Spherical harmonics and wide field imaging

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Introduction

$$V(u,v,w) = \int\int \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{2\pi i(ul+vm+w(\sqrt{1-l^2-m^2}-1)} dldm$$

If $w(\sqrt{1-l^2-m^2}-1) \approx 0$ then it is a 2-D Fourier transform

The term is $\approx \frac{B\lambda}{D^2}$

Worse if low-freq, long baselines, small ant., wide field

If $w(\sqrt{1-l^2-m^2}-1) \gtrsim 1$ how is the problem solved?

- Faceting
- W-projection
- Spherical harmonic transforms?

W-projection and faceting

Faceting: Image plane divided into N x N facets.

Each facet is imaged separately.

W- projection (Cornwell, Golap, Bhatnagar):

$$V(u,v,w) = \int\int \frac{I(l,m)}{\sqrt{1-l^2-m^2}} G(l,m,w) e^{2\pi i(ul+vm)} dldm$$

with $G(l,m,w) = e^{-2\pi i(w\sqrt{1-l^2-m^2}-1)}$

so: $V(u,v,w) = \tilde{G}(u,v,w) * V(u,v,w=0)$

with $\tilde{G}(u,v,w) = \frac{i}{w} e^{-\pi i \frac{u^2+v^2}{w}}$

W-projection vs. faceting

Cornwell, Golap, Bhatnagar,

EVLA Memo 67:

- w-proj about 10x faster
- better for high dynamic range

faceting

w-projection

W PROJECTION: A NEW ALGORITHM FOR NON-COPLANAR BASELINES 15

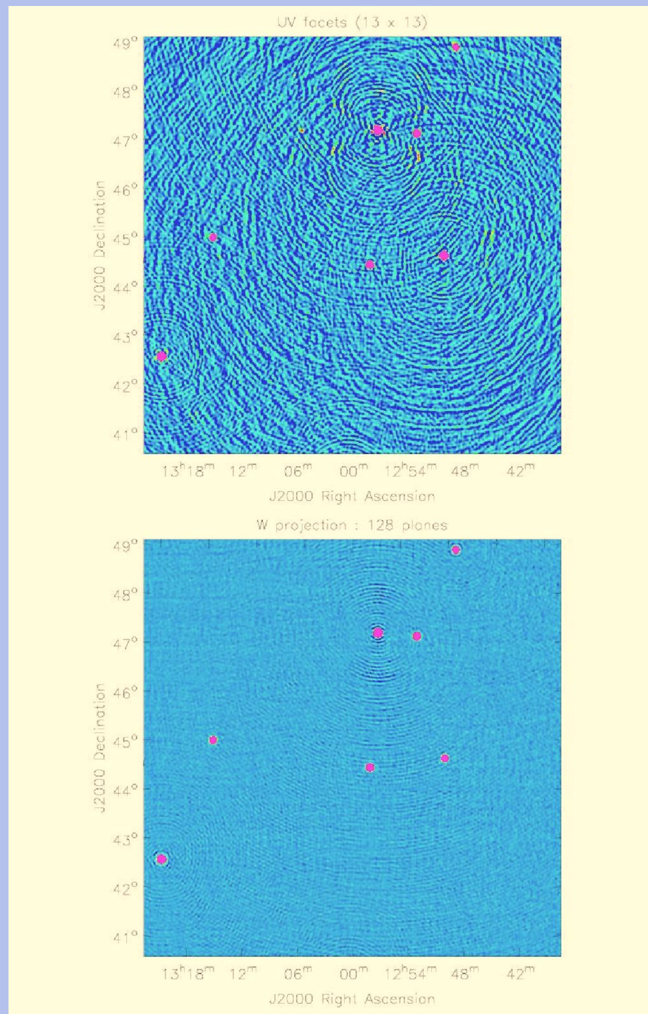

FIGURE 4. Clean images for 74MHz simulation. Top: uv facets (13 x 13), Bottom: w projection (128). The brightness range is -5 to +50 mJy/beam, and the peak brightness should be 16.2Jy. The peak sidelobes seen around the brightest sources in the uv facets image are about 0.3%. Calculation of these images took 2h58m and 28m15s respectively.

Imaging and sph. harm. transforms

$$V(u,v,w) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{2\pi i(ul+vm+w(\sqrt{1-l^2-m^2}-1))} dldm$$

Write this in polar coord. (ignoring $e^{-2\pi i w}$):

$$V(u,v,w) = \iint I(\theta,\ \ ) e^{2\pi i(b\cdot\hat{n})} d^2\Omega$$

$$I(\theta,\ \ ) = \sum_{l,m} i_{lm} Y_{lm}(\theta,\ \ )$$

$$e^{2\pi i b\cdot\hat{n}(\Omega)} = e^{2\pi i|b|\cos\gamma} = \sum_{l=0}^{\infty} i^l j_l(2\pi\,|b|) P_l(\cos\gamma) =$$

$$= \sum_{l=0}^{\infty} i^l j_l(2\pi\,|b|) \frac{4\pi}{2l+1} \sum_{m=-l}^{l} Y_{lm}(\hat{b}) Y_{lm}^{*}(\hat{n})$$

Imaging and sph. harm. transforms

$$V(b) = \sum_{l,m} \frac{i^l i_{lm}}{2l+1} j_l(2\pi b) Y_{lm}(\hat{b})$$

This is the spherical harmonic transform of $V(b)$!

$$i_{lm} = \frac{(-i)^l (2l+1)}{4\pi j_l(2\pi b)} \int V(|b| \cdot b) Y^*_{lm}(\hat{b}) d^2\Omega$$

These i_{lm} are what we want since: $I(\theta, \ ) = \sum_{l,m} i_{lm} Y_{lm}(\theta, \ )$

Potential problems:

- we don't measure all angles
- no 1-1 correspondence with Fourier terms
- for resol~20", $l_{\max}$~5e4, there are 2e9 i_{lm} coefficients...

Practical application

- Include this formalism into existing imaging software
- First step: make use of available spherical harmonic transform package (s2kit, Dartmouth) to test feasibility/speed.
- Compare speed to w-projection (and faceting).
- Thanks to S. Bhatnagar for model and real observations of simple sources.
- Work in progress...