Ultra-Wide Field Imaging 1
a work in progress...

Steven T. Myers

National Radio Astronomy Observatory
Socorro, NM
Ultra-wide field imaging

• low-freq arrays image nearly the whole half-sky
  – LWA, MWA, LOFAR, SKA…
  – plus terrestrial interference on horizon!
• VLA 74 MHz already 10° FWHM!
• tiling the sky is tricky – use single spherical coord. system
  – but need a way to grid (e.g. HEALPIX)
• replace tangent plane coordinates with spherical (angular) coordinates
  – replace uv-plane with spherical harmonic multipoles \( l,m \)?
  – used in CMB (as well as atomic physics)
  – need to relate to interferometer system
The essence of $W$ projection

- Evaluate this integral (and transpose) for regular grid in $(l,m)$ and irregularly spaced samples in $(u,v)$

$$V(u, v, w) = \int \int \frac{d\xi d\eta}{\sqrt{1 - \xi^2 - \eta^2}} I(\xi, \eta) e^{i2\pi[u\xi + v\eta + w(\sqrt{1-\xi^2-\eta^2} - 1)]}$$

- Image space computation = multiplicative function

$$V(u, v, w) = \int \int \frac{d\xi d\eta}{\sqrt{1 - \xi^2 - \eta^2}} G(\xi, \eta, w) I(\xi, \eta) e^{-i2\pi(u\xi + v\eta)}$$

- Fourier space computation = convolution kernel

$$V(u, v, w) = G(u, v, w) \otimes V(u, v, w = 0)$$
Standard sky geometry

- **sky:**
  - unit sphere
  - tangent plane
  - direction cosines
  - $\xi = (\xi, \eta, \zeta)$

- **interferometer:**
  - $u = B / \lambda$
  - $u = (u, v, w)$

- project plane-wave onto baseline vector
  - phase $2\pi \xi \cdot u$
Wavefront correlations

- Rewrite the standard form of the relation as

\[ V(u, v, w) = \int \int \frac{d\xi \, d\eta}{1 + \zeta} \, I(\xi, \eta) \, e^{i2\pi \xi \cdot u} \]

\[ \xi = (\xi, \eta, \zeta) \quad u = (u, v, w) \]

\[ 1 + \zeta = \sqrt{1 - \xi^2 - \eta^2} \]

- Sky is 2-dimensional, but baseline vector 3-d
Whole-sky imaging & transforms

- Celestial spherical coordinates \((\theta, \phi)\)
  - choose pole: celestial, terrestrial, pointing direction
  - coordinates: RA-Dec, Az-ZA, other
  - Intensity field:
    \[
    I(\hat{n}) = (\theta, \phi)
    \]

- Spherical Harmonic Transforms
  \[
  a_{\ell m} = \int d^2\hat{n} Y_{\ell m}(\hat{n}) I(\hat{n})
  \]
  \[
  Y_{\ell m}(\theta, \phi) = P_{\ell} (\cos \theta) e^{-im\phi}
  \]
  - at high \(l\) these become Fourier transforms
  - complete orthogonal harmonic mode basis on sphere
  - there are fast versions and convolvers
Spherical maps

• Need optimized map geometry and fast convolvers:

http://www.eso.org/science/healpix
– see Wandelt & Gorski (astro-ph/0008227) for convolution
WMAP: case study

- HEALpix maps:

  K: 23GHz
  Ka: 33GHz
  Q: 41GHz
  W: 94GHz
Issues

• Is there a simple expression for $V(u,v,w)$ in terms of spherical harmonics $(l,m)$?
• Will the FSHT be fast enough?
• Are fast spherical convolvers fast enough?
• Tiling the sky: is HEALPIX the right way?
• Are there any practical advantages to doing it this way?

to be continued …