Ultra-Wide Field Imaging 1

a work in progress…

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Ultra-wide field imaging

- low-freq arrays image nearly the whole half-sky
  - LWA, MWA, LOFAR, SKA…
  - plus terrestrial interference on horizon!
- VLA 74 MHz already 10° FWHM!
- tiling the sky is tricky – use single spherical coord. system
  - but need a way to grid (e.g. HEALPIX)
- replace tangent plane coordinates with spherical (angular) coordinates
  - replace uv-plane with spherical harmonic multipoles *l,m* ?
  - used in CMB (as well as atomic physics)
  - need to relate to interferometer system

The essence of W projection

- Evaluate this integral (and transpose) for regular grid in *(l,m)* and irregularly spaced samples in *(u,v)*

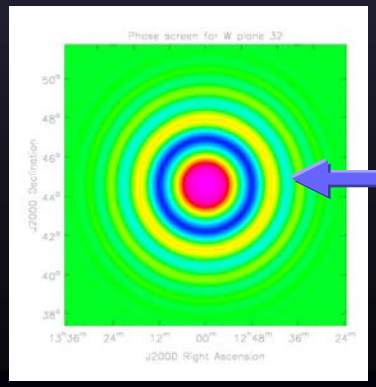
$$V(u,v,w) = \iint \frac{d\xi\, d\eta}{\sqrt{1-\xi^2-\eta^2}} I(\xi,\eta)\, e^{i2\pi[u\xi+v\eta+w(\sqrt{1-\xi^2-\eta^2}-1)]}$$

- Image space computation = multiplicative function

$$V(u,v,w) = \iint \frac{d\xi\, d\eta}{\sqrt{1-\xi^2-\eta^2}} G(\xi,\eta,w) I(\xi,\eta)\, e^{-i2\pi(u\xi+v\eta)}$$

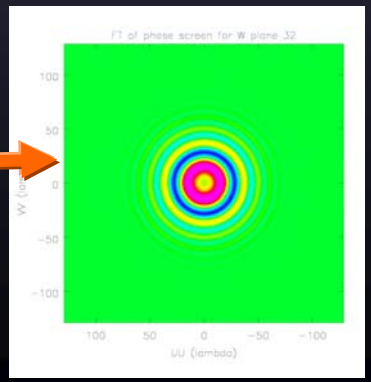
- Fourier space computation = convolution kernel

$$V(u,v,w) = G(u,v,w) \otimes V(u,v,w=0)$$



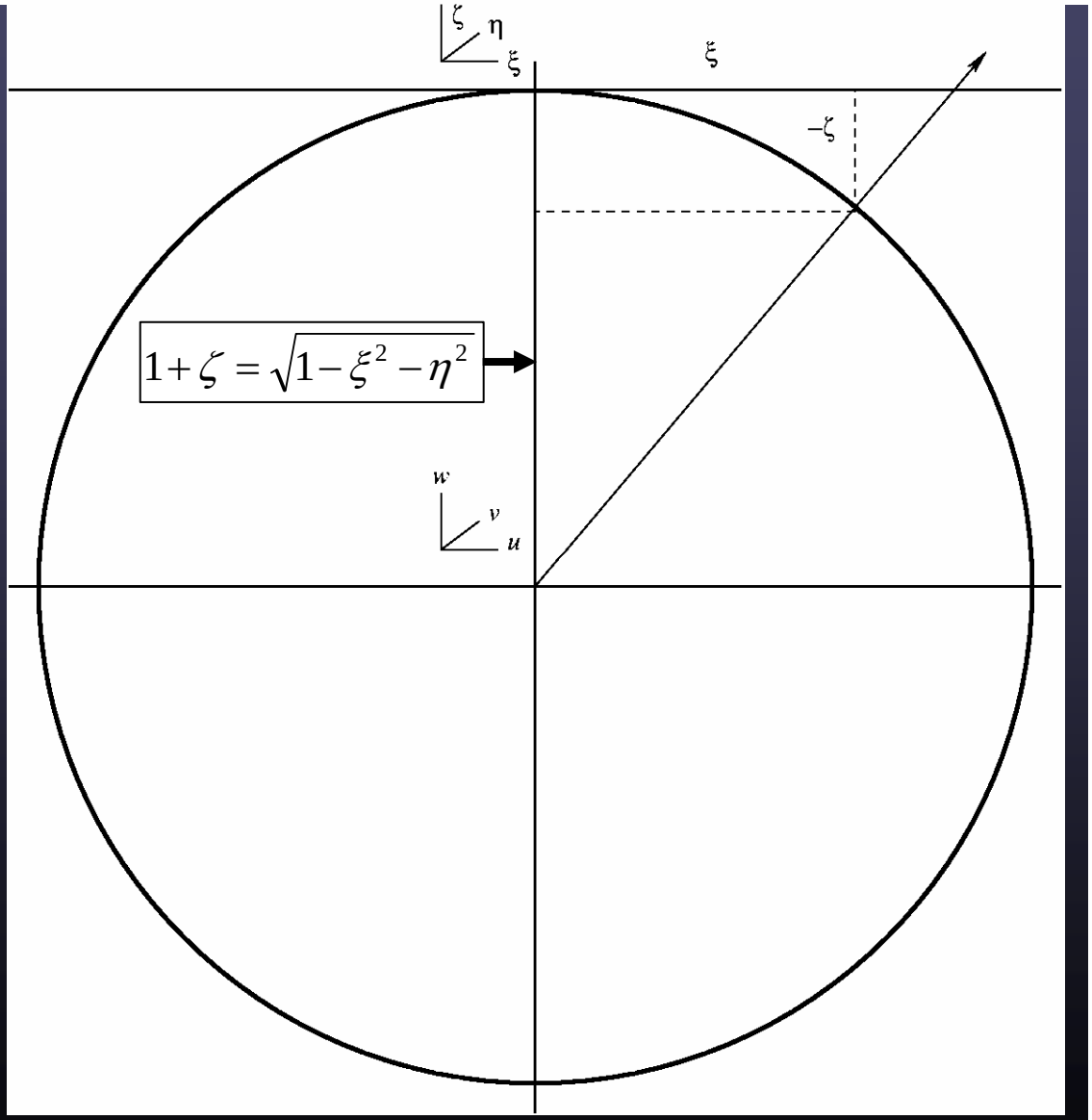
$$e^{j2\pi w\left(\sqrt{1-l^2-m^2}-1\right)}$$

$$\approx e^{-j\pi w(u^2+v^2)}$$



Standard sky geometry

- sky:
  - unit sphere
  - tangent plane
  - direction cosines
  - $\xi = (\xi, \eta, \zeta)$
- interferometer:
  - $\mathbf{u} = \mathbf{B} / \lambda$
  - $\mathbf{u} = (u, v, w)$
- project plane-wave onto baseline vector
  - phase $2\pi\ \xi \cdot \mathbf{u}$



Wavefront correlations

- Rewrite the standard form of the relation as

$$V(u,v,w) = \iint \frac{d\xi\, d\eta}{1+\zeta} I(\xi,\eta)\, e^{i2\pi \boldsymbol{\xi}\cdot\mathbf{u}}$$

$$\boldsymbol{\xi} = (\xi,\eta,\zeta) \qquad \mathbf{u} = (u,v,w)$$

$$1+\zeta = \sqrt{1-\xi^2-\eta^2}$$

- Sky is 2-dimensional, but baseline vector 3-d

Whole-sky imaging & transforms

- Celestial spherical coordinates (θ,ϕ)
  - choose pole : celestial, terrestrial, pointing direction
  - coordinates: RA-Dec, Az-ZA, other
  - Intensity field:

$$I(\hat{\mathbf{n}}) \qquad \hat{\mathbf{n}} = (\theta, \phi)$$

- Spherical Harmonic Transforms

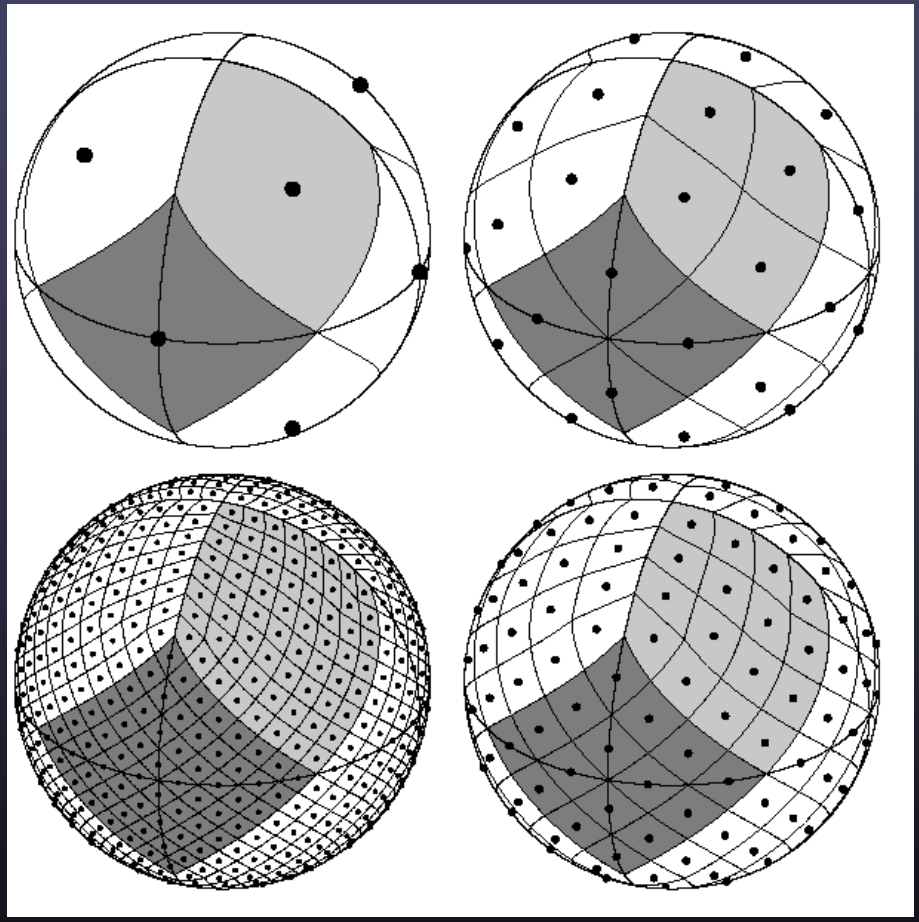
$$a_{\ell m} = \int d^2\hat{\mathbf{n}}\, Y_{\ell m}(\hat{\mathbf{n}})\, I(\hat{\mathbf{n}})$$

$$Y_{\ell m}(\theta,\phi) = P_\ell(\cos\theta) e^{-im\phi}$$

  - at high l these become Fourier transforms
  - complete orthogonal harmonic mode basis on sphere
  - there are fast versions and convolvers

Spherical maps

- Need optimized map geometry and fast convolvers:

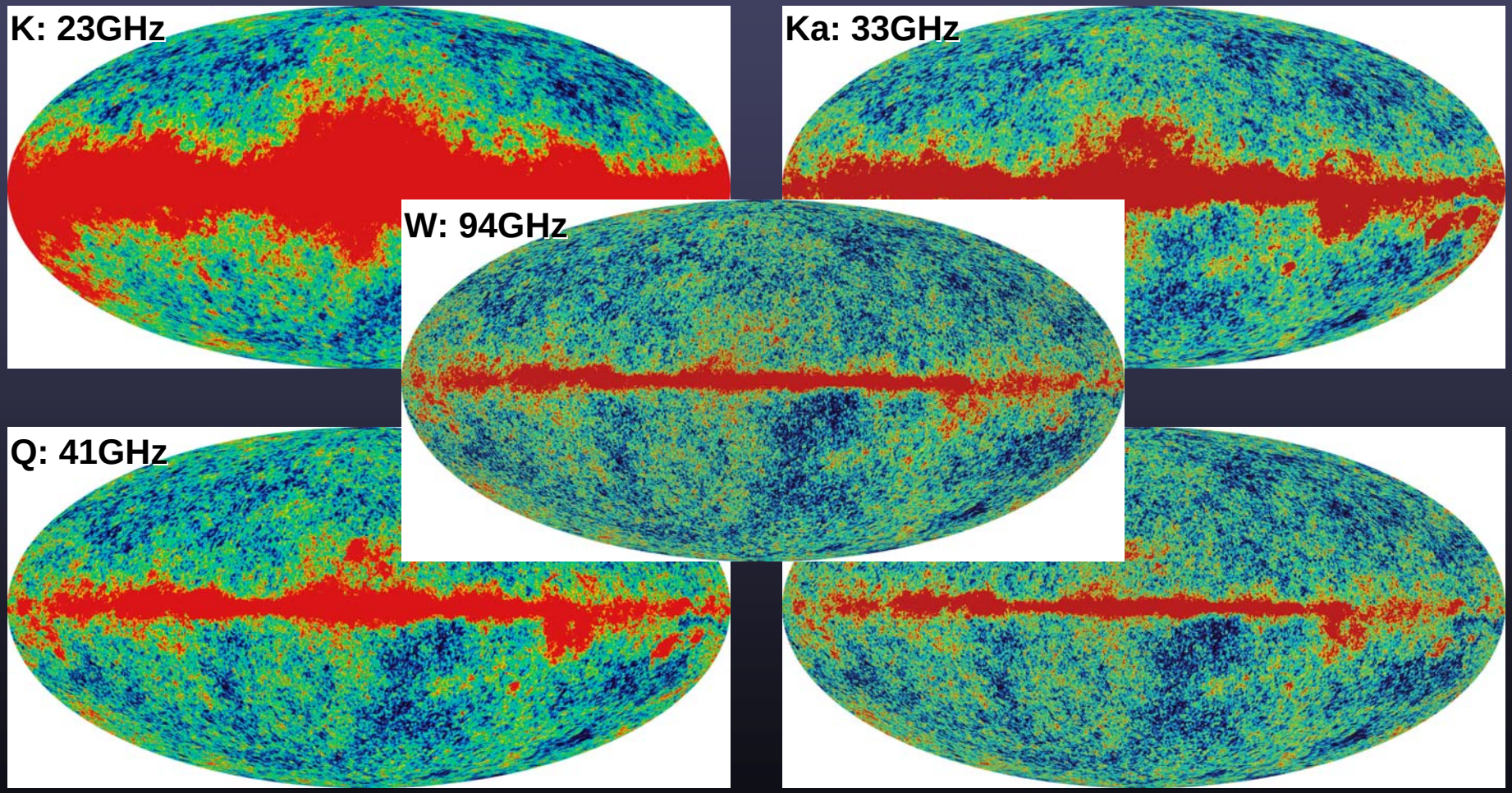


http://www.eso.org/science/healpix

– see Wandelt & Gorski (astro-ph/0008227) for convolution

WMAP: case study

- HEALpix maps:



Issues

- Is there a simple expression for V(*u*,*v*,*w*) in terms of spherical harmonics (*l*,*m*)?
- Will the FSHT be fast enough?
- Are fast spherical convolvers fast enough?
- Tiling the sky: is HEALPIX the right way?
- Are there any practical advantages to doing it this way?

to be continued …