single baselines for a single time. We move on to more practical examples now.

Other Examples of Fidelity Loss

- **Example A: All baselines have an error of ~ ϕ rad at one time.**

  Since each baseline's visibility is gridded in a different place, the errors from each can be considered random in the image plane. Hence, the rms adds in quadrature. The fidelity declines by a factor $\sqrt{N_V} \sim \frac{N}{\sqrt{2}}$

- Thus: $F = \frac{N}{\varepsilon}$ (N = # of antennas)

- So, in a 'snapshot', F ~ 270.

- **Example B: One antenna has phase error ε, at one time.**

  Here, (N-1) baselines have a phase error – but since each is gridded in a separate place, the errors again add in quadrature. The fidelity is lowered from the single-baseline error by a factor $\sqrt{N-1}$, giving

$$F = \frac{N^{3/2}}{\sqrt{2}\varepsilon}$$

- So, for our canonical 'snapshot' example, F ~ 1000

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The Effect of Steady Errors

- **Example C: One baseline has an error of ~ ϕ rad at all times.**

  This case is importantly different, in that the error is not randomly distributed in the (u,v) plane, but rather follows an elliptical locus.

  - To simplify, imagine the observation is at the north pole. Then the locus of the bad baseline is a circle, of radius $q = \sqrt{u^2 + v^2}$
  - One can show (see EVLA Memo 49 for details) that the error pattern is:

$$\Delta I = \frac{2\varepsilon}{N(N-1)} J_0 \left(\pi q \theta \right)$$

- The error pattern consists of rings centered on the source ('bull's eye').
- For large qθ (this is the number of rings away from the center), the fidelity can be shown to be

$$F = \frac{N(N-1)\pi\sqrt{q\theta}}{\sqrt{2}\varepsilon}$$

- So, again taking ε = 0.1, and q$\theta = 100$, F ~ 1.6 x 10^5.

One More Example of Fidelity Loss

- **Example D: All baselines have a steady error of ~ ε at all times.**

  Following the same methods as before, the fidelity will be lowered by the square root of the number of baselines.

Hence, $$F = \frac{N(N-1)\pi\sqrt{q\theta}}{\sqrt{2}\varepsilon}\sqrt{\frac{2}{N(N-1)}} \sim \frac{\pi N\sqrt{q\theta}}{\varepsilon}$$

- So, again taking $\varepsilon = 0.1$, and $q\theta = 100$, $F \sim 8000$.

Time-Variable Errors

- In real life, the atmosphere and/or electronics introduces phase or amplitude variations. What is the effect of these?
- Suppose the phase on each antenna changes by ϕ radians on a typical timescale of Δt hours.
- Over an observation of T hours, we can imagine the image comprising $N_S = \Delta t/T$ individual 'snapshots', each with an independent set of errors.
- The dynamic range on each snapshot is given by

$$F \sim \frac{N}{\varepsilon}$$

- So, for the entire observation, we get

$$F = \frac{N\sqrt{N_S}}{\varepsilon}$$

- The value of N_S can vary from ~100 to many thousands.

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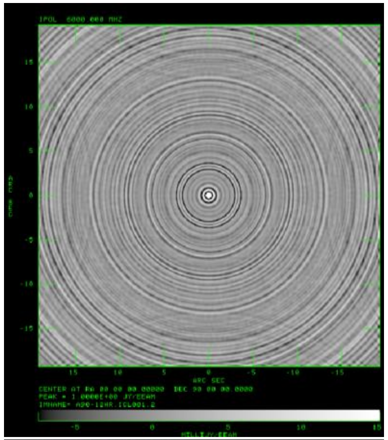
Some Examples: Ideal Data

- I illustrate these ideas with some simple simulations.
- EVLA , ν_0 = 6 GHz, BW = 4 GHz, δ = 90, 'A'-configuration
- Used the AIPS program 'UVCON' to generate visibilities, with S = 1 Jy.



The 'Dirty' Map after a 12 hour observation.

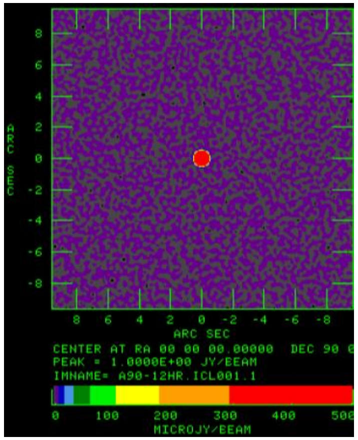
Note the 'reflected' grating rings.

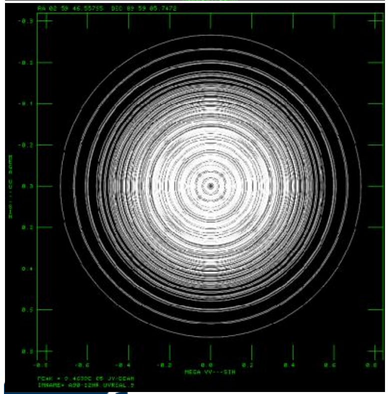
The 'Clean' Map

1σ = 1.3 μJy

Pk = 1 Jy

No artifacts!





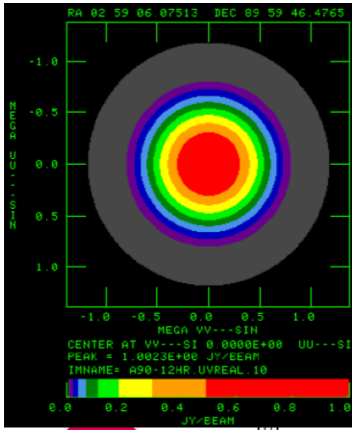
The U-V Coverage after a 12 hour observation.

Variations are due to gridding.

The FT of the 'Clean' map

Note that the amplitudes do *not* match the data!

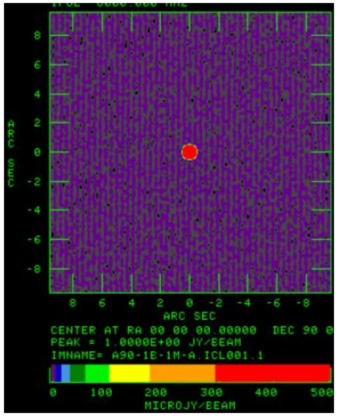
The taper comes from the Clean Beam.

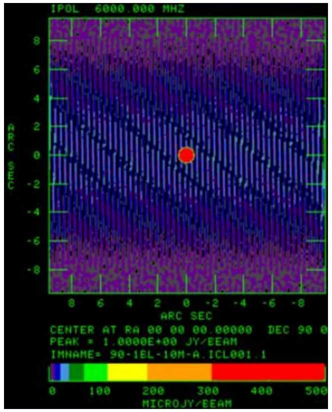


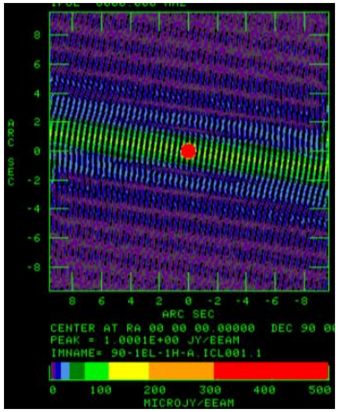
One-Baseline Errors – Amplitude Error of 10%

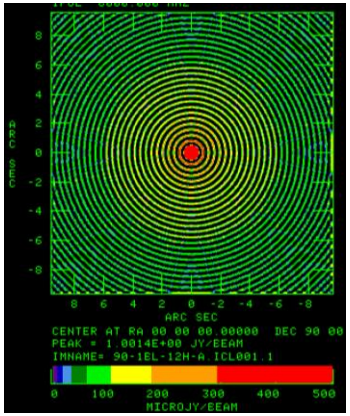
- Examples with a single errant baseline for 1m, 10 m, 1 h, and 12 hours
- N_v ~250,000

1 minute	10 minutes	1 hour	12 hours





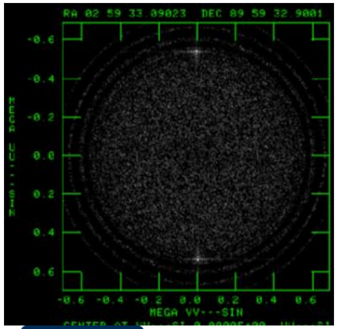


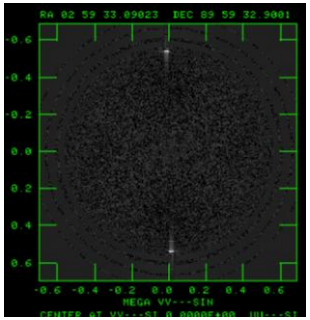


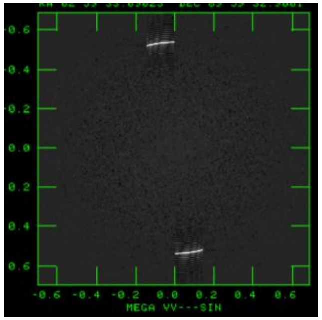
The four 'cleaned' images, each with peak = 1 Jy.

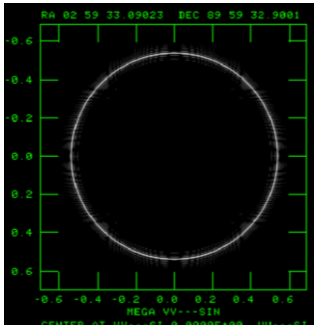
All images use the same transfer function.

1 σ = 1.9 μJy	1 σ = 9.4 μJy	1 σ = 25 μJy	1 σ = 79 μJy









The four U-V plane amplitudes.

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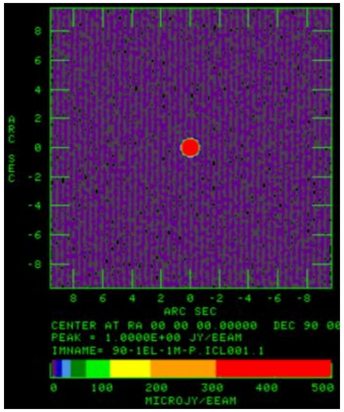
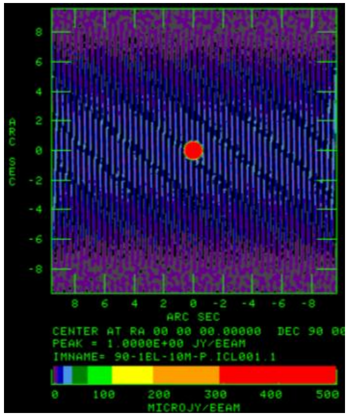
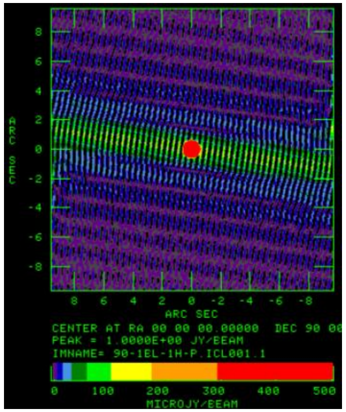
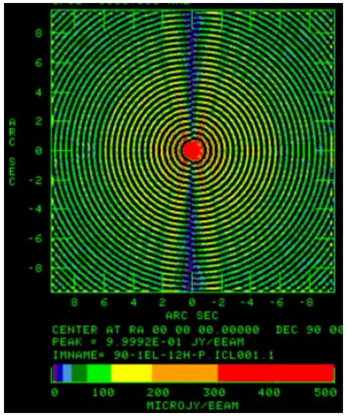




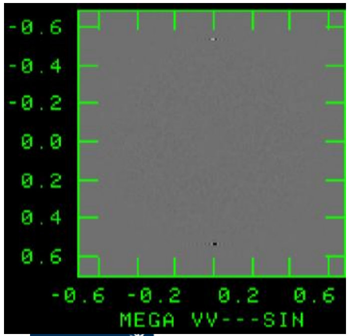


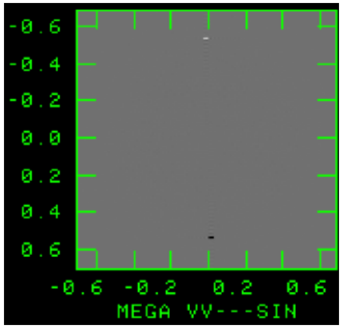


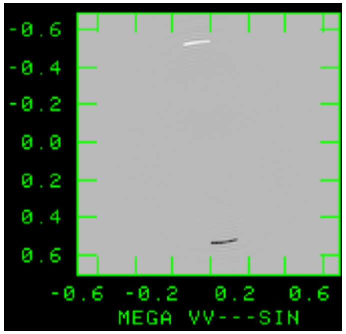
The four 'cleaned' images, each with peak = 1 Jy.

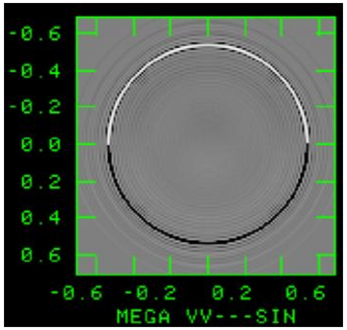
All images use the same transfer function.

1 σ = 2.0 μJy	1 σ = 9.8 μJy	1 σ = 26 μJy	1 σ = 82 μJy









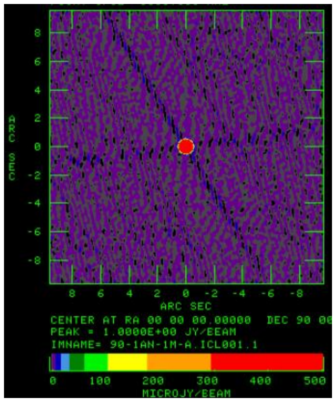
The four U-V plane phases.

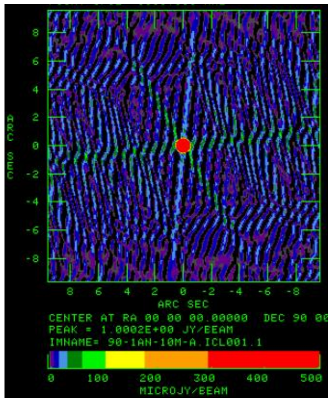
Note the easy identification of the errors.

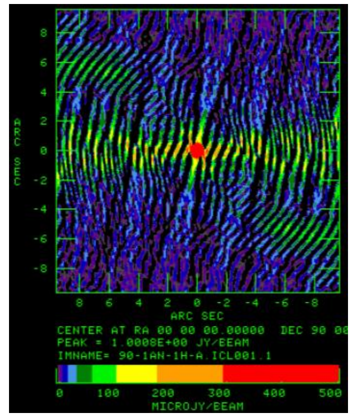
One-Antenna Errors – Amplitude Error of 10%

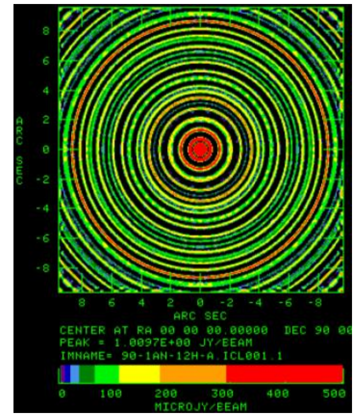
- Examples with a single errant antenna for 1m, 10 m, 1 h, and 12 hours
- N_v ~250,000

1 minute | 10 minutes | 1 hour | 12 hours





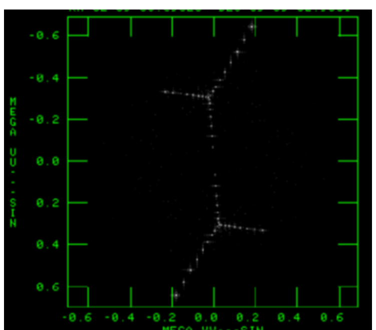


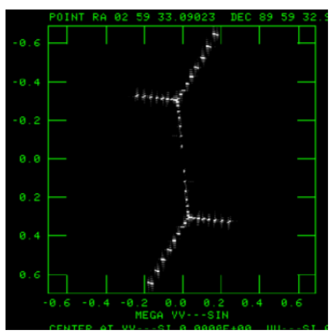


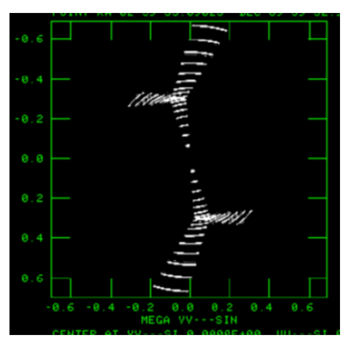
The four 'cleaned' images, each with peak = 1 Jy.

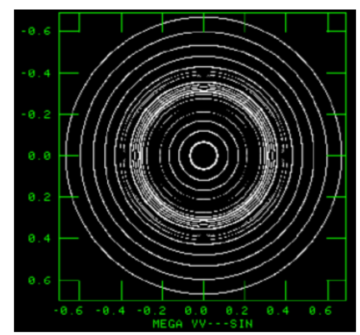
All images use the same transfer function.

1 σ = 2.3 μJy | 1 σ = 16 μJy | 1 σ = 42 μJy | 1 σ = 142 μJy









The four U-V plane amplitudes.

Note the easy identification of the errors.

One-Antenna Errors – Phase Error of 0.1 rad = 6 deg.

- Examples with a single errant antenna for 1m, 10 m, 1 h, and 12 hours
- N_v ~250,000

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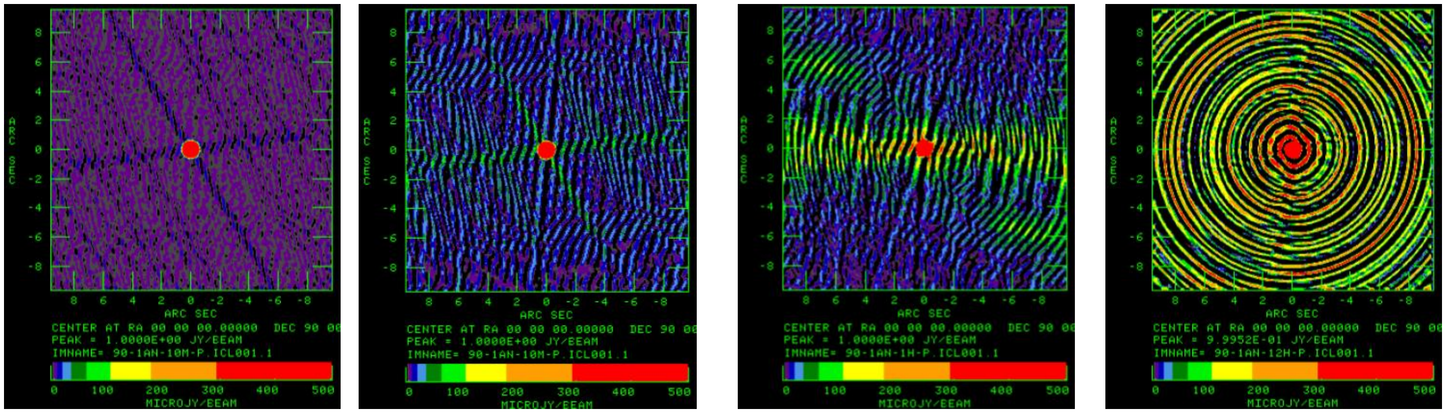
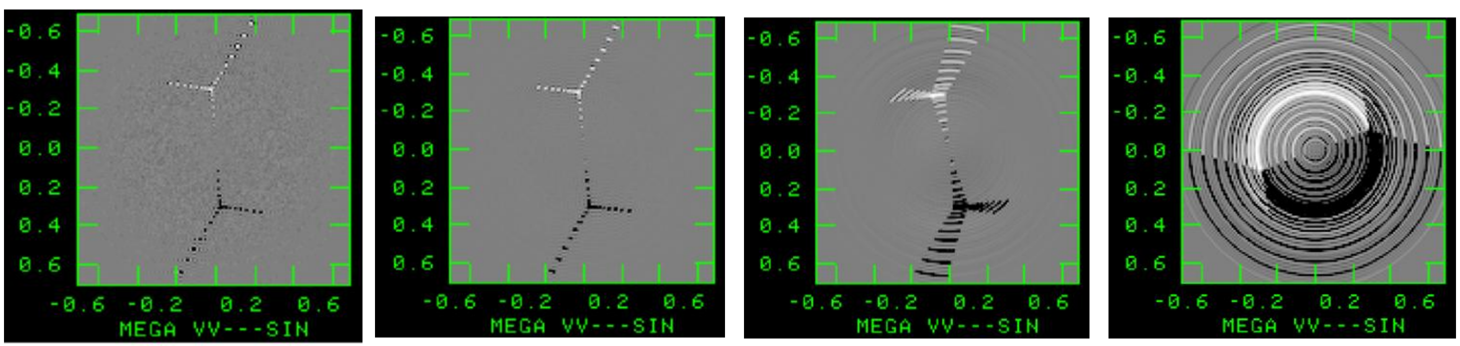


1 σ = 2.9 μJy | 1 σ = 20 μJy | 1 σ = 52 μJy | 1 σ = 147 μJy

The four 'cleaned' images, each with peak = 1 Jy.

All images use the same transfer function.



The four U-V plane phases.

Note the easy identification of the errors.

Finding and Correcting, or Removing Bad Data

- How to find and fix bad data?
- We first must consider the types, and origins, of errors.
- We can write, in general:

$$\tilde{V}_{ij}(t) = g_i(t)g_j^*(t)V_{ij} + g_{ij}(t)V_{ij} + C_{ij}(t) + \varepsilon_{ij}(t)$$

- Here, $\tilde{V}_{ij}(t)$ is the calibrated visibility, and $V_{ij}(t)$ is the observed visibility.

  $g_i(t)$ is an antenna based gain

  $g_{ij}(t)$ is a multiplicative baseline-based gain.

  $C_{ij}(t)$ is an additive baseline-based gain, and

  $\varepsilon_{ij}(t)$ is a random additive term, due to noise.

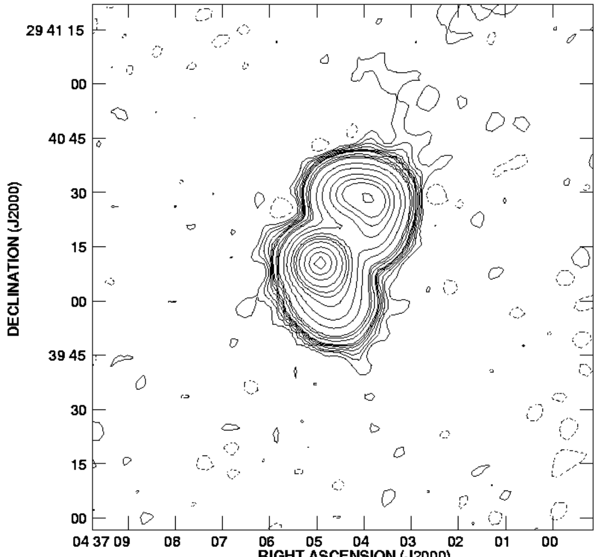
- In principle, the methods of self-calibration are extremely effective at finding and removing all the antenna-based ('closing') errors.
- The method's effectiveness is usually limited by the accuracy of the model.
- In the end, it is usually the 'non-closing' errors which limit fidelity for strong sources.

Finding and Correcting, or Removing Bad Data

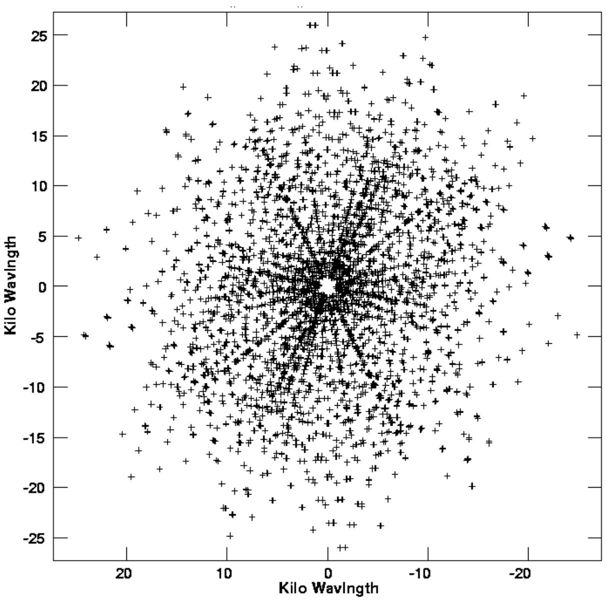
- Self-calibration works well for a number of reasons:
  - The most important errors really are antenna-based (notably atmospheric/instrumental phase.
  - The error is 'seen' identically on *N* - 1 baselines at the same time – improving the SNR by a factor ~ $\sqrt{N-1}$.
  - The N – 1 baselines are of very different lengths and orientations, so the effects of errors in the model are randomized amongst the baselines, improving robustness.
- Non-closing errors can also be calibrated out – but here the process is much less robust! The error is on a single baseline, so not only is the SNR poorer, but there is no tolerance to model errors. The data will be adjusted to precisely match the model you put in!
  - Some (small) safety will be obtained if the non-closing error is constant in time – the solution will then average over the model error, with improved SNR.

Finding and Correcting, or Removing Bad Data – a simple example.

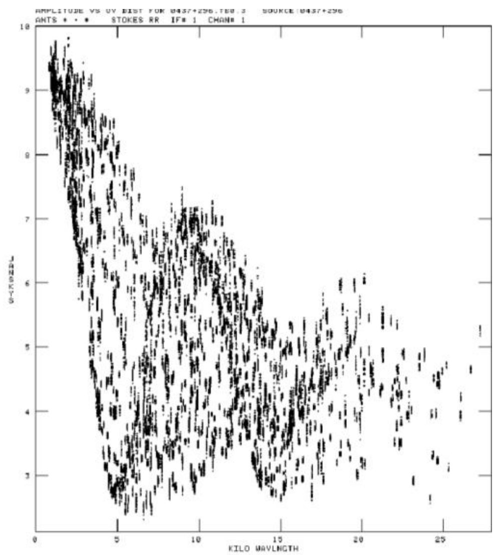
- I show some 'multiple snapshot' data on 3C123, a fluxy compact radio source, observed in D-configuration in 2007, at 8.4 GHz.
- There are 7 observations, each of about 30 seconds duration.
- For reference, the 'best image', and UV-coverage are shown below.
- Resolution = 8.5 arcseconds. Maximum baseline ~ 25 kλ

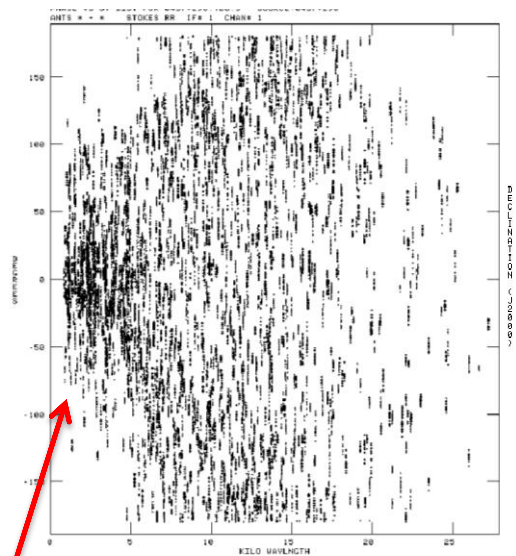


Peak flux = 4.7622E+00 JY/BEAM
Levs = 4.762E-02 * (-0.033, 0.033, 0.067, 0.100, 0.150, 0.200, 0.250, 0.300, 0.500, 0.750, 1, 2.500, 5, 10, 15, 20, 30, 50, 70, 90)

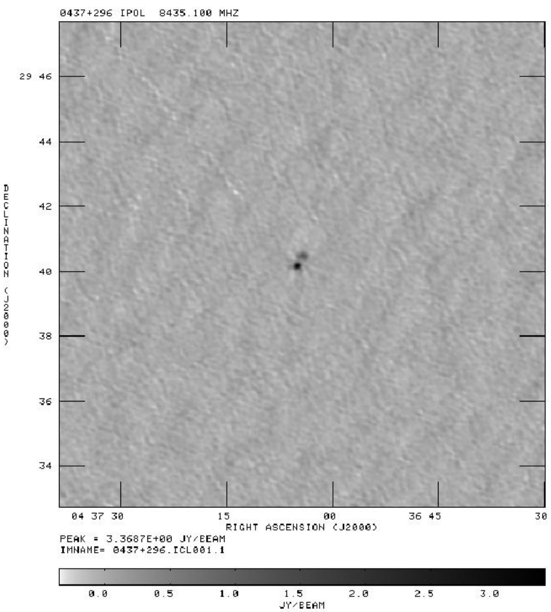


- Following standard calibration against unresolved point sources, and editing the really obviously bad data, the 1-d visibility plots look like this, in amplitude and phase:



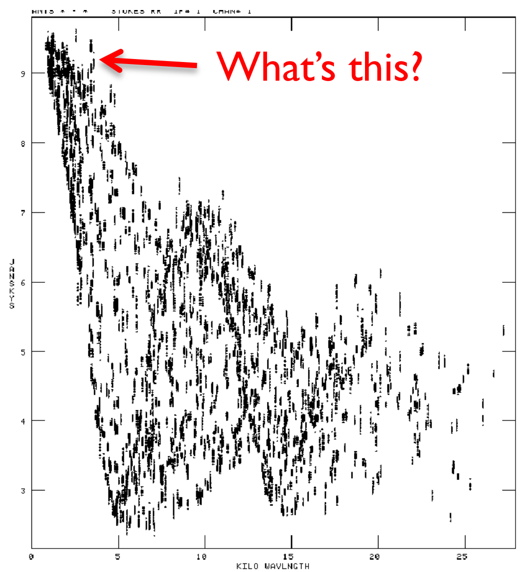


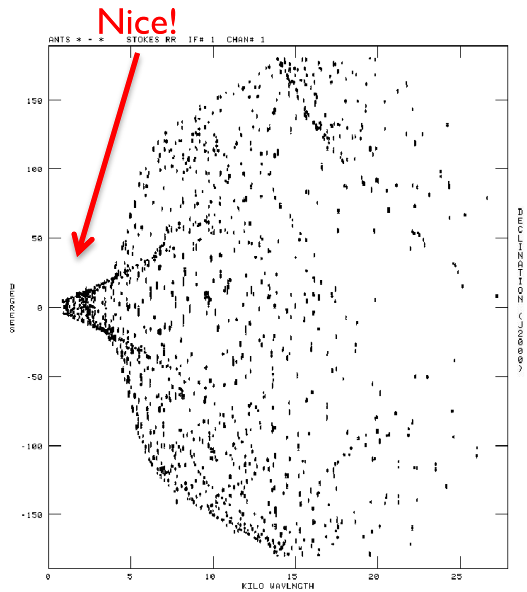
Ugh!

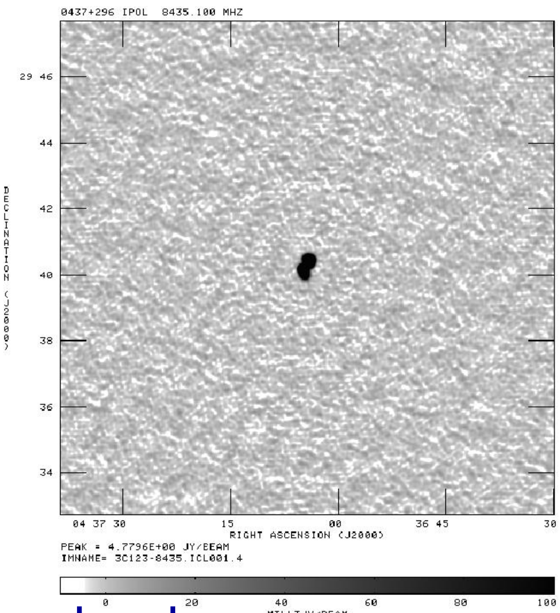


- Note that the amplitudes look quite good, but the phases do not.
- We don't expect a great image.
- Image peak: 3.37 Jy/beam; Image rms = 63 mJy.
- DR = 59 – that's not good!

- Using our good reference image, we do an 'amplitude and phase' self-cal.
- The resulting distributions and image are shown below.

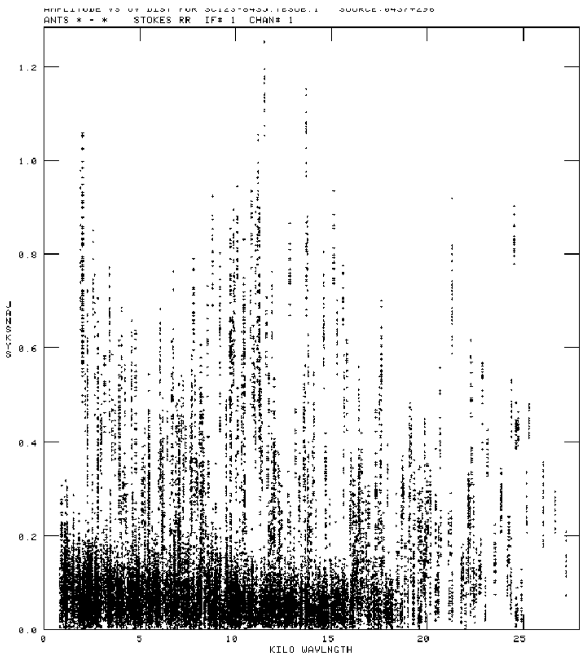






- Note that the amplitudes look much the same, but the phase are much better organized..
- Image peak: 4.77 Jy/beam; Image rms = 3.3 mJy.
- DR = 1450 – better, but far from what it should be…

- When self-calibration no longer improves the image, we must look for more exotic errors.
- The next level are 'closure', or baseline-based errors.
- The usual step is to subtract the (FT) of your model from the data.
- In AIPS, the program used is 'UVSUB'.
- Plot the residuals, and decide what to do …



- If the model matches the data, the residuals should be in the noise – a known value.
- For these data, we expect ~50 mJy.
- Most are close to this, but many are not.

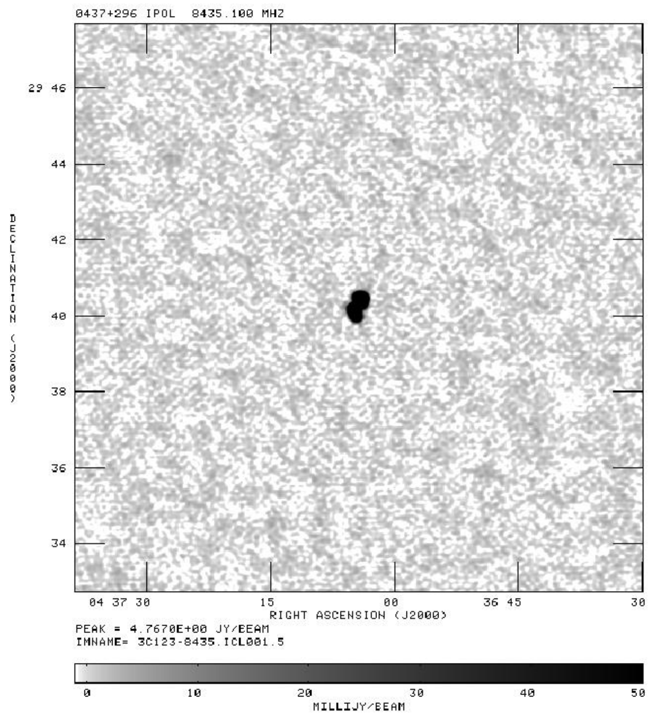
These are far too large

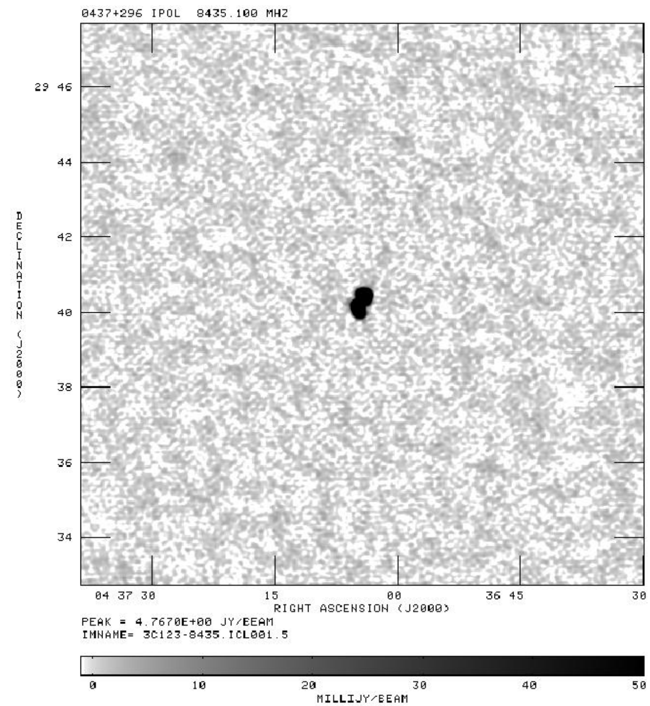
These are about right.

Removing or Correcting Baseline-based Errors

- Once it is determined there are baseline-based errors, the next questions is: What to do about them?
- Solution A: Flag all discrepant visibilities;
- Solution B: Repair them.
- **Solution A:**
  - For our example, I clipped ('CLIP') all residual visibilities above 200 mJy, then restored the model visibilities.
  - Be aware that by using such a crude tool, you will usually be losing some good visibilities, and you will let through some bad ones …
- **Solution B:**
  - Use the model to determine individual baseline corrections.
  - In AIPS, the program is 'BLCAL'. This produces a set of baseline gains that are applied to the data.
  - This is a powerful – but *dangerous* tool …
  - Since 'closure' errors are usually time invariant, use that condition.

- On Left – Image after clipping high residual visibilities. 20.9 kVis used.
- On Right – Image after correcting for baseline-based errors.





Peak = 4.77 Jy s = 1.2 mJy
DR = 3980

Peak = 4.76 Jy s = 0.83 mJy
DR = 5740

Law of Diminishing Returns
or
Knowing When to Quit

- I did not proceed further for this example.
- One can (and many do) continue the process of:
  - Self-calibration (removing antenna-based gains)
  - Imaging/Deconvolution (making your latest model)
  - Visibility subtraction
  - Clipping residuals, or a better baseline calibration.
  - Imaging/Deconvolution
- The process always asymptotes, and you have to give it up, or find a better methodology.
- Note that not all sources of error can be removed by this process.

Sources of Error

- I conclude with a short summary of sources of error.
- This list is necessarily incomplete.
- **Antenna-Based Errors**
  - Electronics gain variations – amplitude and phase – both in time and frequency.
    - Modern systems are very stable – typically 1% in amplitude, a few degrees in phase
  - Atmospheric (Tropospheric/Ionospheric) errors.
    - Attenuation very small at wavelengths longer than ~2 cm – except through heavy clouds (like thunderstorms) for 2 – 6 cm.
    - Phase corruptions can be very large – tens to hundreds of degrees.
    - Ionosphere phase errors dominate for λ > 20cm.
  - Antenna pointing errors: primarily amplitude, but also phase.
  - Non-isoplanatic phase screens

- **Baseline-Based Errors** – this list is much longer
  - System Electronics.
    - Offsets in a particular correlator (additive)
    - Gain (normalization?) errors in correlator (multiplicative)
    - Other correlator-based issues (WIDAR has ‘wobbles’ …)
    - Phase offsets between COS and SIN correlators
    - Non-identical bandpasses, on frequency scales smaller than channel resolution.
    - Delay errors, not compensated by proper delay calibration.
    - Temporal phase winds, not resolved in time (averaging time too long).

- Impure System Polarization $V_r' = V_r + D_r V_l$
  - Even after the best regular calibration, the visibilities contain contaminants from the other polarizations
  - For example, for Stokes 'I', we can write:

$$V_I' = V_I + D_1 V_Q + D_2 V_U + D_3 V_V$$

  - The 'I' visibility has been contaminated by contributions from Q, U, and V, coupled through by complex 'D' factors which describe the leakage of one polarization into the other.
  - This term can be significant – polarization can be 30% or higher, and the 'D' terms can be 5%
  - The additional terms can easily exceed 1% of the Stokes 'I'.
  - Polarization calibration necessary – but note that the antenna beam is variably polarized as a function of angle.

- **Other, far-out effects (to keep you awake at night …)**
- Correlator quantization correction
  - Digital correlators are non-linear – they err in the calculation of the correlation of very strong sources.
  - This error is completely eliminated with WIDAR.
- Non-coplanar baselines.
  - Important when $\frac{\lambda B}{D^2} \geq 1$
  - Software exists to correct this.
- Baseline errors: incorrect baselines leads to incorrect images.
  - Apply baseline corrections to visibility data, perhaps determined after observations are completed.
- Deconvolution Algorithm errors
  - CLEAN, VTESS, etc. do not *guarantee* a correct result!
  - Errors in data, holes in the coverage, absence of long or short spacings will result in incorrect images.
  - Best solution – more data!

- Wide-band data
  - New instruments (like EVLA) have huge fractional bandwidths
  - Image structure changes dramatically
  - Antenna primary beams change dramatically
  - New algorithms are being developed to manage this.
- Distant structure
  - In general, antennas 'sense' the entire sky – even if the distant structure is highly attenuated. (This problem is especially bad at low frequencies …)
  - You are likely interested in only a part of the sky.
  - You probably can't afford to image the entire hemisphere …
  - Some form of full-sky imaging will be needed to remove distant, unrelated visibilities.
  - Algorithms under development for this.

How Good Can It Get?

- Shown is our best image (so far) from the EVLA.
- 3C147, with 'WIDAR0' – 12 antennas and two spectral windows at L-band (20cm).
- Time averaging 1 sec.
- BW averaging 1 MHz
- BW 2 x 100 MHz
- Peak = 21200 mJy
- 2^{nd} brightest source 32 mJy
- Rms in corner: 32 μJy
- Peak in sidelobe: 13 mJy – largest sidelobes are around this!
- DR ~ 850,000!
- Fidelity quite a bit less.