Polarization in Interferometry

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Polarization in Interferometry

• Physics of Polarization
• Interferometer Response to Polarization
• Polarization Calibration & Observational Strategies
• Polarization Data & Image Analysis
• Astrophysics of Polarization
• Examples

• References:
  – Synth Im. II lecture 6, also parts of 1, 3, 5, 32
  – “Tools of Radio Astronomy” Rohls & Wilson

WARNING!

• Polarimetry is an exercise in bookkeeping!
  – many places to make sign errors!
  – many places with complex conjugation (or not)
  – possible different conventions (e.g. signs)
  – different conventions for notation!
  – lots of matrix multiplications

• And be assured…
  – I’ve mixed notations (by stealing slides Ω)
  – I’ve made sign errors Ω (I call it “choice of convention” Ω)
  – I’ve probably made math errors Ω
  – I’ve probably made it too confusing by going into detail Ω
  – But … persevere (and read up on it later) Ω

DON’T PANIC!

Polarization Fundamentals

Physics of polarization

• Maxwell’s Equations + Wave Equation
  – E • B = 0 (perpendicular) ; E₂ = B₂ = 0 (transverse)
• Electric Vector – 2 orthogonal independent waves:
  – Eₓ = E₁ cos (k z + ω t + δ₁) = k = 2π / λ
  – Eᵧ = E₂ cos (k z + ω t + δ₂) = ω = 2π v
  – describes helical path on surface of a cylinder…

  – parameters E₁, E₂, δ₁, δ₂ define ellipse

The Polarization Ellipse

• Axes of ellipse Eₓ, Eᵧ
  – sₒ = E₁² + E₂² = Eₓ² + Eᵧ²  Pointing flux
  – δ phase difference
    – Eₓ = Eₒ cos (t + δ) = Eₓ cos Ψ + Eᵧ sin Ψ
    – Eᵧ = Eₒ sin (t + δ) = -Eₓ sin Ψ + Eᵧ cos Ψ

Rohls & Wilson

Ninth Synthesis Imaging Summer School
Socorro, June 15-22, 2004
The polarization ellipse continued...

- Ellipticity and Orientation
  - $E_1 / E_2 = \tan \alpha$  \tan 2\psi = - \tan 2\alpha \cos \delta$
  - $E_3 / E_4 = \tan \chi$  \sin 2\chi = \sin 2\alpha \sin \delta$
  - handiness (\sin \delta > 0 or \tan \chi > 0 \rightarrow right-handed)

Polarization ellipse – special cases

- Linear polarization
  - $\delta = 0$, $m = 0$, $m = 1$, $m = 2$, ...
  - ellipse becomes straight line
  - electric vector position angle $\Psi = \alpha$

- Circular polarization
  - $\delta = \pi/2$, $m = 0$, $m = 1$, $m = 2$, ...
  - equation of circle $E_x^2 + E_y^2 = E^2$
  - orthogonal linear components:
    - $E_x = E \cos \tau$
    - $E_y = e \cos (\tau - \pi/2)$
  - note quarter wave delay between $E_x$ and $E_y$

Orthogonal representation

- A monochromatic wave can be expressed as the superposition of two orthogonal linearly polarized waves
- An arbitrary elliptically polarized wave can also equally well be described as the superposition of two orthogonal circularly polarized waves
- We are free to choose the orthogonal basis for the representation of the polarization
- NOTE: Monochromatic waves MUST be (fully) polarized – IT’S THE LAW!

Linear and Circular representations

- Orthogonal Linear representation:
  - $E_x = E_0 \cos (\tau + \delta) = E_x \cos \Psi + E_y \sin \Psi$
  - $E_y = E_0 \sin (\tau + \delta) = -E_x \sin \Psi + E_y \cos \Psi$

- Orthogonal Circular representation:
  - $E_x = E_0 \cos (\tau + \delta) = (E_x + E_y) \cos (\tau + \delta)$
  - $E_y = E_0 \sin (\tau + \delta) = (E_x - E_y) \cos (\tau + \delta - \pi/2)$
  - $E_x = E_0 \cos (\tau + \delta)$
  - $E_y = E_0 \sin (\tau + \delta)$

The Poincare Sphere

- Treat 2\psi and 2\chi as longitude and latitude on sphere of radius $S_0$

Stokes parameters

- Spherical coordinates: radius I, axes Q, U, V
  - $S_0 = I = E_x^2 + E_y^2$
  - $S_1 = Q = S_0 \cos 2\chi \cos 2\Psi$
  - $S_2 = U = S_0 \cos 2\chi \sin 2\Psi$
  - $S_3 = V = S_0 \sin 2\chi$
- Only 3 independent parameters:
  - $S_0^2 = S_1^2 + S_2^2 + S_3^2$
  - $I^2 = Q^2 + U^2 + V^2$
- Stokes parameters I,Q,U,V
  - form complete description of wave polarization
  - NOTE: above true for monochromatic wave!
**Stokes parameters and polarization ellipse**

- Spherical coordinates: radius $I$, axes $Q$, $U$, $V$
  - $S_0 = I = E_x^2 + E_y^2$
  - $S_1 = Q = S_0 \cos 2\chi \cos 2\psi$
  - $S_2 = U = S_0 \cos 2\chi \sin 2\psi$
  - $S_3 = V = S_0 \sin 2\chi$
- In terms of the polarization ellipse:
  - $S_0 = I = E_x^2 + E_y^2$
  - $S_1 = Q = E_x E_y$
  - $S_2 = U = 2 E_x E_y \cos \delta$
  - $S_3 = V = 2 E_x E_y \sin \delta$

**Stokes parameters special cases**

- Linear Polarization
  - $S_0 = I = E^2$
  - $S_1 = Q = I \cos 2\psi$
  - $S_2 = U = I \sin 2\psi$
  - $S_3 = V = 0$
- Circular Polarization
  - $S_0 = I = S$
  - $S_1 = Q = 0$
  - $S_2 = U = 0$
  - $S_3 = V = S$ (RCP) or $-S$ (LCP)

**Quasi-monochromatic waves**

- Monochromatic waves are fully polarized
- Observable waves (averaged over $\Delta \nu / \nu << 1$)
  - Analytic signals for $x$ and $y$ components:
    - $E_x(\delta) = a_0(e^{i(\delta - \theta)} - e^{-i\delta})$
    - $E_y(\delta) = a_0 e^{i(\delta - \theta)}$
  - Stokes parameters
    - $S_0 = I = \langle a_0^2 \rangle$
    - $S_1 = Q = \langle a_2 \rangle$
    - $S_2 = U = \langle a_1 \rangle$
    - $S_3 = V = \langle a_3 \rangle$

**Partial polarization**

- The observable electric field need not be fully polarized as it is the superposition of monochromatic waves
- On the Poincare sphere:
  - $S_0^2 \geq S_1^2 + S_2^2 + S_3^2$
  - $S_1^2 \leq S_0^2$
- Degree of polarization $\rho$:
  - $\rho^2 = S_1^2 + S_2^2 + S_3^2$
  - $\rho^4 = Q^2 + U^2 + V^2$

**Stokes parameters and intensity measurements**

- If phase of $E_x$ is retarded by $\epsilon$ relative to $E_y$, the electric vector in the orientation $\theta$ is:
  - $E(\theta, \epsilon) = E_x \cos \theta + E_y e^{i\epsilon} \sin \theta$
- Intensity measured for angle $\theta$:
  - $I(\theta, \epsilon) = E(\theta, \epsilon) E^* (\theta, \epsilon)$
- Can calculate Stokes parameters from 6 intensities:
  - $S_0 = I = I(0^\circ, 0)$
  - $S_1 = Q = I(90^\circ, 0)$
  - $S_2 = U = I(90^\circ, -135^\circ)$
  - $S_3 = V = I(45^\circ, n / 2)$
- this can be done for single dish (intensity) polarimetry

**Summary – Fundamentals**

- Monochromatic waves are polarized
- Expressible as 2 orthogonal independent transverse waves
  - elliptical cross-section
  - polarization ellipse
  - 3 independent parameters
  - choice of basis, e.g. linear or circular
- Poincare sphere convenient representation
  - Stokes parameters $I$, $Q$, $U$, $V$
  - 1 intensity: O.U linear polarization, V circular polarization
- Quasi-monochromatic “waves” in reality
  - can be partially polarized
  - still represented by Stokes parameters
Polarization in Interferometry – S. T. Myers

Interferometer response to polarization

• Stokes parameter recap:
  – intensity I
  – fractional polarization \( (p I)^2 = Q^2 + U^2 + V^2 \)
  – linear polarization Q,U \( (m I)^2 = Q^2 + U^2 \)
  – circular polarization V \( (v I)^2 = V^2 \)
• Coordinate system dependence:
  – I independent
  – V depends on choice of “handedness”
  – V > 0 for RCP
  – Q,U depend on choice of “North” (plus handedness)
  – Q “points” North, U 45° toward East
  – EVPA \( \Phi = \frac{1}{2} \tan^{-1} (U/Q) \) (North through East)

Reflector antenna systems

• Reflections
  – turn RCP \( \rightarrow \) LCP
  – E-field allowed only in plane of surface
• Curvature of surfaces
  – introduce cross-polarization
  – effect increases with curvature (as f/D decreases)
• Symmetry
  – on-axis systems see linear cross-polarization
  – off-axis feeds introduce asymmetries & R/L squint
• Feedhorn & Polarizers
  – introduce further effects (e.g. “leakage”)

Optics – Cassegrain radio telescope

• Paraboloid illuminated by feedhorn:

Optics – telescope response

• Reflections
  – turn RCP \( \rightarrow \) LCP
  – E-field (currents) allowed only in plane of surface
• “Field distribution” on aperture for E and B planes:

Polarization field pattern

• Cross-polarization
  – 4-lobed pattern
• Off-axis feed system
  – perpendicular elliptical
  – linear pol. beams
  – R and L beams diverge (beam squint)
• See also:
  – “Antennas” lecture by P. Napier
Feeds – Linear or Circular?

• The VLA uses a circular feedhorn design
  – plus (quarter-wave) polarizer to convert circular polarization
    from feed into linear polarization in rectangular waveguide
  – correlations will be between R and L from each antenna
  – RR RL LH LH form complete set of correlations
• Linear feeds are also used
  – e.g. ATCA, ALMA (and possibly EVLA at 1.4 GHz)
  – no need for (lossy) polarizer!
  – correlations will be between X and Y from each antenna
  – XX XY YX YY form complete set of correlations
• Optical aberrations are the same in these two cases
  – but different response to electronic (e.g. gain) effects

Example – simulated VLA patterns

• EVLA Memo 58 “Using Grasp8 to Study the VLA Beam” W. Brisken

Example – measured VLA patterns

• frequency dependence of polarization beam:
  – VLA Linear Polarization beam at 1.5 GHz
  – VLA Circular Polarization beam at 1.5 GHz

Example – measured VLA patterns

• AIPS Memo 86 “Widefield Polarization Correction of VLA Snapshot Images at 1.4 GHz” W. Cotton (1994)

Beyond optics – waveguides & receivers

• Response of polarizers
  – convert R & L to X & Y in waveguide
  – purity and orthogonality errors
• Other elements in signal path:
  – Sub-reflector & Feedhorn
    • symmetry & orientation
  – Ortho-mode transducers (OMT)
    • split orthogonal modes into waveguide
  – Polarizers
    • retard one mode by quarter-wave to convert LP \rightarrow CP
    • frequency dependent!
  – Amplifiers
    • separate chains for R and L signals
Back to the Measurement Equation

- Polarization effects in the signal chain appear as error terms in the Measurement Equation
  - e.g. "Calibration" lecture, G. Moellenbrock:

  \[ F = I_{\text{ionospheric Faraday rotation}} \]

  \[ T = I_{\text{ionospheric effects}} \]

  \[ P = I_{\text{parallactic angle}} \]

  \[ E = I_{\text{antenna voltage pattern}} \]

  \[ G = I_{\text{electronic gain}} \]

  \[ B = I_{\text{bandpass response}} \]

  \[ B = \text{Baseline} J \] (outer product)

  \[ J_i(\omega) = B_i(\omega) G_i(\omega) E_i(\omega) F_i(\omega) \]

  \[ = (\tilde{B}_i \otimes \tilde{G}_i) \tilde{E}_i \tilde{F}_i \]

  \[ = \tilde{B}_i \tilde{G}_i \tilde{E}_i \tilde{F}_i \]

  \[ \text{ionospheric Faraday rotation, } F \]

  - The ionosphere is birefringent; one hand of circular polarization is delayed w.r.t. the other, introducing a phase shift:
    \[ \Delta \varphi = 0.15 \frac{\lambda}{d} \left( B_0 - B_i \right) \text{ deg} \] (in cm, \( d \) in cm, \( B_0 \) in G)

    - rotates the linear polarization position angle:
      \[ \tilde{E} = \begin{pmatrix} e^{i\varphi} & 0 \\ 0 & e^{-i\varphi} \end{pmatrix} \tilde{E} = \begin{pmatrix} \cos \Delta \varphi & -\sin \Delta \varphi \\ \sin \Delta \varphi & \cos \Delta \varphi \end{pmatrix} \]

    - more important at longer wavelengths:
      \[ TEC = \int_{0}^{\infty} n_e ds \approx 10^{15} \text{ cm}^{-2} \text{; } B_0 - B_i \text{, } \lambda > 20 \text{ km} \rightarrow \Delta \varphi - 6\theta \]

    - ionosphere most active at solar maximum and sunrise/sunset
    - watch for direction dependence (in-beam)
    - see "Low Frequency Interferometry" (C. Brogan)

Parallactic Angle, \( P \)

- Parallactic angle versus hour angle at VLA:
  - fastest swing for source passing through zenith

Antenna voltage pattern, \( E \)

- Direction-dependent gain and polarization
  - includes primary beam
    - Fourier transform of cross-correlation of antenna voltage patterns
    - includes polarization asymmetry (squint)
      \[ E_{\text{sq}}(u,v) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \]

    - can include off-axis cross-polarization (leakage)
      - convenient to reserve \( B \) for on-axis leakage
      - will then have off-diagonal terms
      - important in wide-field imaging and mosaicing
      - when sources fill the beam (e.g. low frequency)

Ionospheric Faraday Rotation, \( F \)

- Birefringency due to magnetic field in ionospheric plasma

  \[ F = \text{Faraday Rotation} \]

  \[ \text{Plasma} \]

  \[ \text{I} = \text{ionospheric Faraday rotation, } F \]

  - also present in radio sources!

Parallactic Angle, \( P \)

- Orientation of sky in telescope’s field of view
  - Constant for equatorial telescopes
  - Varies for alt-az mounted telescopes:
    \[ X(t) = \text{antenna patterns} \cdot \sin(\lambda_0 \cos(\alpha \cos h(t))) \]
    \[ = \text{latitude, } h(t) = \text{hour angle, } \delta = \text{declination} \]

    - Rotates the position angle of linearly polarized radiation (cf. \( F \))
      \[ \tilde{E} = \begin{pmatrix} e^{i\varphi} & 0 \\ 0 & e^{-i\varphi} \end{pmatrix} \tilde{E} = \begin{pmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{pmatrix} \]

    - defined per antenna (often same over array)

    - \( P \) modulation can be used to aid in calibration
**Polarization Leakage, D**

- Polarizer is not ideal, so orthogonal polarizations not perfectly isolated
  - Well-designed systems have $d < 1\%$
  - A geometric property of the antenna, feed & polarizer design
    - Frequency dependent (e.g., quarter-wave at center $v$)
    - Direction dependent (in beam) due to antenna
  - For RL systems
    - Parallel hands affected as $dQ + dUL$, so only important at high dynamic range (because $Q-U$ & $d$ typically)
    - Cross-hands affected as $dI$ so almost always important

\[
D = \left( \begin{array}{cc} 1 & d^Q \\ d^I & 1 \end{array} \right) \quad \text{Leakage of q into p (e.g., L into R)}
\]

**Coherency vector and correlations**

- Coherency vector:

\[
e = \left( \begin{array}{c} x \otimes x' \\ y \otimes y' \end{array} \right) = \left( \begin{array}{c} x^2 + y^2 \\ x'y' \end{array} \right)
\]

- e.g. for circularly polarized feeds:

\[
e^{\text{circ}} = \left( \begin{array}{c} x^2 - y^2 \\ 2xy \end{array} \right)
\]

**Visibilities and Stokes parameters**

- Convolution of sky with measurement effects:

\[
\tilde{V}_0^{\text{sky}} = \int \left( \tilde{U} \otimes \tilde{I} \right) \delta(l, m) e^{-2\pi j(l+m)v} \, dl \, dm
\]

- Instrumental effects, including 'beam' $E(l,m)$

\[
\tilde{V}_0^{\text{sky}} = \int \left( \tilde{U} \otimes \tilde{I} \right) \delta(l, m) e^{-2\pi j(l+m)v} \, dl \, dm
\]

- Imaging involves inverse transforming these

**Example: RL basis imaging**

- Parenthetical Note:
  - Can make a pseudo-I image by gridding RR+LL on the Fourier half-plane and inverting to a real image
  - Can make a pseudo-V image by gridding RR-LL on the Fourier half-plane and inverting to real image
  - Can make a pseudo-$Q+U$ image by gridding RL to the full Fourier plane (with LR as the conjugate) and inverting to a complex image
  - Does not require having full polarization RR,RL,LL for every visibility
  - More on imaging (& deconvolution) tomorrow!
Leakage revisited…

- Primary on-axis effect is “leakage” of one polarization into the measurement of the other (e.g. \( R \rightarrow L \))
  - but, direction dependence due to polarization beam!
- Customary to factor out on-axis leakage into \( D \) and put direction dependence in “beam”
  - example: expand RL basis with on-axis leakage
  
\[
\begin{align*}
V_{0}^{RL} &= V_{0}^{RR} + d_{i}^{R}V_{0}^{RR} + d_{j}^{R}V_{0}^{RL} + d_{i}^{L}V_{0}^{LR} + d_{j}^{L}V_{0}^{LL} \\
V_{y}^{RL} &= V_{y}^{RR} + d_{i}^{R}V_{y}^{RR} + d_{j}^{R}V_{y}^{RL} + d_{i}^{L}V_{y}^{LR} + d_{j}^{L}V_{y}^{LL}
\end{align*}
\]

- similarly for XY basis

Example: Linearized response

- Dropping terms in \( d, Q, dU, dV \) (and expanding \( G \))
  - For crossed linearly polarized feeds
    \[
    \begin{align*}
    \psi_{\parallel} &= \psi_{\parallel}(I + Q \cos 2\chi + U \sin 2\chi), \\
    \psi_{\perp} &= \psi_{\perp}(I - Q \cos 2\chi - U \sin 2\chi), \\
    \psi_{\chi} &= \psi_{\chi}(I - Q \cos 2\chi - U \sin 2\chi), \\
    \psi_{\varphi} &= \psi_{\varphi}(I + Q \cos 2\chi + U \sin 2\chi). \\
    \end{align*}
    \]
  - for circularly polarized feeds:
    \[
    \begin{align*}
    \psi_{\parallel} &= \psi_{\parallel}(I + V), \\
    \psi_{\perp} &= \psi_{\perp}(I - V), \\
    \psi_{\chi} &= \psi_{\chi}(I - V), \\
    \psi_{\varphi} &= \psi_{\varphi}(I + V). \\
    \end{align*}
    \]
  - warning: using linear order can limit dynamic range!

Example: RL basis leakage

- In full detail:
  \[
  V_{0}^{RL} = \int \frac{E_{0}^{RL}(I, m)(I + V)e^{i(z_{2} - z_{1})}}{\delta_{0}} dtdm + d_{i}^{R}e^{i(z_{2} - z_{1})}(Q - iU) + d_{j}^{R}e^{i(z_{2} - z_{1})}(Q + iU) \\
  + d_{i}^{L}e^{i(z_{2} - z_{1})}(Q - iU) + d_{j}^{L}e^{i(z_{2} - z_{1})}(Q + iU) \\
  V_{y}^{RL} = \int \frac{E_{y}^{RL}(I, m)(I + V)e^{i(z_{2} - z_{1})}}{\delta_{y}} dtdm + d_{i}^{R}e^{i(z_{2} - z_{1})}(Q - iU) + d_{j}^{R}e^{i(z_{2} - z_{1})}(Q + iU) \\
  + d_{i}^{L}e^{i(z_{2} - z_{1})}(Q - iU) + d_{j}^{L}e^{i(z_{2} - z_{1})}(Q + iU)
  \]

Summary – polarization interferometry

- Choice of basis: CP or LP feeds
- Follow the Measurement Equation
  - ionospheric Faraday rotation \( F \) at low frequency
  - parallactic angle \( P \) for coordinate transformation to Stokes
  - “leakage” \( D \) varies with \( \nu \) and/or beam (mix with \( E \))
- Leakage
  - use full (all orders) \( D \) solver when possible
  - linear approximation OK for low dynamic range

So you want to make a polarization map…

Polarization Calibration & Observation

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\[\text{SNTHS IMAGN SUMMR SCHUL}\]

Jupiter

June 20-27, 2000
Socorro, NM, USA

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### Strategies for Polarization Observations

- Follow general calibration procedure (last lecture)
  - will need to determine leakage $D$ (if not known)
  - often will determine $G$ and $D$ together (iteratively)
  - procedure depends on basis and available calibrators
- Observations of polarized sources
  - follow usual rules for sensitivity, uv coverage, etc.
  - remember polarization fraction is usually low! (few %)
  - if goal is to map $E$-vectors, remember to calculate noise in
    \[ \Phi = \frac{1}{2} \tan^{-1} \frac{U}{Q} \]
  - watch for gain errors in $V$ (for CP) or $Q,U$ (for LP)
  - for wide-field high-dynamic range observations, will need to
    correct for polarized primary beam (during imaging)
- **Other strategies**
  - Calibrator of unknown polarization
    - solve for model $IQUV$ and $D$ simultaneously or iteratively
    - need good parallactic angle coverage to modulate sky and
      instrumental signals
    - in instrument basis, sky signal modulated by $e^{i\chi}$
  - With a very bright strongly polarized calibrator
    - can solve for leakages and polarization per baseline
    - can solve for leakages using parallel hands!
  - With no calibrator
    - hope it averages down over parallactic angle
    - transfer $D$ from a similar observation
    - usually possible for several days, better than nothing!
    - need observations at same frequency

### Strategies for Leakage Calibration

- Need a bright calibrator! Effects are low level...
  - determine gains $G$ (mostly from parallel hands)
  - use cross hands (mostly) to determine leakage
  - general ME D solver (e.g. aips++) uses all info
- Calibrator is unpolarized
  - leakage directly determined (ratio to $I$ model), but only to an
    overall constant
  - need way to fix phase $p,q$ (e.g. R-L phase difference), e.g.
    using another calibrator with known EVPA
- Calibrator of known polarization
  - leakage can be directly determined (for $IQUV$ model)
  - unknown $p,q$ phase can be determined (from $U/Q$ etc.)

### Finding Polarization Calibrators

- Standard sources
  - planets (unpolarized if unresolved)
  - $\theta$ Cas, $\theta$ Cet, $\theta$ Hya, $\theta$ Oph (known $IQU$, stable)
  - sources monitored (e.g. by VLA)
  - other bright sources (bootstrap)

### Example: D-term calibration

- D-term calibration effect on RL visibilities:
  - D-term calibration effect in image plane:
    Bad D-Term Solution
    Good D-Term Solution
Example: “standard” procedure for CP feeds

- Parallel correlations sensitive to Stokes I & V
- \( I = \langle a_2 a_2 + a_1 a_1 \rangle \)
- \( V = \langle a_2 a_1 - a_1 a_2 \rangle \)
- Assume \( I = 0 \) for calibrator
- Can separate and solve for gains for \( p \) and \( q \)
- Instrumental \( \psi \) and source polarization \( \langle Q/L \rangle \)
- sum of two vectors:
  \[ I = \langle a_2 a_2 + a_1 a_1 \rangle \]
  \[ V = \langle a_2 a_1 - a_1 a_2 \rangle \]
- Calibrator observations of a range of PA gives close separations.
- Independent gain calibration for \( p \) and \( q \) allows arbitrary phase offset - note all phases to same “reference” antenna.
- \( p-q \) phase difference is that of the reference antenna.
- Need observations of calibrator of known polarization angle via Electric Vector Position Angle (EVPA)

Example: “standard” procedure for LP feeds

- Parallel correlations sensitive to \( I, Q, & U \)
- \( I = \langle a_2 a_2 + a_1 a_1 \rangle \)
- \( Q = \langle a_2 a_1 - a_1 a_2 \rangle \)
- \( U = \langle a_2 a_1 + a_1 a_2 \rangle \)
- Calibrator \( Q \) and \( U \) usually cannot be ignored (few %)
- Phase unaffected by polarization of a point source at the phase center.
- Cannot separate \( p, q \) gains and calibrator polarization.
- \( p - q \) phase offset not known.
- May be unknown orientation error of \( p \) and \( q \)
- Need use of source with known polarization
- \( I = \langle a_2 a_2 + a_1 a_1 \rangle \)
- \( Q = \langle a_2 a_1 - a_1 a_2 \rangle \)
- \( U = \langle a_2 a_1 + a_1 a_2 \rangle \)
- Calibrator \( Q \) and \( U \) effect part of cross pol correlation.
- Calibrator \( V \) affects imaginary part of cross pol correlations but unaffected by PA.

Special Issues

- Low frequency – ionospheric Faraday rotation
  - important for 2 GHz and below (sometimes higher too)
  - \( \lambda^2 \) dependence (separate out using multi-frequency obs.)
  - depends on time of day and solar activity (observatory location)
  - external calibration using zenith TEC (plus gradient?)
  - self-calibration possible (e.g. with snapshots)

- VLBI polarimetry
  - follows same principles
  - will have different parallactic angle at each station!
  - can have heterogeneous feed geometry (e.g. CP & LP)
  - harder to find sources with known polarization
    - calibrators resolved?
    - transfer EVPA from monitoring program

- Observing circular polarization V is straightforward with LP feeds (from Re and Im of cross-hands)
  - With CP feeds:
    - gain variations can masquerade as (time-variable) V signal
    - helps to switch signal paths through back-end electronics
    - R vs. L beam squint introduces spurious V signal
    - limited by pointing accuracy
    - requires careful calibration
    - relative R and L gains critical
    - average over calibrators (be careful of intrinsic V)
    - VLBI somewhat easier
    - different systematics at stations help to average out
### Special Issues – wide field polarimetry

- Actually an imaging & deconvolution issue
  - assume polarized beam D-E is known
- Deal with direction-dependent effects
  - beam squint (R,L) or beam ellipticity (X,Y)
  - primary beam
- Iterative scheme (e.g. CLEAN)
  - implemented in apps++,
  - see lectures by Bhatnagar & Cornwell
- Described in EVLA Memo 62 “Full Primary Beam Stokes IQUV Imaging” T. Cornwell (2003):

### Example: wide field polarimetry (Cornwell 2003)

- Simulated array of point sources
  - 1D beam + squint
  - Full 2D beam

### Example: wide field polarimetry continued…

- Simulated Hydra A image
  - Panels: I Q U V
  - Errors 1D sym. beam
  - Errors full beam

### Summary – Observing & Calibration

- Follow normal calibration procedure (previous lecture)
- Need bright calibrator for leakage D calibration
  - best calibrator has strong known polarization
  - unpolarized sources also useful
- Parallactic angle coverage useful
  - necessary for unknown calibrator polarization
- Need to determine unknown p-q phase
  - CP feeds need EVPA calibrator for R-L phase
  - if system stable, can transfer from other observations
- Special Issues
  - observing CP difficult with CP feeds
  - wide-field polarization imaging (needed for EVLA & ALMA)

### Polarization data analysis

- Making polarization images
  - follow general rules for imaging & deconvolution
  - image & deconvolve in I, Q, U, V (e.g. CLEAN, MEM)
  - note: Q, U, V will be positive and negative
  - in absence of CP, V image can be used as check
  - joint deconvolution (e.g. apps++, wide-field)
- Polarization vector plots
  - use “electric vector position angle” (EVPA) calibrator to set angle (e.g. R-L phase difference)
  - $\Phi = \frac{1}{2} \tan^{-1} U/Q$ for E vectors (B vectors $\perp$ E)
  - plot E vectors with length given by $p$
- Faraday rotation: determine $\Delta \Phi$ vs. $\lambda^2$
### Astrophysical mechanisms for polarization

- Magnetic fields
  - synchrotron radiation → LP (small amount of CP)
  - Zeeman effect → CP
  - Faraday rotation (of background polarization)
  - dust grains in magnetic field
  - maser emission
- Electron scattering
  - incident radiation with quadrupole
  - e.g. Cosmic Microwave Background
- and more…

### Astrophysical sources with polarization

- Magnetic objects
  - active galactic nuclei (AGN) (accretion disks, MHD jets, lobes)
  - protostars (disks, jets, masers)
  - clusters of galaxies IGM
  - galaxy ISM
  - compact objects (pulsars, magnetars)
  - planetary magnetospheres
  - the Sun and other (active) stars
  - the early Universe (primordial magnetic fields???)
- Other objects
  - Cosmic Microwave Background (thermal)
- Polarization levels
  - usually low (<1% to 5-10% typically)

### Example: 3C31
- VLA @ 8.4 GHz
- E-vectors
- Laing (1996)

### Example: Cygnus A
- VLA @ 8.5 GHz
- B-vectors
- Perley & Carilli (1996)

### Example: Blazar Jets
- VLBA @ 5 GHz
- Attridge et al. (1999)

### Example: the ISM of M51
- Neininger (1982)
Example: Zeeman effect

Example: Zeeman in M17

Example: Faraday Rotation

Example: more Faraday rotation

Example: Galactic Faraday Rotation

Example: Stellar SiO Masers