Measures of Antenna Performance

Source and System Temperatures

- What is received power $P$?
- Write $P$ as equivalent temperature of matched termination at receiver input
  - Rayleigh-Jeans limit to Planck law $P = k_B T \times \Delta v$
  - Boltzmann constant $k_B$
  - Observing bandwidth $\Delta v$
- Amplify $P$ by $g^2$ where $g$ is voltage gain
- Separate powers from source, system noise
  - Source antenna temperature $T_s$ => source power $P_s = g^2 \times k_B \times T_s \times \Delta v$
  - System temperature $T_{sys}$ => noise power $P_n = g^2 \times k_B \times T_{sys} \times \Delta v$

System Equivalent Flux Density

- Antenna temperature $T_s = K \times S$
  - Source power $P_s = g^2 \times k_B \times K \times S \times \Delta v$
- Express system temperature analogously
  - Let $T_{sys} = K \times SEFD$
  - $SEFD$ is system equivalent flux density, unit Jy
  - System noise power $P_n = g^2 \times k_B \times K \times SEFD \times \Delta v$
- $SEFD$ measures overall antenna performance
  - $SEFD = T_{sys} / K$
  - Depends on $T_{sys}$ and $K = (\eta_s \times A) / (2 \times k_B)$
  - Examples in Table 9.1

What is Sensitivity & Why Should You Care?

- Measure of weakest detectable radio emission
- Important throughout research program
  - Technically sound observing proposal
  - Sensible error analysis in publication
- Expressed in units involving Janskys
  - Unit for interferometer is Jansky (Jy)
  - Unit for synthesis image is Jy beam$^2$
- $1 \text{ Jy} = 10^{-20} \text{ W m}^{-2} \text{ Hz}^{-1} = 10^{-21} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$
- Common to use milliJy or microJy

Outline

- What is Sensitivity & Why Should You Care?
- What Are Measures of Antenna Performance?
- What is the Sensitivity of a Synthesis Image?

Summary

**SEFD**

- Source and System Temperatures
- Measures of Antenna Performance
- Gain
  - Source power $P_s = g^2 \times k_B \times T_s \times \Delta v$
  - Let $T_s = K \times S$, constant $K$
  - Then $P_s = g^2 \times k_B \times K \times S \times \Delta v$
- But source power also $P_s = \int g^2 \times k_B \times A \times S \times \Delta v$
  - Antenna area $A$, efficiency $\eta_a$
  - Receiver accepts 1/2 radiation from unpolarized source
- Equate (1), (2) and solve for $K$
  - $K = (\eta_s \times A) / (2 \times k_B) = T_s / S$
  - $K$ is antenna’s gain or “sensitivity”, unit degree Jy$^{-1}$
  - $K$ measures antenna performance but no $T_{sys}$

Interferometer Sensitivity

Real Correlator - 1

- Simple correlator with single real output that is product of voltages from antennas $j,i$
  - $SEFD_i = T_{int} / K_i$ and $SEFD_j = T_{int} / K_j$
  - Each antenna collects bandwidth $\Delta v$
- Interferometer built from these antennas has
  - Accumulation time $\tau_{acc}$, system efficiency $\eta_i$
  - Source, system noise powers imply sensitivity $\Delta S$:
- Weak source limit
  - $S \ll SEFD_i$
    - $\Delta S = \frac{1}{\eta_i} \times \sqrt{\frac{SEFD_i \times SEFD_j}{2 \times \Delta v \times \tau_{acc}}}$
Interferometer Sensitivity

**Real Correlator - 2**
- For $SEFD_i = SEFD_o = SEFD$ drop subscripts
  $$\Delta S = \frac{1}{\eta_s} \frac{SEFD}{\sqrt{2\times\Delta V \times \tau_{acc}}}$$
- Units $Jy$
- Interferometer system efficiency $\eta_s$
- Accounts for electronics, digital losses
- Eg: VLA continuum
  - Digitize in 3 levels, collect data 96.2% of time
  - Effective $\eta_s = 0.81 \times \sqrt{0.962} = 0.79$

**Complex Correlator**
- Delivers two channels
  - Real $S_r$, sensitivity $\Delta S$
  - Imaginary $S_i$, sensitivity $\Delta S$
- Eg: VLBA continuum
  - Figure 9-1 at 8.4 GHz
  - Observed scatter $S_d(t), S_f(t)$
  - Predicted $\Delta S = 69$ mJy
  - $\Delta S = \frac{1}{\eta_s} \frac{SEFD}{\sqrt{2\times\Delta V \times \tau_{acc}}}$
  - Resembles observed scatter

Interferometer Sensitivity

**Measured Amplitude**
- Measured visibility amplitude $S_m = \sqrt{S_a^2 + S_i^2}$
  - Standard deviation (sd) of $S_a$ or $S_i$ is $\Delta S$
- True visibility amplitude $\Delta S$
- Probability $Pr(S_m / \Delta S)$
  - Figure 9-2
  - Behavior with true $S / \Delta S$
    - High: Gaussian, sd $\Delta S$
    - Zero: Rayleigh, sd $\Delta S / (2^{1/2})$
    - Low: Rice, $S_m$ gives biased estimate of $S$. Use unbias method.

**Measured Phase**
- Measured visibility phase $\theta_m = \arctan (S_i / S_a)$
- True visibility phase $\theta$
- Probability $Pr(\theta - \theta_m)$
  - Figure 9-2
  - Behavior with true $S / \Delta S$
    - High: Gaussian
    - Zero: Uniform
- Seek weak detection in phase, not amplitude

Image Sensitivity

**Single Polarization**
- Simplest weighting case where visibility samples
  - Have same interferometer sensitivities $\Delta S = \frac{1}{\eta_s} \frac{SEFD}{\sqrt{2\times\Delta V \times \tau_{acc}}}$
  - Have same signal-to-noise ratios $\sigma$
  - Combined with natural weight (W=1), no taper (T=1)
- Image sensitivity is sd of mean of $L$ samples, each with sd $\Delta S$, i.e., $\Delta S = \Delta S / \sqrt{L}$
  - No. of interferometers $\frac{1}{2}N(N-1)$
  - No. of accumulation times $t_{acc} / \tau_{sec}$
  - So $\Delta S = \frac{1}{\eta_s} \frac{SEFD}{\sqrt{2\times\Delta V \times \tau_{sec}}} \times \sqrt{N(N-1)\times\tau_{sec}}$

**Dual Polarizations - 1**
- Single-polarization image sensitivity $\Delta I$
- Dual-polarization data $\Rightarrow$ image Stokes $I,Q,U,V$
  - Gaussian noise in each image
  - Mean zero, $\Delta I = \Delta Q = \Delta U = \Delta V = 0$
- Polarized flux density $P = \sqrt{Q^2 + U^2}$
  - Rayleigh noise, sd $\Delta Q = \Delta U = \Delta V = \Delta I / \sqrt{2}$
  - Cf. visibility amplitude, Figure 9-2
- Polarization position angle $\chi = \frac{1}{2} \arctan (Q/U)$
  - Uniform noise between $\pm \pi / 2$
  - Cf. visibility phase, Figure 9-2
**Image Sensitivity**

**Dual Polarizations – 2**

- Eg: VLBA continuum
  - Figure 9-3 at 8.4 GHz
  - Observed
    - T: Stokes I, simplest weighting
    - B: Gaussian noise \( \Delta I = 90 \text{ microJy beam}^{-1} \)
  - Predicted
    \[
    \Delta I = I_{\text{sys}} \sqrt{\frac{1}{E}} = \Delta S \sqrt{\frac{1}{F}}
    \]
    \[
    I_{\text{sys}} = \frac{1}{N} \times (1/N) \times (I_{\text{sys}})
    \]
    - Previous eq \( \Delta S \)
    - Plus here \( I = 77,200 \)
    - So: \( \Delta I = 88 \text{ microJy beam} \)

**Image Sensitivity**

**Dual Polarizations – 3**

- Eg: VLBA continuum
  - Figure 9-3 at 8.4 GHz
  - Observed
    - T: I_{\text{sys}} = 2 \text{ milliJy beam}^{-1}
    - B: Gaussian noise \( \Delta I = 90 \text{ microJy beam}^{-1} \)
  - Position error from sensitivity?
    \[
    \Delta I \ll \frac{1}{2} \times (I_{\text{sys}}) \times \frac{1}{I_{\text{sys}}} \]
    - Gaussian beam \( \theta_{\text{beam}} = 1.5 \text{ milliarcsec} \)
    - Then \( \Delta I = 34 \text{ microarcsec} \)
    - Other position errors dominate

**Image Sensitivity**

**Dual Polarizations – 4**

- Eg: VLA continuum
  - Figure 9-4 at 1.4 GHz
  - Observed
    - Q, U images, simplest weighting
    - Gaussian noise \( \Delta Q = \Delta U = 17 \text{ microJy beam}^{-1} \)
  - Predicted
    \[
    \Delta Q = \Delta U = \frac{1}{E} \times \frac{1}{\sqrt{F}} = \Delta I \sqrt{\frac{1}{F}}
    \]
    \[
    I_{\text{sys}} = \frac{1}{N} \times (1/N) \times (I_{\text{sys}})
    \]
    - So: \( \Delta Q = \Delta U = 16 \text{ microJy beam} \)

**Image Sensitivity**

**Dual Polarizations – 5**

- Eg: VLA continuum
  - Figure 9-4 at 1.4 GHz
  - Observed
    - Q, U images, simplest weighting
    - Gaussian noise \( \Delta Q = \Delta U = 17 \text{ microJy beam}^{-1} \)
    - Form image of \( P = \sqrt{Q^2 + U^2} \)
    - Rayleigh noise in \( P \)
    - \( \Delta P = 31 \text{ microJy beam}^{-1} \)
  - Predicted
    \[
    \Delta Q \times \Delta U = \sqrt{(Q^2 + U^2)} \approx \Delta P \times \Delta P
    \]
    - So: \( \Delta Q = \Delta U = 17 \text{ microJy beam}^{-1} \)

**Image Sensitivity**

**Dual Polarizations – 6**

- Eg: VLA continuum
  - Figure 9-4 at 1.4 GHz
  - Observed
    - Q, U images, simplest weighting
    - Gaussian noise \( \Delta Q = \Delta U = \Delta I = \Delta I_{\text{sys}} \)
  - I, Q, U will have same \( \Delta \) if each is limited by sensitivity
    - Recall \( \Delta I = \Delta Q = \Delta U = \Delta I_{\text{sys}} \times \frac{1}{E} \)
    - Other factors can increase \( \Delta I \)
  - Suspect dynamic range as \( \Delta I_{\text{sys}} = 10,000 \)
    - Lesson: Use sensitivity as tool to diagnose problems

**Sensitivity**

**Summary – 1**

- One antenna
  - System temperature \( T_{\text{sys}} \)
  - Gain \( K \)
  - Overall antenna performance is measured by system equivalent flux density \( SEFD \)
    \[
    SEFD = \frac{T_{\text{sys}}}{K}
    \]
    - Units ly
Sensitivity

**Summary - 2**

- Connect two antennas to form interferometer
  - Antennas have same SEFD, observing bandwidth $\Delta v$
  - Interferometer system efficiency $\eta_i$
  - Interferometer accumulation time $\tau_{acc}$
- Sensitivity of interferometer
  \[
  \Delta S = \frac{1}{\eta_i} \times \frac{\text{SEFD}}{\sqrt{2 \times \Delta v \times \tau_{acc}}}
  \]
  - Units Jy

**Summary - 3**

- Connect $N$ antennas to form array
  - Antennas have same SEFD, observing bandwidth $\Delta v$
  - Array has system efficiency $\eta_r$
  - Array integrates for time $t_{int}$
  - Form synthesis image of single polarization
- Sensitivity of synthesis image
  \[
  \Delta I_{int} = \frac{1}{\eta_r} \times \frac{\text{SEFD}}{\sqrt{N \times (N-1) \times \Delta v \times t_{int}}}
  \]
  - Units Jy beam$^{-1}$