Mm-Wave Interferometry
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Why a special lecture on mm interferometry?
• everything about interferometry is more difficult at high frequencies
• some of the problems are unique to the mm/submm, and affect the way observations are carried out

Why do we care about mm/submm?
• unique science can be done at mm/submm wavelengths, because of the sensitivity to thermal emission from dust and molecular lines
  e.g. @ λ = 1 mm (ν = 300 GHz) hν/k~14 K
  • probe of cool gas and dust in:
    • molecular clouds
    • dust in dense regions
    • star formation in our Galaxy and in the high-redshift universe
    • protoplanetary disks
    • etc...

Science at mm/submm wavelengths: dust emission
In the Rayleigh-Jeans regime, hν ≈ kT,

\[ S_ν = \frac{2kTν^2}{c^2} \text{ W m}^{-2}\text{Hz}^{-1} \]

dust opacity \( \propto ν^2 \)
so for optically-thin emission, flux density

\[ S_ν \propto ν^4, \quad T_B \propto ν^2 \]

⇒ emission is brighter at higher frequencies

Star-forming galaxies in the early universe
(figures from C. Carilli)

Science at mm/submm wavelengths: molecular line emission
• most of the dense ISM is H₂, but H₂ has no permanent dipole moment ⇒ use trace molecules
• lines from heavy molecules → mm
• lighter molecules (e.g. hydrides) → submm

| Table 3. The (νJ→νJ-1) Rotational Transitions of Single-Deep Sources |
|------------------|------------------|------------------|
| Species          | νJ              | νJ-1             | J = 12(12)       |
| CO (1-0)         | 230.53GHz       | 230.53GHz        | 10⁻¹² m²/Hz      |
| CS (2-1)         | 133.56GHz       | 133.56GHz        | 10⁻¹² m²/Hz      |
| HCN (3-2)        | 308.15GHz       | 308.15GHz        | 10⁻¹² m²/Hz      |
| H2S (2-1)        | 337.00GHz       | 337.00GHz        | 10⁻¹² m²/Hz      |
| HCO+ (3-2)       | 360.21GHz       | 360.21GHz        | 10⁻¹² m²/Hz      |
| SO (2-1)         | 295.60GHz       | 295.60GHz        | 10⁻¹² m²/Hz      |

+ many more complex molecules (CH₃CN, CH₃CHO, CH₃COOH, etc.)
• probe kinematics, density, temperature
• abundances, interstellar chemistry, etc...
• for an optically-thin line in turns out that

\[ S_ν \propto ν^4, \quad T_B \propto ν^2 \quad (\text{cf. dust}) \]

Spectrum of molecular emission from Orion at 345 GHz
Problems unique to the mm/submm

- atmospheric opacity: raises $T_{\text{sys}}$, attenuates source
  - opacity vs frequency and altitude; typical values
  - calibration techniques, rapid calibration
- atmospheric phase fluctuations
  - cause of the fluctuations: variable $H_2O$
  - current and planned calibration schemes
- antennas
  - pointing accuracy, surface accuracy
  - baseline determination
- instrument stability

Problems, continued...

- millimeter/submm receivers (will not be discussed further)
  - SIS mixers, cryogenics
- local oscillators
- IF bandwidths
- correlators (will not be discussed further)
  - need high-speed (high bandwidth) for spectral lines: $\Delta V = 300$ km s$^{-1}$
  - 1.4 MHz @ 1.4 GHz, 230 MHz @ 230 GHz
  - broad bandwidth also needed for sensitivity to thermal continuum
- existing and future arrays
  - small field of view, need for mosaicing: FWHM of 10 m antenna @ 230 GHz is ~ 30''
  - limited m-coverage, small number of elements

Atmospheric opacity

- due to the troposphere, $h \approx 7$–10 km
- constituents of the troposphere = dry air ($N_2$, $O_2$, Ar, CO$_2$, Ne, He, Kr, CH$_4$, $H_2$, $N_2O$)
  - $H_2O$ abundance is highly variable but is < 1% in mass, mostly in the form of water vapor
  - particulates

Transmission of the atmosphere from 0 to 1000 GHz for the ALMA site in Chile, and for the VLA site in New Mexico

⇒ atmosphere little problem for $\lambda >$ cm (most VLA bands)

Effect of atmospheric noise on $T_{\text{sys}}$

- consider a simple cascaded amplifier system, with one component:
  
  $S_{\text{in}} = N_1$
  
  output = $G(S_{\text{in}} + N_1)$
  
  output noise relative to $S_{\text{in}}$: $N_{\text{out}} = G(N_{\text{in}}G + N_1)$

- now consider two components:
  
  $S_{\text{in}} = N_1 + N_2$
  
  output = $G_2[G_1(N_{\text{in}}G_1 + N_2)]$

  divide by $G_1G_2$ to find noise relative to $S_{\text{in}}$, then
  
  $N_{\text{out}}^2 = N_1 + N_2$

  and in general, $N_{\text{out}}^2 = N_1 + N_2 + N_3 + \ldots$

Optical depth of the atmosphere at the VLA site
Atmospheric opacity, continued…

Now consider the troposphere as the first element of a cascaded amplifier system:

- $T_{atm} = T_e^{atm} \times (1-e^{-\tau})$, where $T_e^{atm}$ = physical temperature of the atmosphere, ~ 300 K
- “effective” system noise temperature scaled to the top of the atmosphere (i.e., relative to the unattenuated celestial signal) is:
  $T_{sys}^{eff} = e^{\tau} \times (T_{atm}(1-e^{-\tau}))$

*ignoring spillover terms, etc.

Atmospheric opacity, continued…

- example: typical 1.3 mm conditions at OVRO
  - $T_e = 0.2$, elevation = 30° => $T = 0.4$
  - $T_{sys}^{(DSB)} = 1.5 (100 + 50) = 225 K$
  - dominated by the atmosphere
  - if receiver is double side band and sideband gain ratios are unity, then
    $T_{sys}^{(SSB)} = 2 T_{sys}^{(DSB)} = 450 K$
  - very noisy
  - i.e., atmosphere is noisy and is often the dominant contribution to $T_{sys}$; it is a function of airmass and changes rapidly, so need to calibrate often

Calibration of $T_{sys}$

- systems are linear $\Rightarrow P_{sys} = m \times (T_{sys} + T_{inp})$
- if $P_{inp} = 0$ then $T_{sys} = -T_{inp}$

\[
T_{sys} = \frac{(T_e - T_{atm}) P_1 - T_1}{(P - P_1)}
\]

Calibration of $T_{sys}$, continued…

- at cm wavelengths loads $T_1$ and $T_2$ are the 3 K cosmic background radiation and a noise source with known noise temperature switched into the signal path
- at mm wavelengths we need two known loads above the atmosphere!
  (1) 3 K cosmic background radiation
  (2) $T_{atm}$ obtained from a load placed in front of the feed at $T_{load} = T_{atm}$

\[
T_{sys} \rightarrow \text{atmosphere} \rightarrow \text{load at } T_{load} = T_{sys}
\]

Absolute gain calibration

- there are no non-variable quasars in the mm/submm for setting the absolute flux scale; instead, have to use:
  - planets: roughly black bodies of known size and temperature, e.g., Uranus @ 230 GHz has $T_e = 37 Jy$
  - diameter ~ 4°
- problem: if the planet is resolved by the array, have to use single-dish (total power) calibration
- if the planet is resolved by the primary beam, have to know its sidelobe pattern
- $T_e$ is derived from models, can be uncertain by ~ 10%
- stars: black bodies of known size
  - e.g., the Sun at 10 pc: $T_e = 1.3$ K @ 230 GHz, diameter ~ 1 mas
- problem: very faint! not possible for current arrays, but will be useful for ALMA

Atmospheric phase fluctuations

- at mm wavelengths variable atmospheric propagation delays are due to tropospheric water vapor (ionosphere is important for v < 1 GHz)
- the phase change experienced by an electromagnetic wave is related to the refractive index of the air and the distance traveled by
  \[
  \phi_n = 2 \pi n x D
  \]
  or in terms of an “electrical pathlength”, $L_e = \lambda \phi_n = n x D$
- for water vapor
  \[
  n = \frac{\gamma}{D T_{atm}}
  \]
  so
  \[
  L_e = \frac{6.3 x 10^6}{\gamma} \text{cm}
  \]
  and
  \[
  \phi_n = \frac{12.6 x 10^6}{\gamma} \text{rad}
  \]
Atmospheric phase fluctuations, continued…

• variations in the amount of precipitable water vapor therefore cause
  – pointing offsets, both predictable and anomalous
  – delay offsets
  – phase fluctuations, which are worse at shorter wavelengths,
    and result in
    • low coherence (loss of sensitivity)
    • radio “seeing”, typically 1–3″ at λ = 1 mm
  • effect of structure in the water vapor content of the atmosphere on different scales:

Phase fluctuations: loss of coherence

\[
\langle V \rangle = V_0 \times \exp(-\phi_{\text{rms}}^2/2) = V_0 \times \exp(-(Kb^2/\lambda^2)/2)
\]

– measured visibility decreases with \( b \)
– source appears resolved, convolved with “seeing” function

Phase fluctuations: radio “seeing”
Dependence of radio seeing on $\lambda$

Consider observations at two frequencies, but the same resolution:

$$\lambda_1, b_1$$

$$\lambda_2, b_2 = b_1 (\lambda_2 / \lambda_1)$$ for the same resolution

then

$$(\phi_{rms})_1 = b_1 \alpha / \lambda_1 = b_2 \lambda_2 / \lambda_1$$

for example, $\alpha = 0.5$, $\lambda_1 = 1$ mm, $\lambda_2 = 6$ cm:

$$(\phi_{rms})_{1mm} = 8$$

$$(\phi_{rms})_{6cm}$$

⇒ phase fluctuations are severe at mm/submm wavelengths, correction methods are needed

• Self-calibration: OK for bright sources that can be detected in a few seconds
• Fast switching: used at the VLA for high frequencies. Calibrate in the normal way using a calibration cycle time, $t_{cyc}$, short enough to reduce $\phi_{rms}$ to an acceptable level. Effective for $t_{cyc} < b/v_w$.  
• Paired array calibration: divide array into two separate arrays, one for observing the source, and another for observing a nearby calibrator. Note:
  - this method will not remove fluctuations caused by electronic phase noise
  - only works for arrays with large numbers of antennas (e.g., VLA)

• Radiometry: measure fluctuations in $T_B^{atm}$ with a radiometer, use these to derive the fluctuations in $pwv$, and convert this into a phase correction using $\phi_e = 12.6 \pi \times pwv$$ / \lambda$

(from Bremer)

Monitor: 22 GHz H$_2$O line (OVRO, BIMA, VLA)
183 GHz H$_2$O line (CSO-JCMT, SMA, ALMA)
total power (IRAM, BIMA)

Examples of phase correction: 22 GHz Water Line Monitor at OVRO

From Carpenter, Woody, & Scoville 1999

Examples of phase correction: 22 GHz Water Line Monitor at OVRO, continued…

"Before" and "after" images from Woody, Carpenter, & Scoville 2000

Examples of phase correction: 183 GHz Water Vapor Monitor at the CSO-JCMT

Phase fluctuations are reduced from 60° to 26° mm
(from Wiedner et al. 2001)
Antenna requirements

- Observing strategy: depends on the strength of your source
  - Stronger (≥ 0.1 Jy) on the longest baseline for continuum observations, stronger for spectral line: can apply self-calibration, use short integration times, no need for fast switching
  - Weak: external phase calibrator needed, use short integration times and fast switching, especially in A & B configurations
- Sources with a strong maser feature within the IF bandpass: monitor the atmospheric phase fluctuations using the maser, and apply the derived phase corrections to a continuum channel or spectral line channels, use short integration times, calibrate the instrumental phase offsets between the IFs being used every 30 mins or so

Baseline determination: phase errors due to errors in the positions of the telescopes are
\[ \Delta \phi = \frac{2\pi}{\lambda} \times \Delta b \times \Delta \theta \]

Aperture efficiency: Ruze formula gives
\[ \eta = \exp(-\frac{4\Delta\theta^2}{\pi^2 \lambda^2}) \]

\[ \eta = 50\% \] at 350 GHz, need a surface accuracy of \( \Delta \)mm

\( \Delta \phi < \Delta \theta \) need \( \Delta b < \lambda / 2\pi \)

Calibrators at 22 and 43 GHz
- Flux: 3C48/3C138/3C147/3C286. All are extended, but there are a separate, stronger source to track amplitude variations
- Calibrators at 22 GHz: 3C138/3C147/3C286. All are extended, but there are a separate, stronger source to track amplitude variations
- Calibrators at 43 GHz: 3C286 or other strong source

- Pointing errors: 3″ error => ΔGains at pointing center = 5%
- Gain fluctuations at half power point = 22%
  - => need pointing accurate to 1″

Instrument stability
- Everything is more critical at shorter wavelengths.
  - Transmission line for the local oscillator should be stable to \( \lambda \)
  - Needs to be temperature controlled
  - Round-trip path measurements can be ~ 1 turn/day, but quicker at sunrise/sunset
  - Calibrate instrumental phase every 20 to 30 mins

Practical aspects of observing at high frequencies with the VLA

Note: details may be found at http://www.aoc.nrao.edu/vla/html/highfreq/

- Observing strategy: depends on the strength of your source
  - Strong (≥ 0.1 Jy) on the longest baseline for continuum observations, stronger for spectral line: can apply self-calibration, use short integration times, no need for fast switching
  - Weak: external phase calibrator needed, use short integration times and fast switching, especially in A & B configurations
- Sources with a strong maser feature within the IF bandpass: monitor the atmospheric phase fluctuations using the maser, and apply the derived phase corrections to a continuum channel or spectral line channels, use short integration times, calibrate the instrumental phase offsets between the IFs being used every 30 mins or so

- Pointing: for a 10 m antenna operating at 350 GHz the primary beam is ~ 18″
  - 3″ error => ΔGain at pointing center = 5%
  - ΔGain at half power point = 22%
  - => need pointing accurate to 1″

- Aperture efficiency: Ruze formula gives
  \[ \eta = \exp(-\frac{4\Delta\theta^2}{\pi^2 \lambda^2}) \]
  - => for \( \eta = 50\% \) at 350 GHz, need a surface accuracy of \( \Delta \)mm

- Baseline determination: phase errors due to errors in the positions of the telescopes are
  \[ \Delta \phi = \frac{2\pi}{\lambda} \times \Delta b \times \Delta \theta \]

Technical requirements

<table>
<thead>
<tr>
<th>Telescope</th>
<th>Altitude (m)</th>
<th>Diam. (m)</th>
<th>No.</th>
<th>A (m m^2)</th>
<th>V_{max} (GHz)</th>
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</thead>
<tbody>
<tr>
<td>BIMA</td>
<td>1,400</td>
<td>6</td>
<td>10</td>
<td>280</td>
<td>250</td>
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<tr>
<td>OVRO</td>
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<td>10</td>
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<td>250</td>
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<td>3.5/6/10</td>
<td>23</td>
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<tr>
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<td>10</td>
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<td>250</td>
</tr>
<tr>
<td>IRAM PdB</td>
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<td>15</td>
<td>1060</td>
<td>250</td>
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<tr>
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<td>10015</td>
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<td>650</td>
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<tr>
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<td>6</td>
<td>8</td>
<td>230</td>
<td>850</td>
</tr>
<tr>
<td>ALMA</td>
<td>10,000</td>
<td>12</td>
<td>3</td>
<td>9200</td>
<td>850</td>
</tr>
</tbody>
</table>

Note: Existing millimeter instruments are on sites at 1,000 to 2,400 m altitude, with typically a few millimeters of precipitable H₂O

- Primary beam (field of view) = 40″ (IRAM) to 120″ (BIMA) at 115 GHz, resolution 1 to 2″. Note:
  - Very small fields of view
  - Not sensitive to extended emission on scales ≳ Δb/λ
  - Mosaicing necessary for imaging even moderate-sized areas
  - Small number of antennas make it hard to build up good coverage = not many independent pixels in the image plane

Practical aspects, continued...

- Referenced pointing: pointing errors can be a significant fraction of a beam at 43 GHz
  - Point on a nearby source at 8 GHz every 45–60 mins, more often when the airmass is changing rapidly. Pointing sources should be compact with F_{maj} > 0.5 Jy
  - Calibrators at 22 and 43 GHz
    - Phase: the spatial structure of water vapor in the troposphere requires that you find a phase calibrator < 3″ from your source, if at all possible; for phase calibrators weaker than 0.5 Jy you will need a separate, stronger source to track amplitude variations
    - Flux: 3C40/3C336/3C147/3C286. All are extended, but there are good models available for 22 and 43 GHz
Practical aspects, continued...

• Opacity corrections and tipping scans
  - Can measure the total power detected as a function of elevation, which has contributions
    \[ T_{\text{sys}} = T_0 + T_{\text{atm}}(1-e^{-\tau_0 a}) + T_{\text{spill}}(a) \]
  and solve for \( \tau_0 \).
  - Or, make use of the fact that there is a good correlation between the surface weather and \( \tau_0 \) measured at the VLA (Butler 2002):
  and apply this opacity correction using FILM in AIPS

Practical aspects, continued...

• If you have to use fast switching
  - Quantify the effects of atmospheric phase fluctuations (both temporal and spatial) on the resolution and sensitivity of your observations by including measurements of a nearby point source with the same fast-switching settings: cycle time, distance to calibrator, strength of calibrator (weak/strong)
  - If you do not include such a "check source" the temporal (but not spatial) effects can be estimated by imaging your phase calibrator using a long averaging time in the calibration
  • During the data reduction
    - Apply phase-only gain corrections first, to avoid decorrelation of amplitudes by the atmospheric phase fluctuations

The Atmospheric Phase Interferometer at the VLA

Accessible from http://www.aoc.nrao.edu/vla/html/PhaseMonitor/phasemon.html

Summary

• Atmospheric emission can dominate the system temperature
  - Calibration of \( T_{\text{sys}} \) is different from that at cm wavelengths
• Tropospheric water vapor causes significant phase fluctuations
  - Need to calibrate more often than at cm wavelengths
  - Phase correction techniques are under development at all mm/submm observatories around the world
  - Observing strategies should include measurements to quantify the effect of the phase fluctuations
• Instrumentation is harder for mm/submm
  - Observing strategies must include pointing measurements to avoid loss of sensitivity
  - Need to calibrate instrumental effects on timescales of 10s of mins, or more often when the temperature is changing rapidly