# RFI factorization: potential problems

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## 1 Formulation

Let $V_{ij}^s$ and $g_i$ be the visibility due to the target source and the antenna based complex gain (assumed to be constant across the delay beam) respectively, and $E_i^k$ be the electric field vector at antenna $i$ from the $k^{th}$ RF interferer with a complex gain $a_i^k$. The Measurement Equation for the baseline $i$-$j$, with an integration time constant of $T$ in the presence of ground based fixed RFI sources can then be written as:

$$V_{ij} = g_i g_j^\star V_{ij}^s + \left\langle \left[ \sum_k a_i^k E_i^k e^{\iota u_i^k l_i^k} \right] \left[ \sum_k a_j^k E_j^k e^{\iota u_j^k l_j^k} \right]^\star \right\rangle_T \tag{1}$$

where $u_i^k$ is the antenna co-ordinate projected in the direction of the $k^{th}$ RFI source and $l_i^k$ is the angular co-ordinate of the $k^{th}$ RFI source (with respect to the direction of the phase tracking center) as seen by the $i^{th}$ antenna.

## 2 Few points to note

1. $u_i^k$ in the above equation is independent of time (Hour angle) since the interferer is fixed in the antenna frame (fixed on the Earth). It is however different for the various RFI sources. Also, the phase at each baseline will be different for the various RFI sources - implying that they will not *all* appear at the North Pole. The uv-coverage for each RF interferer will be significantly different from that for the sources around the phase center.

2. $l_i^k$ in the above equation is the direction cosine of the angle between the phase center and the RFI source. For large separation from the

phase center, this should produce similar effects as the wide field of view case.

3. If $E_i^k$ from different RFI sources has coherence time greater than $T$, cross terms will survive (problem of imaging coherent sources?). **For a channel width of $\Delta\nu$,** $1/\Delta\nu << T <<$ Max. fringe rate **corresponding to the RFI sources?!**

Assuming that $\left\langle E_i^n E_j^{m\star} \right\rangle_T = 0$, the RFI term in Eq. 1 will be:

$$V_{ij}^R = \left\langle \sum_k a_i^k a_j^{k\star} A^k e^{\iota\left(u_i^k l_i^k - u_j^k l_j^k\right)} \right\rangle_T \quad (2)$$

where $A^k$ is the amplitude of the $k^{th}$ interferer (assumed to be unresolved).

# 3 Potential problems

1. The exponent in Eq.2 cannot be cast as the Fourier kernel unless $l_i^k = l_j^k$ (**near field problems cannot be ignored?**).

2. Under the far field assumption, Eq. 2 reduces to

$$V_{ij}^R = \left\langle \sum_k a_i^k a_j^{k\star} A^k e^{\iota u_{ij}^k l^k} \right\rangle_T \quad (3)$$

This now looks like the Measurement Equation for a resolved source, but with a fixed uv-coverage as a function of time for each component. Even if all the above assumptions were to be true and Eq. 3 usable, each RFI component will potentially have a (slightly?) different PSF. This needs to be taken into account while building the RFI-model. For $a_i^k = 1\ \forall i, k$, the various RFI sources will line up along an arc at the North Pole (since they will have similar elevation from the phase tracking center and roughly along an arc in the azimuthal direction). This probably mimics a time-smeared source. In which case, $a_i^k$s cannot be factorized correctly into antenna based quantities without the knowledge of the RFI source distribution.

3. Knowing the RFI distribution, which is not impossible, *and* if the condition $\left\langle E_i^n E_j^{m\star} \right\rangle_T = 0$ holds, one can plug-in Eq. 2 for the RFI

term in Eq. 1 and solve for $g_i$s and $a_i^k$s if the number of interferers is small compared to the number of antennas.

Is this then an extreme and slightly modified case of angle dependent gain problem (non-isoplanatic calibration - modified due to the fact that the "sources" needing angle dependent gains are fixed in the array reference frame)?