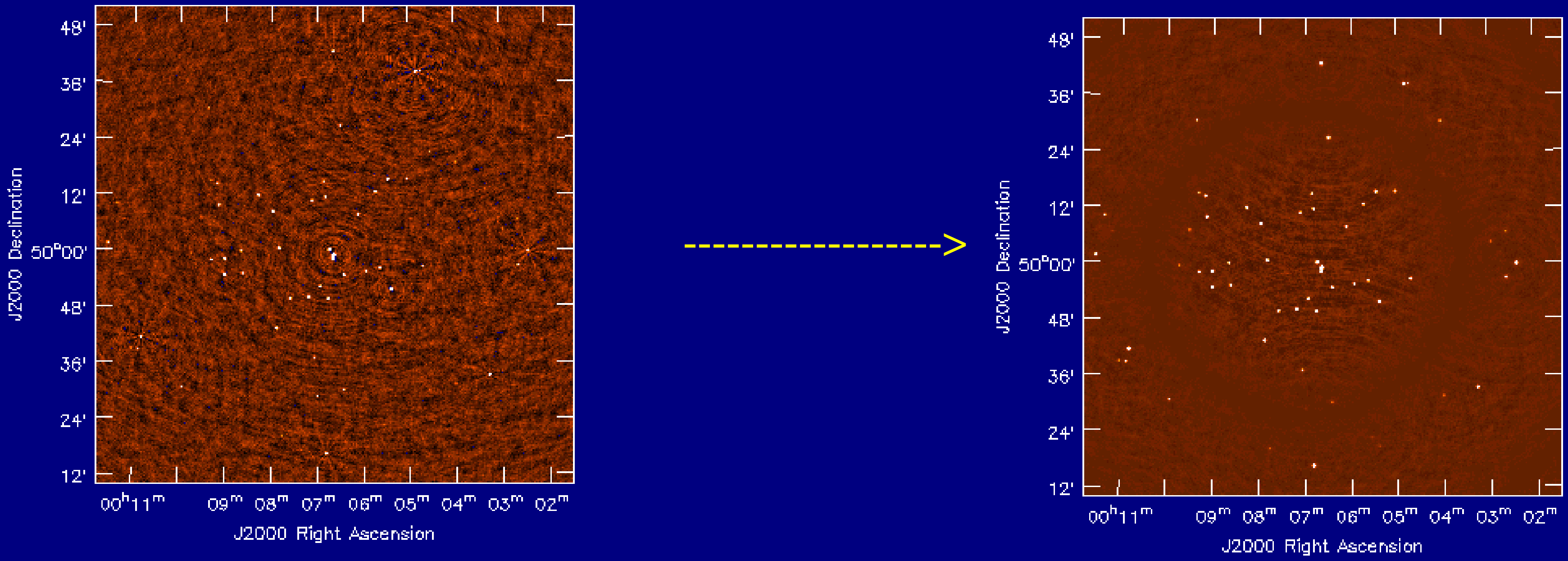
High Dynamic Range Imaging



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The Measurement Equation

- Generic Measurement Equation:

$$V_{ij}^{Obs}(\nu)=J_{ij}(\nu,t)\int J_{ij}^{S}(S,\nu,t)\ \ I(S)\ \ e^{\iota S.B_{ij}}\ \ dS$$

Data **Corruptions** **Sky**

$J_{ij}=J_i \otimes J_j^*$ **:direction independent corruptions.**

$J_{ij}^s=J_i^s \otimes J_j^{s*}$ **:image plane errors (direction dependent).**

- $V_{ij}^{Obs}=J_{ij}W_{ij}E_{ij}V^o$ where $E_{ij}=F\,J_{ij}^s F^T$
  - J_{ij} is multiplicative in the Fourier domain
  - J^s_{ij} is multiplicative in the Image domain *only if* $J^s_i = J^s_j$

Hierarchy of algorithms

- Unknowns of the problem: J_{ij}, J^{s}_{ij}, *and* I^{M}.
- $J^{s}_{i} = J^{s}_{j}$ and *independent* of time
  - Keep J^{s}_{ij}, and I^{M} fixed and solve for J_{i} : Calibration
  - Keep J_{ij} and J^{s}_{ij} fixed and solve for I^{M} : Image reconstruction

  Imaging and calibration as orthogonal operations
- $J^{s}_{i}(t) = J^{s}_{j}(t)$ (Poln. squint, PB correction, etc.) (EVLA Memo 62)
  - J^{s}_{ij} is multiplicative in the image plane for appropriate ∇T

$$\Delta I^{D} \propto \Re\left[\sum_{n} J^{s^{T}}(n\nabla T)\sum_{ij}[\Delta V_{ij}(\nabla T)e^{\iota S.B_{ij}}]\right]$$

Hierarchy of algorithms

- $J^s_i(t) \neq J^s_j(t)$ (Pointing offsets, PB variations, etc.)
  - Image plane effects not known a-priori
    - Solve for E_{ij} for a given I^M:

$$min : |V^{Obs}_{ij} - E_{ij}.V^M_{ij}|^2 \text{ w.r.t. parameterized } E_{ij}$$

E.g. Pointing selfcal (EVLA Memo 84, paper in preparation)

  - Correct for J^s_{ij} during image deconvolution
    - Use pre-computed visibility plane filters for effects known a-priori (e.g. w-term) (EVLA Memo 67, paper in press)
    - Compute the filters on-the-fly. (In preparation)
- Simultaneous solver for J_{ij}, J^s_{ij}, *and* I^M*!!*

General structure of imaging algorithms

- $\vec{V}^{Obs} = A\vec{I}^{o}$ A: The Measurement Matrix

- Solve the normal equation $A^T[\vec{V} - A\vec{I}] = 0$
  - Compute the approx. update direction: $\Delta I^D = A^T[\Delta V^R]$
  - Update the model: $I_i^M = I_{i-1}^M + \alpha\, max(\Delta I^D)$

    (Steepest Descent minimization: Clean algorithm)
  - Compute residuals: $\vec{V} - A\overrightarrow{I^M}$
- Transform implemented using FFT: $V^M = C[A\vec{I}^M]$
- Incorporate the image plane effects in the transform operator: Forward/inverse transforms: EA and $A^T E^T$
- Iteratively solve the modified normal equation:

$$B[\vec{V} - A\vec{I}] = 0 \quad \text{where } B \sim A^T$$

Motivation

- Single pointing L-Band observations limited due to pointing/PB asymmetries ~10-20microJy/beam.
  - Next generation telescopes hope to do >10x better
- Mosaicking dynamic range limited by pointing errors.

- Use of pixel basis for image representation: deconvolution errors: > 10mircoJy/beam

- Frequency dependence of the sky & the instrument: 10-15microJy/beam

Pieces of the puzzle

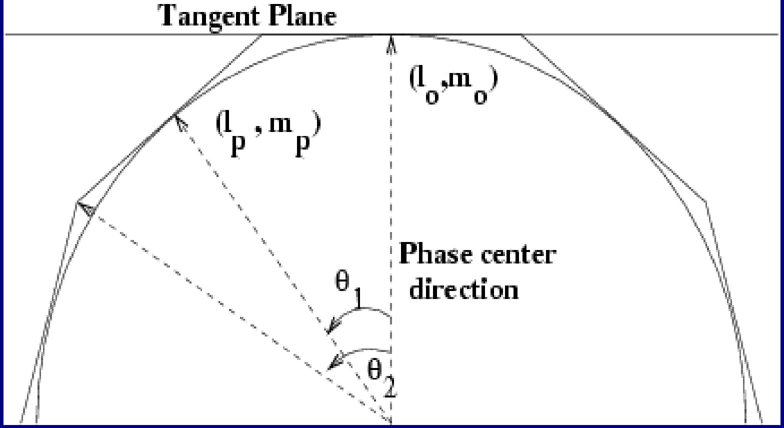
- Efficient algorithms to correct for image plane effects
  - Approximate inverse transform (Vis -> Image)
  - Forward transform (accurate)
- Decomposition of the sky in an appropriate basis
  - Frequency sensitive (next talk)
- Solvers for the “unknown” image plane effects
  - As expensive as imaging!
- Larger computers! (More memory, CPU power, fast I/O)

Known direction dependent effects

- Non-coplanar baselines

$$V(u,v,w)=\iint I(l,m)G(l,m,w)\mathrm{e}^{2\pi\iota(ul+vm)}\frac{dl\,dm}{\sqrt{1-l^2-m^2}}$$

- Traditional approach: Faceting



- W-projection: Visibility filtering (>10x faster)

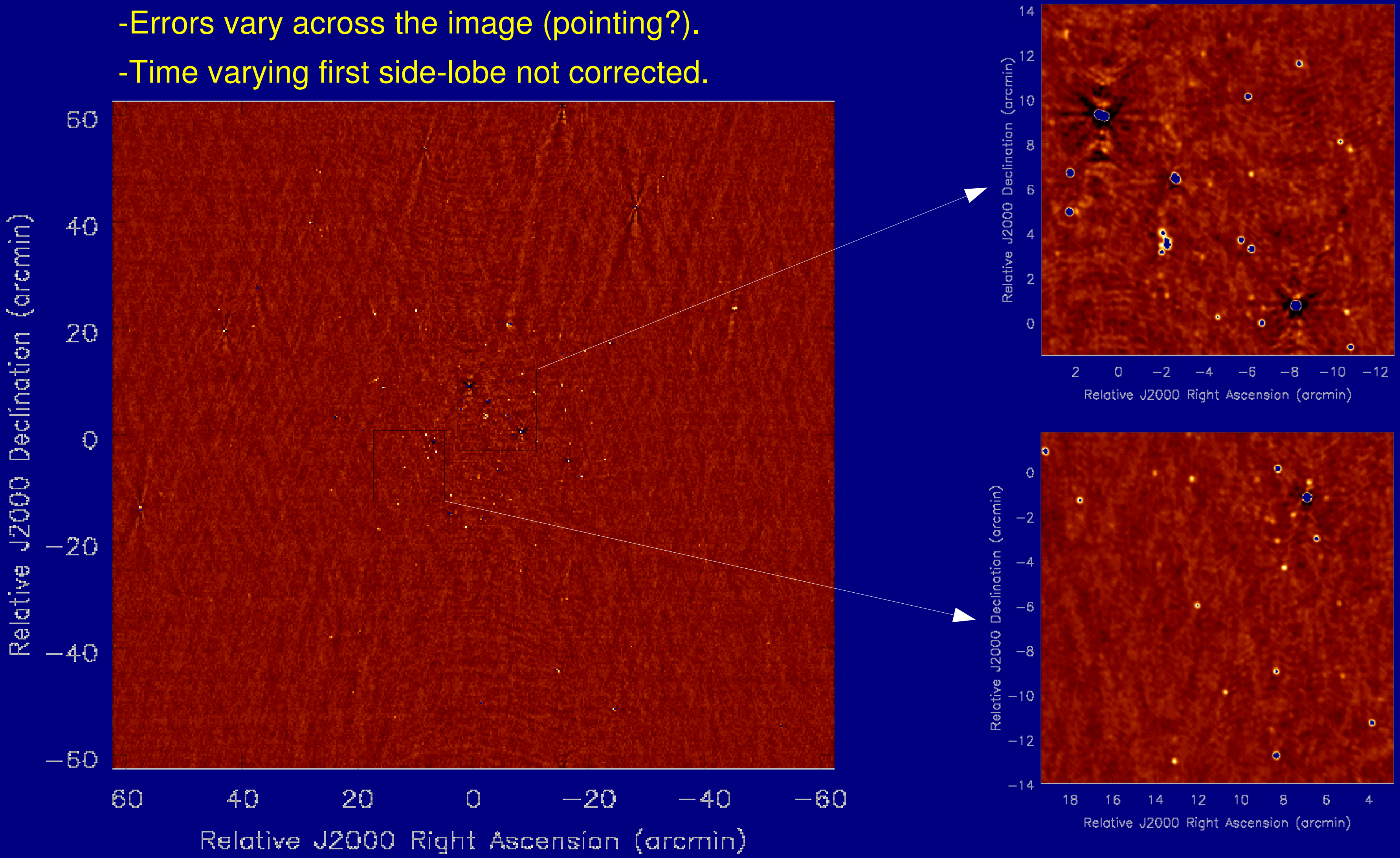
W-projection: Example

L-Band, VLA C-array, ~40hr. integration (Fomalont et al.)

-RMS: ~15microJy, Peak: 40mJy

-Errors vary across the image (pointing?).

-Time varying first side-lobe not corrected.



Measured direction dependent effects

- E_{ij} as a function of direction is measured a-priori

$$V_{ij}^{M} = \boldsymbol{E}_{\boldsymbol{ij}}[\boldsymbol{A}\,\boldsymbol{I}^{M}]_{ij} \quad where \quad E_{ij}(l_i, l_j, u_{ij}; p_i, p_j)$$

- Aperture Function: E_i *different* for each poln. product *pq*

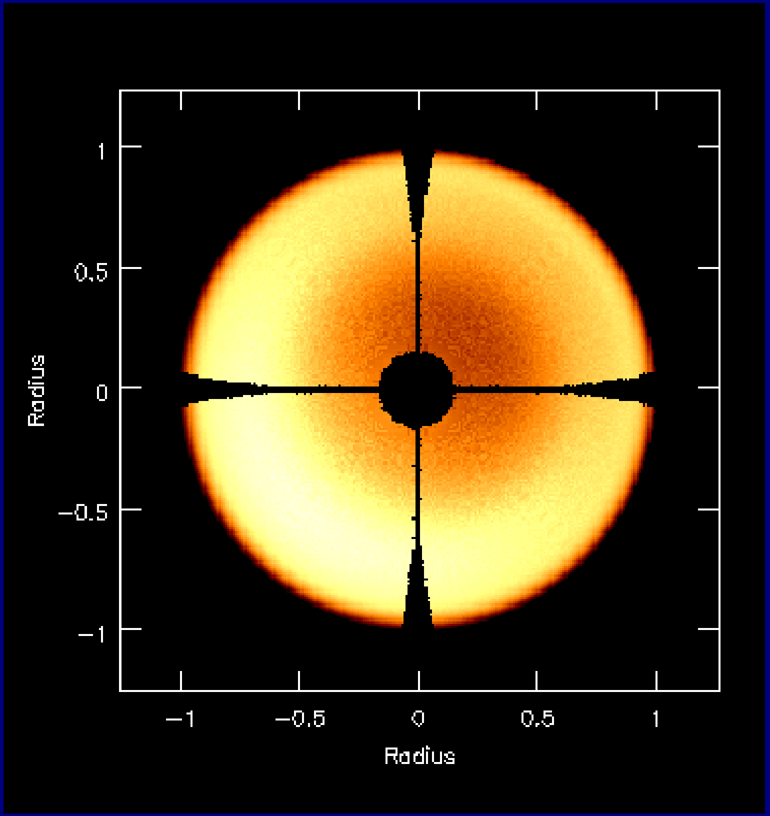
$$E_{ij}^{pq} = E^{pq^{o}} f(\phi_i - \phi_j)\, e^{\iota(\phi_i + \phi_j)}$$

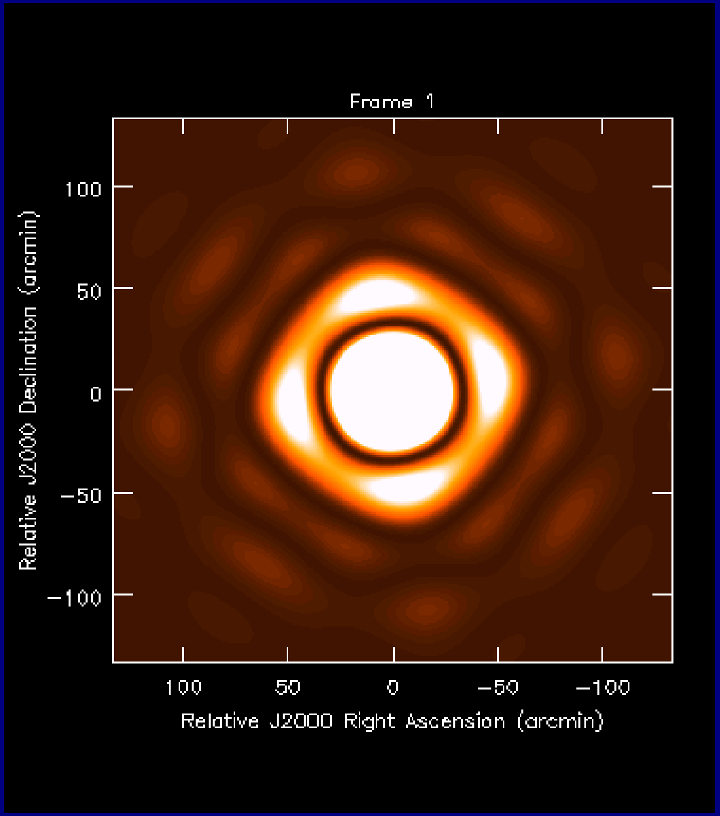
Needs a solver: Pointing SelfCal

- Asymmetric Primary Beams:

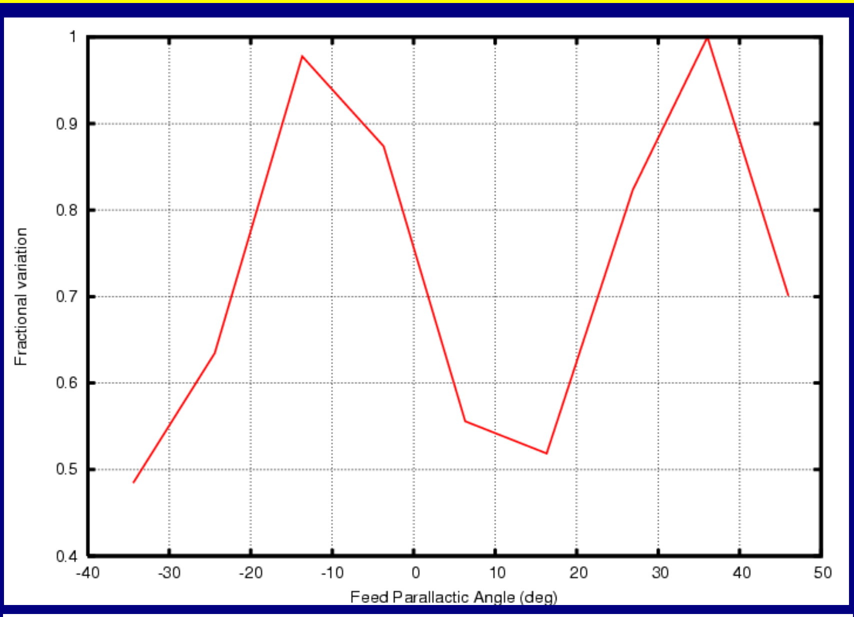
Aperture function

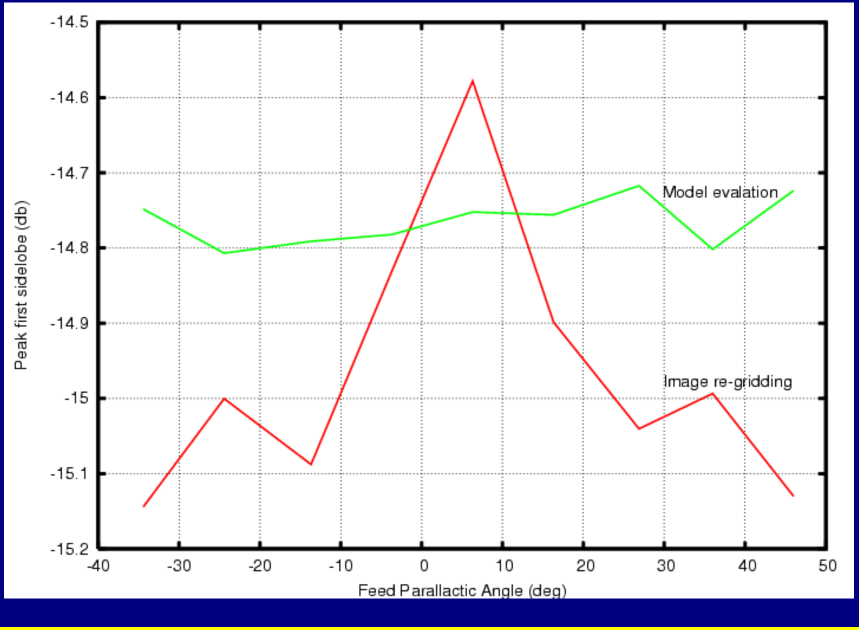
Model for VLA antenna illumination at L-band (courtesy W.Brisken)





Variable side-lobe gain

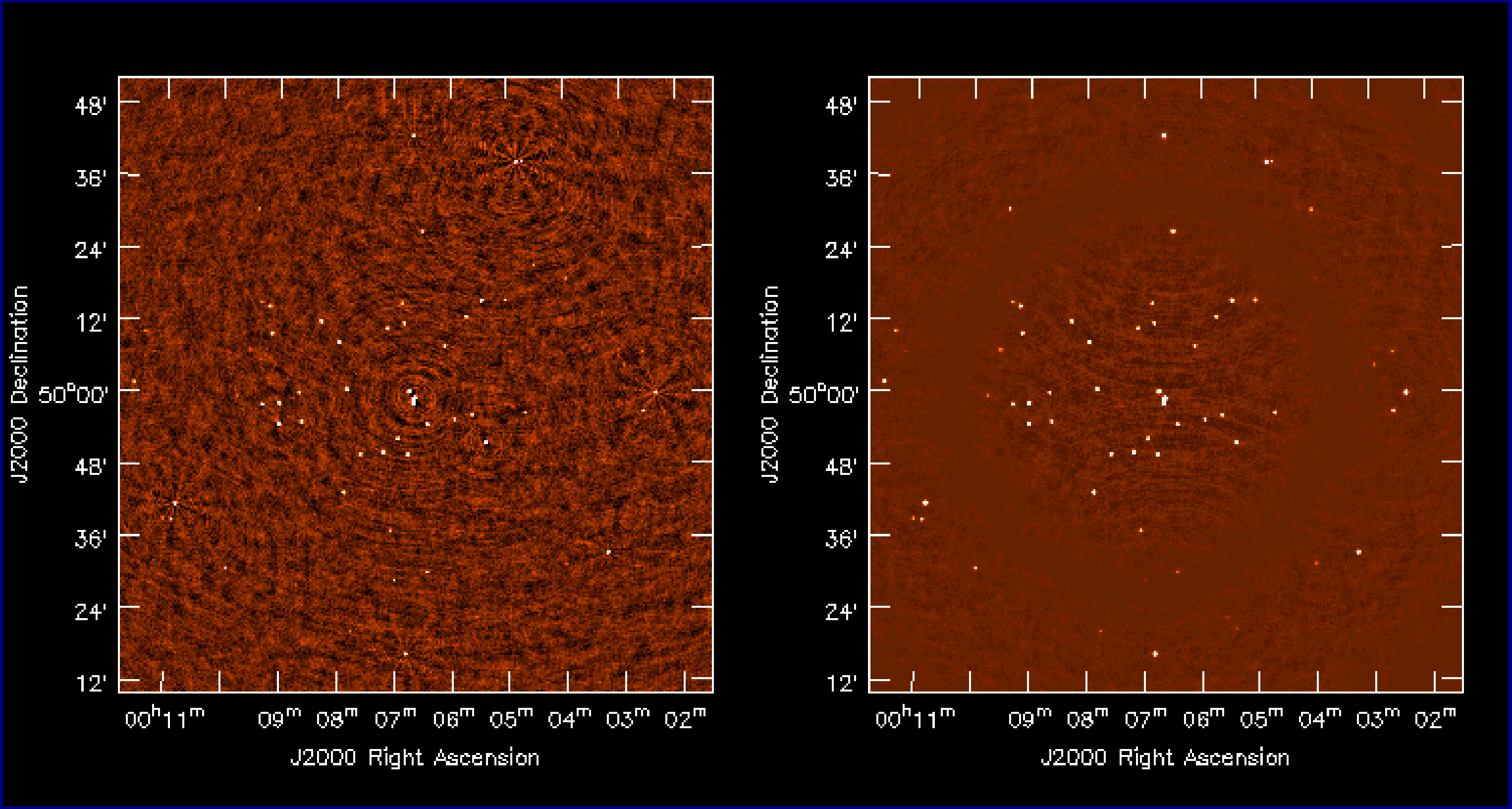




PB-projection: Stokes-I

Stokes-I imaging with and without PB effects

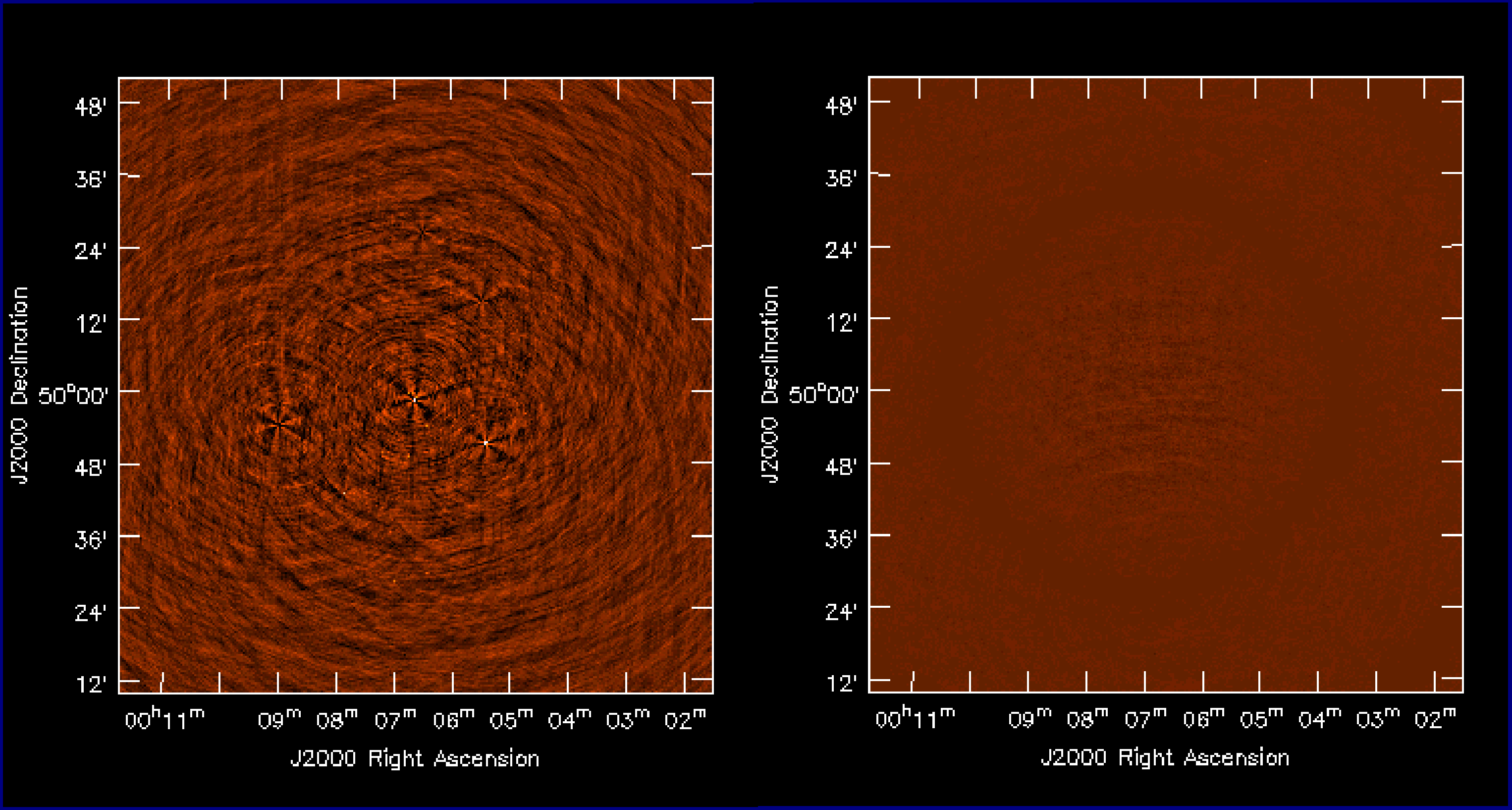
(Polarization squint, Pointing offsets, PB rotation)



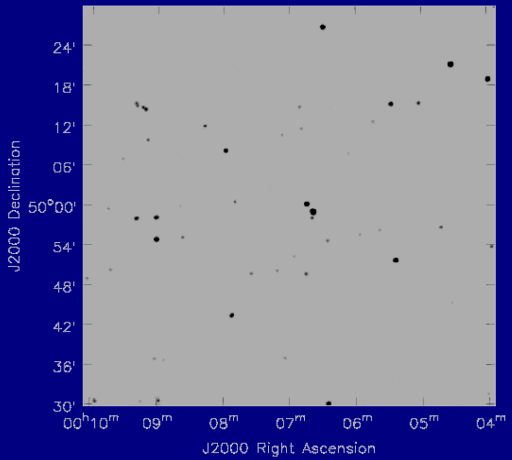
PB-Projection: Stokes-V

Stokes-V imaging with and without PB effects

(Polarization squint, Pointing offsets, PB rotation)



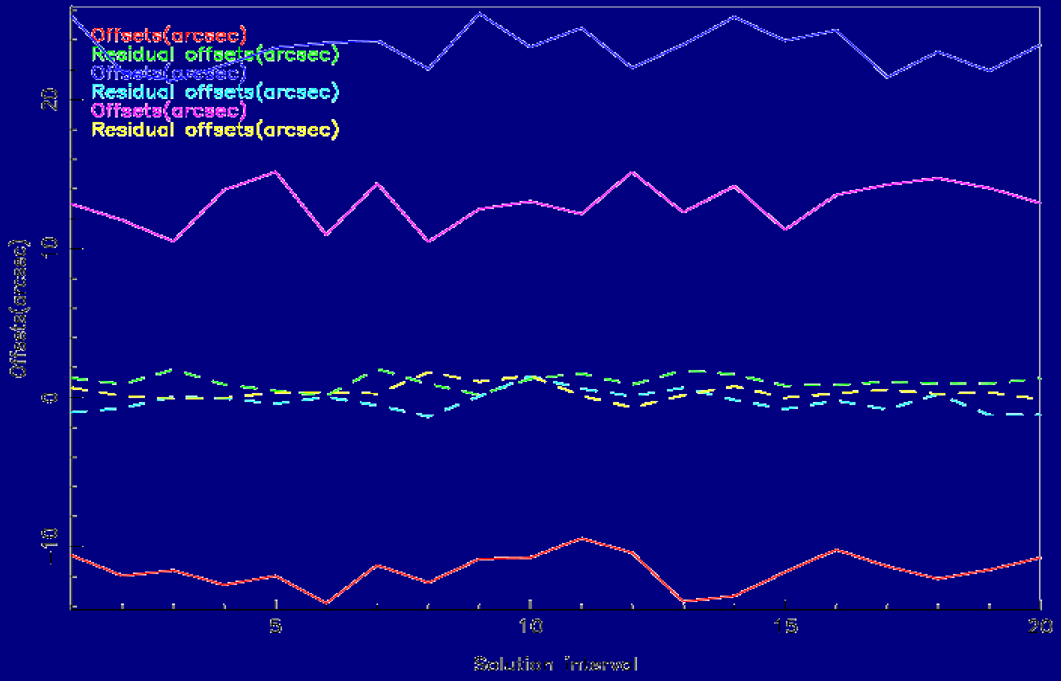
Pointing correction



Model image using 59 sources from NVSS.
Flux range ~2-200 mJy

Details in EVLA Memo 84 (2004)

http://www.aoc.nrao.edu/evla/geninfo/memoseries/evlamemo84.pdf

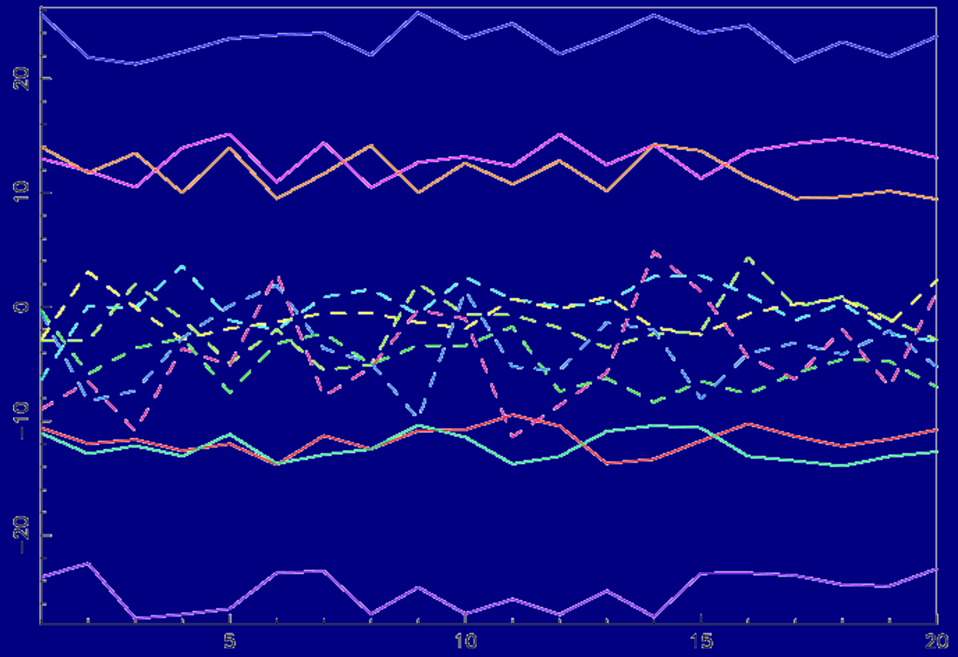


Continuous lines: Typical antenna pointing offsets for VLA as a function of time (Mean between +/- 25” and RMS of 5”).

Dashed lines: Residual pointing errors. RMS ~1”.

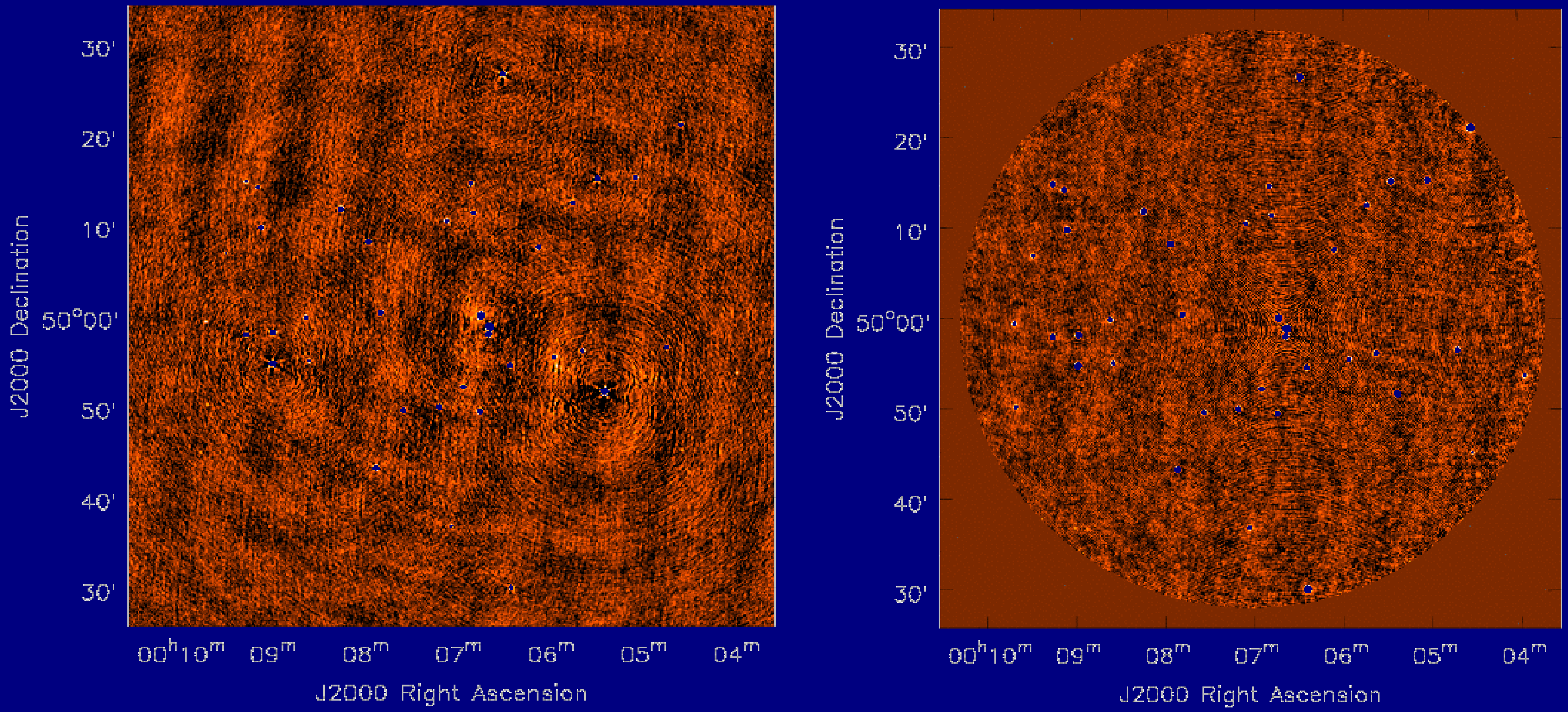
Pointing SelfCal

- Model image: deconvolved using entire data
- Pixelated model image



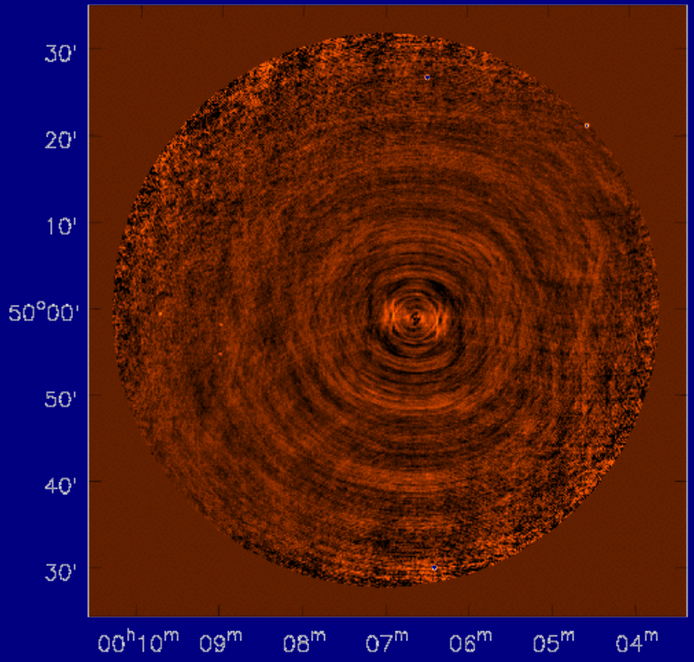
Pointing SelfCal

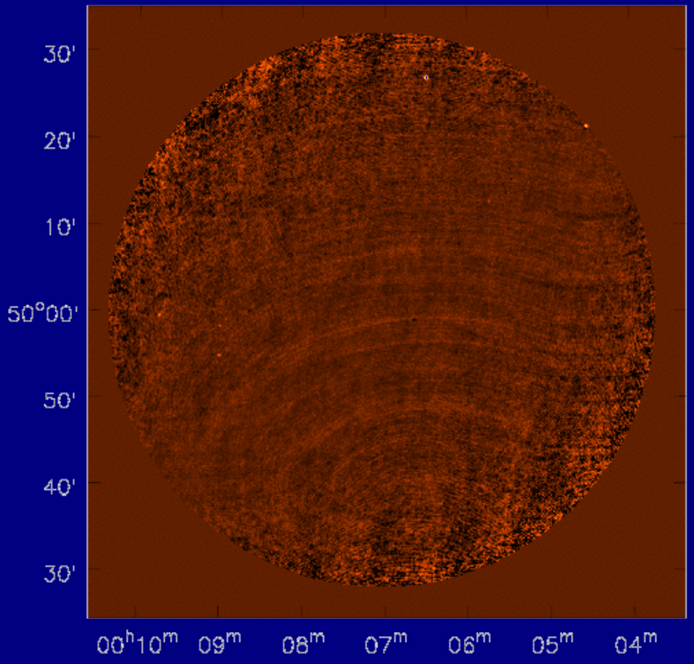
- Stokes-I imaging: Before and after pointing correction



Pointing SelfCal

- Stokes-V imaging: Need to use component imaging?





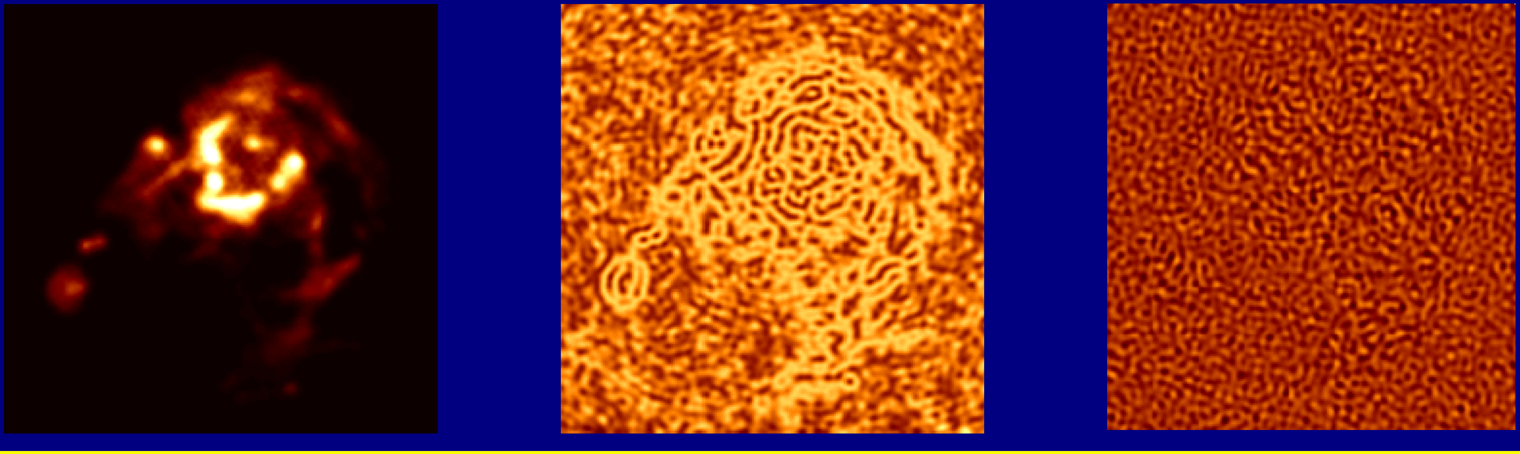
Scale sensitive imaging: Asp-Clean

- Pixel-to-pixel noise in the image is correlated

$$I^D = PI^o + PI^N \quad \text{where} \quad P = \text{Beam Matrix}$$

- The scale of emission *fundamentally* separates signal *(I^o)* from the noise *(I^N).*

- Asp-Clean (Bhatnagar & Cornwell, A&A,2004)
  - Search for local scale, amplitude and position



Progress so far

- Eliminate aperture function re-gridding [Done]
- SelfCal <-> imaging iterations [Testing]
- Component image model (Asp-Clean + PB-Projection + W-Projection) [In progress]

- Is current deep L-band imaging pointing-error limited?
- Mosaicking dynamic range limited by pointing errors?

- Wide-band imaging (EVLA memo #?, next talk)
  - Use PB-projection to correct for PB scaling
  - MSF extensions: Freq. sensitive image plane modeling (Component based imaging)

Computing and I/O costs

- Significant increase in run-time due to more sophisticated parameterization
  - Deconvolution: Fast evaluation of $B * \sum_k A_k P(Scale_k, Pos_k)$
    - E.g. limits the use of MCMC approach (http://www.aoc.nrao.edu/~rurvashi)
  - Calibration: Fast evaluation of $E_{ij} * V_{ij}$
- Cost of computing residual visibilities is dominated by I/O costs for large datasets (~200GB for EVLA)
  - Deconvolution: Approx. 20 access of the entire dataset
  - Calibration: Each trial step in the search accesses the entire dataset
- Solutions: Analytical approximations, caching, Parallel computing and I/O,...

References

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3. Bhatnagar & Cornwell, 2004, Scale sensitive deconvolution of interferometric images, A&A, 426, 747-754, 2004 (astro-ph/0407225)
4. Cornwell, Golap & Bhatnagar, 2003, W-Projection: A new algorithm for non-coplanar baselines, Tech. rep., EVLA Memo 67
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6. Bhatnagar, Cornwell & Golap, 2004, Solving for the antenna based pointing errors, Tech. rep., EVLA Memo 84
7. Bhatnagar, Cornwell & Golap, in prep., Image plane corrections

http://www.aoc.nrao.edu/~sbhatnag/talks.html