Interferometric Polarization Imaging & Calibration

Steven T. Myers

National Radio Astronomy Observatory

Socorro, NM

Outline

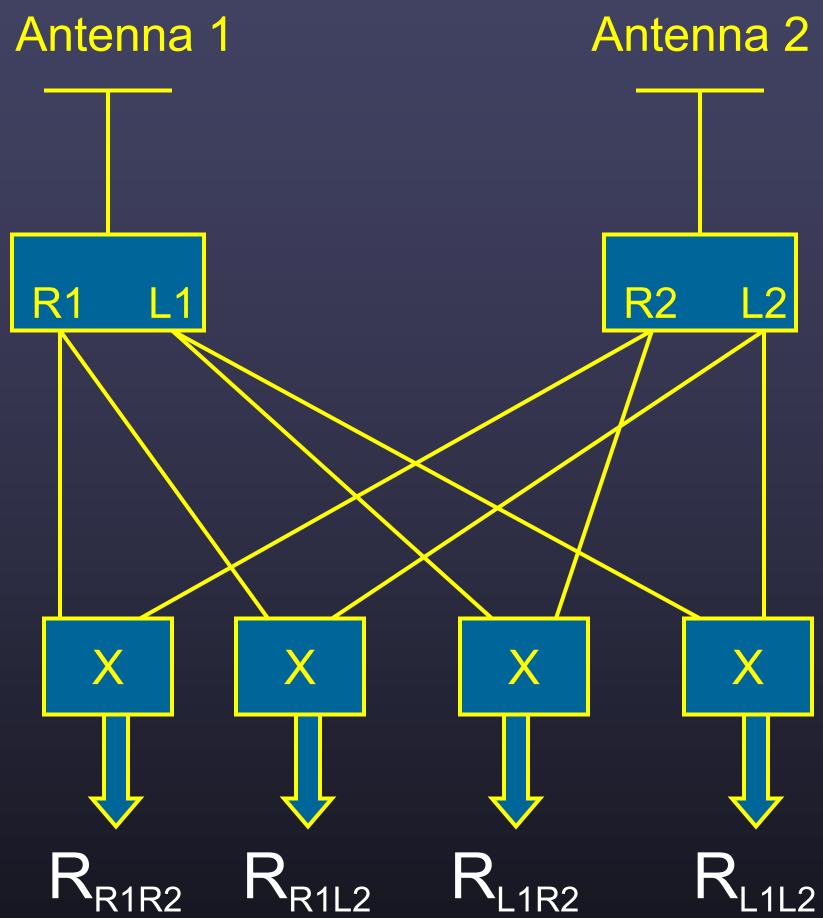
- I WILL
  - describe general polarization effects in interferometry
  - indulge in some imaging pedagogy
  - give an example of polarization imaging
  - speculate on what to do about polarization issues
- I will NOT
  - give beginner introduction to polarization or imaging
  - tell you how to solve for calibration
  - go into details of the measurement equation etc.
  - promise to use consistent notation or keep sign convention
- for reference material see:
  - my Synthesis Imaging 2008 lecture (and others)
  - various papers by Hamaker et al.
  - Myers et al. 2003 ApJ, 591, 575-598 (2003); astro-ph/0205385
  - Sievers et al. 2007 ApJ, 660, 976-987 (2007); astro-ph/0509203

Intro to Polarization Interferometry

- Cribbed from my Synthesis Imaging 2008 lecture
- I will go fast to get to the more interesting stuff…
- You can refer back to this later at your leisure

Complex Polarization Correlations

- Two antennas, each with two differently polarized outputs, possibly produce four complex correlations
- From these four outputs, we want to possibly make four Stokes Images (IQUV)
- Antennas can observe in RL or XY basis



Correlation products: coherency vector

- Coherency vector: outer product of 2 antenna vectors as averaged by correlator

$$\vec{v}_{ij} = \left\langle \vec{E}_i \otimes \vec{E}_j^{\,*} \right\rangle = \left\langle \begin{pmatrix} E^p \\ E^q \end{pmatrix}_i \otimes \begin{pmatrix} E^p \\ E^q \end{pmatrix}_j^* \right\rangle = \begin{pmatrix} \left\langle E_i^p \cdot E_j^{*p} \right\rangle \\ \left\langle E_i^p \cdot E_j^{*q} \right\rangle \\ \left\langle E_i^q \cdot E_j^{*p} \right\rangle \\ \left\langle E_i^q \cdot E_j^{*q} \right\rangle \end{pmatrix} = \begin{pmatrix} v^{pp} \\ v^{pq} \\ v^{qp} \\ v^{qq} \end{pmatrix}_{ij}$$

  - these are essentially the uncalibrated *visibilities* **v**
    - circular products RR, RL, LR, LL
    - linear products XX, XY, YX, YY
  - need to include corruptions before and after correlation

Coherency vector and Stokes vector

- Maps (perfect) visibilities to the Stokes vector **s**
- Example: circular polarization (e.g. VLA)

$$\vec{v}_{circ} = \mathbf{S}_{circ}\vec{s} = \begin{pmatrix} v^{RR} \\ v^{RL} \\ v^{LR} \\ v^{LL} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & i & 0 \\ 0 & 1 & -i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+V \\ Q+iU \\ Q-iU \\ I-V \end{pmatrix}$$

- Example: linear polarization (e.g. ALMA, ATCA)

$$\vec{v}_{lin} = \mathbf{S}_{lin}\vec{s} = \begin{pmatrix} v^{XX} \\ v^{XY} \\ v^{YX} \\ v^{YY} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \\ 1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+Q \\ U+iV \\ U-iV \\ I-Q \end{pmatrix}$$

Corruptions: Jones Matrices

- Antenna-based corruptions
  - pre-correlation polarization-dependent effects act as a matrix muliplication. This is the Jones matrix:

$$\vec{E}^{out} = \mathbf{J}\,\vec{E}^{in} \qquad \mathbf{J} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \qquad \vec{E} = \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}$$

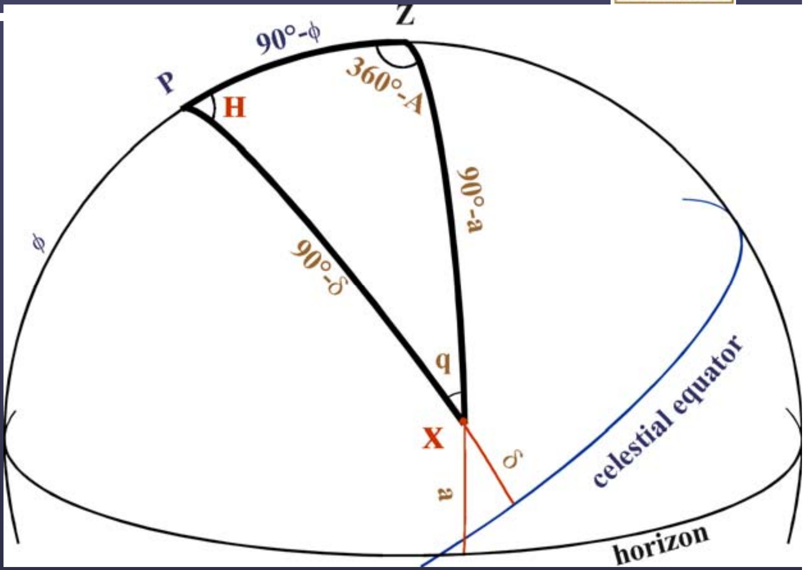
  - form of **J** depends on basis (RL or XY) and effect
    - off-diagonal terms J_{12} and J_{21} cause corruption (mixing)
  - total **J** is a string of Jones matrices for each effect

$$\mathbf{J} = \mathbf{J}_F\,\mathbf{J}_E\,\mathbf{J}_D\,\mathbf{J}_P$$

    - Faraday, polarized beam, leakage, parallactic angle

Parallactic Angle, *P*

- Orientation of sky in telescope's field of view
  - Constant for equatorial telescopes
  - Varies for alt-az telescopes
  - Rotates the position angle of linearly polarized radiation (R-L phase)



$$\mathbf{J}_P^{RL} = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}; \ \mathbf{J}_P^{XY} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

  - defined per antenna (often same over array)

$$\phi(t) = \arctan\left(\frac{\cos(l)\sin(h(t))}{\sin(l)\cos(\delta) - \cos(l)\sin(\delta)\cos(h(t))} \right)$$

$$l = \text{latitude},\ h(t) = \text{hour angle},\ \delta = \text{declination}$$

  - P modulation can be used to aid in calibration

Visibilities to Stokes on-sky: RL basis

- the (outer) products of the parallactic angle (P) and the Stokes matrices gives

$$\vec{v} = \mathbf{J}_P \mathbf{S} \vec{s}$$

- this matrix maps a sky Stokes vector to the coherence vector representing the four perfect (circular) polarization products:

$$\begin{pmatrix} v^{RR} \\ v^{RL} \\ v^{LR} \\ v^{LL} \end{pmatrix} = \begin{pmatrix} e^{-i(\phi_i-\phi_j)} & 0 & 0 & e^{-i(\phi_i-\phi_j)} \\ 0 & e^{-i(\phi_i+\phi_j)} & ie^{-i(\phi_i+\phi_j)} & 0 \\ 0 & e^{i(\phi_i+\phi_j)} & -ie^{i(\phi_i+\phi_j)} & 0 \\ e^{i(\phi_i-\phi_j)} & 0 & 0 & -e^{i(\phi_i-\phi_j)} \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} \xrightarrow[\phi_i=\phi_j=\phi]{} \begin{pmatrix} I+V \\ (Q+iU)e^{-i2\phi} \\ (Q-iU)e^{i2\phi} \\ I-V \end{pmatrix}$$

Circular Feeds: linear polarization in cross hands, circular in parallel-hands

Basic Interferometry equations

- An interferometer naturally measures the transform of the sky intensity in uv-space convolved with aperture
  - cross-correlation of aperture voltage patterns in uv-plane
  - its transform on sky is the primary beam **A** with FWHM ~ λ/D

$$V(\mathbf{u}) = \int d^2\mathbf{x}\, A(\mathbf{x}-\mathbf{x}_p)\, I(\mathbf{x})\, e^{-2\pi i\, \mathbf{u}\cdot(\mathbf{x}-\mathbf{x}_p)} + \mathrm{n}$$

$$= \int d^2\mathbf{v}\, \tilde{A}(\mathbf{u}-\mathbf{v})\, \tilde{I}(\mathbf{v})\, e^{2\pi i\, \mathbf{v}\cdot\mathbf{x}_p} + \mathrm{n}$$

  - The "tilde" quantities are Fourier transforms, with convention:

$$\tilde{T}(\mathbf{u}) = \int d^2\mathbf{x}\, e^{-i2\pi\mathbf{u}\cdot\mathbf{x}}\, T(\mathbf{x}) \qquad \mathbf{x} = (x, y) \leftrightarrow \mathbf{u} = (u, v)$$

$$T(\mathbf{x}) = \int d^2\mathbf{u}\, e^{i2\pi\mathbf{u}\cdot\mathbf{x}}\, \tilde{T}(\mathbf{u})$$

Polarization Interferometry : Q & U

- Parallel-hand & Cross-hand correlations (circular basis)
  - visibility k (antenna pair ij , time, pointing x, channel ν, noise n):

$$V_k^{RR}(\mathbf{u}_k) = \int d^2\mathbf{v}\, \tilde{A}_k^{RR}(\mathbf{u}_k - \mathbf{v})[\tilde{I}_\nu(\mathbf{v}) + \tilde{V}_\nu(\mathbf{v})] e^{2\pi i \mathbf{v}\cdot\mathbf{x}_k} + \mathrm{n}_k^{RR}$$

$$V_k^{RL}(\mathbf{u}_k) = \int d^2\mathbf{v}\, \tilde{A}_k^{RL}(\mathbf{u}_k - \mathbf{v})[\tilde{Q}_\nu(\mathbf{v}) + i\tilde{U}_\nu(\mathbf{v})] e^{-i2\phi_k} e^{2\pi i \mathbf{v}\cdot\mathbf{x}_k} + \mathrm{n}_k^{RL}$$

$$V_k^{LR}(\mathbf{u}_k) = \int d^2\mathbf{v}\, \tilde{A}_k^{LR}(\mathbf{u}_k - \mathbf{v})[\tilde{Q}_\nu(\mathbf{v}) - i\tilde{U}_\nu(\mathbf{v})] e^{i2\phi_k} e^{2\pi i \mathbf{v}\cdot\mathbf{x}_k} + \mathrm{n}_k^{LR}$$

$$V_k^{LL}(\mathbf{u}_k) = \int d^2\mathbf{v}\, \tilde{A}_k^{LL}(\mathbf{u}_k - \mathbf{v})[\tilde{I}_\nu(\mathbf{v}) - \tilde{V}_\nu(\mathbf{v})] e^{2\pi i \mathbf{v}\cdot\mathbf{x}_k} + \mathrm{n}_k^{LL}$$

  - where kernel A is the aperture cross-correlation function, ϕ is the parallactic angle, and **Q+iU=P** is the complex linear polarization

$$\tilde{P}(\mathbf{v}) = \tilde{Q}(\mathbf{v}) + i\tilde{U}(\mathbf{v}) = \left|\tilde{P}(\mathbf{v})\right| e^{i2\varphi(\mathbf{v})}$$

    - the phase of **P** is φ (the R-L phase difference)

Example: RL basis imaging

- Parenthetical Note:
  - can make a pseudo-I image by gridding RR+LL on the Fourier half-plane and inverting to a real image
  - can make a pseudo-V image by gridding RR-LL on the Fourier half-plane and inverting to real image
  - can make a pseudo-(Q+iU) image by gridding RL to the full Fourier plane (with LR as the conjugate) and inverting to a complex image
  - does not require having full polarization RR,RL,LR,LL for every visibility
- More on talks imaging (& deconvolution) later!

Polarization Leakage, *D*

- Polarizer is not ideal, so orthogonal polarizations not perfectly isolated
  - Well-designed systems have *d* < 1-5% (but some systems >10% ☹)
  - A geometric property of the antenna, feed & polarizer design
    - frequency dependent (e.g. quarter-wave at center ν)
    - direction dependent (in beam) due to antenna
  - For *R,L* systems
    - parallel hands affected as d•Q + d•U , so only important at high dynamic range (because *Q,U~d*, typically)
    - cross-hands affected as d•I so almost always important

$$\mathbf{J}_D^{pq} = \begin{pmatrix} 1 & d^p \\ d^q & 1 \end{pmatrix}$$

Leakage of q into p (e.g. L into R)

Leakage revisited…

- Primary on-axis effect is "leakage" of one polarization into the measurement of the other (e.g. R ⇔ L)
  - but, direction dependence due to polarization beam!
- Customary to factor out on-axis leakage into D and put direction dependence in "beam"
  - example: expand RL basis with on-axis leakage

$$\hat{V}_{ij}^{RR} = V_{ij}^{RR} + d_i^R V_{ij}^{LR} + d_j^{*R} V_{ij}^{RL} + d_i^R d_j^{*R} V_{ij}^{LL}$$

$$\hat{V}_{ij}^{RL} = V_{ij}^{RL} + d_i^R V_{ij}^{LL} + d_j^{*L} V_{ij}^{RR} + d_i^R d_j^L V_{ij}^{LR}$$

  - also for LR and LL (similarly for XY basis)
- Can "correct" visibilities by applying adjoint of $\mathbf{J}_D$
  - will introduce correlations (e.g. if non-orthogonal)
  - REMINDER: correcting visibilities only for "on-axis" leakage

Example: RL basis leakage

- In full detail:

$$V_{ij}^{RR} = \int_{sky} dxdy\, \tilde{A}_{ij}^{RR}(x,y)[(\mathrm{I}+\mathrm{V})e^{i(\chi_i-\chi_j)}$$

$$+ d_i^R e^{-i(\chi_i+\chi_j)}(\mathrm{Q}-i\mathrm{U}) + d_j^{*R} e^{i(\chi_i+\chi_j)}(\mathrm{Q}+i\mathrm{U})$$

$$+ d_i^R d_j^{*R} e^{-i(\chi_i-\chi_j)}(\mathrm{I}-\mathrm{V})](x,y)e^{-i2\pi(u_{ij}l+v_{ij}m)}$$

$$V_{ij}^{RL} = \int_{sky} dxdy\, \tilde{A}_{ij}^{RL}(l,m)[(\mathrm{Q}+i\mathrm{U})e^{i(\chi_i+\chi_j)}$$

$$+ d_i^R(\mathrm{I}-\mathrm{V})e^{-i(\chi_i-\chi_j)} + d_j^{*L}(\mathrm{I}+\mathrm{V})e^{i(\chi_i-\chi_j)}$$

$$+ d_i^R d_j^{*L}(\mathrm{Q}-i\mathrm{U})e^{-i(\chi_i+\chi_j)}](x,y)e^{-i2\pi(u_{ij}l+v_{ij}m)}$$

“true” signal

2nd order: D•P into I

2nd order: D^2•I into I

1st order: D•I into P

3rd order: D^2•P* into P

Example: linearized leakage

- RL basis, keeping only terms linear in I,Q±iU,d:

$$V_{ij}^{RL} = (\mathrm{Q} + i\mathrm{U})e^{-i(\phi_i + \phi_j)} + \mathrm{I}\,(d_i^R\, e^{i(\phi_i - \phi_j)} + d_j^{*L}\, e^{-i(\phi_i - \phi_j)})$$

$$V_{ij}^{LR} = (\mathrm{Q} - i\mathrm{U})e^{i(\phi_i + \phi_j)} - \mathrm{I}\,(d_i^L\, e^{-i(\phi_i - \phi_j)} + d_j^{*R}\, e^{i(\phi_i - \phi_j)})$$

- Likewise for XY basis, keeping linear in I,Q,U,V,d,sin(ϕ_i-ϕ_j)

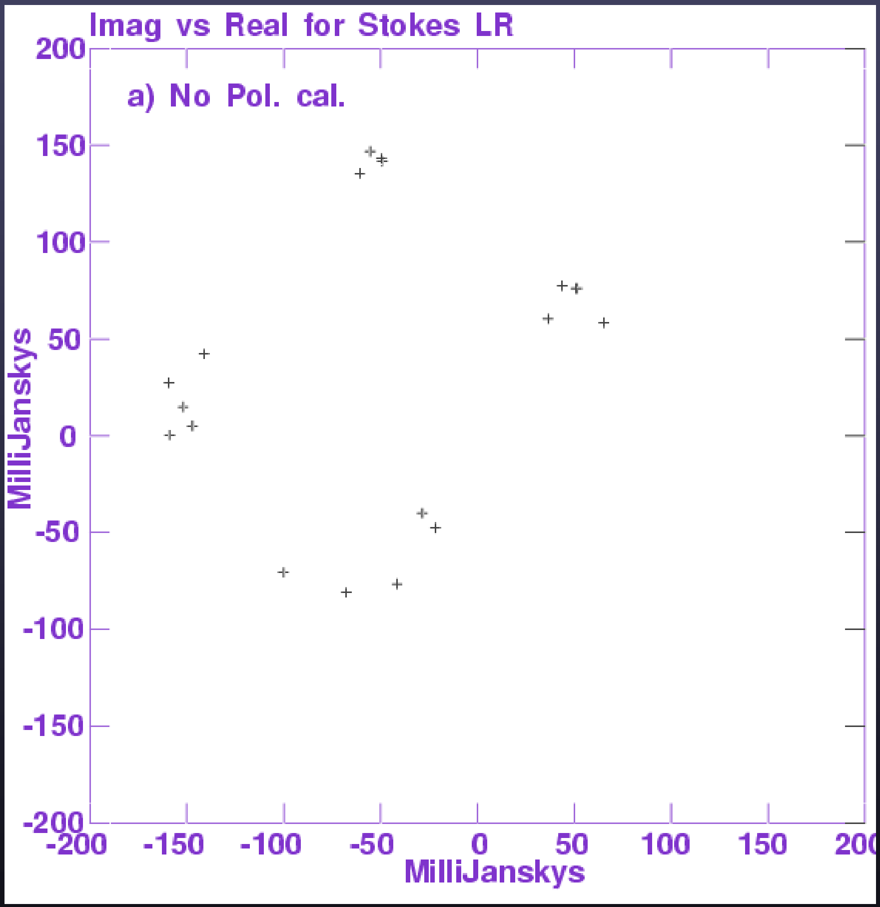
$$V_{ij}^{XY} = \mathrm{Q}\sin(\phi_{\mathrm{i}} + \phi_j) + \mathrm{U}\cos(\phi_i + \phi_j) + i\,\mathrm{V} + [(d_i^X + d_j^{*Y})\cos(\phi_i - \phi_j) - \sin(\phi_i - \phi_j)]\mathrm{I}$$

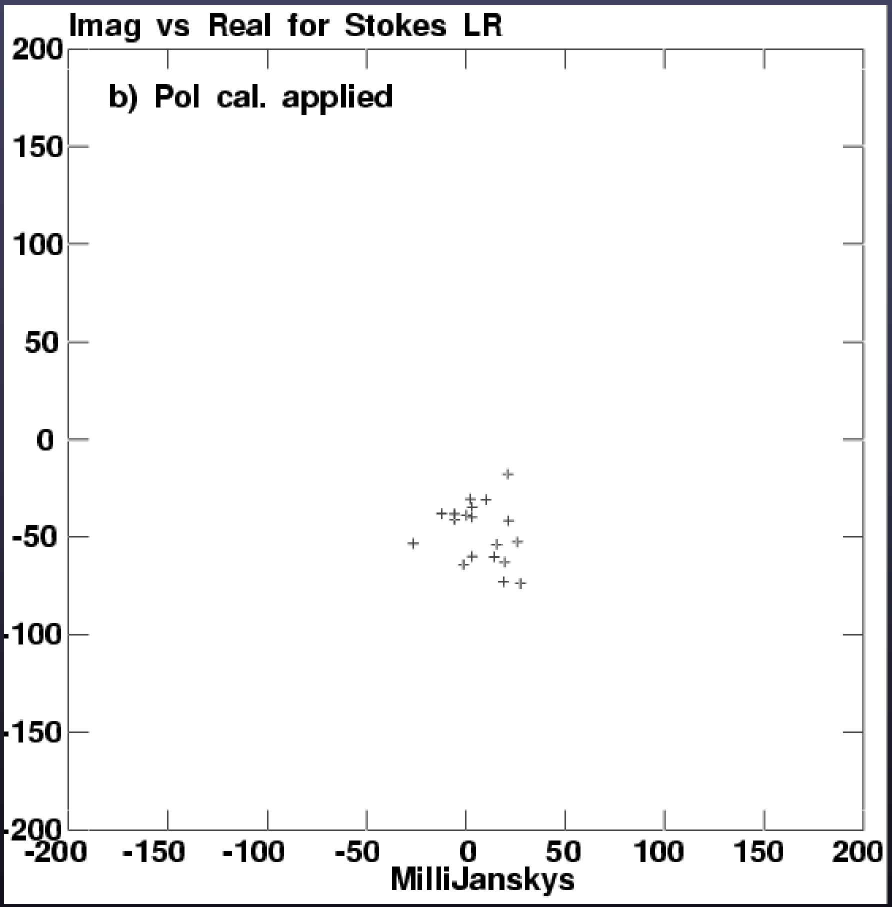
$$V_{ij}^{YX} = \mathrm{Q}\sin(\phi_{\mathrm{i}} + \phi_j) + \mathrm{U}\cos(\phi_i + \phi_j) + i\,\mathrm{V} + [(d_i^Y + d_j^{*X})\cos(\phi_i - \phi_j) + \sin(\phi_i - \phi_j)]\mathrm{I}$$

WARNING: Using linear order will limit dynamic range!

Example: D-term calibration

- D-term calibration effect on RL visibilities (should be Q+iU):

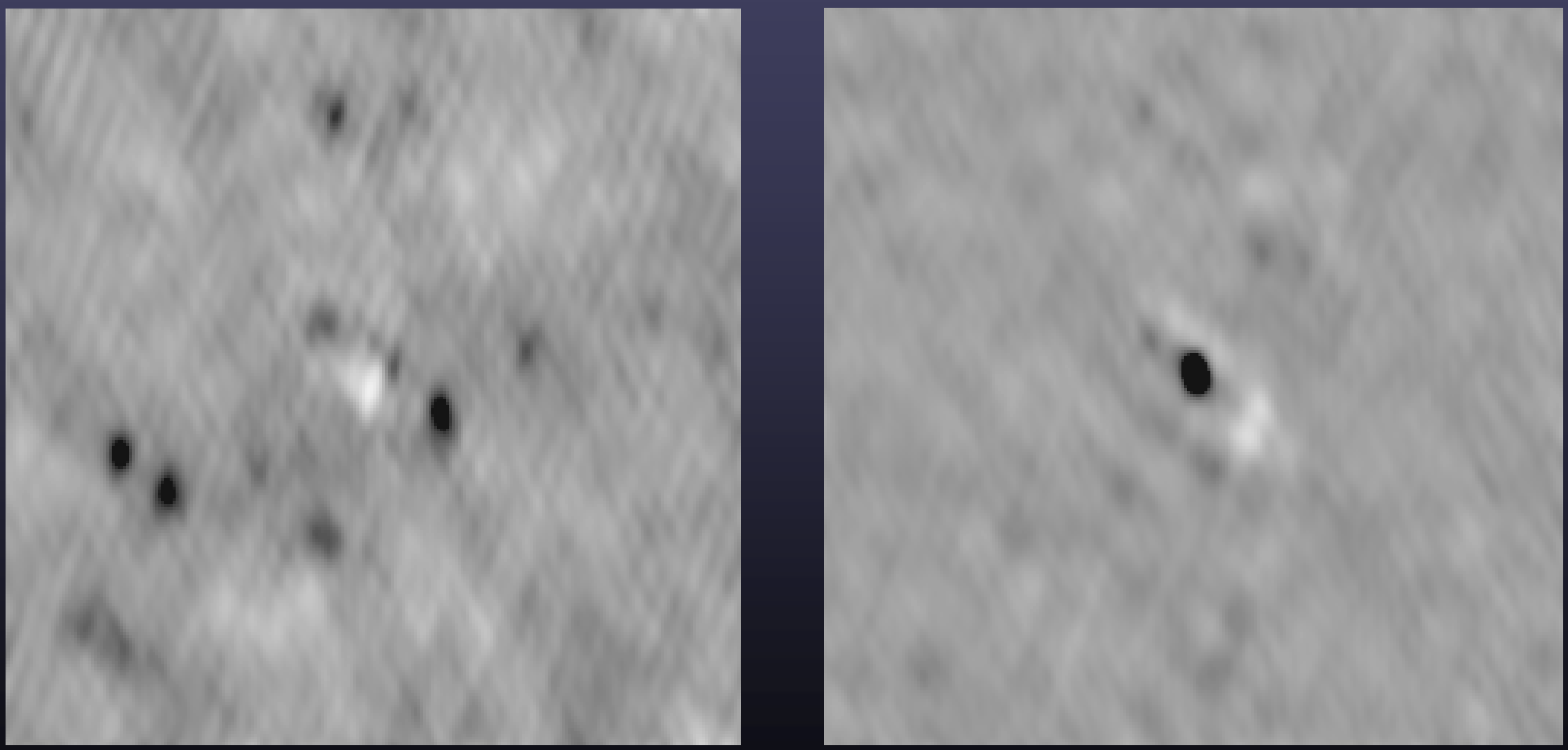




Example: D-term calibration

- D-term calibration effect in image plane :

Bad D-term solution   Good D-term solution



Antenna voltage pattern, E

- Direction-dependent gain and polarization
  - includes primary beam
    - Fourier transform of cross-correlation of antenna voltage patterns
    - includes polarization asymmetry (squint)

$$\mathbf{J}_E^{pq} = \begin{pmatrix} e^{pp}(x,y) & e^{pq}(x,y) \\ e^{qp}(x,y) & e^{qq}(x,y) \end{pmatrix}$$

  - includes off-axis cross-polarization (leakage)
    - convenient to reserve D for on-axis leakage
  - important in wide-field imaging and mosaicing
    - when sources fill the beam (e.g. low frequency)

Polarization Imaging

- What I really want to talk about…

From sky to Fourier domain

- Write as linear operators acting on vectors
  - think of it as pixelizing the sky and uv-plane
  - underlined vectors and operators in uv domain
- The Fourier Transform Operator
  - the sky in the image domain $\mathbf{x}_i = (x_i, y_i)$

$$\mathrm{s} \quad : \quad \mathrm{s}_i = \mathrm{s}(\mathbf{x}_i)$$

  - the Fourier domain ("uv-plane") $\mathbf{u}_l = (u_l, v_l)$

$$\underline{\mathrm{s}} \quad : \quad \underline{\mathrm{s}}_l = \underline{\mathrm{s}}(\mathbf{u}_l)$$

  - the Fourier kernel

$$\mathrm{s} = \mathbf{F}\,\underline{\mathrm{s}} \quad \Leftrightarrow \quad \underline{\mathrm{s}} = \mathbf{F}^{-1}\,\mathrm{s}$$

$$F_{il} = e^{2\pi i \mathbf{u}_l \cdot \mathbf{x}_i} \Leftrightarrow F_{li}^{-1} = e^{-2\pi i \mathbf{u}_l \cdot \mathbf{x}_i}$$

Forward: Image to Visibility

- know how to compute "visibility" given sky model
- imaging equation:
  - "visibility" indexed by k
    - includes time t, antenna pair *ab* polarization product *pq* $k = tpqab$

$$V_k(\mathbf{u}_k) = \int dx\, dy\, A_k\left(x, y, u_k, v_k\right) I\left(x, y\right) e^{2\pi i \mathbf{u}_k \cdot \mathbf{x}}$$

  - kernel ***A*** time, antenna, polarization, direction dependent
    - polarization effects (leakage, Faraday)
    - plus direction dependence (squint, polarization beam)
    - plus u,v dependence (w-term)
    - plus time dependence (gains, $\boldsymbol{u}_k$ rotation, parallactic angle, …)
  - ***A*** is the difficult thing to compute
    - usually pretend we are sampling field $\underline{\boldsymbol{v}}_k(u,v)$

Example Effect: w-term

- the usual equation (aperture and w-term)

$$V(u,v,w) = \int dx\,dy \frac{A(x,y)}{\sqrt{1-x^2-y^2}} I(x,y) e^{2\pi i\left[ux+vy+w\left(\sqrt{1-x^2-y^2}-1\right)\right]}$$

$$= \int dx\,dy\, G(x,y,u,v,w) I(x,y) e^{2\pi i(ux+vy)}$$

- use kernel ***G*** :

$$G(x,y,u,v,w) = \frac{A(x,y)}{\sqrt{1-x^2-y^2}} e^{2\pi i w\left(\sqrt{1-x^2-y^2}-1\right)}$$

  - in terms of operators:
    - compute transform $\underline{\mathbf{G}} = \mathbf{F}^{-1}\,\mathbf{G}\,\mathbf{F}$ in "planes" in *w*
    - this is "aperture w-projection" (if A=1 then just "w-projection")

Example Effect: mosaicing

- the usual equation (aperture and offset term)

$$V(u,v,w) = \int dx\,dy\; A(x - x_k, y - y_k) I(x,y) e^{2\pi i [u(x-x_k) + v(y-y_k)]}$$
$$= \int dx\,dy\, B(x,y,u,v) I(x,y) e^{2\pi i (ux+vy)}$$

  – note that we assume "phased up" at each pointing $\boldsymbol{x}_k$!

- use kernel ***B*** :

$$B(x,y,u,v) = A(x - x_k, y - y_k) e^{-2\pi i (ux_k + vy_k)}$$

  – this is the "mosaicing" kernel
    - note similarity to AW-kernel (can include if needed)
    - if offset $\boldsymbol{x}_k$ is unknown, then this is a "pointing error"
    - offset for R and L polarizations is the "squint" term
    - the transform $\underline{\boldsymbol{B}} = \boldsymbol{F}^{-1}\, \boldsymbol{B}\, \boldsymbol{F}$ is equivalent to Fourier shift theorem

The Calibration Part

- unknown factors in the equation

$$V_{pqij}(u,v) = \int dxdy\, g_i g_j^* D_{pi} D_{qj}^* A(x-x_k, y-y_k) I(x,y) e^{2\pi i(ux+vy)}$$
$$= \int dxdy\, B_{pqij}(x,y,u,v) I(x,y) e^{2\pi i(ux+vy)}$$

  - includes leakage, pointing/squint, gain factors
- calibration: solve for unknown factors
  - if they are “known” then correct for them during imaging
- formalism: measurement equation (Hamaker et al.)
  - e.g. use polarization terms in “meq-trees” (Smirnov)
  - e.g. pointing calibration in Sanjay’s talk
- complication: integrate into “self-cal” cycle
- upshot: if you have bright isolated sources should be OK

The Imaging Part

- Visibilities and the Sky including noise

$$\underline{v} = \mathbf{F}^{-1} \boldsymbol{A}\, s + \underline{n} = \mathbf{F}^{-1} \boldsymbol{A}\, \mathbf{F}\, \mathbf{F}^{-1}\, s + \underline{n}$$

$$\underline{v} = \underline{\boldsymbol{A}}\, \underline{s} + \underline{n} \qquad \underline{\boldsymbol{A}} = \mathbf{F}^{-1} \boldsymbol{A}\, \mathbf{F}$$

  - $\underline{\boldsymbol{A}}$ contains known corruptions and unknown calibration effects
  - however: $\underline{\boldsymbol{A}}$ is <u>not invertible</u>
  - instrumental noise $\underline{n}$ is a <u>random variable</u>
- The mathematical issues:
  - unknown random noise $\underline{n}$
  - convolution due to size of $\underline{\boldsymbol{A}}$ in uv domain (finite FOV)
  - incomplete sampling of uv-plane by visibilities (deconvolution)
- The real issue:
  - how to compute $\underline{\boldsymbol{A}}$ efficiently
    - implementation as important as "algorithm"

Maximum Likelihood Reconstruction

- The noise and its covariance

$$\underline{n} = \underline{v} - \underline{\mathbf{A}}\ \underline{s} \quad : \quad \underline{\mathbf{N}} = \langle \underline{n}\ \underline{n}^T \rangle$$

  - if noise is uncorrelated (Gaussian) then $\underline{\mathbf{N}}$ is diagonal

$$\underline{N}_{kk'} = \sigma_k^2\ \delta_{kk'}$$

- The likelihood function

$$L(\underline{s} \mid \underline{v}) = \det(2\pi\underline{\mathbf{N}}^{-1}) \exp[-½\ (\underline{v} - \underline{\mathbf{A}\ s})^T\ \underline{\mathbf{N}}^{-1}\ (\underline{v} - \underline{\mathbf{A}\ s})]$$

  - find map $\underline{m}$ that maximizes L

$$dL/d\underline{s}\ |_{\underline{s}=\underline{m}} = 0$$

  - Maximum Likelihood Estimate (MLE) :

$$m_{MLE} = (\ \underline{\mathbf{A}}^T\ \underline{\mathbf{N}}^{-1}\ \underline{\mathbf{A}})^{-1}\ \underline{\mathbf{A}}^T\ \underline{\mathbf{N}}^{-1}\ \underline{v}$$

  - this is the least-squares solution, not practical due to inverse
    - but note that without the inverse term this is just uv gridding

The Dirty Map

- Grid onto sampled uv-plane

$$\underline{d} = \underline{\mathbf{H}}\,\underline{\mathbf{v}} = \underline{\mathbf{H}}\,\underline{s} + \underline{n}_d$$

  - $\underline{\mathbf{H}}$ should be close to $\underline{\mathbf{H}}_{MLE}$, e.g.

$$\underline{\mathbf{H}} = \underline{\mathbf{B}}^T\,\underline{N}^{-1} \quad : \quad \underline{\mathbf{B}} \sim \underline{\mathbf{A}}$$

  - $\underline{\mathbf{B}}^T$ should sample onto suitable grid in uv-plane
- Invert onto sky ➔ "dirty image"

$$\mathrm{d} = \mathbf{F}\,\underline{d} = \mathbf{R}\,\mathrm{s} + \mathrm{n}_d \qquad \mathbf{R} = \mathbf{F}\,\underline{\mathbf{R}}\,\mathbf{F}^{-1}$$

  - image is "dirty" as it contains artifacts
    - noise
    - convolution by "point spread function" (columns of **R**)
    - multiplication by response function (diagonal of **R**)
    - **R** contains the polarization corruptions and calibration errors

Image, uv, and Data Spaces

- image plane ⇔ uv-plane ⇔ visibilities
  - operators **F** , $\underline{\boldsymbol{H}}$, $\underline{\boldsymbol{A}}$ handle these transformations
  - not all operators have inverses ($\underline{H}$ and $\underline{\boldsymbol{A}}$ do not)
- example: model image m
  - first transform sky model to uv-plane

$$\underline{m} = \mathbf{F}^{-1}\, m$$

  - then project onto the visibilities (data space)

$$\underline{v}_{m} = \underline{\boldsymbol{A}}\, \underline{m} = \underline{\boldsymbol{A}}\, \mathbf{F}^{-1}\, m$$

  - form residual

$$\delta\underline{v}_{m} = \underline{v} - \underline{v}_{m} = \underline{\boldsymbol{A}}\, (\, \underline{s} - \underline{m}\,) + \underline{n}$$

  - finding “best model” will involve minimizing this residual
    - basis of CLEAN algorithm, solve iteratively
  - Imaging: Find the most accurate model ***m*** (not image ***d***)
    - IMPORTANT: all relevant corruptions included in $\underline{\boldsymbol{A}}$!

Imaging and Calibration

- where to build in corruption effects
  - (1) when making dirty image (in gridding kernel)

$$\underline{d} = \underline{\mathbf{H}}\,\underline{\mathbf{v}} \qquad\qquad \underline{\mathbf{H}} = \underline{\mathbf{B}}^{T}\,\underline{N}^{-1} \;:\; \underline{\mathbf{B}} \sim \underline{\mathbf{A}}$$

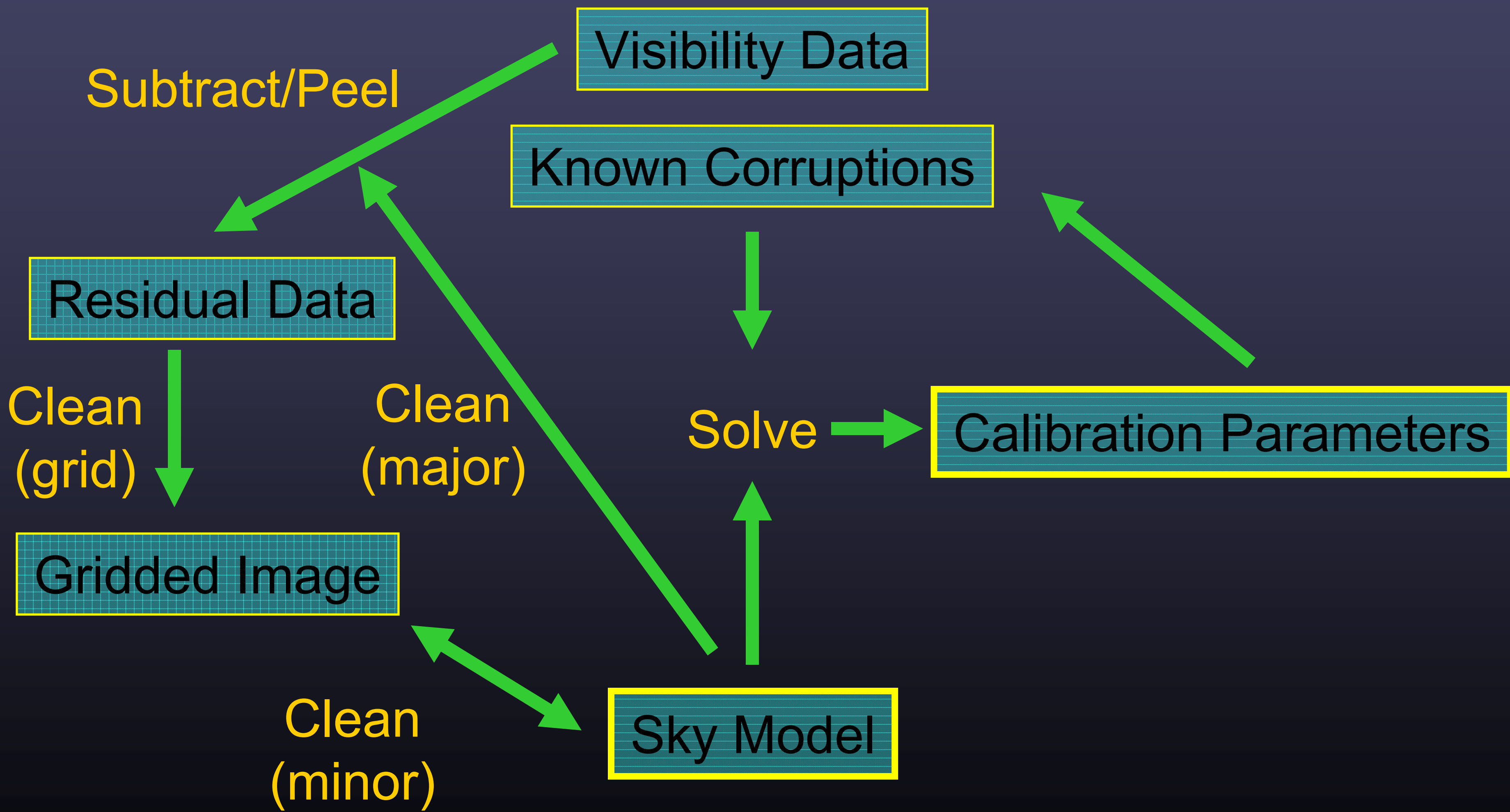
  - (2) when you form residual (in "degridding" step)

$$\underline{\delta v}_{m} = \underline{v} - \underline{v}_{m} = \underline{\mathbf{A}}\,(\,\underline{s} - \underline{m}\,) + \underline{n}$$

  - including best knowledge in (2) is more important
    - want to iterate towards uncorrupted model
    - residuals will include effects at lower order
    - complicated (diffuse) images may need (1) using good kernel
  - example: squint correction
- after imaging
  - use best model ***m*** for "self-calibration"

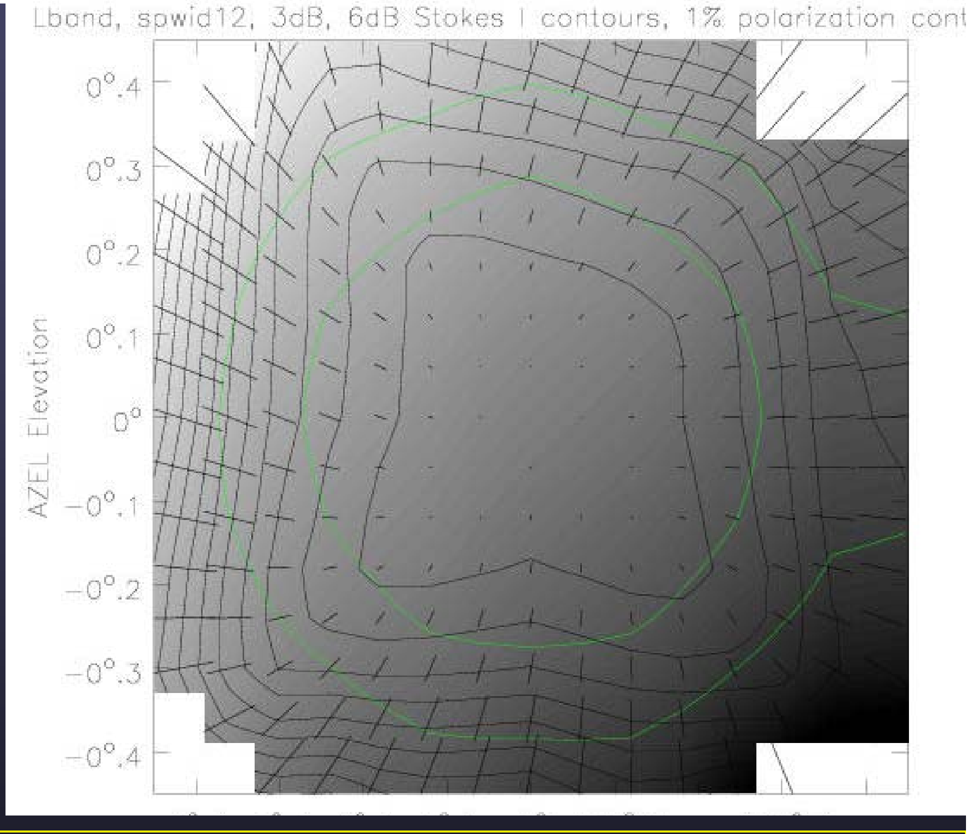
Imaging & Calibration “Flow”

- The imaging and calibration process



Example: Polarization Primary Beam

- VLA primary beams
  - Beam squint (off-axis optics)
  - off-axis polarization structure
  - AZ-EL telescopes, RL basis
- Instrumental polarization patterns rotate on sky with parallactic angle
  - Limits polarization imaging
  - Limits Stokes I dynamic range (via second order terms)
  - must implement during imaging
- Predicted vs. Measured beam
  - W. Brisken EVLA memo 58



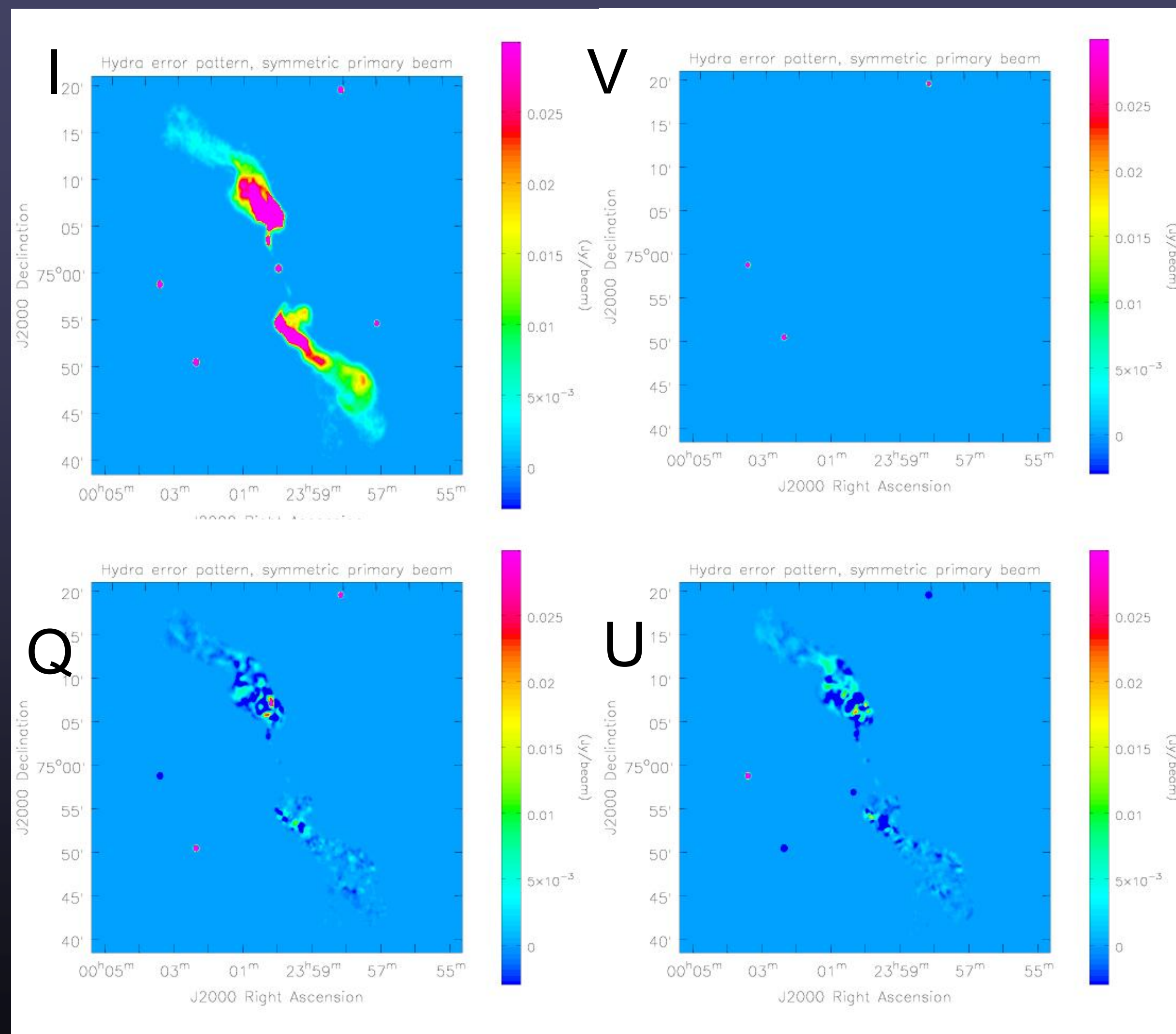
green contours: Stokes I 3dB, 6dB
black contours: fractional polarization 1% and up
vectors: polarization position angle
raster: Stokes V

Polarization Imaging “Algorithm”

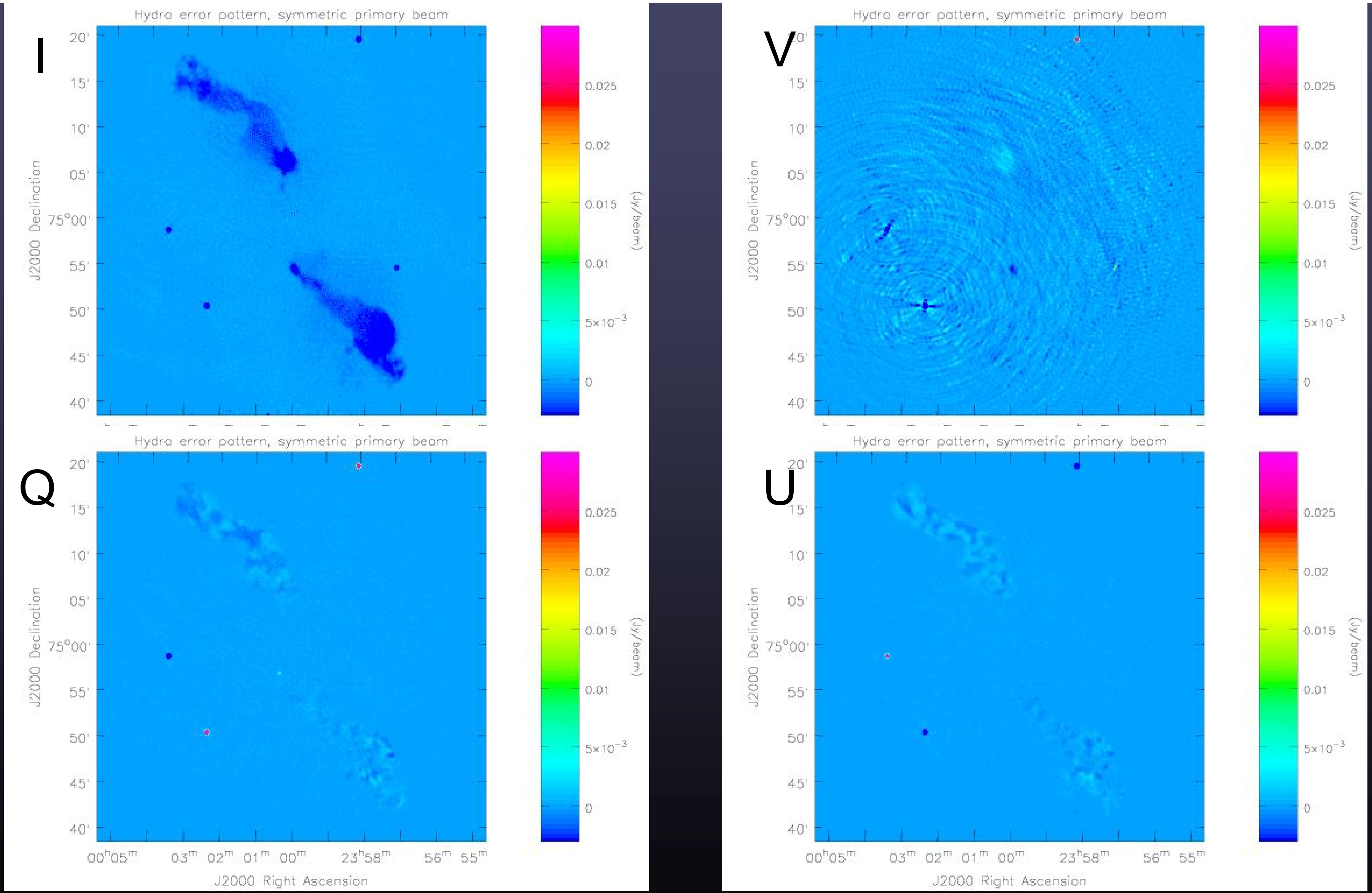
- Minor cycle: Hogbom CLEAN
  - modified for spatially variant SNR (due to primary beam)
- Major cycle: Accurate residual calculation
  - For each parallactic angle bin, multiply image model by full polarized primary beam, Fourier Transform, subtract from observed coherence, Inverse Fourier transform, and multiply by adjoint of full polarized primary beam
  - Must model I,Q,U, and V
  - Extra cost ~ number of parallactic angle bins (~ 10 - 30)
  - Apply exactly to component model
  - Follows standard MeasurementEquation formulation
- include in existing calibration & imaging procedures
- See EVLA memo 62 (Cornwell)
  - Example: courtesy Tim Cornwell

Simulations on a complex model

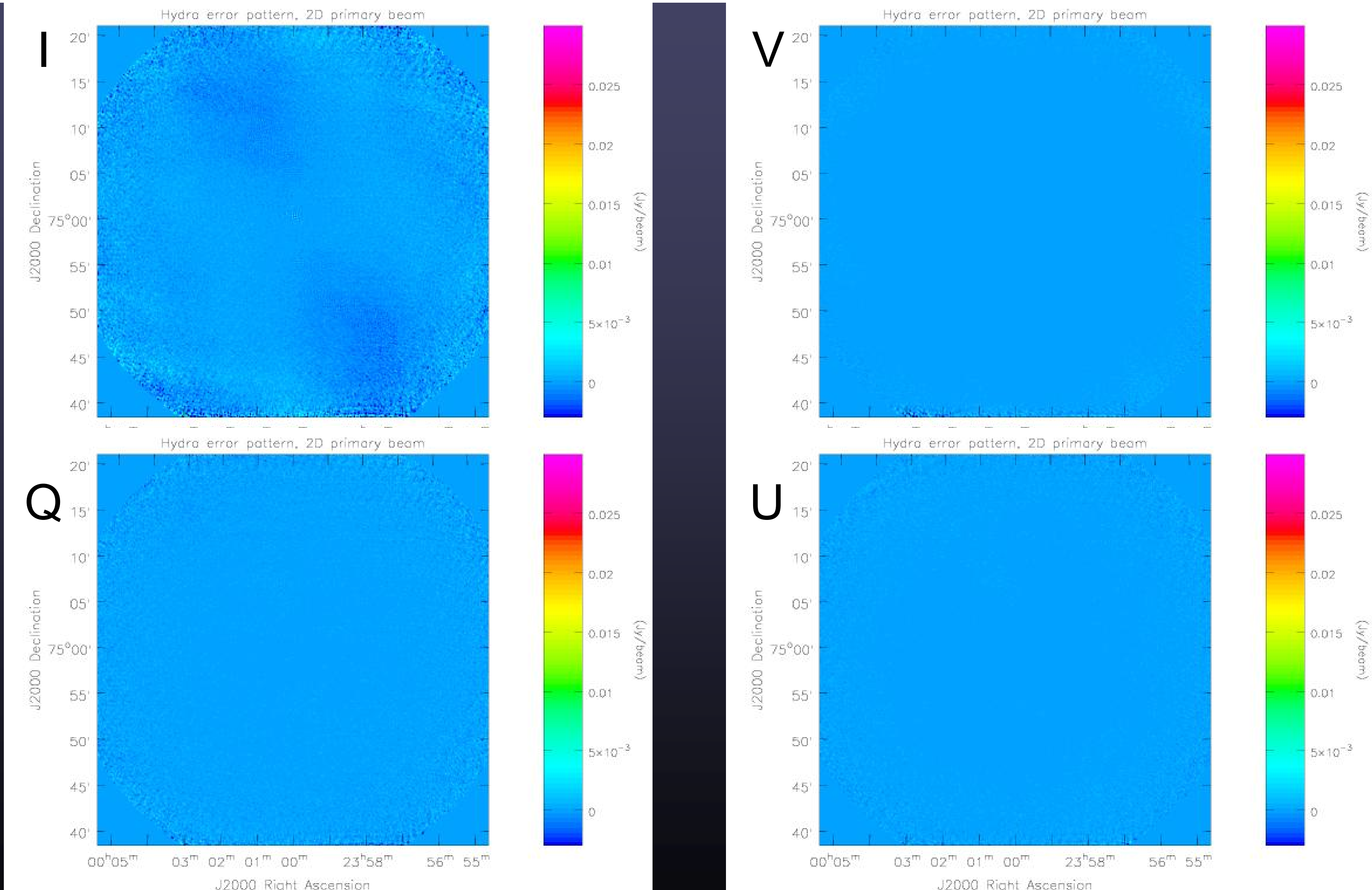
- VLA simulation of ~ 1 Jy point sources + large source with complex polarization (“Hydra A”)
- Long integration with full range of parallactic angles
- equivalent to weak 1.4GHz source observed with EVLA
- Antenna primary beam model by W. Brisken
- See EVLA memo 62 (Cornwell)



1D symmetric beam ~ 200:1

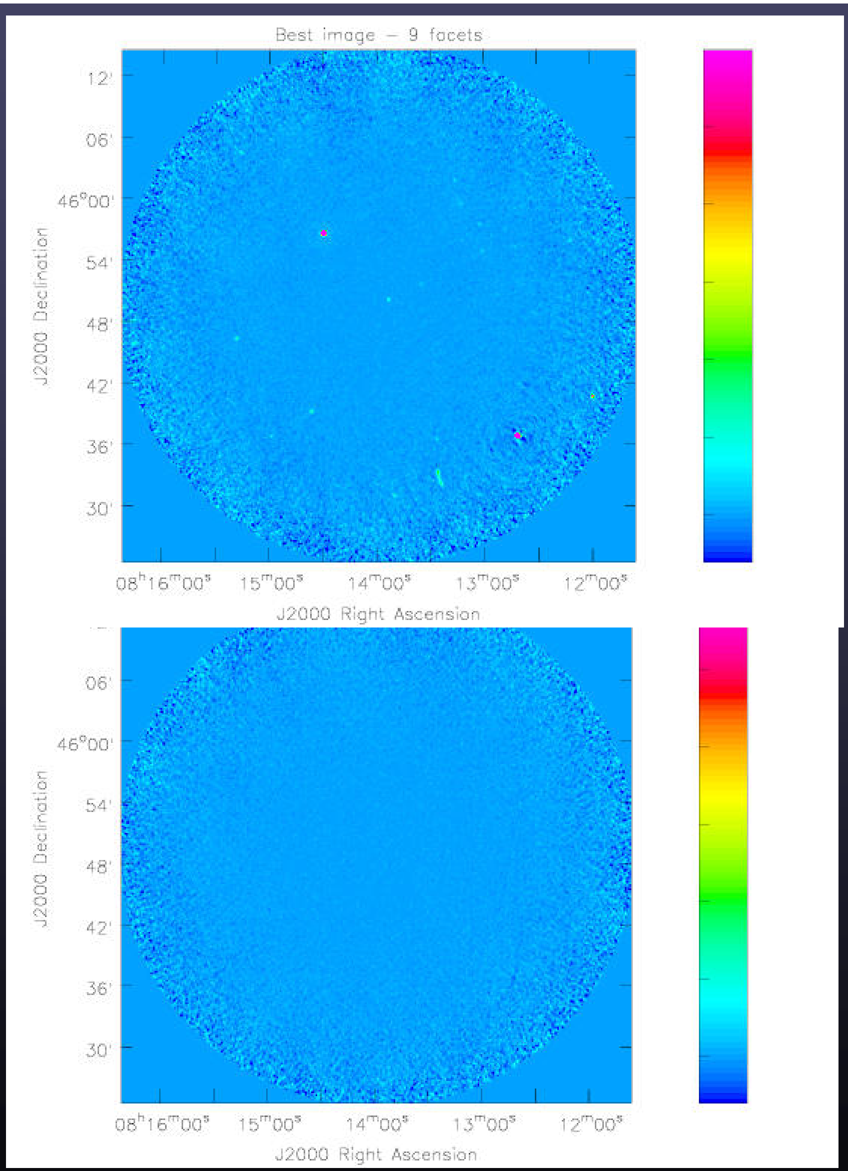


2D beam model ~ 10,000:1



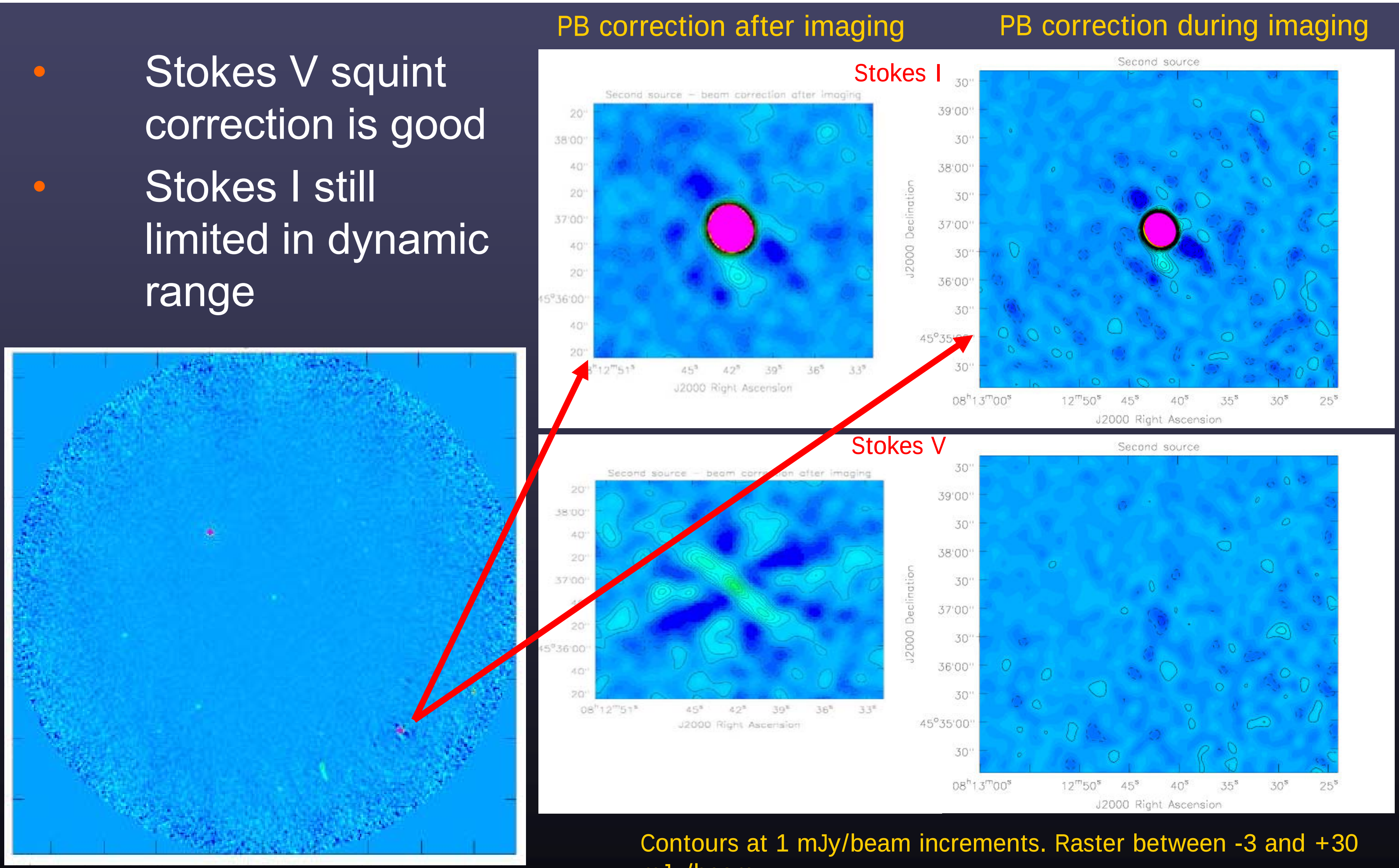
Test on real data: IC2233

- VLA C configuration long integration at 20cm
- Used wide-field imaging plus antenna beam polarization correction
- Stokes V very clean
- Stokes I shows ~ 1% error at 30% sensitivity point
- Helps a bit but not enough
- Errors vary across the beam
  - Due to pointing errors?



Correction after or during imaging

- Stokes V squint correction is good
- Stokes I still limited in dynamic range

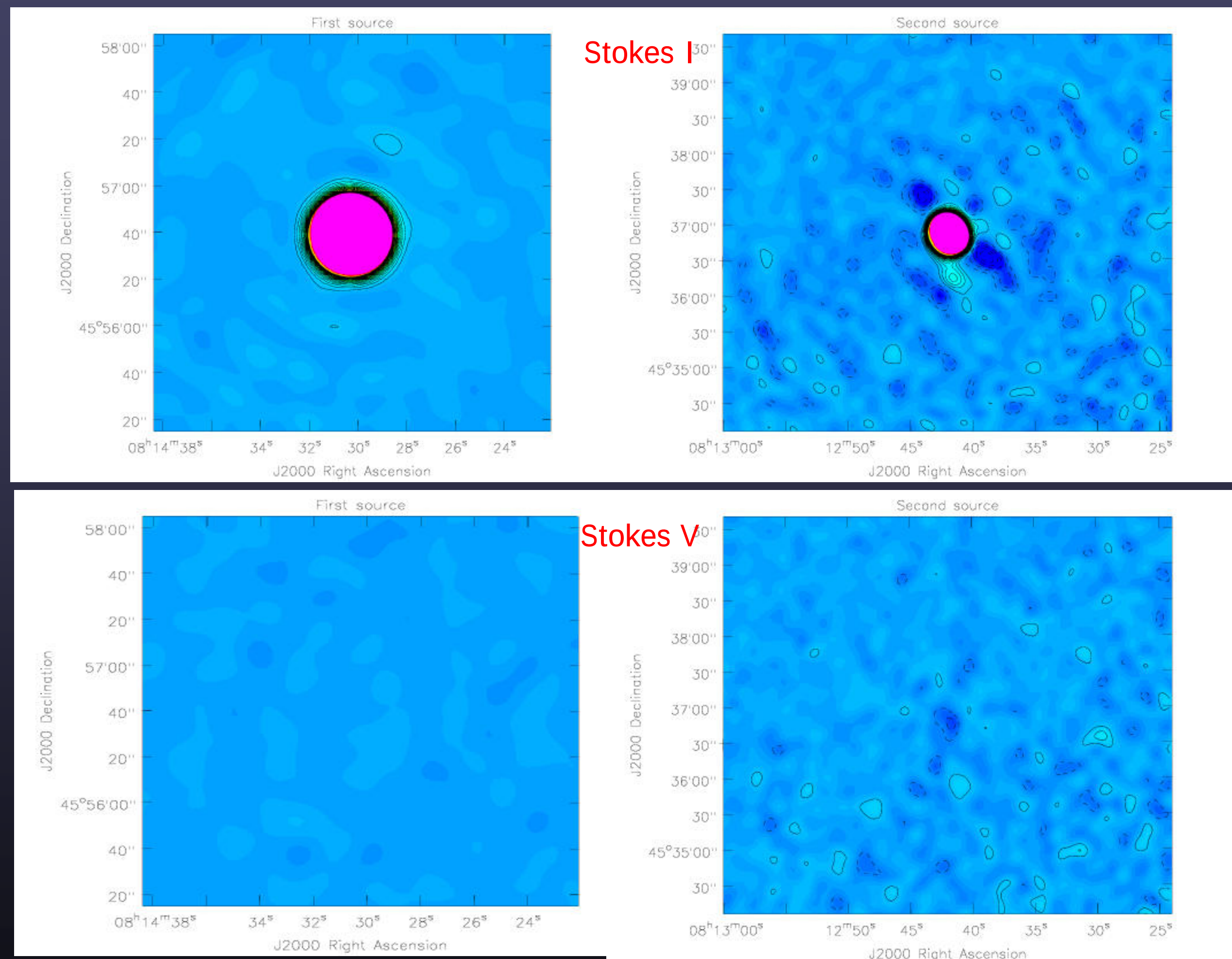


Contours at 1 mJy/beam increments. Raster between -3 and +30 mJy/beam

Comparison of brightest and second-brightest sources

- Error in Stokes I is non-isoplanatic : only present on second source
- Not due to atmospheric phase errors
- Due to pointing errors?
- VLA L band antennas not well collimated
- Example of problem likely to afflict EVLA, ALMA, and SKA

1.1Jy source 0.41Jy source



Contours at 1 mJy/beam increments. Raster between -3 and +30 mJy/beam

Summary: Key algorithmic issues

polarimetric imaging & calibration

- basic problem
  - instrumental basis (RL,XY) to Stokes (IQUV)
  - compact and diffuse (polarization-dependent) structure
- on-axis instrumental polarization
  - “leaks” one instrumental polarization into another
  - dominant term, will affect intensity at high dynamic range
  - frequency dependent instrumental polarization
- beam squint
  - XY phase gradients $\Rightarrow$ RL beams offset on-sky (spurious V)
- full-beam polarimetry
  - high dynamic range ($>10^4$:1)
  - frequency & direction-dependent instrumental polarization
  - critical for diffuse polarized sources

More key algorithmic issues

general imaging & calibration

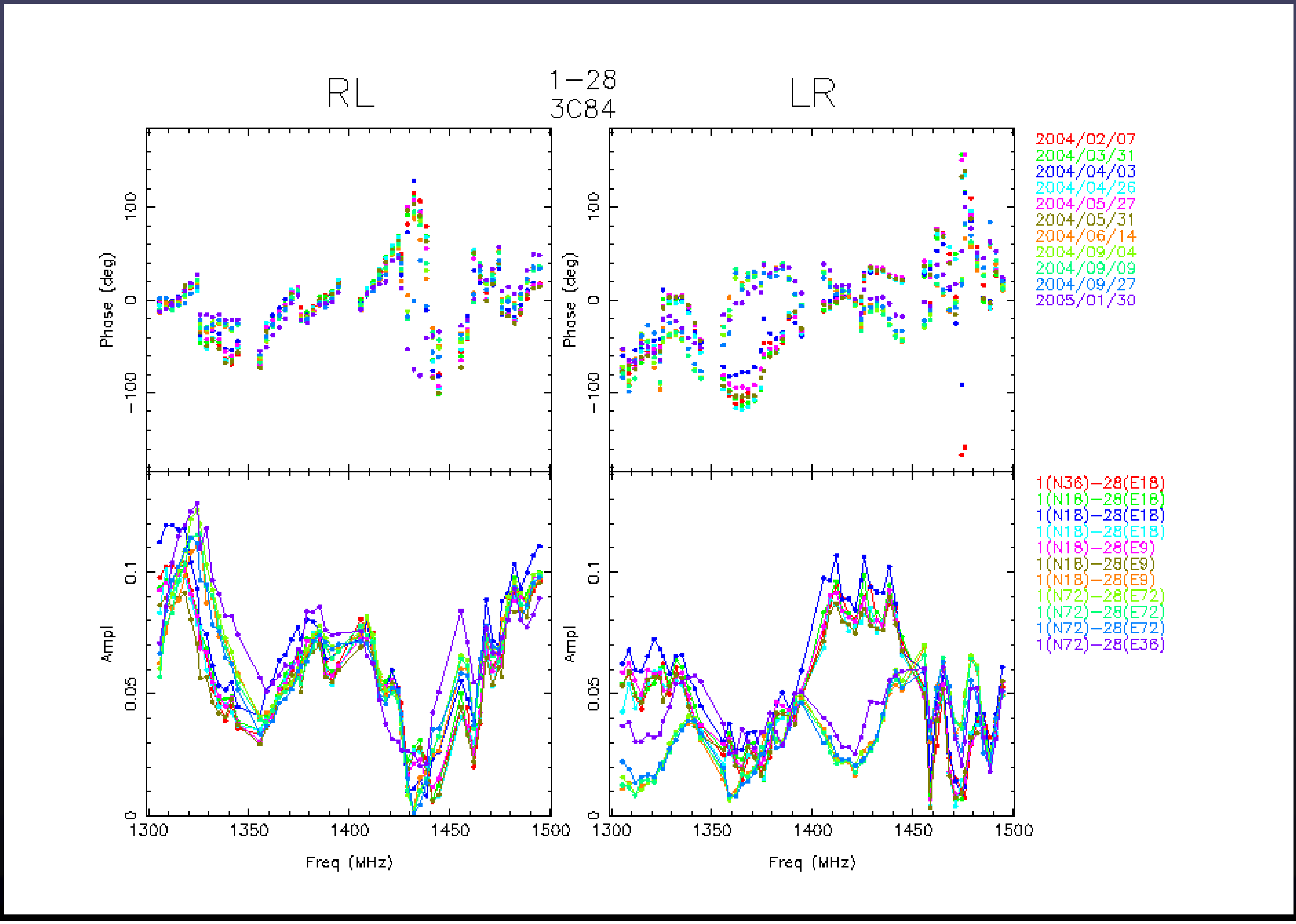
- polarization is not a “special” case for imaging
- inherits all the other general imaging issues
  - wide-field imaging
    - w-term issue at low frequencies
    - mosaicing at high frequencies
  - wide-band (continuum) imaging
    - important in all bands with >30% bandwidth
    - image spectral index & Faraday rotation
  - ionospheric correction (low frequencies)
    - faraday rotation also
- what makes these more difficult
  - high dynamic range (bright sources in field)
  - high fidelity (accurate reconstruction)
  - large data volumes (worry about I/O load and CPU cost)

Conclusions and Speculation

- polarization calibration & imaging
  - EM radiation polarized, not a special case
  - must include from beginning
    - IF you want to image polarization
    - IF you want high dynamic range and fidelity
- how hard do you have to work?
  - on-axis polarization calibration
    - straightforward and affordable
  - beam squint (RL)
    - straightforward and mostly affordable
  - full-beam polarization calibration and imaging
    - both imaging and calibration impact, known and unknown
    - complicated and expensive
- can we get by doing accurate residuals?
  - how good is good enough? ERROR CONTROL

Extra: VLA leakage freq dependence

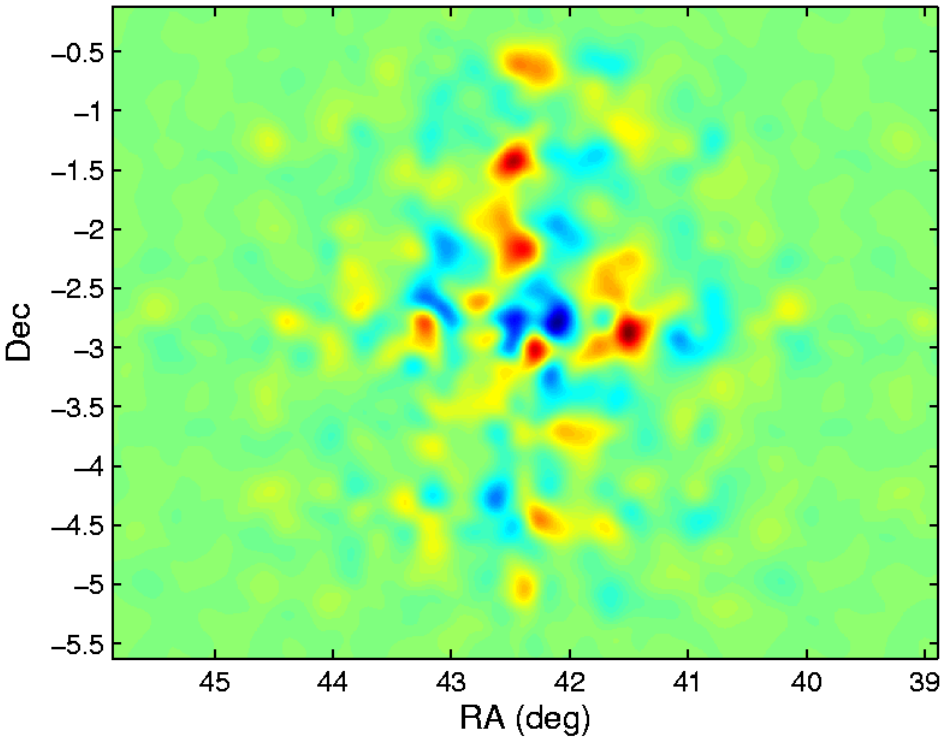
- Courtesy G. Moellenbrock

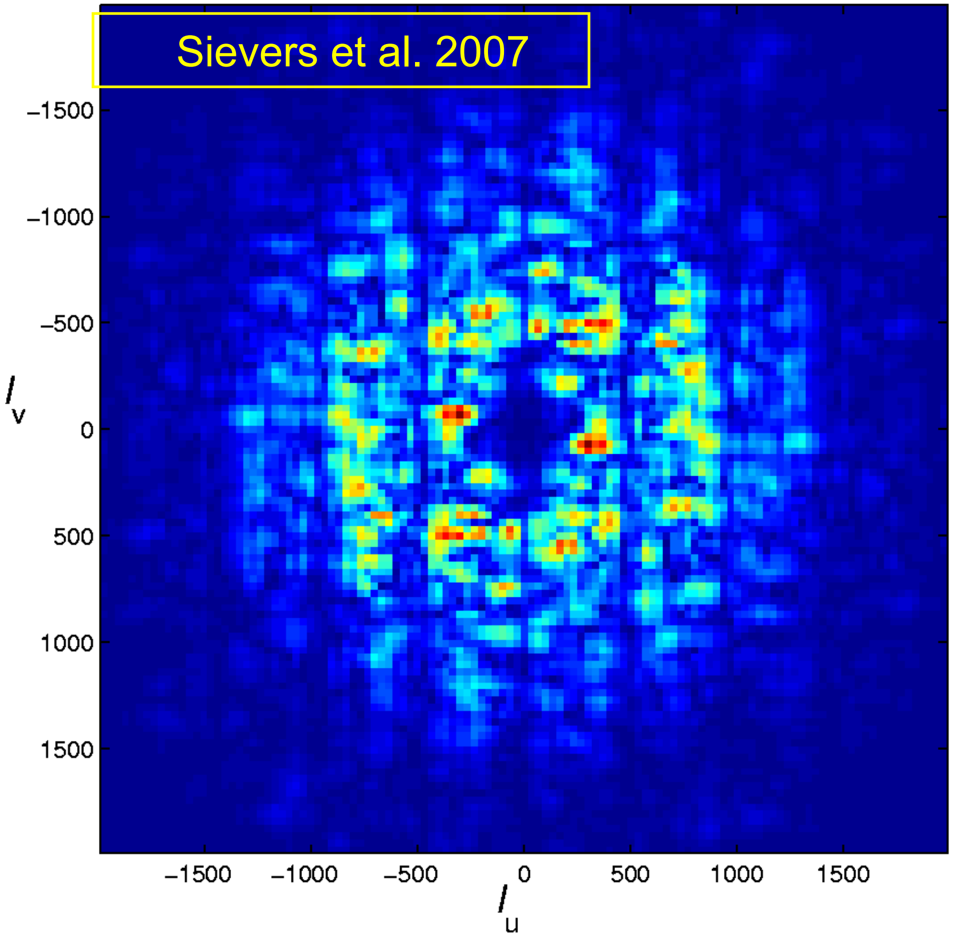


Extra: CMB ℓ-space maps

- use gridded visibilities to reconstruct T,E,B in ℓ-space

CBI 02h 6x6 field T mosaic





T image ⇔ ℓ T_ℓ

➔test for non-Gaussianity in ℓ-space

ℓ -space CLEAN deconvolved!

ℓ-space maps

use gridded visibilities to reconstruct T,E,B in ℓ-space

linear Wiener filtered reconstruction

sub-Nyquist mosaic pattern ➔ "sidelobes" in ℓ-space