Spherical Harmonic Interferometric Imaging: Implementation

Practical Concerns

1. **What is the maximum value of l needed? Is it too larger?**
2. **How do we handle computing Legendre Polynomials over wide range of l's?**
3. **How can we make a dirty image or compute the dirty beam for CLEAN?**

The Maximum Value of l

Recall that we are approximating the sky with spherical harmonics:

$$I(\theta,\phi) = \sum_{l=0}^{l_{max}} \sum_{m=-l}^{l} i_{l,m} Y_l^m(\theta,\phi) \tag{1}$$

also recall that $Y_l^m(\theta,\phi) = e^{-im\phi} X_l^m(\theta)$ **where** $X_l^m(\theta) \propto P_l^m(\mathbf{cos}\theta)$

$\rightarrow$ **need** l_{max} **(and** m_{max}**)** $\sim$**206265/**α **where** α **is angular resolution in arcseconds**

$\rightarrow$ **for descent angular resolution, need** l_{max} $\sim$**100,000!**

Fortunately, for $l >> \theta$ **and** $l >> m$**,** $X_l^m(\theta)$ **is very well approximated by**

$$X_l^m(\theta) = \frac{1}{\pi\sqrt{\mathbf{sin}\theta}} \mathbf{cos}\left[\left(l + \frac{1}{2}\right)\theta - \frac{\pi}{4} + \frac{m\pi}{2} + \left(m^2 - \frac{1}{4}\right)\frac{\mathbf{cot}\theta}{2l+1}\right] \tag{2}$$

For larger values of m**, have recursion relations that can be used for fixed** l

How can we make a dirty image with SH?

1. Formally solve for $i_{l,m}$:

$$i_{l,m} = \frac{\int V(u,v,w) Y_l^m(\theta_b,\phi_b) du dv dw}{4\pi \int j_l(2\pi b) b^2 db} \tag{3}$$

Drawbacks: involves a volume integral $\rightarrow$ can't use fast SHT; can only analytically do integral of $j_l(2\pi b)b^2$ for $b >> l$

2. Don't bother inverting equation and use optimization to find set of $i_{l,m}$ values that best matches observed visibilities

Drawbacks: number of $i_{l,m}$ values given by $\sum_{l=0}^{l_{max}} (2l+1) = (l_{max}+1)^2$

$\rightarrow$ for $l_{max} = 10^5$, number of $i_{l,m}$ values is about 10^{10}

$\rightarrow$ unless some simplifying assumptions can be made, number of free parameters is too large

Initial Try at Making a Dirty Image

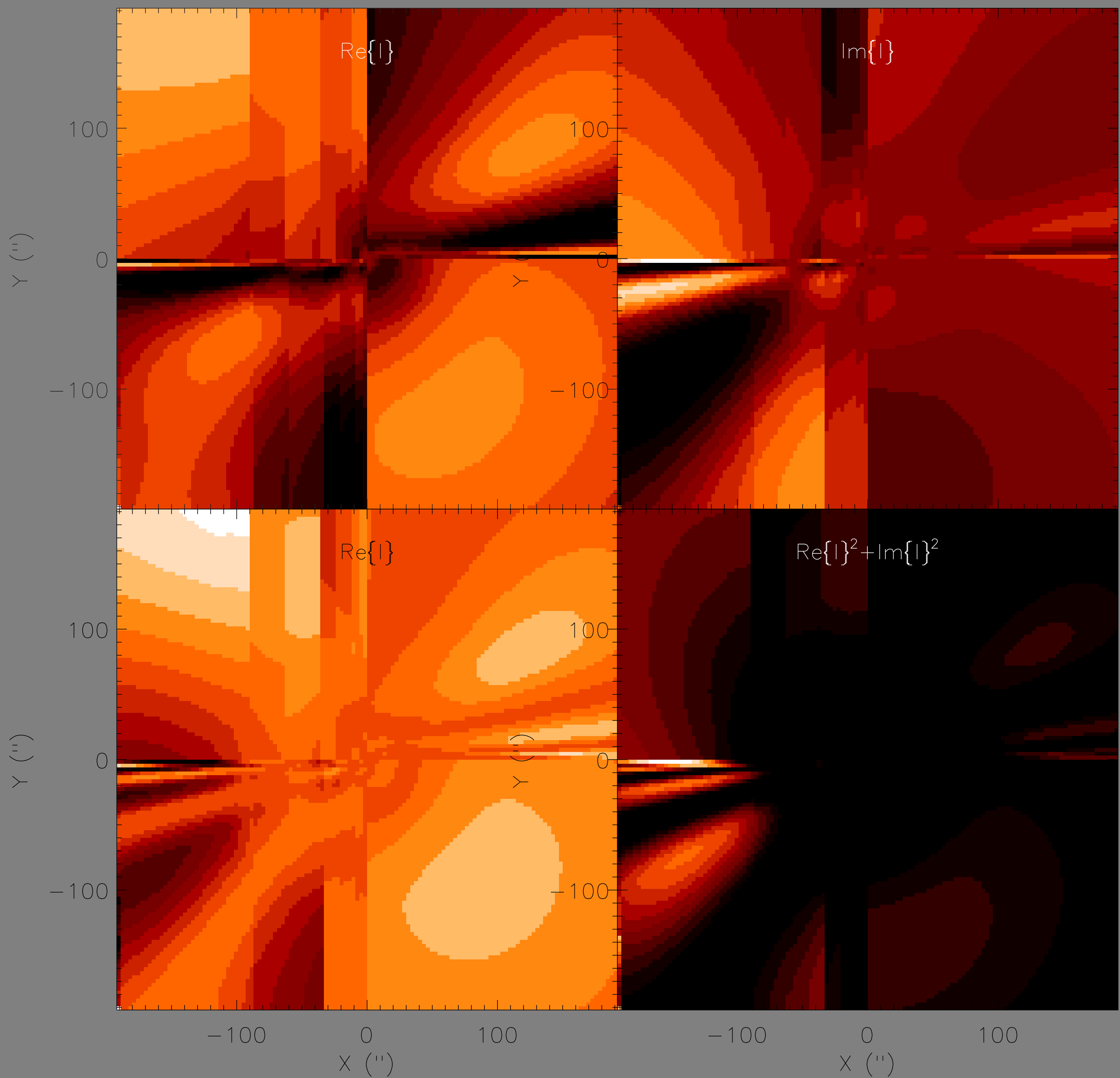
"brute force" method: bin the visibilities in cubic volume elements with sides of length $\delta \rightarrow i_{l,m} = \delta^3 A(l) \sum_{i=1}^{N_B} < V(u_i, v_i, w_i) > Y_l^m(\theta_{b,i}, \phi_{b,i})$

where N_B **is the number of non-empty volume bins,** $< V(u_i, v_i, w_i) >$ **is the average visibility within each volume bin, and** $A(l) = \left[4\pi \int_0^{b_{max}} J_l(2\pi b) b^2 db \right]^{-1}$

Have computed $< V(u_i, v_i, w_i) >$ **for data from S. Bhatnagar for 1**$^\circ$ **FOV centered on IC 2233 for this purpose; attempted at dirty image with** l_{max}**=30** $\rightarrow$ **illustrates need for large** l **and reliable methods for computing** $X_l^m(\theta, \phi)$ **and** $j_l(2\pi b)$

To do list:

1. **implement and test the quality and speed of methods for computing** Y_l^m **for large values of** l **and** $j_l(2\pi b)$ **for large** l **and** b
2. **adapt fast SHT software already available (s2kit) to do the inverse transform to go from** $i_{l,m}$ **to** $I(\theta, \phi) \rightarrow$ **current version of software will only do this over the entire sphere**
3. **explore approximations that might be used to speed up the determination of** $i_{l,m}$ **values**



Re{I}
Im{I}
Re{I}
Re{I}²+Im{I}²
Y (")
X (")
100
0
−100