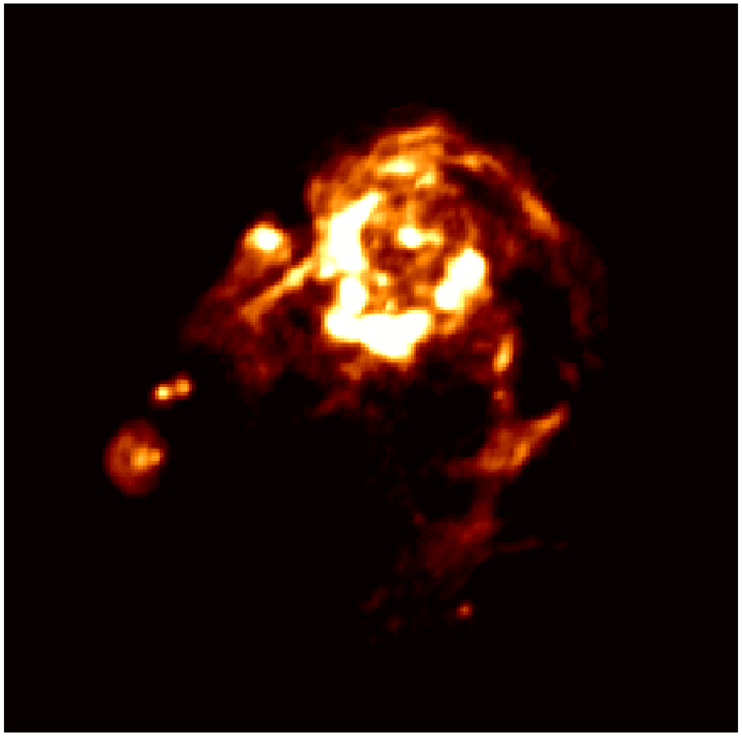
Challenges in advanced imaging and deconvolution



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Challenges

- Explicitly incorporate the scale information in the deconvolution algorithms.
  - Widely separated pixels are coupled due to the sidelobes of the Point Spread Function (PSF). Fast computation of this coupling is a challenge.
  - Decoupling the various scales in the image, or controlling the dimensionality of the search space is a challenge.
- Solving for direction dependent corruptions as a function of time, frequency and polarization.
- Incorporate these direction dependent effects while predicting the model visibilities.
- Modeling the sky as a function of frequency and polarization

Basic Interferometry

- Interferometers measure the source coherence function (the Visibility function) $V(u_{ij}, v_{ij}, w_{ij}) = \langle E_i E_j^* \rangle$

  E_i is the electric field measured at antenna *i*

  u,v,w are the projected separation between the antennas *i* and *j*

- In terms of the sky brightness distribution ($I^o(l,m)$)

$$V(u_{ij}, v_{ij}, w_{ij}) = \int\int I^o(l,m) e^{-2\pi\iota(u_{ij}l + v_{ij}m + w_{ij}\sqrt{1-l^2-m^2})} \frac{dl\, dm}{\sqrt{1-l^2-m^2}}$$

- In the small angle approximation, sky is the 2D Fourier transform of the Visibility function (van-Citter Zernike Theorem)

$$V(u_{ij}, v_{ij}) = \int\int I^o(l,m) e^{-2\pi\iota(u_{ij}l + v_{ij}m)} dl\, dm$$

$$I^o = FT[V]$$

Basic Imaging

- Visibility function is measured at discrete points only

$$V^{Obs}(u,v,w) = S(u,v,w)\,V(u,v,w)$$

The sampling function S(u,v,w) = 1 at the measured points, and 0 otherwise.

- The Dirty Image (I^D) is the convolution of the true image and the PSF (*B*): $I^D(l,m) = B(l,m) * I^o(l,m)$

- Image deconvolution: Given the PSF and the measured visibilities, find a model of the sky (I^M) such that residuals are noise-like.

$$Minimize: \sum_{ij} |V_{ij}^{obs} - FT[B * I^M]_{ij}|^2 \quad w.r.t.\ I^M$$

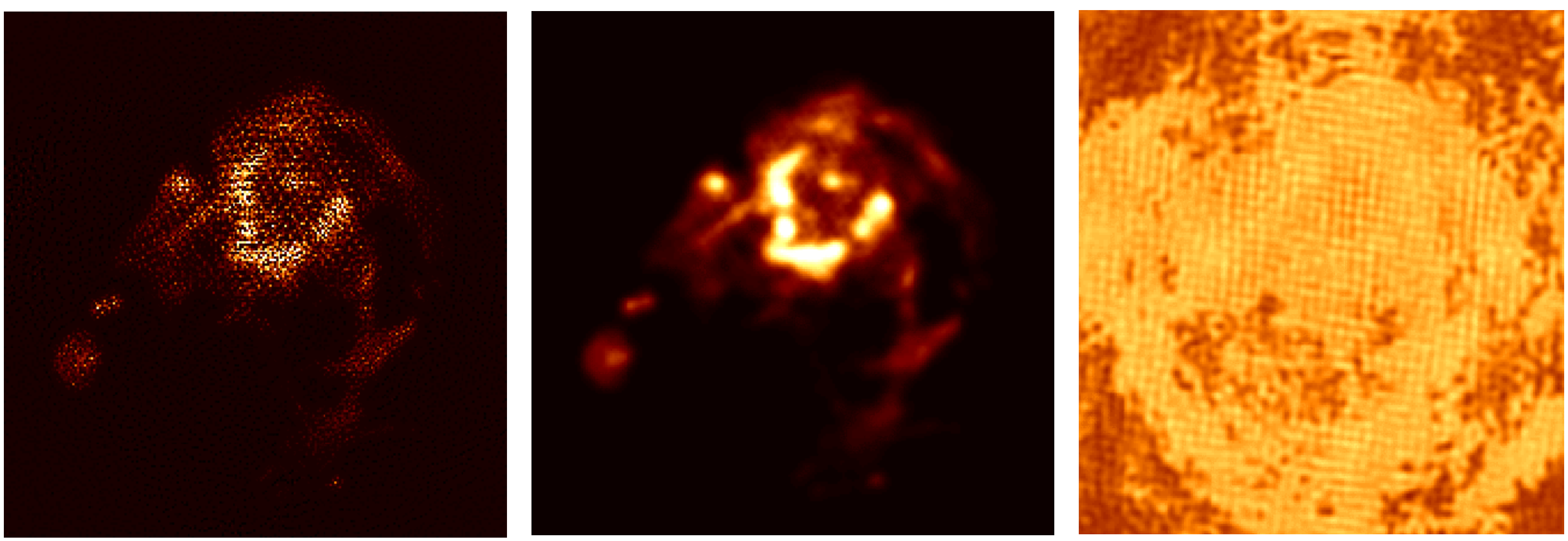
- Estimation of I^M is a non-linear inverse problem.

Deconvolution

- Currently used algorithms generate a sky-model in the pixel basis (Clean, MEM):

$$I^M(l) = \sum_k A_k \delta(l - l_k)$$

- Inherent coupling of the pixels due to the large scale structures in I^o is ignored. Leads to correlated residuals (large scale emission is poorly reconstructed).

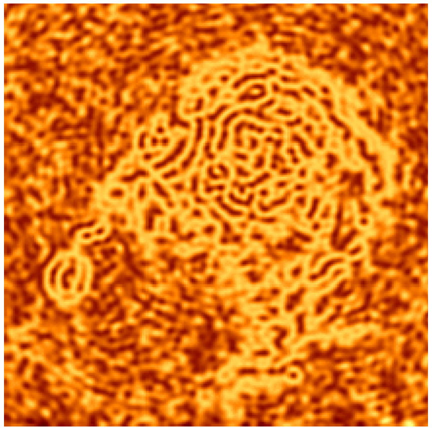


Scale sensitive deconvolution-I

- Pixel-to-pixel noise in the image is correlated at the scale of the resolution element

$$I^D = B * I^o + B * I^N \quad where\ I^N = FT\left[Visibility\ Noise\right]$$

- The scale of emission *fundamentally* separates signal (I^o) from the noise (I^N).
- Multi-Scale Clean (Cornwell & Holdaway, 2004, in prep)
  - Decompose the sky in a set of components at few scales $I^M = \sum_{k=1}^{N} A_k\, P(Scale_k)$
  - Computing cost is independent of N (no. of scales).
  - Assumes that the space of P_k's is orthogonal (ignore coupling of P_k's).
  - Large scale emission is better reconstructed.
  - Non-symmetric structures are not optimally reconstructed.
  - Difficult to incorporate frequency dependence (wide-band imaging).



Scale sensitive deconvolution-II

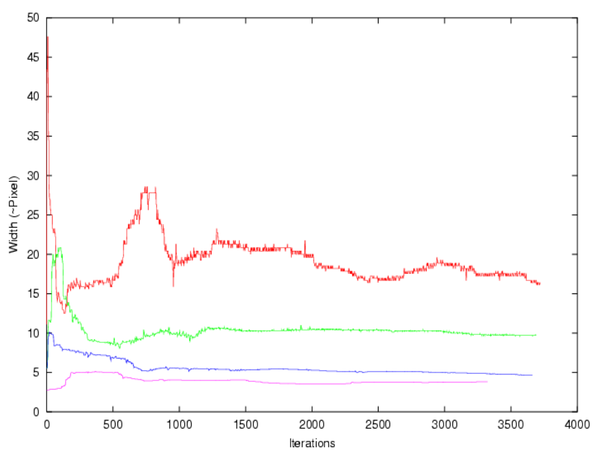
- Asp-Clean (Bhatnagar & Cornwell, A&A, in press)
  - Explicitly solve for the local scale, position and amplitude of the pixel model

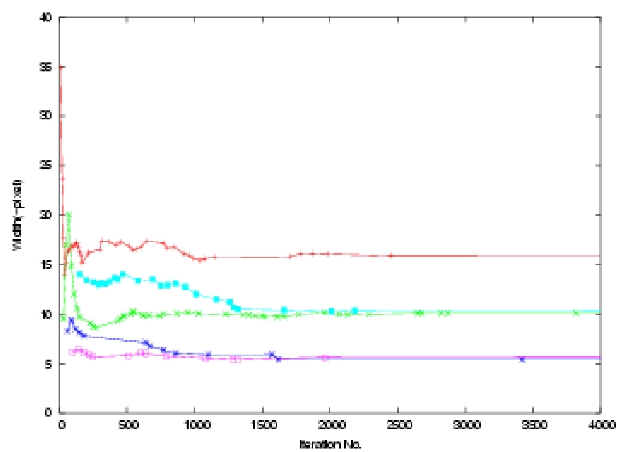
$$I^{M} = \sum_k A_k \, P(Scale_k, Pos_k)$$

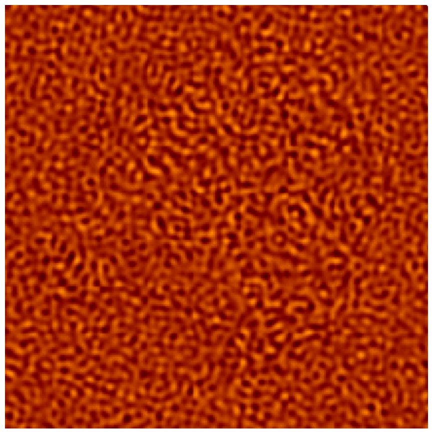
  - Large scale emission and asymmetric structures are better reconstructed
  - Computationally expensive: cost increases with the no. of components

$$V^{R} = V^{Obs} - B * P(Scale, Position)$$
$$\nabla V^{R} = B * \nabla P(Scale, Position)$$

  - Acceleration: Solve in a sub-space; adaptively determine the sub-space







Imaging and calibration errors

- Most data corruptions are separated in antenna based quantities

$$V_{ij}^{Obs}(\nu,t) = G_{ij}(\nu,t)\left(\int\int P_{ij}(\nu,t) I^{M}(l,m) e^{2\pi\iota(u_{ij}l + v_{ij}m)} dl\, dm\right)$$

Data Corruptions Sky

$G_{ij} = G_i G_j^*$ where G_i is the complex antenna based gains (direction independent)

$P_{ij} = P_i(l,m) P_j(l,m)$ where P_i is the image plane errors (direction dependent).

- Assuming $E_{ij}=1$, direction independent terms can be solved by minimizing: $\sum_{ij} |V^{Obs} - G_i G_j^* V^M|^2 \;\; w.r.t. \;\; G_i's$
- Direction dependent terms remain separable in the visibility domain, but more expensive to apply

$$V_{ij}^{Obs} = E_{ij} * V_{ij}^{M} \quad where \quad E_{ij} = E_i * E_j^* \;\; ; \;\; E_i = FT[P_i]$$

Correction for image plane effects

- V^{Obs} does not regularly sample the *(u,v)* plane. $FT[I^D]$ using FFT is on a regular grid. V^M is computed by re-sampling the grid using a Gridding Convolution Function (GCF)

$$V^M(u_{ij}, v_{ij}) = (GCF(u,v) * FT[I^D])(u_{ij}, v_{ij})$$

- Image plane effects can be applied by using E_{ij} as the GCF ==> potentially different GCF for each baseline!
- Pre-compute *all* E_{ij}'s (memory demanding)

OR

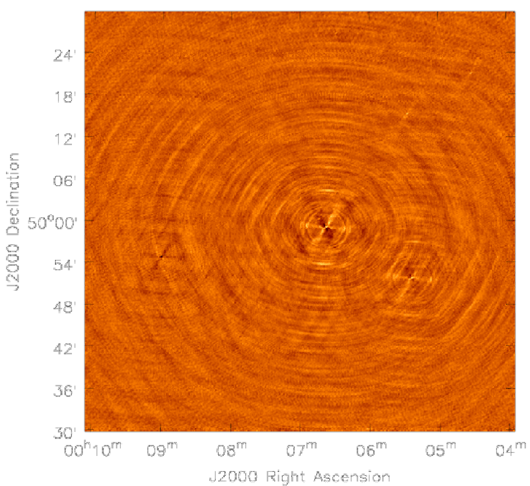
- $E_{ij} = E^o[1 + \Delta E_{ij}]$ compute E^o and parameterize ΔE_{ij}

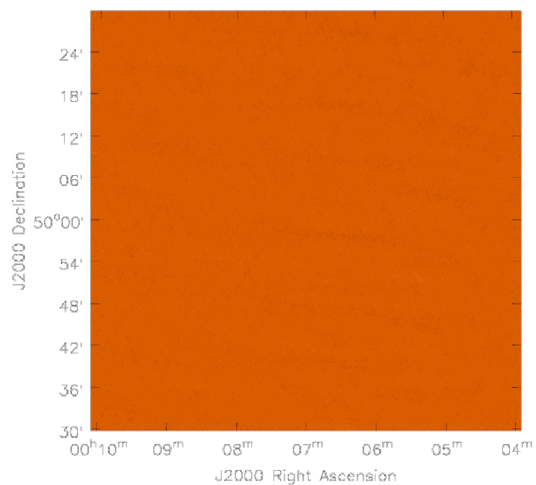
Pointing offset calibration

- $E_{ij}(l; l_i, l_j) = E^{o}_{ij}(l)\, e^{-(\frac{l_i + l_j}{2})}\, e^{-\pi \iota (l_i - l_j)}$ (l_i *is the pointing offset*)

  and minimize: $\sum_{ij} |V^{Obs}_{ij} - E_{ij} * V^{M}|^2$ $w.r.t.$ l_i

  (Bhatnagar et al., 2004, EVLA Memo 84)





- Compute $V^{R}_{ij} = V^{Obs}_{ij} - E_{ij} * FT[I^{D}]$ during image deconvolution.

Computing and I/O costs

- Increase in computing due to more sophisticated parameterization
  - Deconvolution: Fast evaluation of $B * \sum_k A_k P(Scale_k, Pos_k)$
  - Calibration: Fast evaluation of $E_{ij} * V^M$
- Cost of computing residual visibilities is dominated by I/O costs for large datasets (~200GB for EVLA)
  - Deconvolution: Approx. 20 access of the entire dataset
  - Calibration: Each trial step in the search accesses the entire dataset
- Solutions: Analytical approximations, caching, Parallel I/O,...
- Joint solver for deconvolution and calibration!
- Solutions: Sub-spaces, MCMC, Parallel computing,...

Wide band continuum imaging

- EVLA bandwidth ratio of 2:1
  - $V(u_{ij}, v_{ij}) = \sum_{\nu_k} V(u_{ij}, v_{ij}; \nu_k) = \sum_{\nu_k} P_{ij}(\nu_k) FT[I^D(\nu_k)]$

  Sky emission, the Primary Beams, etc. become a function of frequency.

  Ideas: Apply PB effects during predict. Sky model parameterized in frequency as well (MFS; Asp-Clean, direction dependent calibration)

- Other primary beam effects
  - Rotating non-symmetric PBs, polarized PBs, polarization squint
- Combining Scale sensitive + frequency sensitive deconvolution with image plane corrections