Steps towards correction of image plane effects

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Basic Interferometry

- Interferometers measure the source coherence function (the Visibility function) \( V(u_{ij}, v_{ij}, w_{ij}) = \langle E_i E_j^* \rangle \)

  \( E_i \) is the electric field measured at antenna \( i \)

  \( u, v, w \) are the projected separation between the antennas \( i \) and \( j \)

- In terms of the sky brightness distribution \( (I^o(l,m)) \)

  \[ V(u_{ij}, v_{ij}, w_{ij}) = \int \int I^o(l, m) e^{-2\pi \imath (u_{ij}l + v_{ij}m + w_{ij}n)} \frac{dl \, dm}{n} \]

- In the small angle approximation, sky is the 2D Fourier transform of the Visibility function (van Cittert-Zernike Theorem)

  \[ V(u_{ij}, v_{ij}) = \int \int I^o(l, m) e^{-2\pi \imath (u_{ij}l + v_{ij}m)} \, dl \, dm \]

\[ I^o = FT[V] \]
Imaging and calibration errors

- Generic Measurement Equation:
  \[ V_{ij}^{\text{Obs}}(\nu, t) = G_{ij}(\nu, t) \left[ \int \int X_{ij}(\nu, t) I^M(l, m) e^{2\pi i (lu_{ij} + mv_{ij})} \right] dl \, dm \]

  \[ G_{ij} = G_i G_j^* \text{ where } G_i \text{ are antenna based gains (direction independent)} \]

  \[ X_{ij} = X_i(l,m)X_j(l,m) \text{ where } X_i \text{ are image plane errors (direction dependent)} \]

- \[ V_{ij}^{\text{Obs}} = G_{ij} \left[ E_{ij} * V_{ij}^M \right] \text{ Unknowns are: } G_{ij}, E_{ij}, \text{ and } I^M \]

- **SelfCal**: Given \( I^M \) and knowing \( E_{ij} \), \( G_{ij} \) is the dominant error term.

- **Imaging/deconvolution**: Knowing \( G_{ij} \) and \( E_{ij} \), solve for \( I^M \)
Visibility inversion

- Visibility inversion is done using the FFT algorithm which operates on a regular grid.

- $V^{\text{Obs}}$ does not regularly sample the $(u,v)$ plane. $V^{\text{M}}(u,v)$ is computed by re-sampling the grid using a Gridding Convolution Function (GCF)

  $$V^{\text{M}}(u_{ij}, v_{ij}) = (GCF(u, v) * \text{FFT}[I^{\text{M}}])(u_{ij}, v_{ij})$$

- Image plane effects can be efficiently applied by incorporating them in the GCF.

- **Challenge**: Fast evaluation of the modified GCF.
Known direction dependent errors

- Non co-planar baselines
  \[ V(u, v, w) = \iint I(l, m) e^{2\pi i \left( ul + vm + w \sqrt{1 - l^2 - m^2} \right)} \frac{dl \, dm}{\sqrt{1 - l^2 - m^2}} \]

- Traditional approach: Image plane faceting

- **W-projection**
  \[ V_{ij}^{\text{Obs}} (u, v, w) = E_{ij} \ast V_{ij}^M (u, v, w=0) \text{ where } E_{ij} = \mathcal{F}T \left[ e^{2\pi i w \sqrt{1 - l^2 - m^2}} \right] \]
W-projection

- $\rho_{12} = \langle E_1(u, v, w=0) E_2^*(0,0,0) \rangle$

$E_1 = E'_1(u, v, w)$ propagated using Fresnel diffraction theory. The above convolution equation is reproduced with $r_F/\lambda \approx \sqrt{w}$.

- A $w \neq 0$ interferometer is not a device to measure a single Fourier component.
W-projection algorithm
(CGB, EVLA Memo #67)

- Pre-compute $GCFs$ with uniform sampling in $\sqrt{w}$ such that aliasing effects are less than the required dynamic range.

Scaling laws:

**W-projection:**

$\left( N^2_{wproj} + N^2_{GCF} \right) N_{vis}$

**UV-facet:**

$N^2_{facets} N^2_{GCF} N_{vis}$

**Ratio:**

$\approx N^2_{GCF}$ for large no. of facets
W-projection: Examples

GMRT P-band image of TXS2342+342 (Kanekar et al.)

Galactic Plane at P-band – VLA B,C,D (Brogan et al.)
Measured direction dependent effects

- $E_{ij}$ as a function of direction is measured a priori

$$V_{ij}^{Obs} = G_{ij}[E_{ij} \ast V^M] \text{ where } E_{ij}(l_i, l_j, u_{ij}; p_i, p_j)$$

- Primary beam effects
  - **Asymmetric Primary Beams:**
    $$E_{ij} = E_i^o \ast E_j^o \text{ where } E_i^o = FT\left[\text{Measured } PB_i\right]$$
  - Polarized primary beam: **Beam squint**
    - $E_i$ separately measured for the two polarizations
    - Applied with the known squint
Unknown direction dependent effects

- $E_{ij}$ as function of direction is not known a priori
  
  $V_{ij}^{\text{Obs}} = G_{ij} \cdot [E_{ij} \ast \text{FFT}[I^M]]$ where $E_{ij}(l_i, l_j, u_{ij}; p_i, p_j)$

- $E_{ij}$ is parameterized ($p_i$). Needs a solver

- Primary beam effects

  - Pointing offsets:
    
    $E_{ij} = E^0[1 + \Delta E_{ij}(l_i, l_j; t)]$  
    
    $l_i$ is the pointing offset

- Ionospheric/atmospheric effects: non-isoplanatic phases

- Source structure as a function of polarization and frequency: $I^M$ is also parameterized (spectral index effects)

  $I^M = \sum_k \text{Components}(\nu, \text{Poln}, \text{Position}, \text{Scale})$
Pointing offset calibration

\[ E_{ij}(l; l_i, l_j) = E_{ij}^0(l) e^{-\frac{(l_i-l_j)^2}{2}} e^{-\pi l u_{ij}(l_i+l_j)} \text{ (} l_i \text{ is the pointing offset)} \]

- Visibility prediction:
  - Gridded model \( V^{M,\text{Grid}} = \text{FFT}[I^M] \)
  - Re-sample on measured \((u, v)\) using \( E_{ij} \) as the GCF for baseline \(i-j\):
    \( V^M_{ij} = E_{ij} \ast V^{M,\text{Grid}} \)
  - \textit{GCF is different for each baseline!}
- Use lookup tables

OR

- Approximation: \( E_{ij} = E_{ij}^0 [1 - (l_i-l_j)^2 \sigma^2 / 2 - \pi l u_{ij}(l_i+l_j) + ...] \)
Pointing SelfCal

• Pointing errors for single pointing observation of typical L-Band field limits the RMS noise to ~10microJy.
  • EVLA L-band sensitivity: 1microJy/beam
• Mosacing dynamic range limited by pointing errors.
  • Significant fraction of ALMA observation will be mosacing observations.
• This implies significantly increased computing:
  • Each iteration involves expensive visibility prediction
Pointing SelfCal: Example

Model image using 59 sources from NVSS.
Flux range ~2-200 mJy/beam

Continuous lines: Typical antenna pointing offsets for VLA as a function of time (Mean between +/- 20arcsec and RMS of 5arcsec).

Dashed lines: Residual pointing errors. RMS ~ 1arcsec.
Pointing SelfCal: Example

Ref: BCG, EVLA Memo #84

Residual image without pointing correction:
\[ \text{FT}[V_{\text{Obs}} - V_M] \]
Peak flux \(~250\) microJy/beam
RMS \(~15\) microJy

Residual image after pointing correction:
\[ \text{FT}[V_{\text{Obs}} - E_{ij} \times V_M] \]
Peak flux \(~5\) microJy/beam
RMS \(~1\) microJy
Wide band continuum imaging

• EVLA bandwidth ratio of 2:1
  \[ V(u_{ij}, v_{ij}) = \sum_{v_k} V(u_{ij}, v_{ij}; v_k) = \sum_{v_k} P_{ij}(v_k) FT[I^D(v_k)] \]

Sky emission, the Primary Beams, etc. become a function of frequency.

Ideas: Apply PB effects during predict. Sky model parameterized in frequency as well.

• Combining Scale sensitive + frequency sensitive deconvolution with image plane corrections.

Scale sensitive deconvolution-I

- Pixel-to-pixel noise in the image is correlated at the scale of the resolution element

\[ I^D = B \ast I^o + B \ast I^N \text{ where } I^N = FT\left[ \text{Visibility Noise} \right]; B = PSF \]

- The scale of emission *fundamentally* separates signal \((I^o)\) from the noise \((I^N)\).

- Multi-Scale Clean (Cornwell & Holdaway, 2004, in prep)

  - Decompose the sky in a set of components at few scales \( I^M = \sum_{k=1}^{N} A_k P(Scale_k) \)
Scale sensitive deconvolution-II

  - Explicitly solve for the local scale, position and amplitude of the pixel model
    \[ I^M = \sum_k A_k P(Scale_k, Pos_k) \]
  - Large scale emission and asymmetric structures are better reconstructed
  - Computationally expensive: cost increases with the no. of components
  - Acceleration: Solve in a sub-space; adaptively determine the sub-space
Ionospheric phase correction

- \( V_{ij}^{\text{Obs}}(\nu, t) = \int \int K_{ij}(l, m; \nu, t) I^M(l, m) e^{2\pi i (u_{ij} l + v_{ij} m)} \, dl \, dm \)
  where \( K_{ij} \) is the ionospheric, direction dependent phase

- The general form for residuals: \( V_{ij}^R = V_{ij}^{\text{Obs}} - X_{ij} \ast FT[I^M] \)
  where \( X_{ij} = W_{ij}E_{ij}K_{ij} \) (and other direction dependent terms)

- Express ionospheric phase in the antenna beam using ionospheric physics/geometrical effects/primary beam effects (de-focusing, time varying illumination, etc.)

- \( X_{ij} \) is separable into antenna based quantities \( X(p_i) \ast X^*(p_j) \)
  Solve for \( p_i \) 's
Computing and I/O costs

- Increase in computing due to more sophisticated parameterization
  - Deconvolution: Fast evaluation of $B \sum_k A_k P(Scale_k, Pos_k)$
  - Calibration: Fast evaluation of $E_{ij} * V^M$

- Cost of computing residual visibilities is dominated by I/O costs for large datasets (~200GB for EVLA)
  - Deconvolution: Approx. 20 access of the entire dataset
  - Calibration: Each trial step in the search accesses the entire dataset

- Solutions: Analytical approximations, caching, Parallel computing and I/O,...