Noise and Interferometry
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VLA 5e9 Hz
VLTI 5e14 Hz

Radio vs. Optical Interferometry

Radio: heterodyne + correlator
Optical: mirrors + direct detector (CCD)

The Delay Lines

- **The Paranal-Express**
  - Stroke: 60 m (24 km in OPL)
  - Resolution: < 50 m
  - Stab. velocity: < 0.5 m/s
  - Stability (jitter): < 14 mm rms
  - Power dissipation: < 15 W
  - Stability = 1e8

References

- ‘Noise and interferometry,’ V. Radhakrishnan 1999, SIRA II
- ‘Radio and infrared interferometry,’ E. Purcell 2000, Caltech
- ‘The intensity interferometer,’ Brown and Twiss 1974, (Taylor-Francis)
- ‘Multiple telescope infrared interferometry,’ Townes and Sutton 1981, (ESO conference, eds. Ulich and van de Stadt)
- ‘Coherent vs. incoherent detection for interferometry,’ de Graauw 1981, (ESO conference, eds. Ulich and van de Stadt)
- ‘Measurement of power spectra,’ Blackman and Tuckey (Dover)
- ‘Celestial masers,’ Cook (Cambridge Univ. Press) chap 4
- ‘Fundamentals of statistical physics,’ Reif (McGraw-Hill) chap 9

Photon statistics

- Bose-Einstein statistics for gas without number conservation (indistinguishable particles or wave function symmetric under particle exchange, spin 0, Reif Chap 9)
- Thermal equilibrium => Planck distribution function
  \[ n_s = \frac{1}{e^{s/kT} - 1} \]
  \[ \langle n_s \rangle = n_s \]
  \[ \langle n_s^2 \rangle = \langle n_s \rangle + \langle n_s \rangle \]
- Photon noise: Fluctuations in # of photons arriving each second in free space beam
  \[ \langle \Delta n^2 \rangle = \langle n_s \rangle (e^{s/kT} - 1) \]

Electron statistics

- Fermi-Dirac (indistinguishable particles, but number of particles in each state = 0 or 1, or antisymmetric wave function under particle exchange, spin 1/2)
  \[ \langle n_s \rangle = \left( e^{s/kT} + 1 \right)^{-1} \]
  \[ \langle n_s^2 \rangle = \left( e^{s/kT} - 1 \right)^{-1} \]
  \[ \langle n_s^2 \rangle - \langle n_s \rangle = \left( e^{s/kT} - 1 \right)^{-2} \]
  \[ e.g. \text{maximum} \langle n_s \rangle = 1 \Rightarrow \text{all states are filled} \]
  \[ \text{variance} = 0 \]
Disclaimer on Wave noise

Richards 1994: ‘The first term in equation 11 can be obtained more directly. For Poisson statistics the mean square fluctuation in the number of photons arriving in 1s is just equal to the number of photons arriving. This term has been verified experimentally in many experiments. The second term, by contrast, has not been measured unambiguously.’

Zmuidzinas 2000: ‘Richards has recently discussed the second term of Eq. 4.1, raising questions about the theoretical and experimental justification for this term. However, as discussed in section III, the second term is needed in order to recover the Dickey radiometer equation for single mode detectors in the high background limit; we therefore disagree that there is no empirical justification for this term.’

Origin of wave noise I: Young’s 2 slit experiment

Origin of wave noise II: ‘Bunching of Bosons’ in phase space (time and frequency) allows for interference (i.e. coherence).

Bosons can, and will, occupy the exact same phase space if allowed, such that interference (destructive or constructive) will occur. Restricting phase space (i.e. narrowing the bandwidth and sampling time) leads to interference within the beam. This naturally leads to fluctuations that are proportional to intensity (= wave noise).

Origin of wave noise III

Origin of wave noise III

‘Think then, of a stream of wave packets each about c/Δν long, in a random sequence. There is a certain probability that two such trains accidentally overlap. When this occurs they interfere and one may find four photons, or none, or something in between as a result. It is proper to speak of interference in this situation because the conditions of the experiment are just such as will ensure that these photons are in the same quantum state. To such interference one may ascribe the ‘abnormal’ density fluctuations in any assemblage of bosons.

Were we to carry out a similar experiment with a beam of electrons we should find a suppression of the normal fluctuations instead of an enhancement. The accidental overlapping wave trains are precisely the configurations excluded by the Pauli principle.’ Purcell 1959
Origin of wave noise IV

Photon arrival time: normalized probability of detecting a second photoelectron after time interval \( t \) in a plane wave of linearly polarized light with Gaussian spectral profile of width \( \Delta \nu \) (Mandel 1963). \(^2\) Exac \( \) s the same factor 2 as in Young’s slits!

Relevant timescale = \( 1/\Delta \nu \)

Origin of wave noise V

“If we were to split a beam of electrons by a nonpolarizing mirror, allowing the beams to fall on separate electron multipliers, the outputs of the latter would show a negative cross-correlation. A split beam of classical particles would, of course, show zero cross-correlation. As usual in fluctuation phenomena, the behavior of fermions and bosons deviate in opposite directions from that of classical particles. The Brown-Twiss effect is thus, from a particle point of view, a characteristic quantum effect.”

Purcell 1959

Intensity Interferometry: rectifying signal with square-law detector (‘photon counter’) destroys phase information. Cross correlation of intensities still results in a finite correlation, proportional to the square of the field correlation coefficient as measured by a ‘normal’ interferometer. Exact same phenomenon as increased correlation for \( t < 1/\Delta \nu \) with \( \gamma \) correlation coefficient as measured by counter’ destroys phase information. Cross correlation of intensities still results in a finite

Intensity Interferometry

Signal-to-Noise II

\[ \text{Variance of measurement} = \left( \frac{\Delta \nu}{2} \right)^2 \]

\[ \text{SNR} = \frac{S}{\Delta \nu} \]

\[ \text{Wavenoise} = \frac{1}{\Delta \nu} \]

\[ n_s = A \frac{1}{\Delta \nu} \text{photons/Hz} \]

\[ n_s < 1 \text{ (counting rate)} \]

\[ n_s \approx n \approx \gamma \]

\[ \text{SNR} \approx \frac{S}{\Delta \nu} \]

\[ n_s \approx \gamma \]

\[ \text{Strong signal} \]

Signal to Noise II

\[ A \sim \Delta \nu \]

\[ n \sim 1/\Delta \nu \]

\[ n \sim \gamma \]

\[ \text{SNR} \sim S/\Delta \nu \]

\[ n_s \approx \gamma \]

\[ \text{Strong signal} \]

Photon occupation number I: bright radio source

\[ \text{Cygnus A: } S_1 \approx 1400 \text{ Jy, } z = 0.057 \]

\[ \text{VLA (25 cm): } T_a = 140 \text{ K} \]

\[ n_s = \frac{h^2}{kT_a} = 0.0005 \]

\[ n_s \approx (S/\Delta \nu) \sim 2000 \text{ Hz}^{-1} \sec^{-1} \]

\[ n_s \text{ wave noise dominated} \]

\[ n_s \approx A \frac{1}{\Delta \nu} \text{ photons/Hz} \]

\[ n_s \approx \gamma \]

\[ \text{SNR} \approx S/\Delta \nu \]

\[ n_s \approx \gamma \]

\[ \text{Strong signal} \]
Photon occupation number II: optical source

Betrugse
HST: $T_e = 3000 K \Rightarrow h \kappa T_e = 8 \times 10^{14} Hz$
$\Rightarrow n_e = 0.0001 Hz^{-1} \text{sec}^{-1}$
\(\cdot \) 'counting noise' dominated

C. Carilli, Synthesis Summer School, 24 June 2002

Photon occupation number III: faint source

Betrugeuse
HST: $T_e = 3000 K \Rightarrow h \kappa T_e = 8 \times 10^{14} Hz$
$\Rightarrow n_e = 0.0001 Hz^{-1} \text{sec}^{-1}$
\(\cdot \) 'counting noise' dominated

C. Carilli, Synthesis Summer School, 24 June 2002

Even the feeble microwave background ensures that the occupation number at most radio frequencies is already high. In other words, even though the particular contribution to the signal that we seek is very very weak, it is already in a classical sea of noise and if there are benefits to be derived from retaining the associated aspects, we would be foolish to pass them up. " Radhakrishnan 1998

Quantum noise I: Commutation relations

Quantum noise II: Coherent Amplifiers

Quantum noise III: 2 slit paradox

Quantum noise IV: classical regime

Which slit does the photon enter? With a phase conserving amplifier it seems one could both detect the photon and 'build-up' the interference pattern (which we know can't be correct). But quantum noise dictates that the amplifier introduces 1 photon/mode noise, such that:

$$ I_{tot} = 1 \pm 1 $$

and we still cannot tell which slit the photon came through.
Quantum noise IV: Einstein Coefficients

Einstein Coefficients:

- Stimulated emission: \( B^+ \), Spontaneous Emission: \( A^- \), with \( g_p \) and \( h \)
- Stimulated Absorption: \( B^- \)
- Radiative Transfer:
  \[ \frac{\partial}{\partial s} \left( h \frac{\partial}{\partial h} B^+ \right) + \frac{1}{4} \left( h \frac{\partial}{\partial h} B^- + h \frac{\partial}{\partial h} A^- \right) \]

- Spontaneous:
  \[ h \frac{\partial}{\partial h} B^+ = \frac{1}{2} \frac{\partial}{\partial h} (2kT) \]

- Stimulated: \( B^- \)

Quantum noise IVb: Quasar as a quantum limited amplifier as dictated by the Einstein coefficients (Zmuidzinas 2000)

Quantum noise V: Radio vs. Optical Interferometry

Quantum noise: \( n_e = 1 \) Hz^{-1} sec^{-1}

Optical: \( n_o \ll 1 \) is QN disastrous.

- Better touse mirrors and director detectors (CCDs)
- Advantages: Rx noise is = 0, large bandwidths
- Disadvantages: adding elements means splitting signal \( \rightarrow \) lower SNR complex, precise optics, delays

Radio: \( n_r \gg 1 \) is QN irrelevant.

- Might as well use phase conserving electronics
- Advantages: adding antennas = full pol doesn’ affect SNR easy to electronic
- Disadvantages: high Rx noise

Quantum limit VI: Heterodyne vs. direct detection interferometry

\[ \frac{\text{SNR}_{\text{HET}}}{\text{SNR}_{\text{DD}}} = \left( \frac{n_e}{n_r} \right)^{1/2} \]

Betelgeuse with HST at 5e14 Hz:

\[ \frac{\text{SNR}_{\text{HET}}}{\text{SNR}_{\text{DD}}} = \left[ (5000) \right]^{1/2} \Rightarrow \text{DD wins} \]

Cygnus A with VLA at 1.4 GHz:

\[ \frac{\text{SNR}_{\text{HET}}}{\text{SNR}_{\text{DD}}} = \left[ (2000)^N \right]^{1/2} \Rightarrow \text{Heterodyne wins} \]

Quantum noise Vb: Radio vs. Optical Interferometry

(Sources: A = Radio, B = Optical)

Quantum limit Vb: On the border

Orions 145GHz CO(3 – 2):

\[ T_e = 10K \Rightarrow h \beta/kT = 1.7 \Rightarrow n_e = 1/4 \]

\[ \frac{\text{SNR}_{\text{HET}}}{\text{SNR}_{\text{DD}}} = \left[ (N/4)^N \right]^{1/2} \Rightarrow \text{toss-up} \]