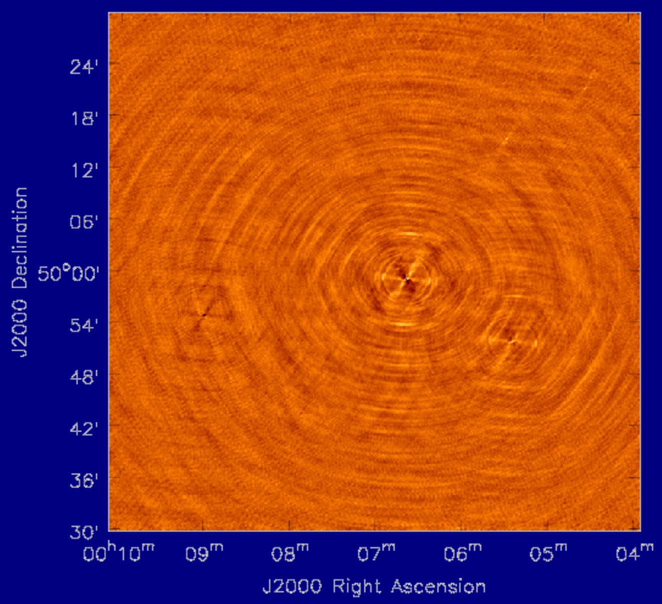
Image plane corrections



Pointing Correction

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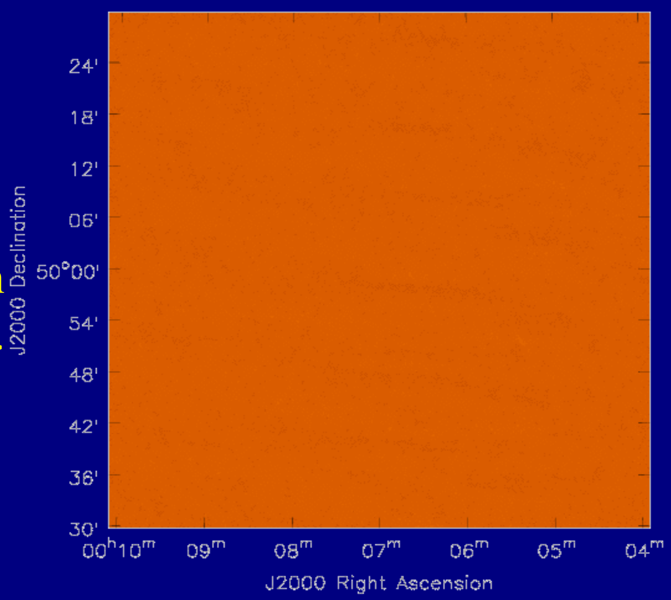


S. Bhatnagar (NRAO, Socorro)

T.J.Cornwell (ATNF), K.Golap (NRAO, Socorro)

The Measurement Equation

- Generic Measurement Equation:

$$V_{ij}^{Obs}(\nu) = J_{ij}(\nu, t) \int J_{ij}^{S}(S, \nu, t) I^{o}(S) e^{\iota S.B_{ij}} dS$$

Data **Corruptions** **Sky**

***J_{ij}* are the direction independent corruptions.**

***J^{S}_{ij}* are the image plane errors (direction dependent).**

- $V_{ij}^{Obs} = J_{ij}.\left[E_{ij} * V_{ij}^{o}\right]$ where E_{ij} = FT[J^{S}_{ij}]. Unknowns: J_{ij}, J^{S}_{ij}, *and* I^{M}.
- SelfCal: Given I^{M} *and* knowing E_{ij} $\quad min: |V_{ij}^{Obs} - J_{ij}.V_{ij}^{M}|^2$ w.r.t. J_i
- Deconvolution: Given J_{ij} *and* E_{ij}, $\quad min: |J_{ij}^{-1} V_{ij}^{Obs} - V^{M}|^2$ w.r.t. I^{M}

Visibility correction

- In the absence of direction dependent errors $V_{ij}^{Obs} = J_{ij} V_{ij}^{o}$
- Observed data is corrected as $V_{ij}^{Corr} = J_{ij}^{-1} V_{ij}^{Obs}$
- Imaging is done using the corrected visibilities

  Imaging and calibration are decoupled

- Antenna independent direction dependent corruptions (e.g. Primary beam effects) can be written as multiplicative terms in the image domain. $V_{ij}^{M} = FT[J^{S} I^{M}]_{ij}$

- This can be used in an iterative deconvolution scheme to make I^M which is corrected for J^S (VLA Poln. squint correction, mosaicking, etc.)

$$\vec{V}^{Obs} = E * \vec{V}^{M} \quad \text{and} \quad \Delta I^{D} \propto \Re \sum_{ij} [J_{ij}^{S^{*T}} \Delta V_{ij} e^{\iota S.B_{ij}}]$$

Visibility correction

- For effects which are separable as antenna based effects

$$J^S_{ij} = J^S_i \otimes J^{S*}_j$$

- When $J^S_i \neq J^S_j$, effects of J^S_{ij} cannot be reduced to the convolution of the visibility plane by a single function.

$$V^{Obs}_{ij} = E_{ij} * V^o_{ij}$$

- Visibility domain filters vary across the visibility plane.

Visibility inversion

$$\vec{V}^{Obs} = E[A\vec{I}^{o}]$$

V,I : The visibility and image vectors

A: The Measurement Matrix

E: The direction dependent effect in the Fourier plane

- To use FFT for Fourier transforming, re-sampling is done by convolutional gridding $V_{ij}^{M} = \left| C[A\vec{I}^{M}] \right| (u_{ij}, v_{ij})$
- Major cycle: $\Delta \vec{I}^{D} = A^{T}[C^{T}\vec{V}^{R}]$
- Minor cycle: $\vec{I}_{i}^{M} = \vec{I}_{i-1}^{M} + \alpha \Delta \vec{I}^{D}$
- Direction dependent effects can be incorporated in imaging by using *EA* as the transform operator

Pieces of the puzzle

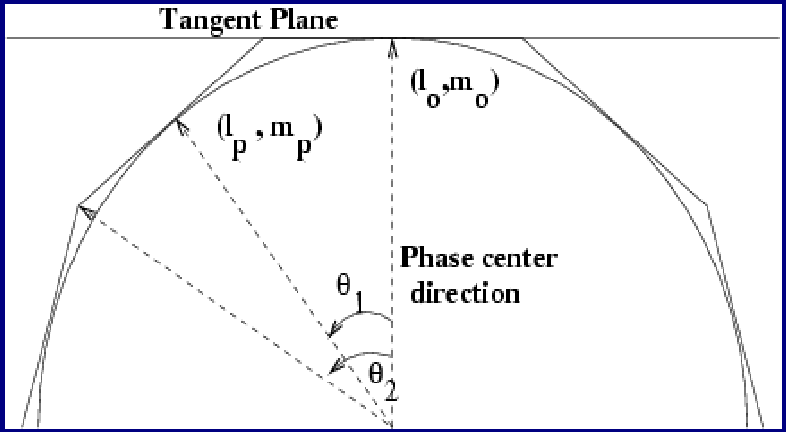
- Efficient algorithms to include image plane effects during deconvolution
  - Forward transform (Vis -> Image) (no less accurate than the PSF used for deconvolution)
  - Inverse transform (accurate)
- Decomposition of the sky in an appropriate basis
- Solvers for the “unknown” image plane effects
  - Larger computers! (More memory, CPU power)

Known direction dependent effects

- Non-coplanar baselines

$$V(u,v,w)=\iint I(l,m)\, e^{2\pi\iota\left(ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right)} \frac{dl\,dm}{\sqrt{1-l^2-m^2}}$$

- Traditional approach: Faceting



$$V_{ij}^{Obs}(u,v,w)=E_{ij}*V_{ij}^{M}(u,v,w=0) \quad where \quad E_{ij}=FT\left[e^{2\pi\iota w\left(\sqrt{1-l^2-m^2}\right)}\right]$$

- W-projection: Visibility filtering (>10x faster)

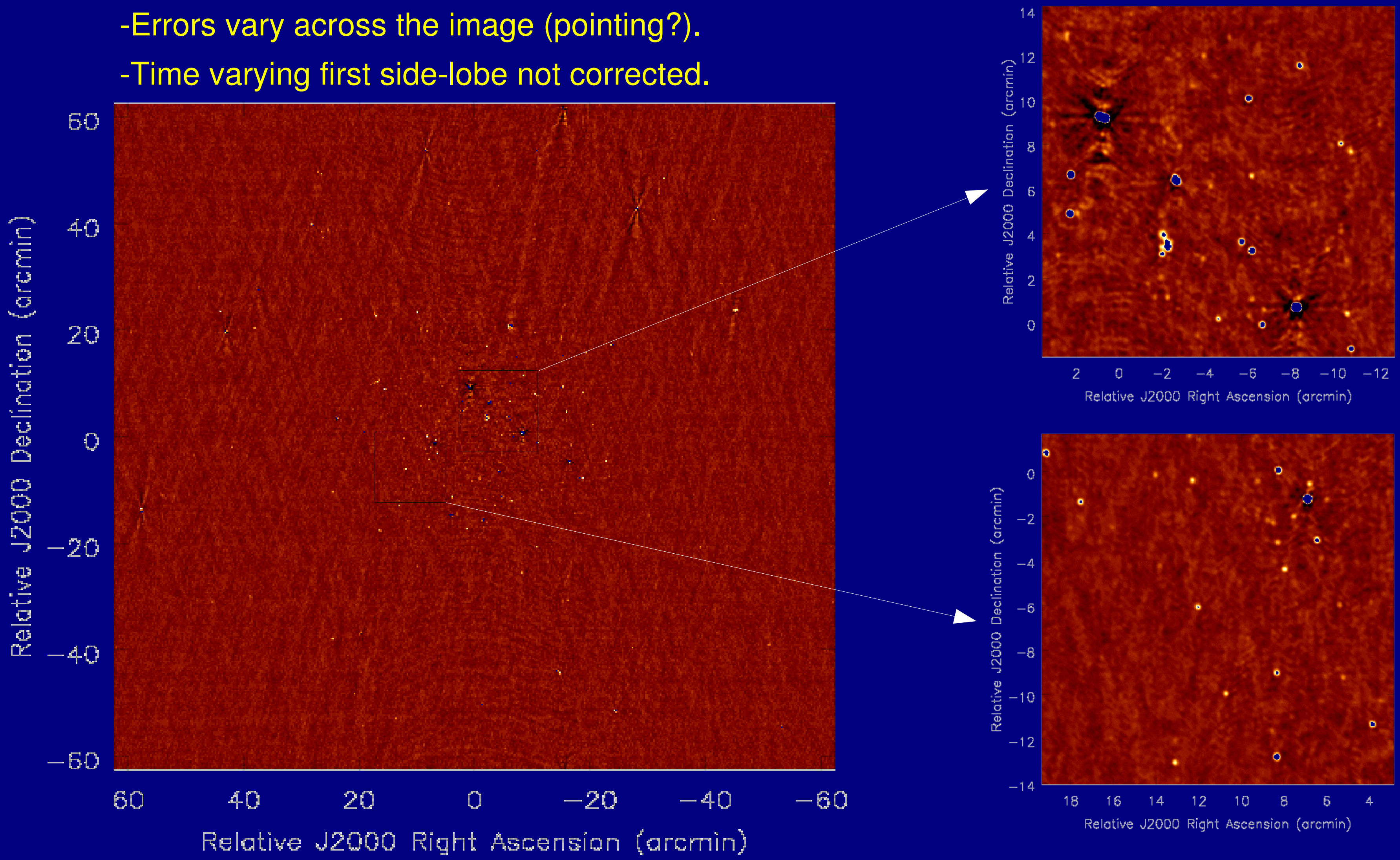
W-projection: Example

L-Band, VLA C-array, ~40hr. integration (Fomalont et al.)

-RMS: ~15microJy, Peak: 40mJy

-Errors vary across the image (pointing?).

-Time varying first side-lobe not corrected.



Measured direction dependent effects

- E_{ij} as a function of direction is measured a priori

$$V_{ij}^{M} = [\boldsymbol{E}_{ij} * [\boldsymbol{A}\,\boldsymbol{I}^{M}]_{ij}] \quad where \quad E_{ij}(l_i, l_j, u_{ij}; p_i, p_j)$$

- Aperture Function: E_{ij} *separate* for each poln. product *pq*

$$E_{ij}^{pq} = E^{pq^{o}} f(\phi_i - \phi_j) e^{\iota(\phi_i + \phi_j)}$$

Needs a solver: Pointing SelfCal

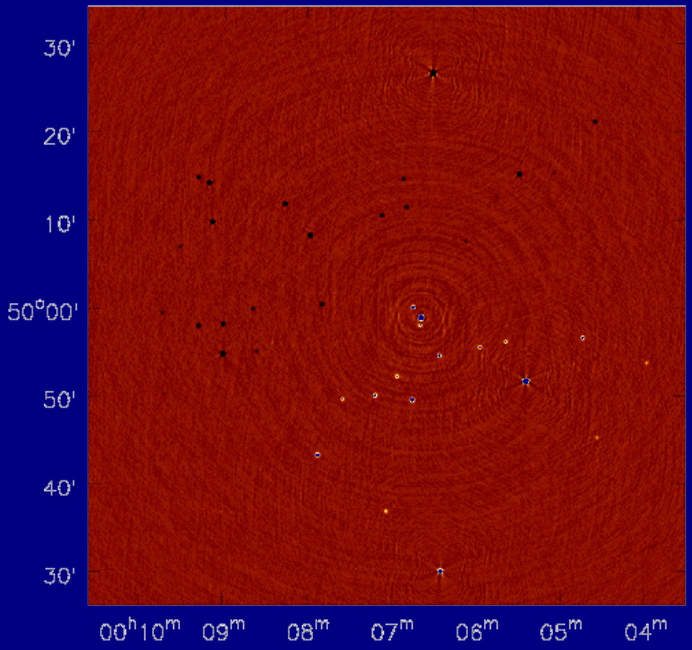
- Asymmetric Primary Beams:

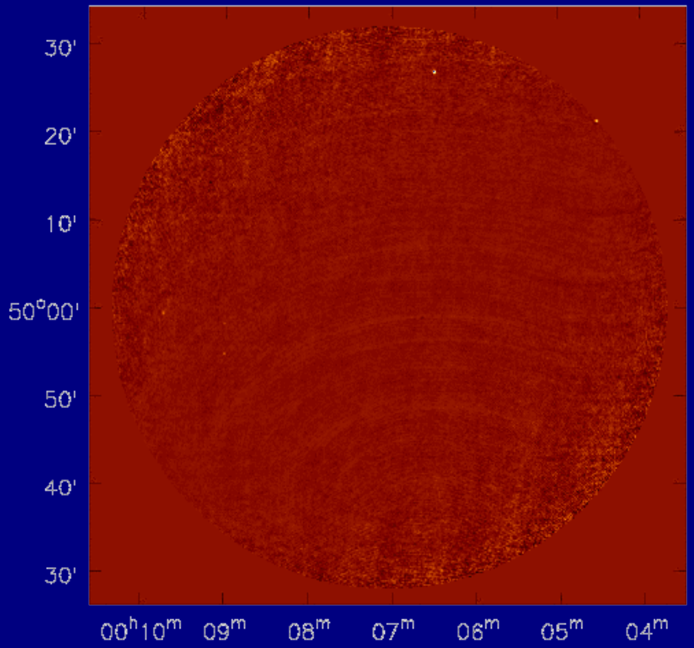
$$E_{ij} = E_i^{o} * E_j^{o} \quad where \quad E_i^{o} = \boldsymbol{FT}[Measured\, PB_i]$$

PB-projection

Full beam polarimetry: Squint corrected Stokes-V imaging

Typical NVSS field: Peak 190mJy



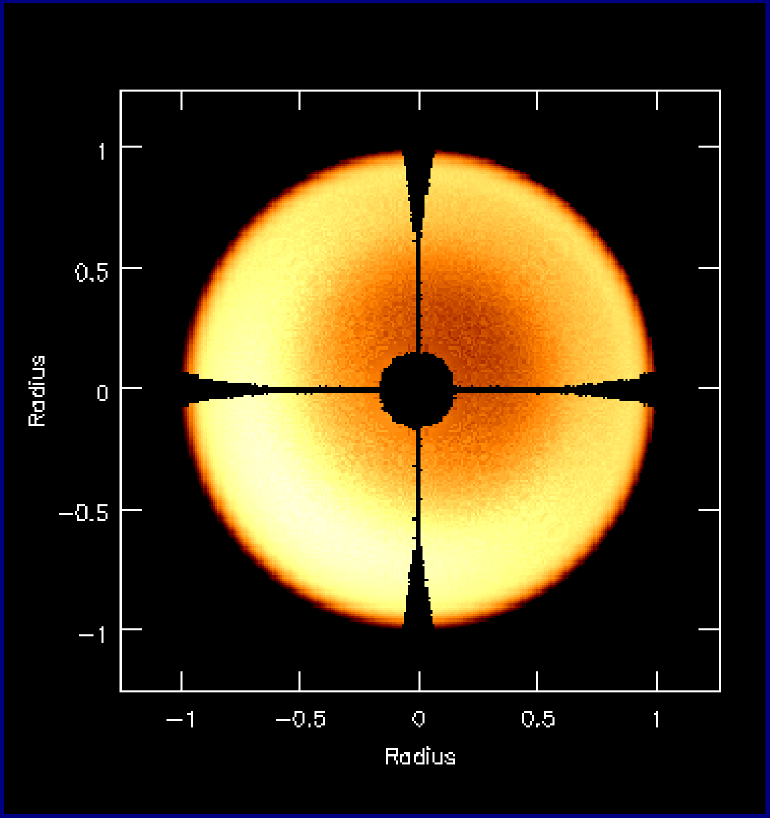


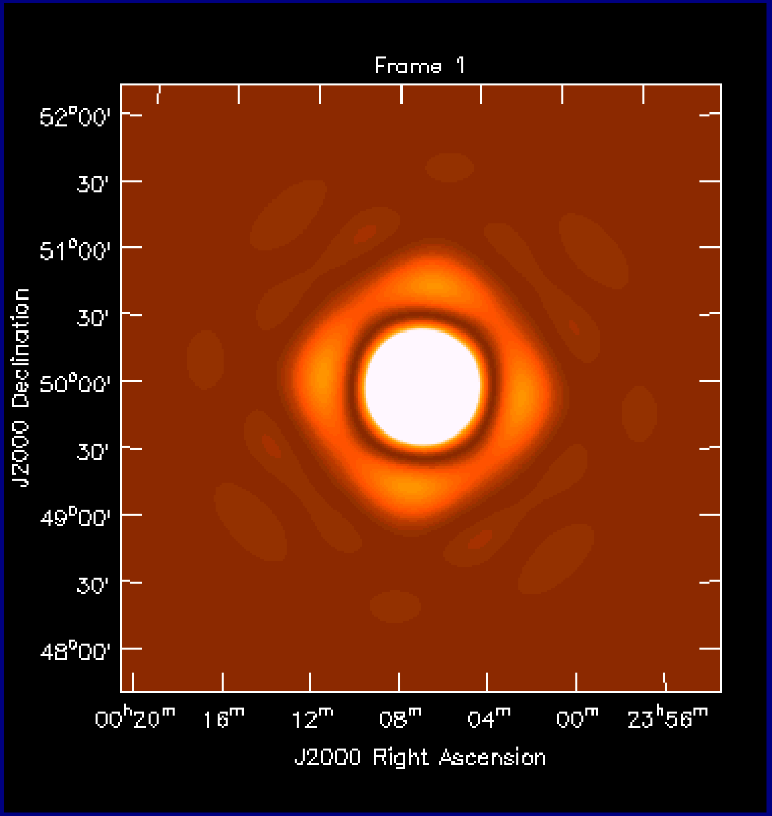
Visibility filters computed for 15° PA increments.

Azimuthally symmetric Primary Beams.

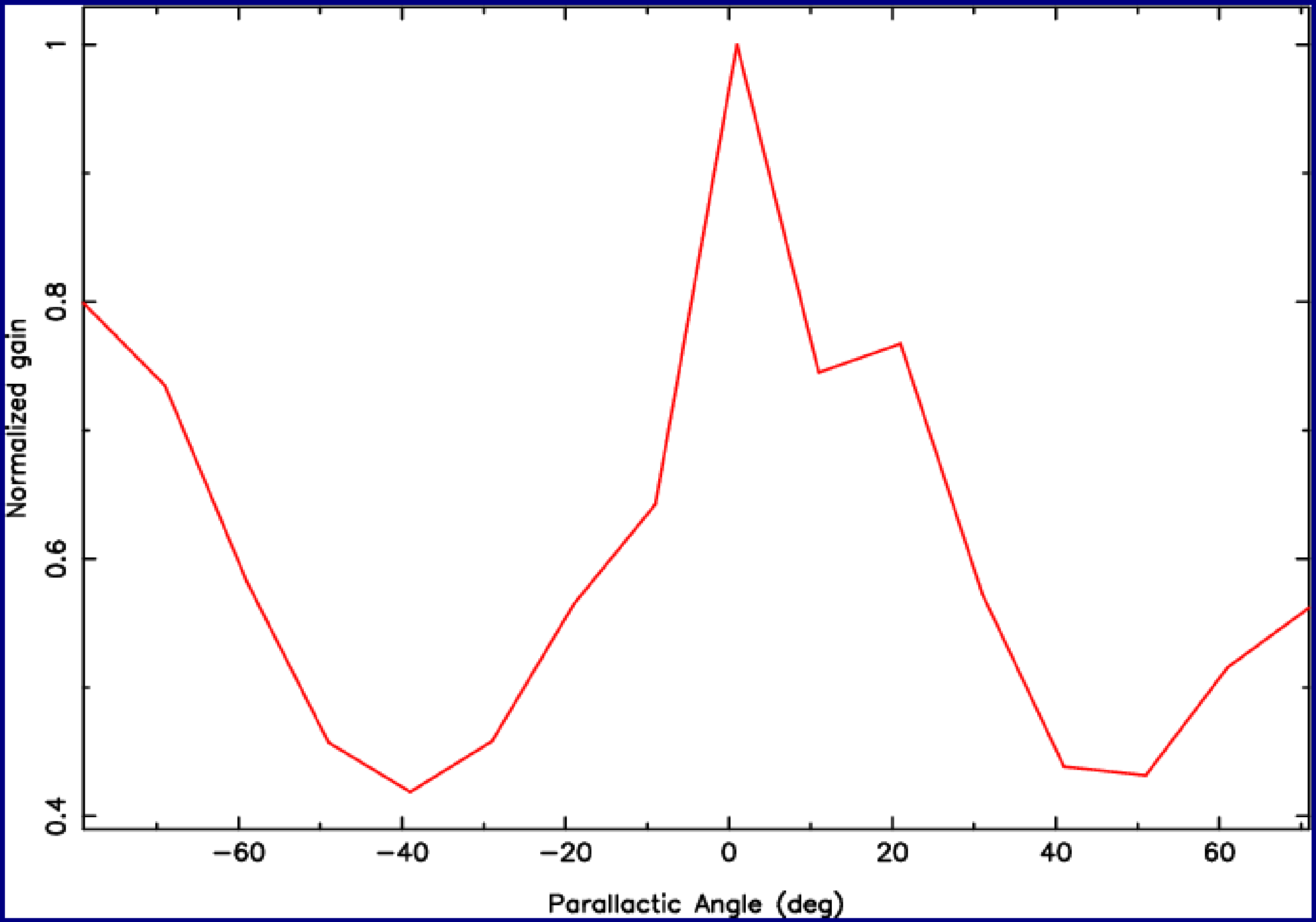
Aperture function

Model for VLA antenna illumination at L-band
(courtesy W.Brisken)





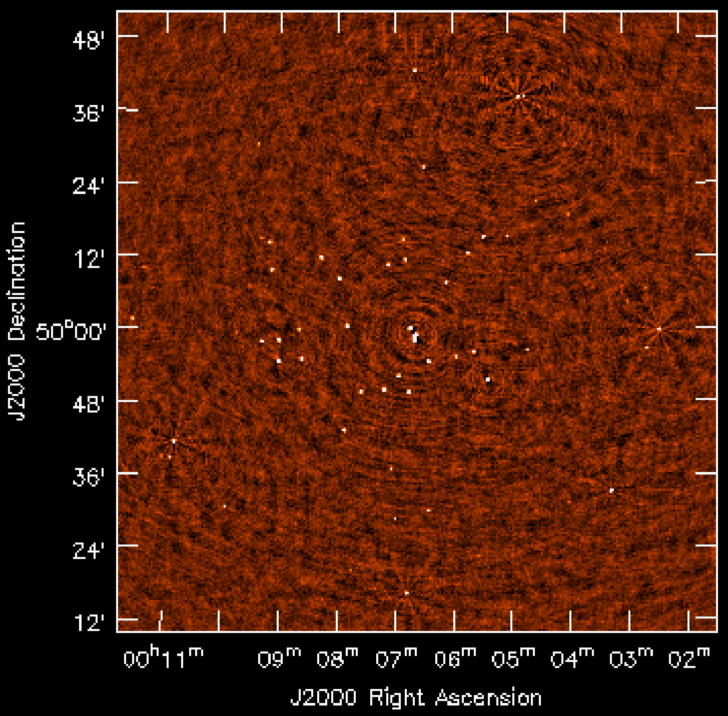
Variable side-lobe gain

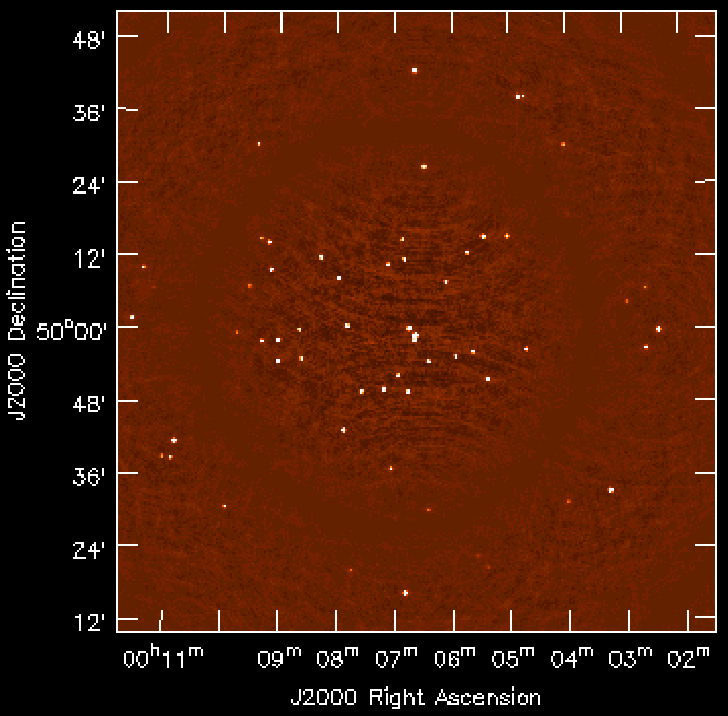


PB-projection: Stokes-I

Stokes-I imaging with and without PB effects

(Polarization squint, Pointing offsets, PB rotation)

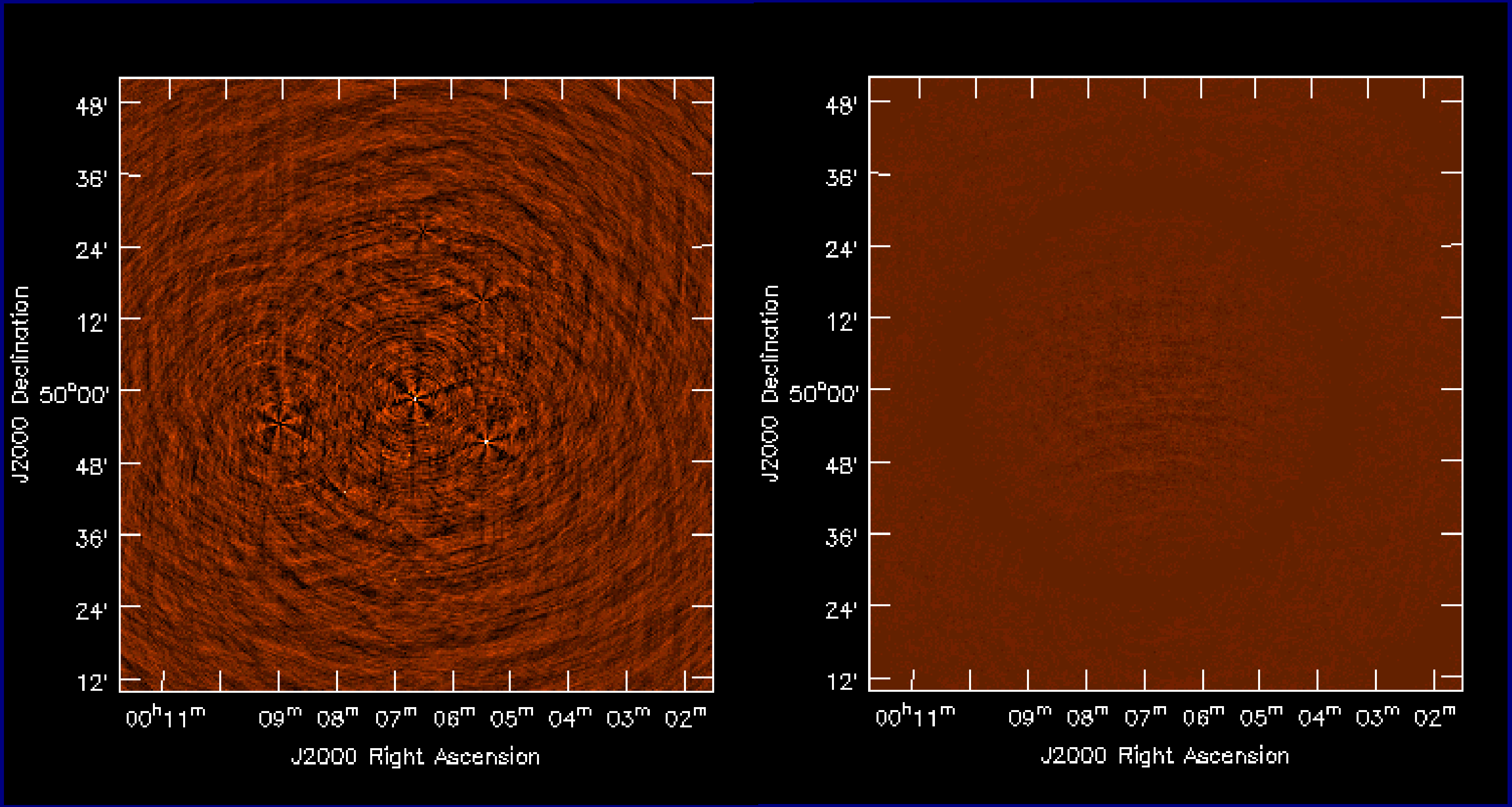




PB-Projection: Stokes-V

Stokes-V imaging with and without PB effects

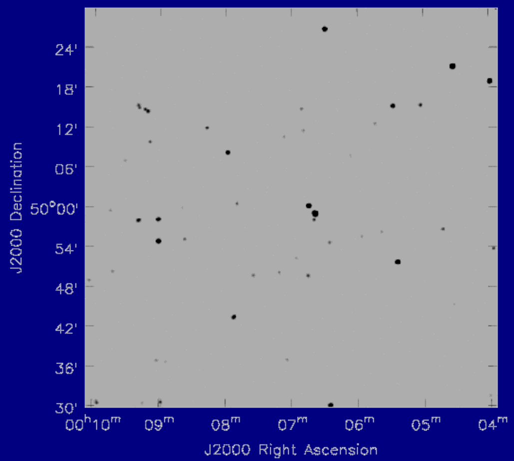
(Polarization squint, Pointing offsets, PB rotation)



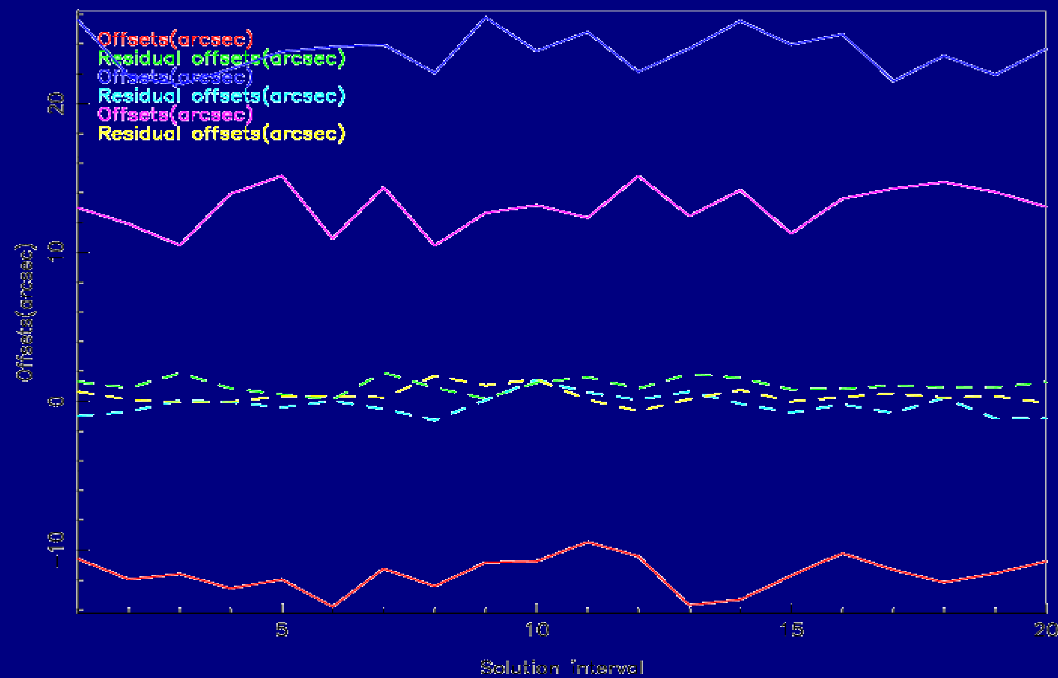
Pointing Solver: Motivation

- Single pointing L-Band observations limited due to pointing ~10microJy/beam.
  - EVLA L-band sensitivity: 1microJy/beam
- Mosaicking dynamic range limited by pointing errors.
  - Significant fraction of ALMA observation will be mosaicking observations.
- Significantly increased computing:
  - Each iteration involves expensive visibility prediction

Pointing correction



Model image using 59 sources from NVSS.
Flux range ~2-200 mJy



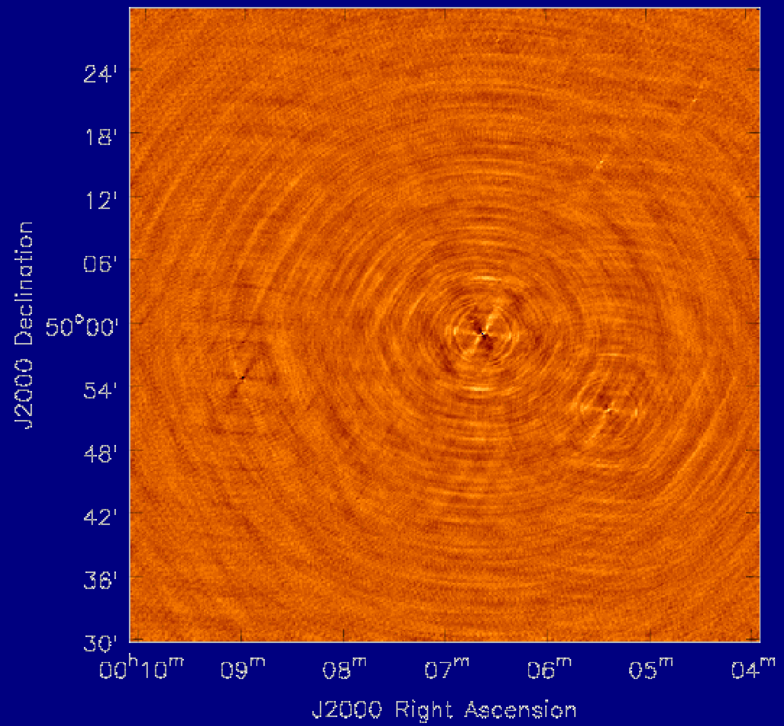
Continuous lines: Typical antenna pointing offsets for VLA as a function of time (Mean between +/- 25” and RMS of 5”).

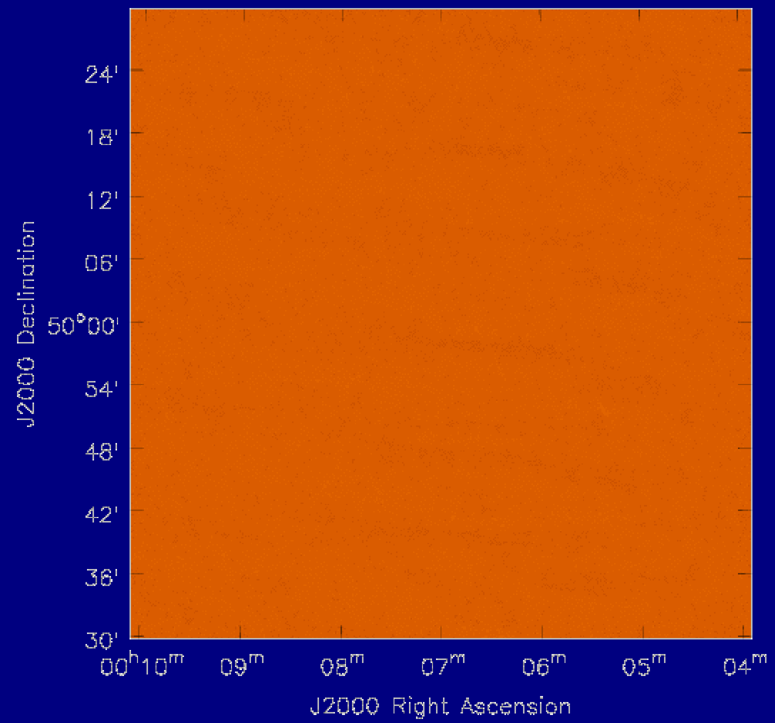
Dashed lines: Residual pointing errors. RMS ~1”.

Pointing correction

Details in EVLA Memo 84 (2004)

http://www.aoc.nrao.edu/evla/geninfo/memoseries/evlamemo84.pdf





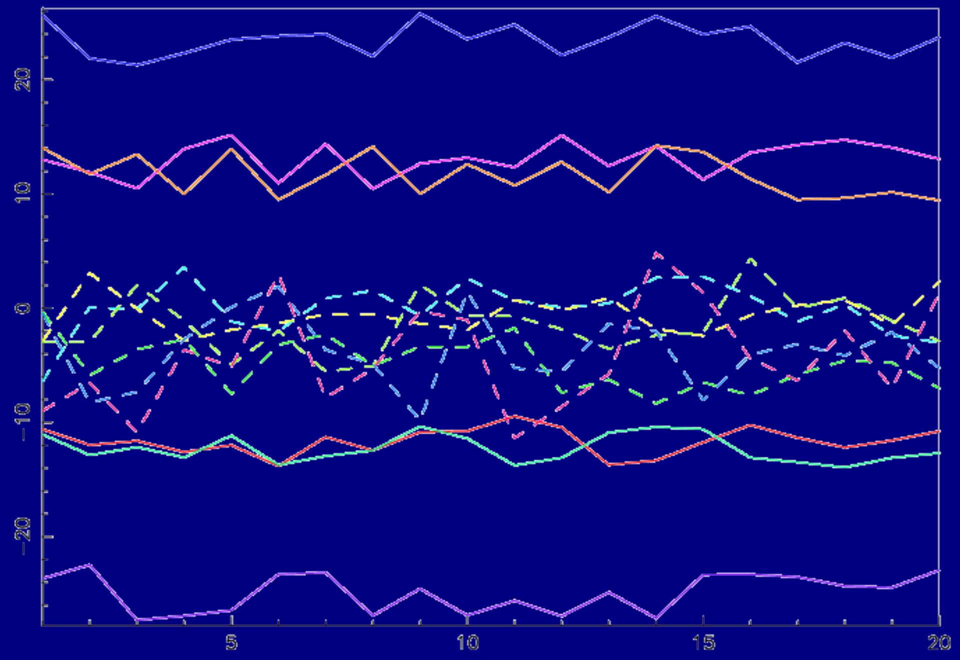
RMS: 15microJy/beam vs. 1microJy/beam

Peak: 250microJy vs. 5 microJy

Model image as components

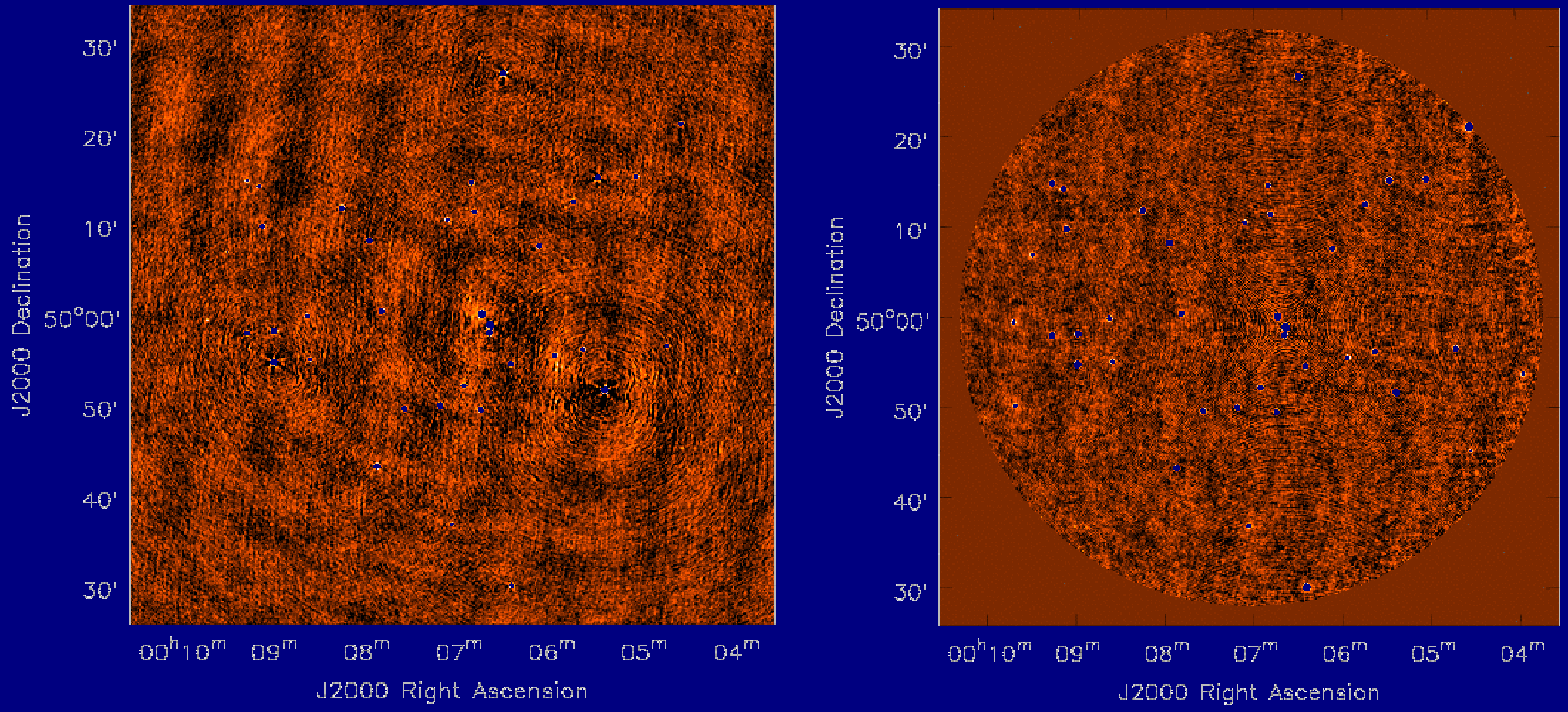
Pointing SelfCal

- Model image: deconvolved using entire data
- Pixelated model image



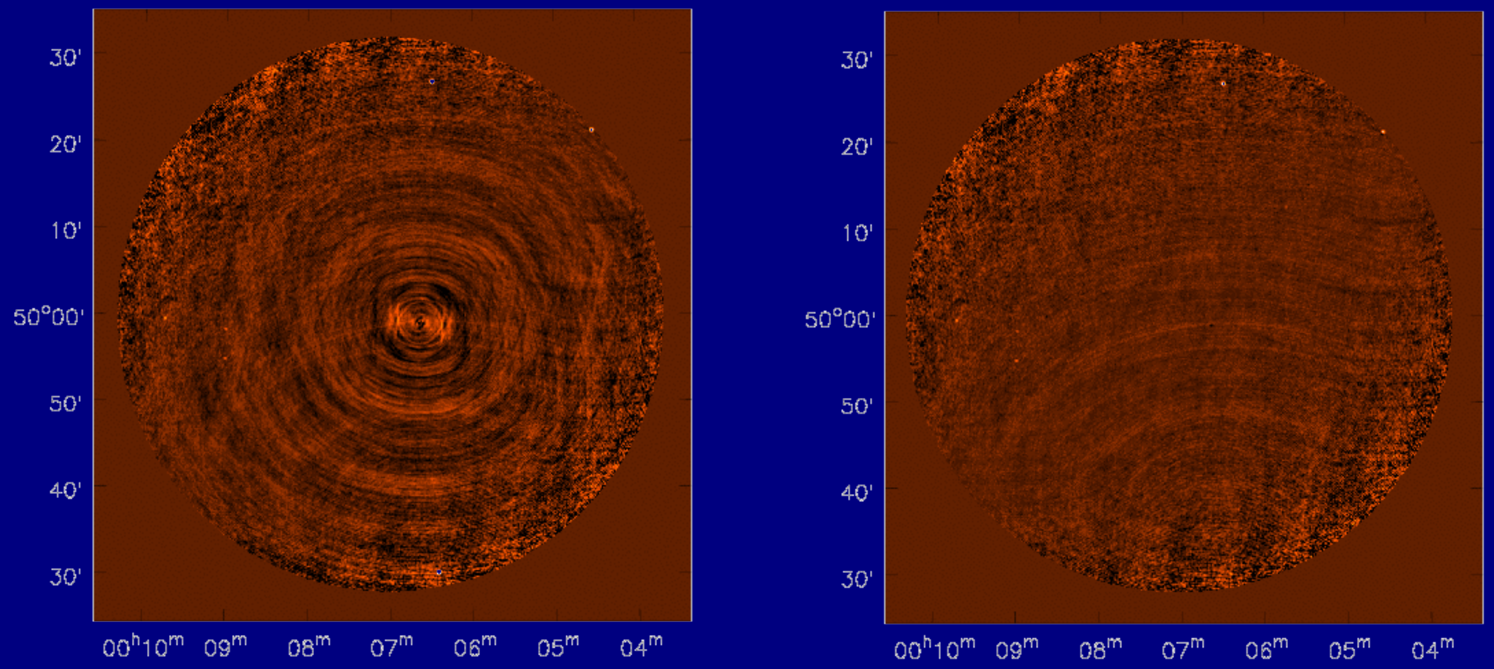
Pointing SelfCal

- Stokes-I imaging: Before and after pointing correction



Pointing SelfCal

- Stokes-V imaging: Need to use component imaging?
- Use illumination patterns (work in progress)



Where to from here...

- Eliminate aperture function re-gridding
- SelfCal <-> imaging iterations
- Component based imaging and prediction (Asp-Clean + PB-Projection + W-Projection)
- Is current deep L-band imaging pointing-error limited?

- Mosaicking dynamic range limited by pointing errors?
- Wide-band imaging (EVLA memo #?)
  - Use PB-projection to correct for PB scaling
  - MSF extensions: Freq. sensitive image plane modeling (Component based imaging?)

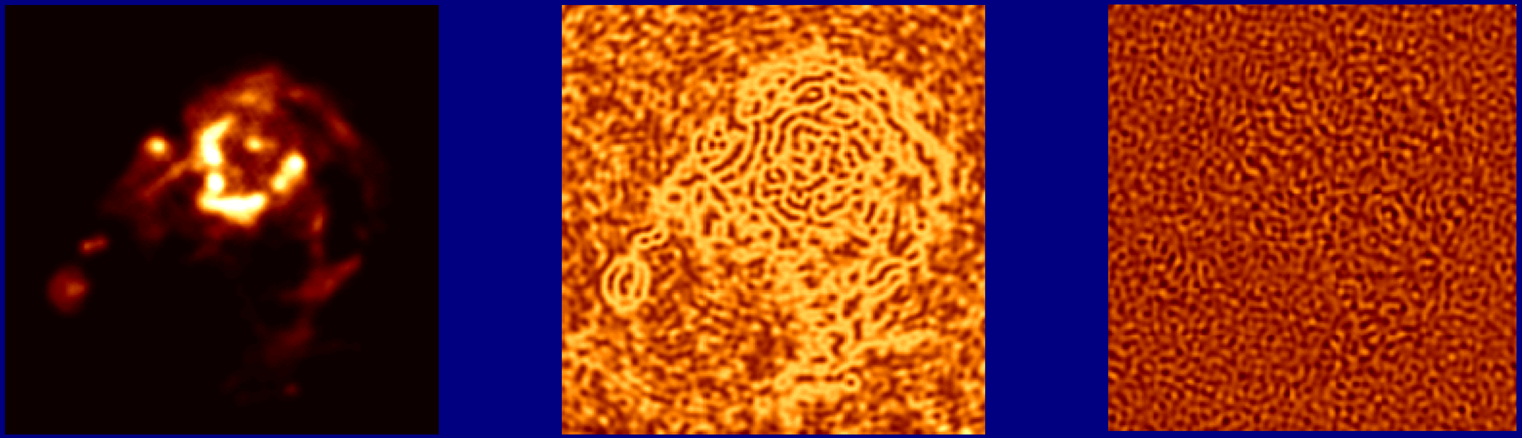
Scale sensitive imaging: Asp-Clean

- Pixel-to-pixel noise in the image is correlated

$$I^D = B * I^o + B * I^N \quad \text{where } B = PSF$$

- The scale of emission *fundamentally* separates signal *(I^o)* from the noise *(I^N).*

- Asp-Clean (Bhatnagar & Cornwell, A&A,2004)
  - Search for local scale, amplitude and position



Wide-band continuum imaging

- EVLA bandwidth ratio of 2:1

$$V(u_{ij}, v_{ij}) = \sum_{\nu_k} V(u_{ij}, v_{ij}; \nu_k) = \sum_{\nu_k} P_{ij}(\nu_k) FT[I^D(\nu_k)]$$

Sky emission, the Primary Beams, etc. are a function of frequency.

Ideas: Apply PB effects during predict using pb-projection.

Parameterized sky model for prediction.

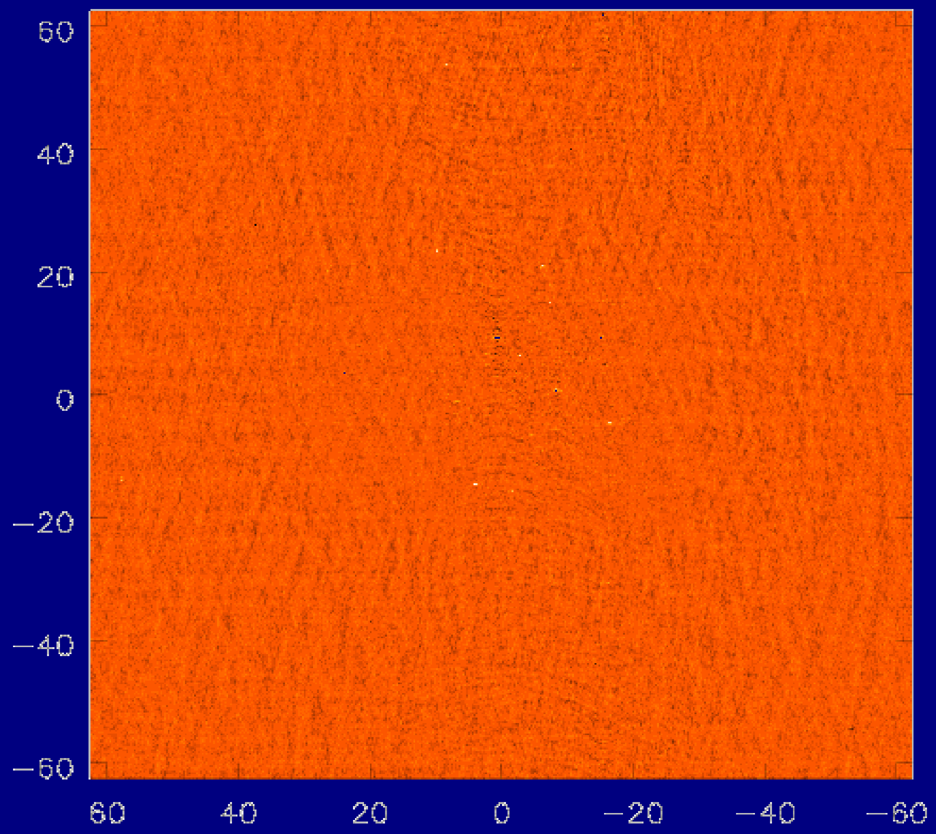
- Combining Scale sensitive + frequency sensitive deconvolution with image plane corrections.

Computing and I/O costs

- Significant increase in run-time due to more sophisticated parameterization
  - Deconvolution: Fast evaluation of $B * \sum_k A_k P(Scale_k, Pos_k)$
  - Calibration: Fast evaluation of $E_{ij} * V_{ij}$
- Cost of computing residual visibilities is dominated by I/O costs for large datasets (~200GB for EVLA)
  - Deconvolution: Approx. 20 access of the entire dataset
  - Calibration: Each trial step in the search accesses the entire dataset
- Solutions: Analytical approximations, caching, Parallel computing and I/O,...

Wide-band imaging: The problem

- IF1 – IF2: 50MHz apart
  - Spectral index? Frequency dependence of the PBs?



- EVLA bandwidth ratio of 2:1

General approach for imaging

- Design filters to be used as Gridding Convolution Functions (GCF) for accurate forward transform (major cycle)

$$V^M(u_{ij}) = \Big[E_{ij}(u) * [A\, I^M](n\,\Delta u)\Big](u_{ij})$$

- Use an approximation for the inverse transform to compute the update direction (minor cycle)

- Major cycle: $\vec{I}^R = B[\vec{V}^{Obs} - A'\vec{I}^M] \quad where \quad B \approx A'^T$

- Minor cycle: $\vec{V}^M = A'\vec{I}^M$