Imaging and deconvolution

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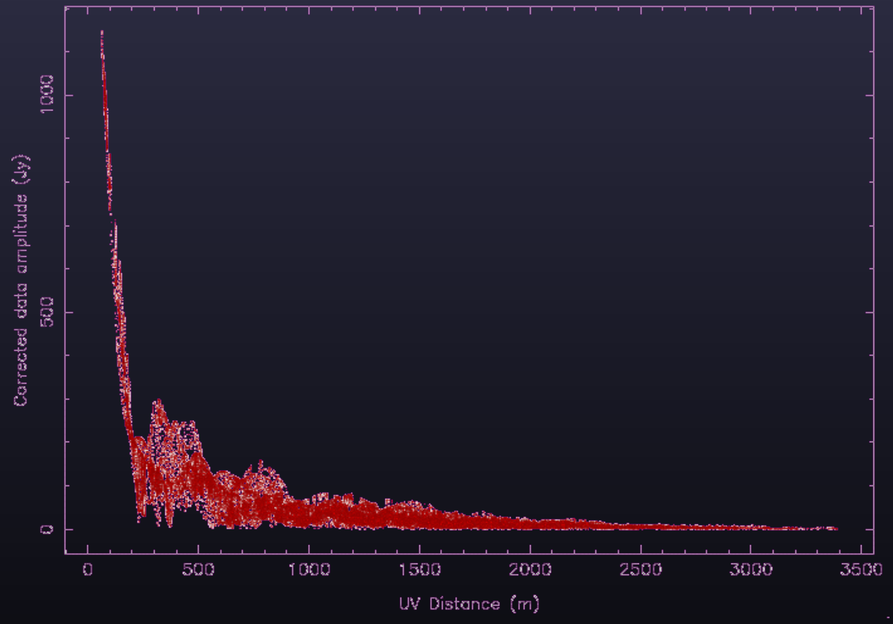
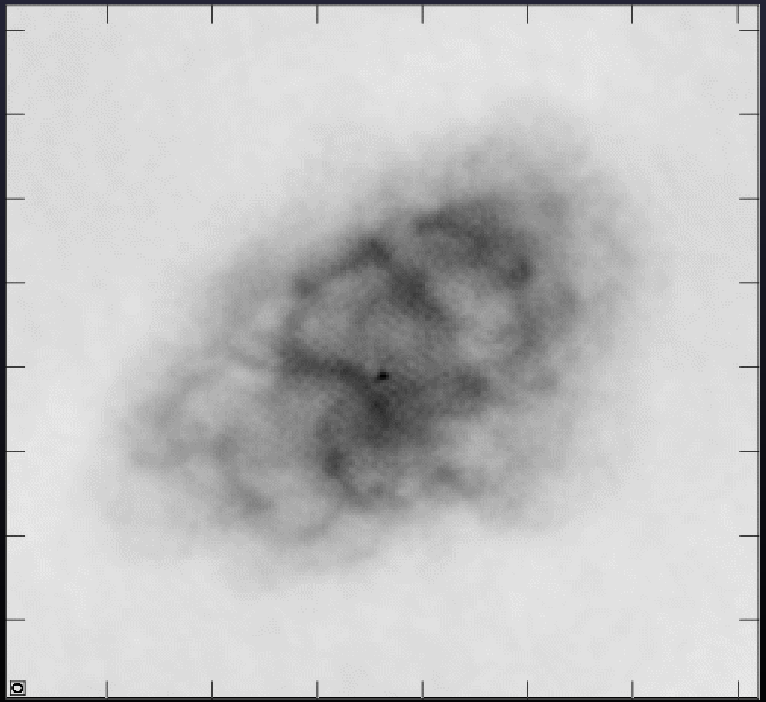




Ninth Synthesis Imaging Summer School

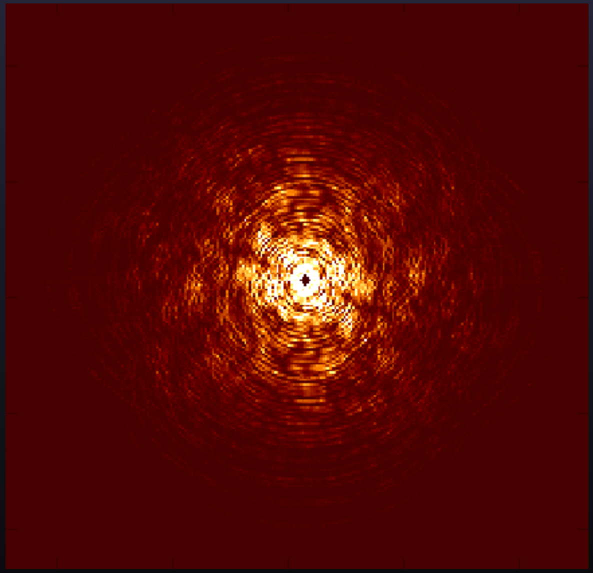
Socorro, June 15-22, 2004

Plan for the lecture-I

How do we go from the measurement of the coherence function (the Visibilities) to the images of the sky?

- First half of the lecture: Imaging

Measured Visibilities <--> Dirty Image



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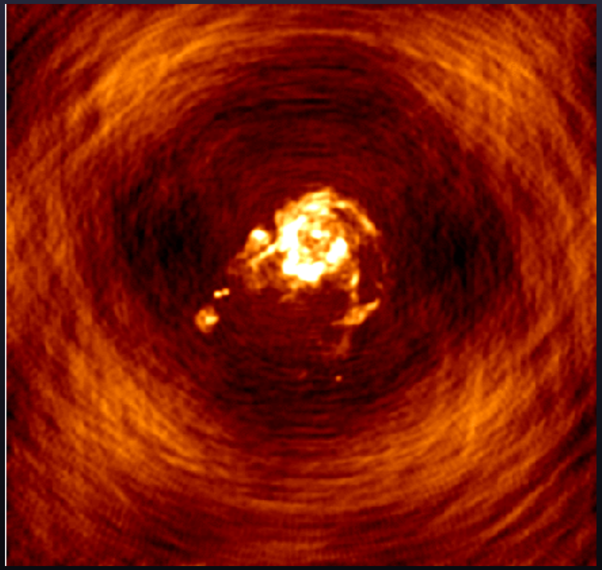
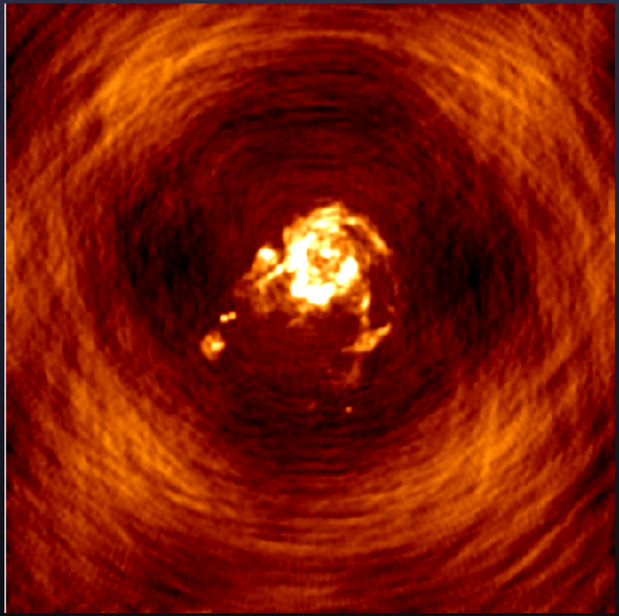


Plan for the lecture-II

- **Second half of the lecture: Deconvolution**

Dirty image <--> Model of the sky



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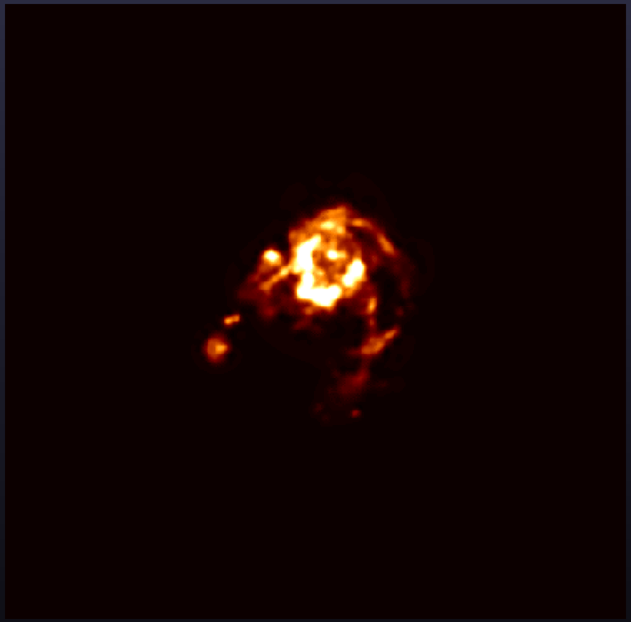


Imaging

- **Interferometers are indirect imaging devices**

$$V^\circ(u,v,w) = \int\int I(l,m) e^{2\pi\iota\left[ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right]} \frac{dl\,dm}{\sqrt{1-l^2-m^2}-1}$$

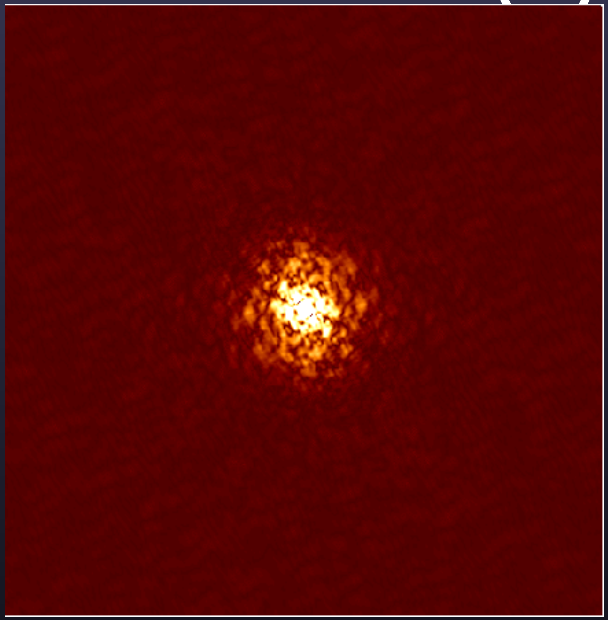
- **For small w (small max. baseline) or small field of view *(l^2 + m^2 << 1)* *I(l,m)* is 2D Fourier transform of *V (u,v)***

$$V^\circ(u,v) = \int\int I(l,m) e^{2\pi\iota[ul+vm]} dl\,dm$$

$$V^\circ(u,v) \rightleftharpoons I(l,m)$$

Imaging: Ideal 2D Fourier relationship

Ideal visibilities(*V*)



FT

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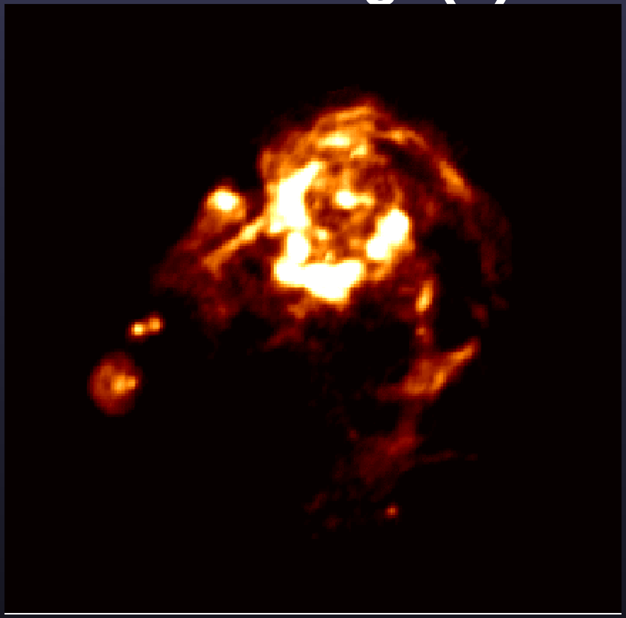
True image(*I*)



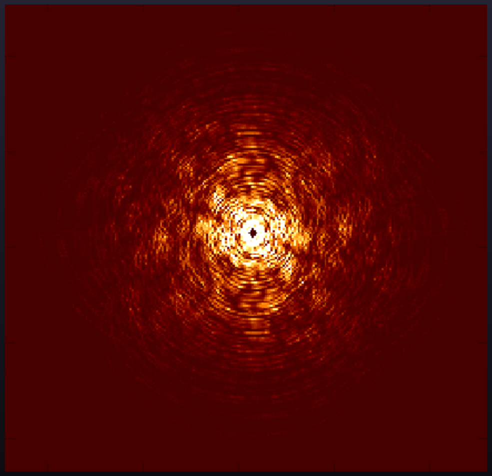
- **This is true ONLY if *V* is measured for all *(u,v)*!**

Imaging: UV-plane sampling

- With limited number of antennas, the uv-plane is sampled at discrete points:

$$S(u,v) = \sum_k \delta(u_k, v_k)$$

$$V^M(u,v) = S(u,v) V^\circ(u,v)$$



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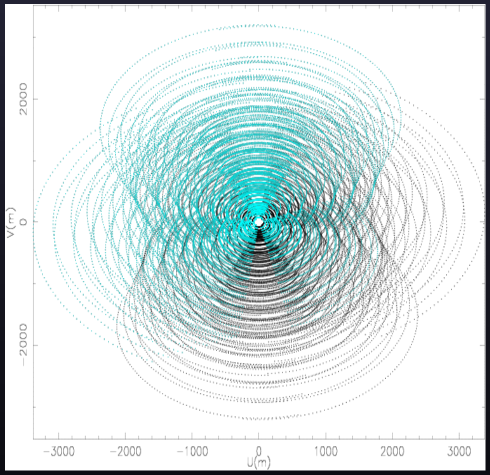
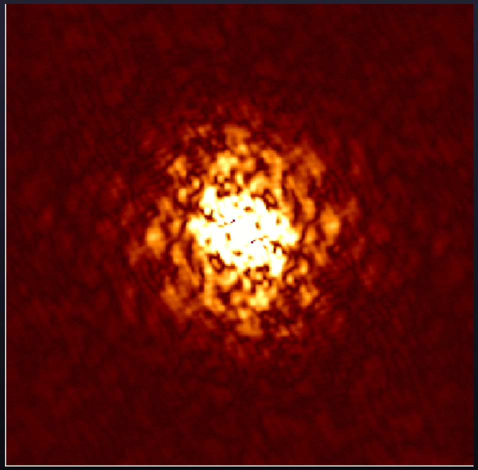


X



V^M S V^o

Convolution with the PSF

- **Effect of sampling the uv-plane:**

$$I^d(l,m) = FT^{-1}\left[V^\circ(u,v)S(u,v)\right]$$

- **Using the Convolution Theorem:**

$$I^d(l,m) = B(l,m) \star I^\circ(l,m)$$

The Dirty Image (I^d) is the convolution of the True Image (I^o) and the Dirty Beam/Point Spread Function (B)

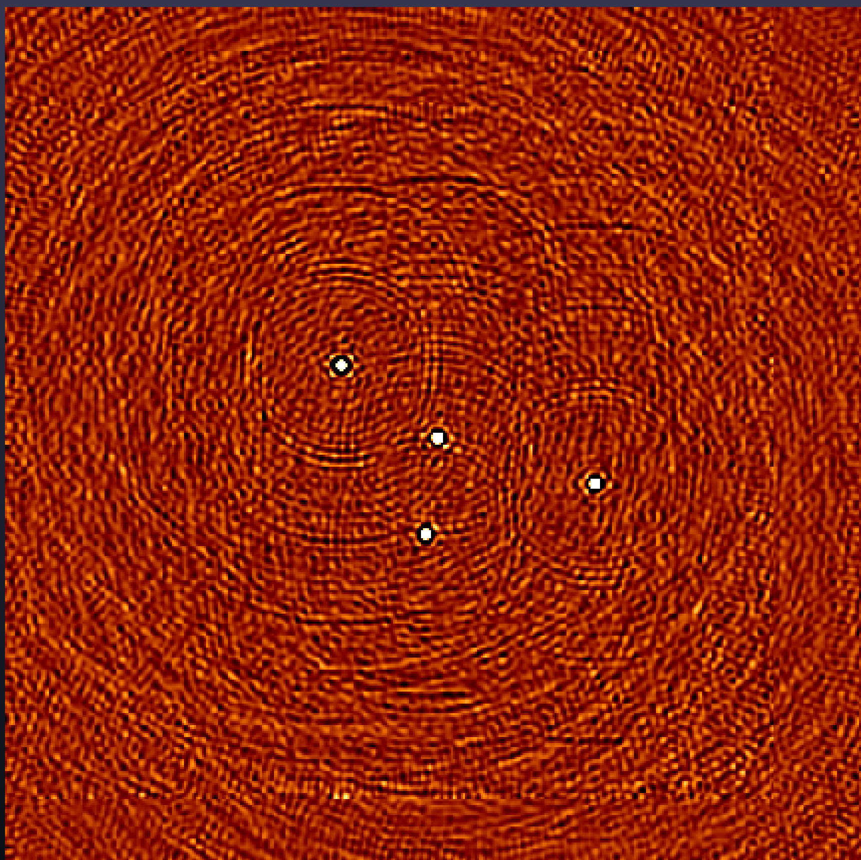
$$B = \mathrm{FT}^{-1}(S)$$

- **In practice**

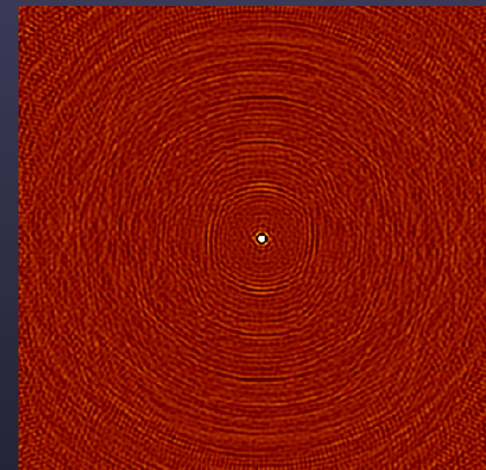
$$I^d = B * I^o + B * I^N \quad \text{where} \quad I^N = \mathrm{FT}^{-1}(\text{Vis. Noise})$$

- **To recover I^o, we must deconvolve B from I^d. The algorithm must also separate $B*I^o$ from $B*I^N$.**

Convolution



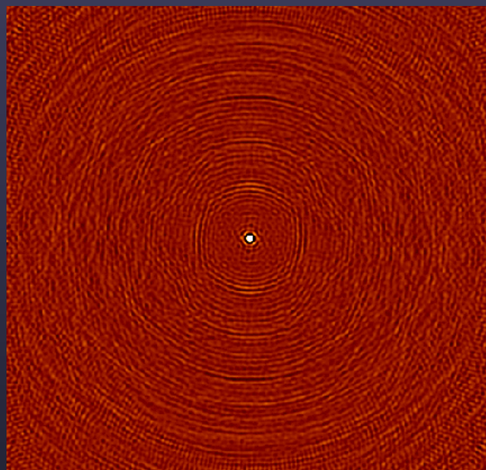
The PSF



$$= I(x_0)B(x-x_o) + I(x_1)B(x-x_1) + \ldots$$

The Dirty Image

The PSF



FT

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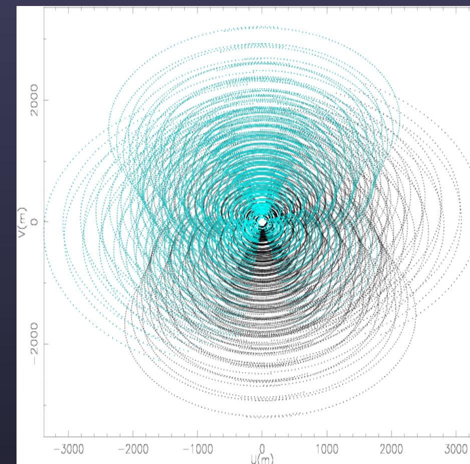
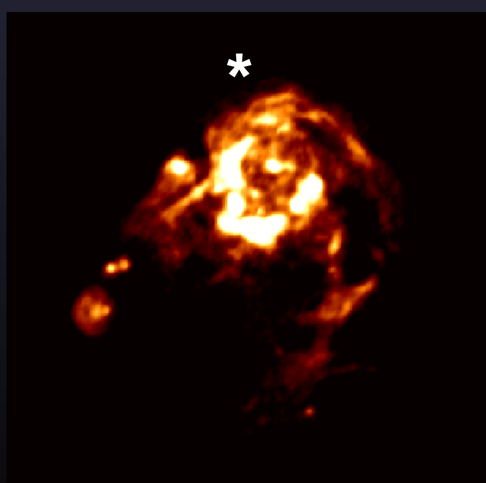


UV-coverage



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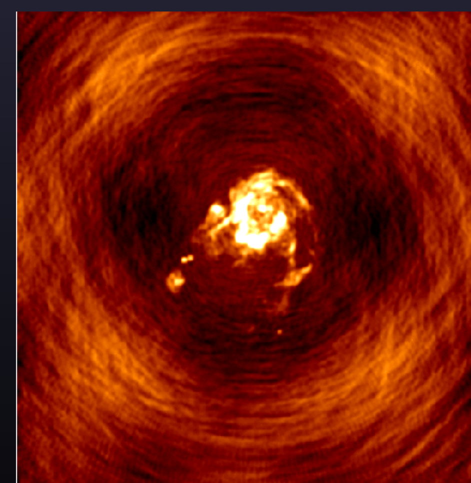


The

Dirty Image

Making of the Dirty Image

- **Fast Fourier Transform (FFT) is used for efficient Fourier transformation. It however requires regularly spaced grid of data.**
- **Measured visibilities are irregularly sampled (along uv-tracks).**
- **Convolutional gridding is used to effectively interpolate the visibilities everywhere and then re-sample them on a regular grid (the Gridding operation)**

$$V^S = V^M * C = (V^o S) * C \implies I^d \cdot FT^{-1}(C)$$

- ***C* is designed to have desirable properties in the image domain.**

Dirty Beam: Interesting properties

- **PSF is a weighted sum of cosines corresponding to the measured Fourier components:**

$$B(l,m) = \frac{\sum_k W_k cos(u_k l + v_k m)}{\sum_k W_k}$$

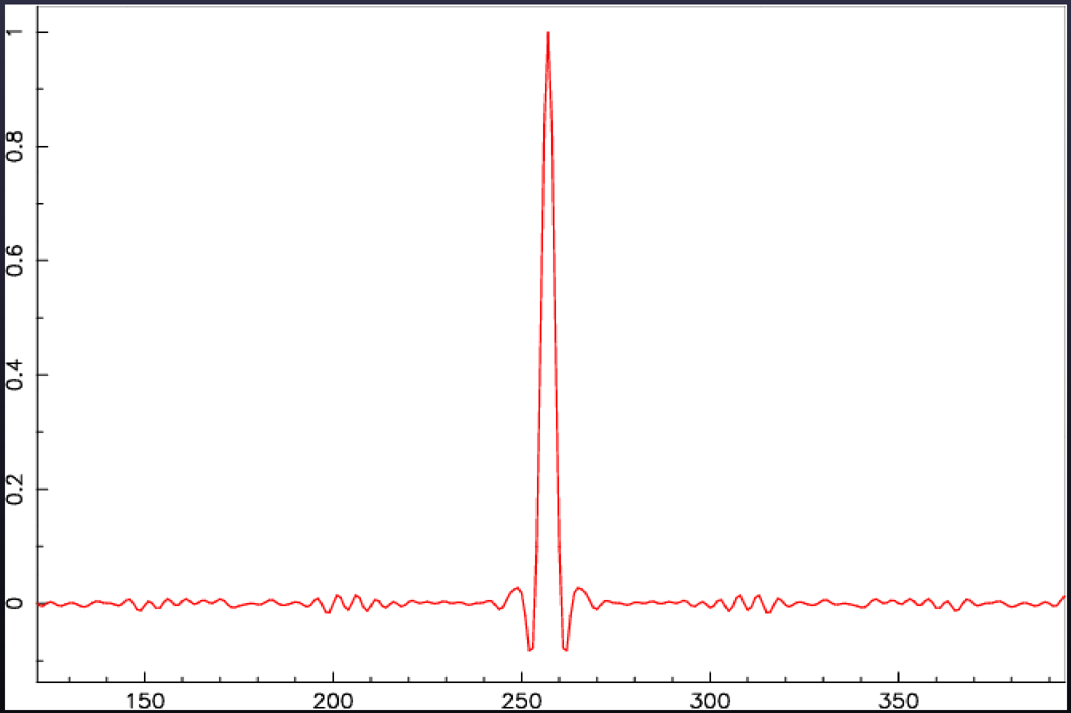
Visibility weights (*w_i*) are also gridded on a regular grid and FFT used to compute the Dirty Beam (or the PSF).

- **The peak of the PSF is normalized to 1.0**
- **The 'main lobe' has a size** $x \sim 1/u_{max}$ **and** $y \sim 1/v_{max}$

This is the 'diffraction limited' resolution (the Clean Beam) of the telescope.

Dirty Beam: Interesting properties

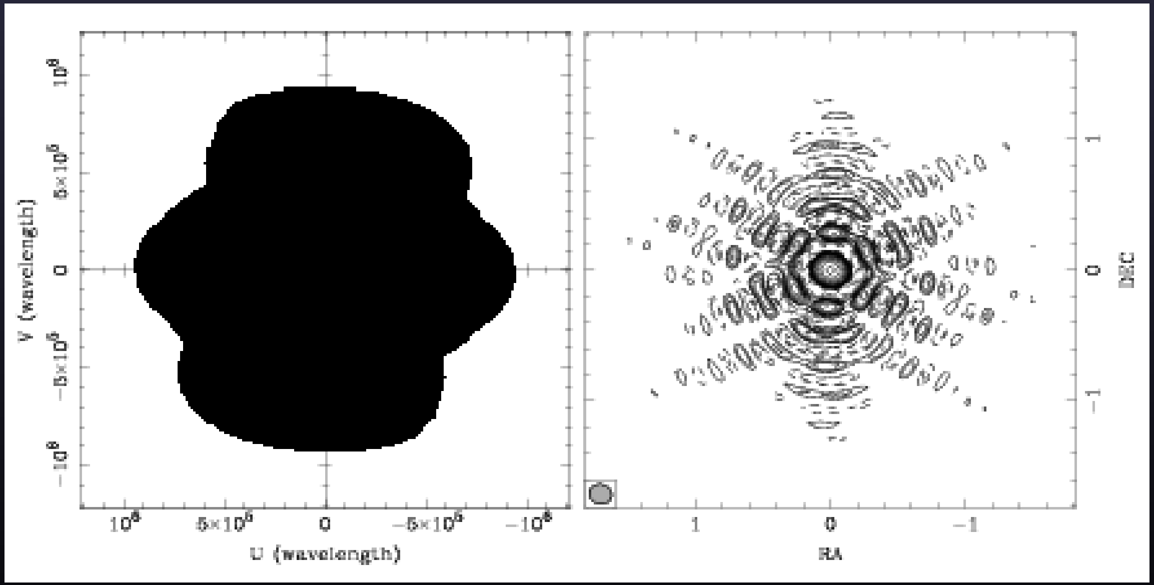
- **Side lobes extend indefinitely**
- **RMS ~ *1/N* where *N* = No. of antennas**



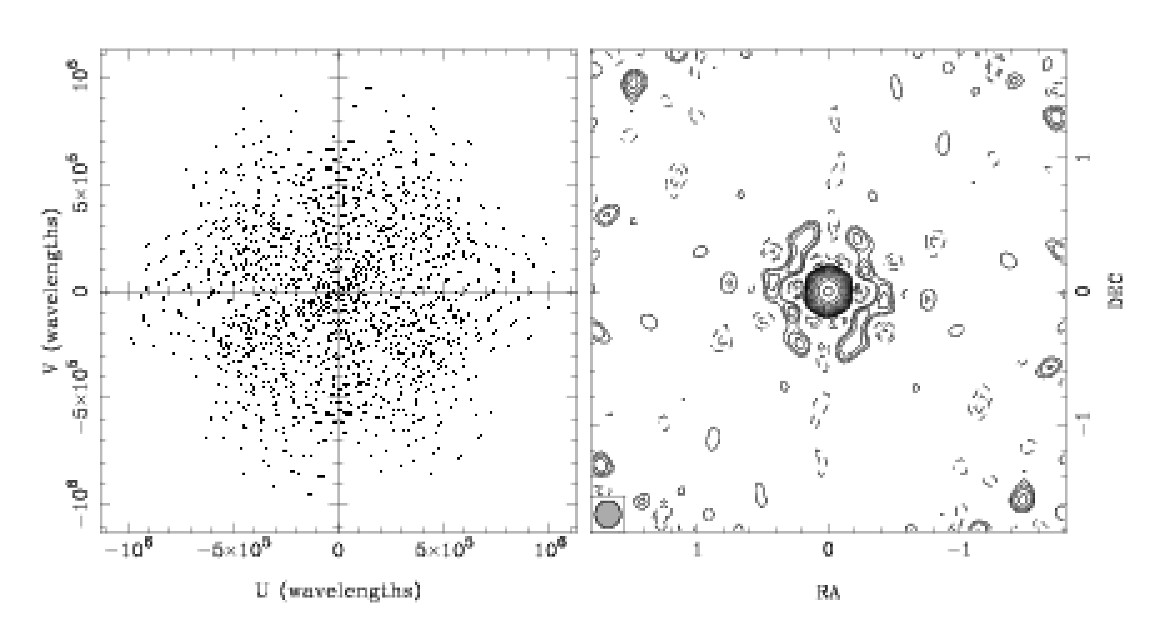
Close-in side lobes of the PSF

- **Close-in side lobs of the PSF are controlled by the uv- coverage envelope.**

E.g., if the envelop is a circle, the side lobes near the main lobe must be similar to the FT of a circle: Bessel function/Radius



Close-in side lobes: VLA uv-coverage



PSF forming: Weighting...

- Weighting function (W_k) can be chosen to modify the side lobes

$$B(l,m) = \frac{\sum_k W_k cos(u_k l + v_k m)}{\sum_k W_k}$$

- Natural Weighting

  $W_k = 1/\sigma_k^2$ where σ_k^2 is the RMS noise

- Best RMS across the image.
- Large scales (smaller baselines) have higher weights.
- Effective resolution less than the inverse of the longest baseline.

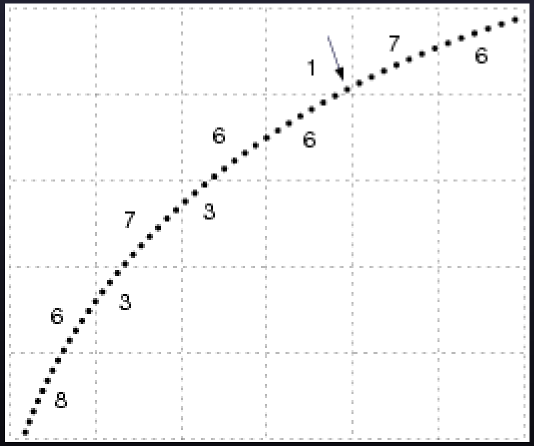
...Weighting...

- **Uniform weighting**

  $W_k = 1/\rho(u_k, v_k)$ where $\rho(u_k, v_k)$ is the density of uv-points in the k^{th} cell.
- **Short baselines (large scale features in the image) are weighted down.**
- **Relatively better resolution**
- **Increases the RMS noise.**
- **Super uniform weighting:**

  Consider density over larger region.

  Minimize side lobes locally.

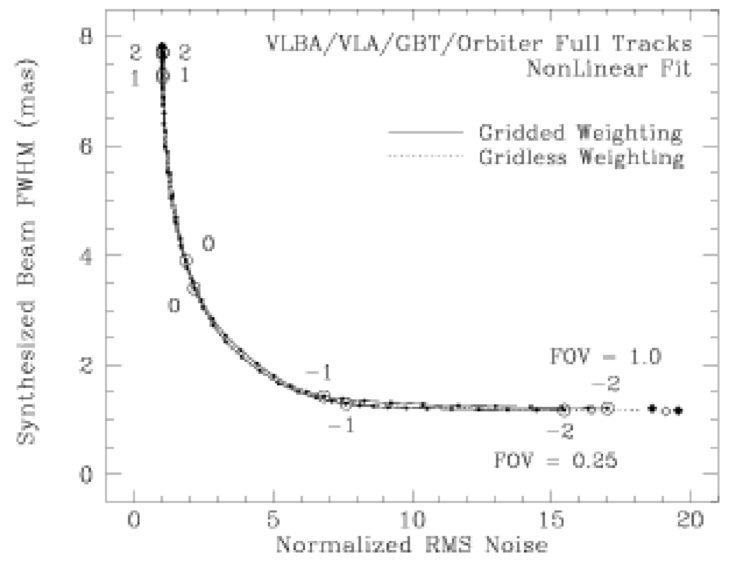


...Weighting

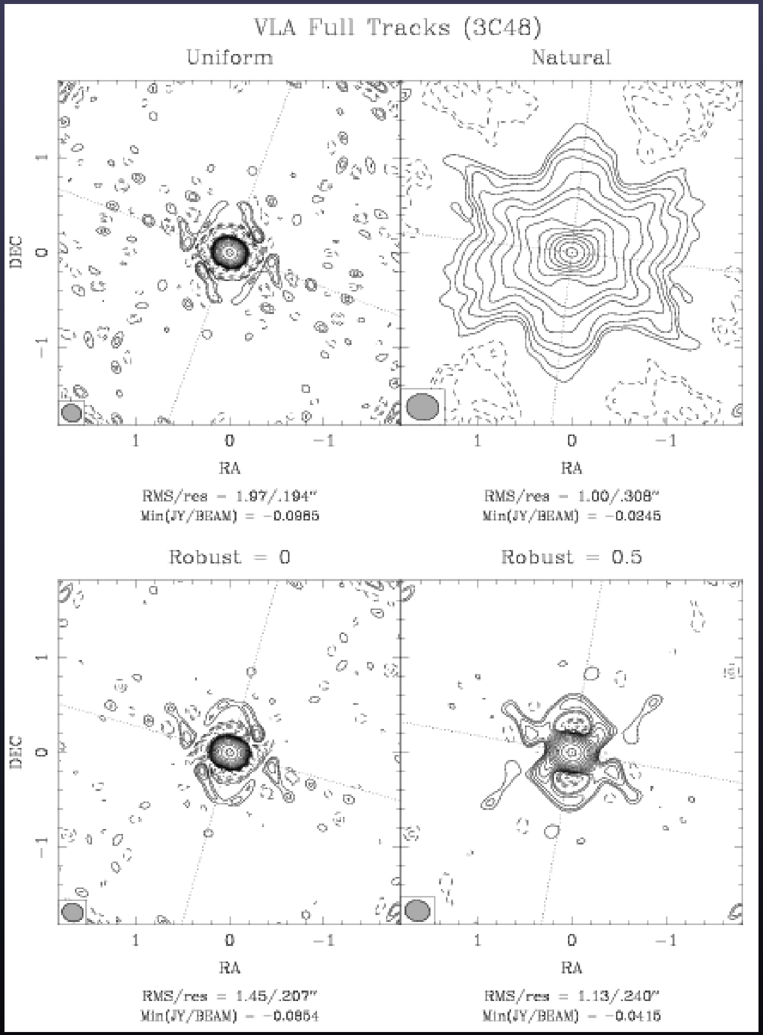
- **Robust/Briggs weighting:**

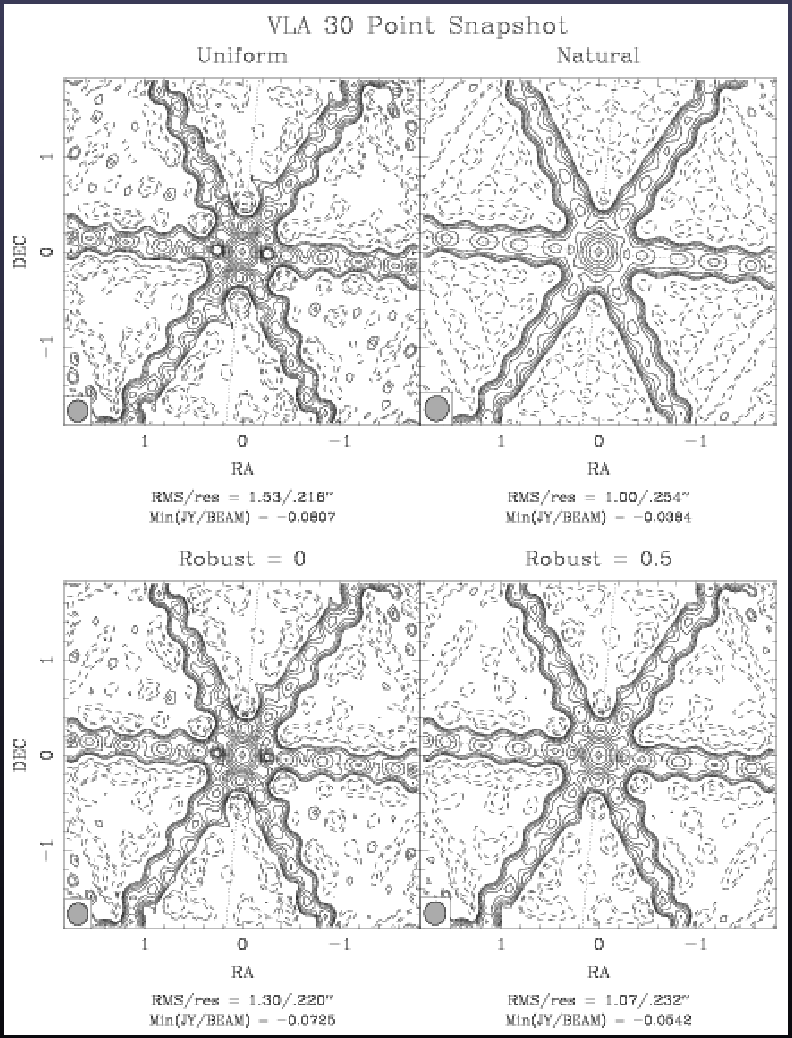
$$W_k = 1/[S.\,\rho(u_k\ ,\, v_k) + \sigma_k^2]$$

- **Parameterized filter – allows continuous variation between optimal resolution (uniform weighting) and optimal noise (natural weighting).**



Examples of weighting





PSF Forming: Tapering

- **The PSF can be further controlled by applying a tapering function on the weights (e.g. such that the weights smoothly go to zero beyond the maximum baseline).**

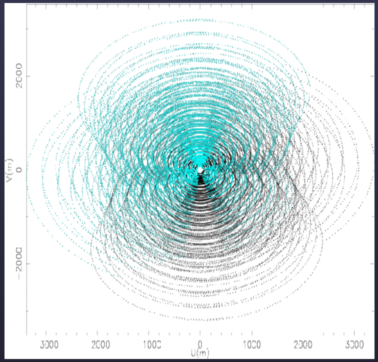
$$W'_k = T(u_k, v_k)\, W_k(u_k, v_k)$$

- **Bottom line on weighting/tapering:**

These help a bit, but imaging quality is limited by the deconvolution process!

The missing information

- **As seen earlier, not all parts of the uv-plane are sampled – the 'invisible distribution'**

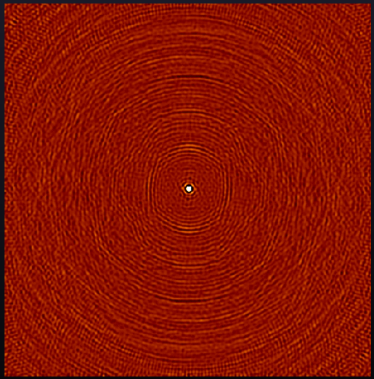


1. "Central hole" below u_{min} and v_{min}:

- Image plane effect: Total integrated power is not measured.

- Upper limit on the largest scale in the image plane.

2. No measurements beyond u_{max} and v_{max}:



- Size of the main lobe of the PSF is finite (finite resolution).

3. Holes in the uv-plane:

- Contribute to the side lobes of the PSF.

More on missing information

- Missing 'central hole' means that the total flux, integrated over the entire image is zero.

$$V(u=0, v=0) = \int\int I^d(l,m)\; dl dm = 0$$

- Total flux for scales corresponding to the Fourier components between u_{max} and u_{min} can be measured.
- In the presence of extended emission, the observations must be designed keeping in mind:

  - the required resolution ==> maximum baseline

  - the largest scale to be reliably reconstructed ==> minimum baseline

Recovering the missing information

- **For information beyond the max. baseline, one requires extrapolation. That's unphysical (unconstrained).**
- **Information corresponding to the "central hole": possible, but difficult (need extra information).**
- **Information corresponding to the uv-holes: requires interpolation. The measurements provide constraints – hence possible. But non-linear methods necessary.**

If *Z* is the unmeasured distribution, then *B*Z=0*. If I^M *is a solution to* $I^d=B*I^M$*, then so is* $I^M + \alpha Z$ *for any value of* α.

Deconvolution = interpolation in the visibility plane.

Prior knowledge about the sky

- ***What can we assume about the sky emission:***
  1. ***Sky does not look like cosine waves***
  2. ***Sky brightness is positive (but there are exceptions)***
  3. ***Sky is a collection of point sources (weak assertion)***
  4. ***Sky could be smooth***
  5. ***Sky is mostly blank (sometimes justifies "boxed" deconvolution)***
- ***Non-linear deconvolution algorithms search for a model image I^M such that the residual visibilities $V^R=V^o-V^M$ are minimized, subject to the constraints given by the (assumed) prior knowledge.***

Small digression: Vector notation

- ***Let***

  A = Measurement matrix to go from the image domain to the visibility domain (the measurement domain).

  I = Vector of the image pixel values

  V = Vector of visibilities

  B = Operator (matrix) for convolution with the PSF

  N = The noise vector

- ***Then,***

$$I^d = BI^o + BI^N \text{ where } BI^N = A^T AN$$

$$V^M = AI^M \text{ and } V^o = AI^o + N$$

$$V^R = V^o - AI^M$$

Some observations

- ***A is rectangular (not square) and is a collection of sines and cosines corresponding to only the measured Fourier components.***

  A is singular ==> A^{-1} does not exist

  $I^M = A^{-1} V^M$ not possible ==> non-linear methods needed

- ***N is independent gaussian random process. Noise in the image domain = BI^N***

  Pixel-to-pixel noise in the image is not independent

...more observations

- **For successful recovery of I^o given I^d, prior knowledge must fundamentally separate *BI^o and BI^N.***
- ***^{2} is the optimal estimator. Deconvolution then is equivalent to:***

$$\text{Minimize : } \chi^2 = \left|V^M - AI^M\right|^2 \quad \text{where} \quad I^M = \sum_k P_k \quad ; \quad P_k \equiv \text{Pixel Model}$$

Deconvolution is equivalent to function minimization

- ***Algorithms differ in the parameterization of P_k, the type of constraints and the way the constraints are applied.***

Deconvolution algorithms

- ***Scale-less algorithms:***

$$P_k = A_k \delta(x - x_k)$$

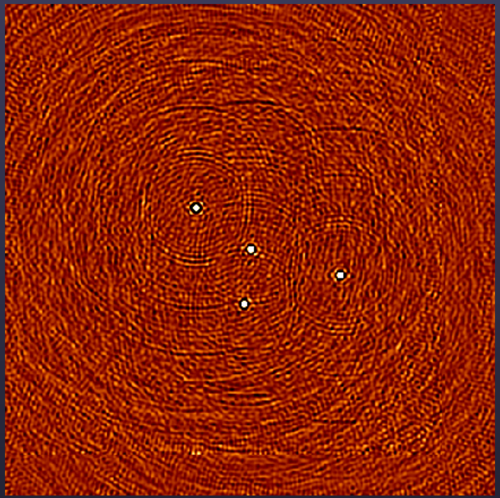
Popular ones: Clean, MEM and their variants

- ***Scale-sensitive algorithms (new turf!):***

$$P_k = A_k f(Position,\ Scale)$$

Existing ones: Multi-scale Clean, Asp-Clean

The classic Clean algorithm (Hogbom, 1974)



- **Prior knowledge about the sky:**
  - **is composed of point sources**
  - **is mostly blank**
- **Algorithm:**
  1. **Search for the peak in the dirty image.**
  2. **Add a fraction *g* (loop gain) of the peak value to I^M.**
  3. **Subtract a scaled version of the PSF from the position of the peak.**

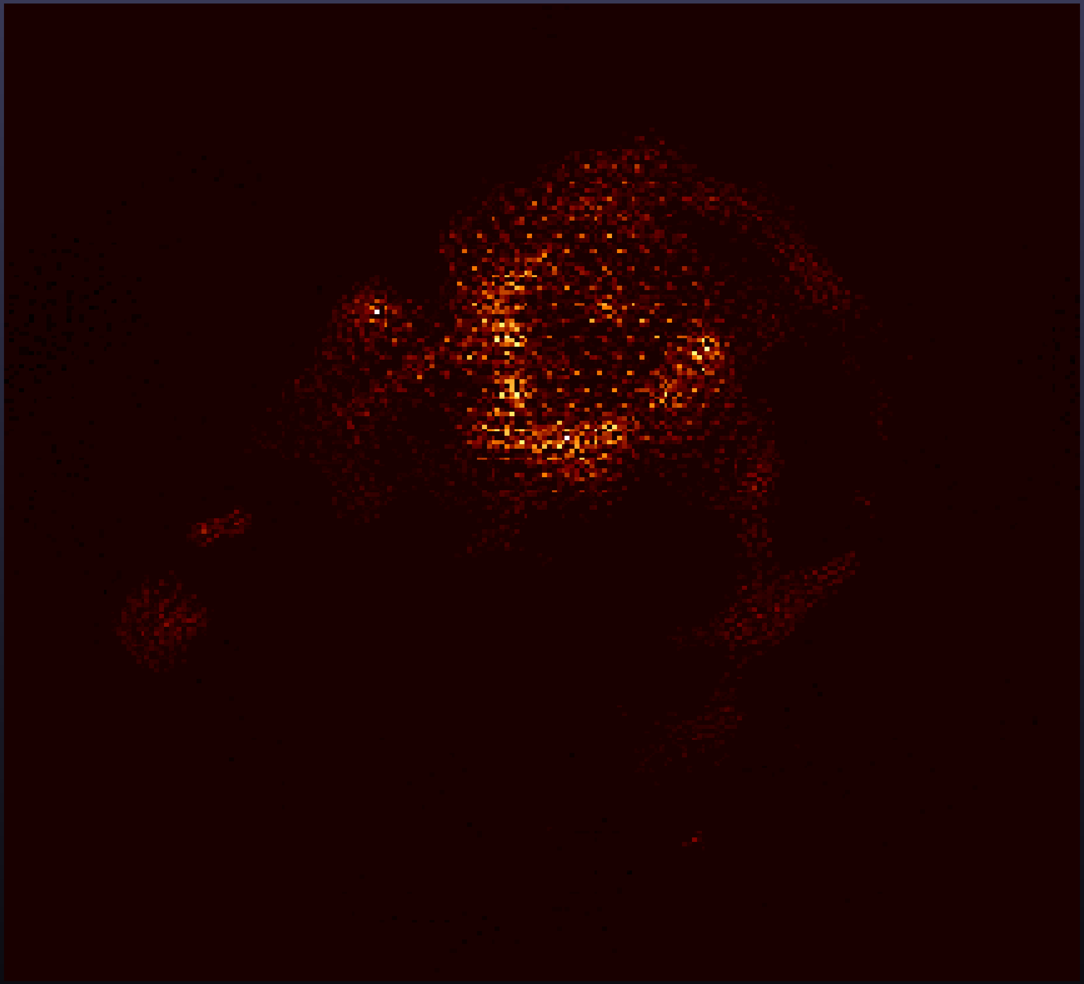
     $$I^R_{i+1} = I^R_i - g\,B\,\max(I^R_i)$$

  4. **If residuals are not "noise like", goto 1.**
  5. **Smooth I^M by an estimate of the main lobe (the "clean beam") of the PSF and add the residuals to make the "restored image"**

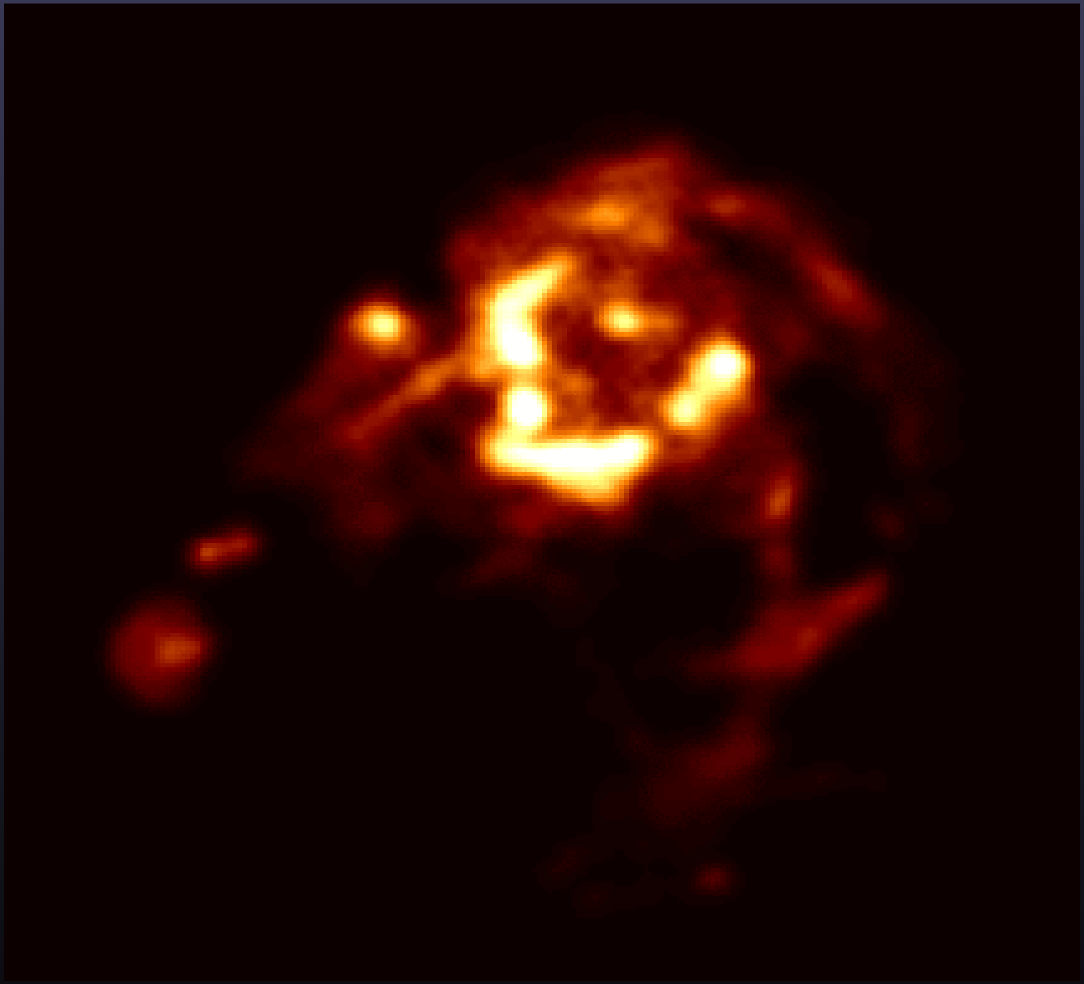
Details of Clean

- **It is a steepest descent minimization.**
- **Model image is a collection of delta functions – a scale insensitive algorithm.**
- **A least square fit of sinusoids to the visibilities which is proved to converge (Schwarz 1978).**
- **Stabilized by keeping a small loop gain (usually g=0.1-0.2).**
- **Stopping criteria: either the max. iterations or max. residuals some multiple of the expected peak noise.**
- **Search space constrained by user defined windows.**
- ✗ **Ignores coupling between pixels (extended emission) – assumes an orthogonal search space.**

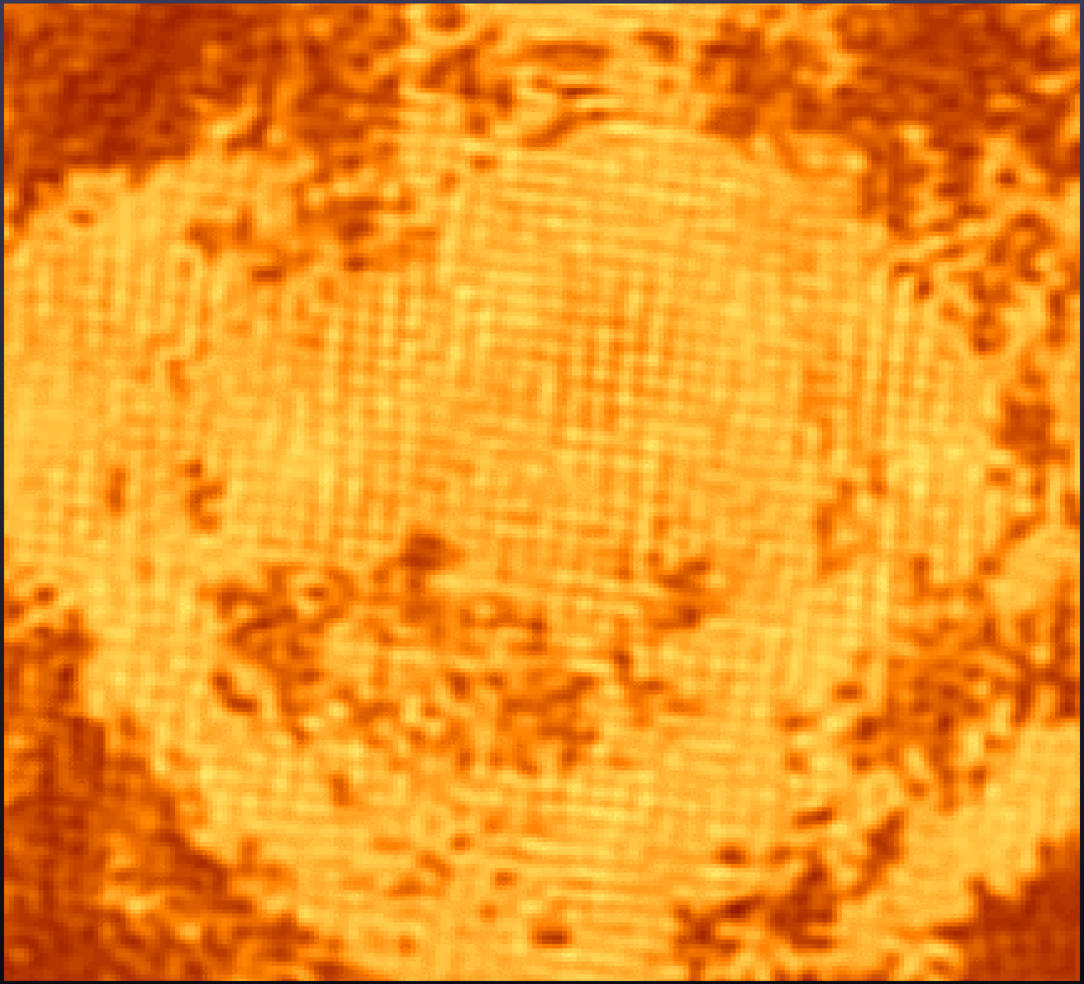
Clean: Model



Clean: Restored

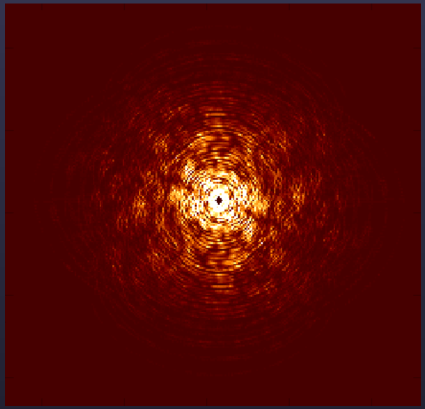


Clean: Residual

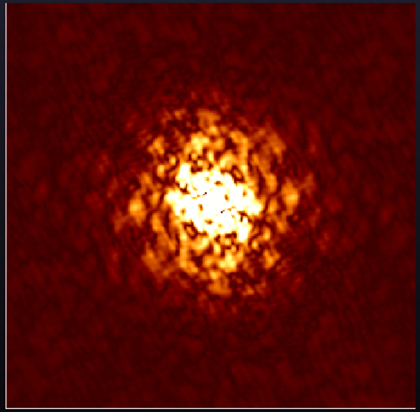


Clean: Model visibilities

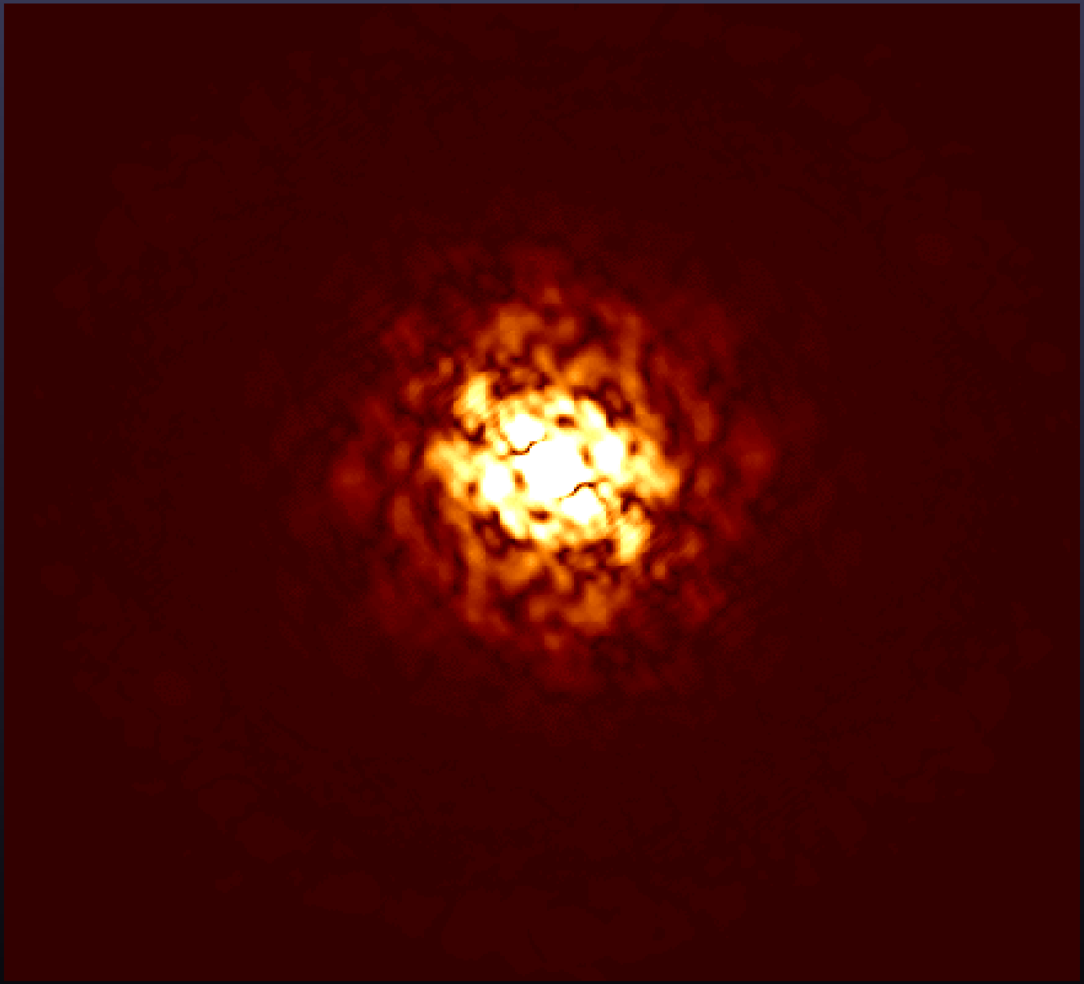
Sampled Vis



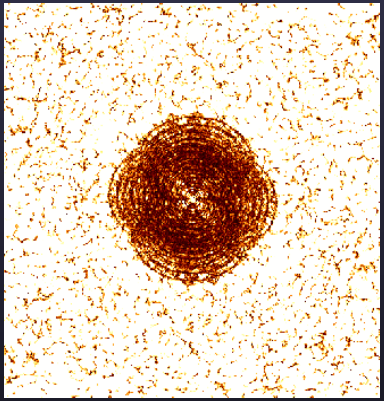
True Vis.



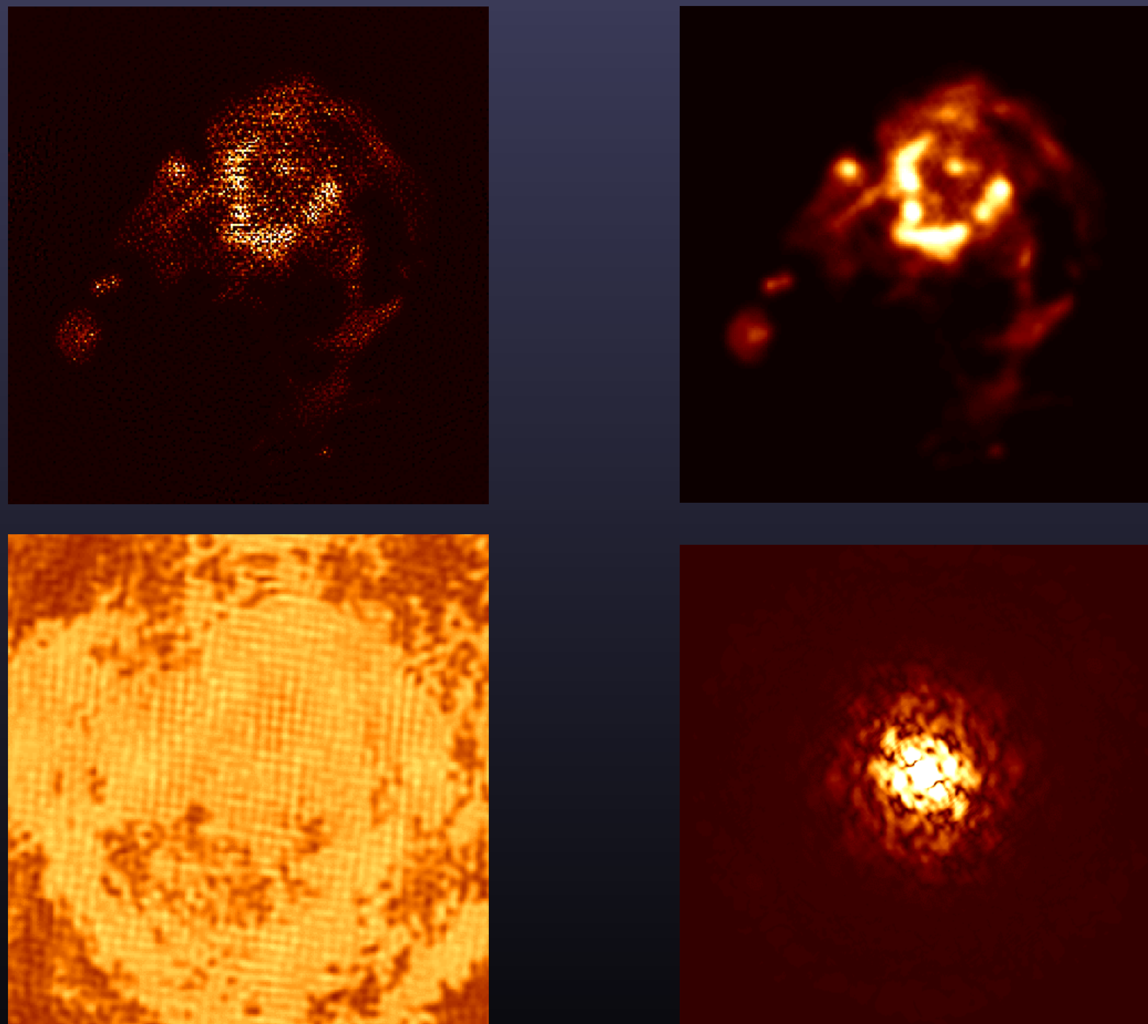
Model Vis.



Residual Vis.



Clean: Example



Variants of the Classic Clean

- **Clark Clean – uses FFT to speed up**

  Minor cycle(inexpensive) : Clean the brightest points using an approximate PSF to gain speed

  Major cycle(expensive): Use FFT convolution to accurately remove the point sources found in the minor cycle

- **Cotton-Schwab Clean: A variant of Clark Clean**

  Subtract the point sources from the visibilities directly.

  Sometimes faster and always more accurate then Clark Clean.

  Easy to adapt for multiple fields.

Deconvolution algorithms: MEM

- MEM is a constrained minimization algorithm.
- Fast non-linear optimization algorithm due to Cornwell&Evans(1983).
- Solve the convolution equation, with the constrain of smoothness via the 'entropy'

$$H(I) = -\sum_k I_k - log(I_k/m_k)$$

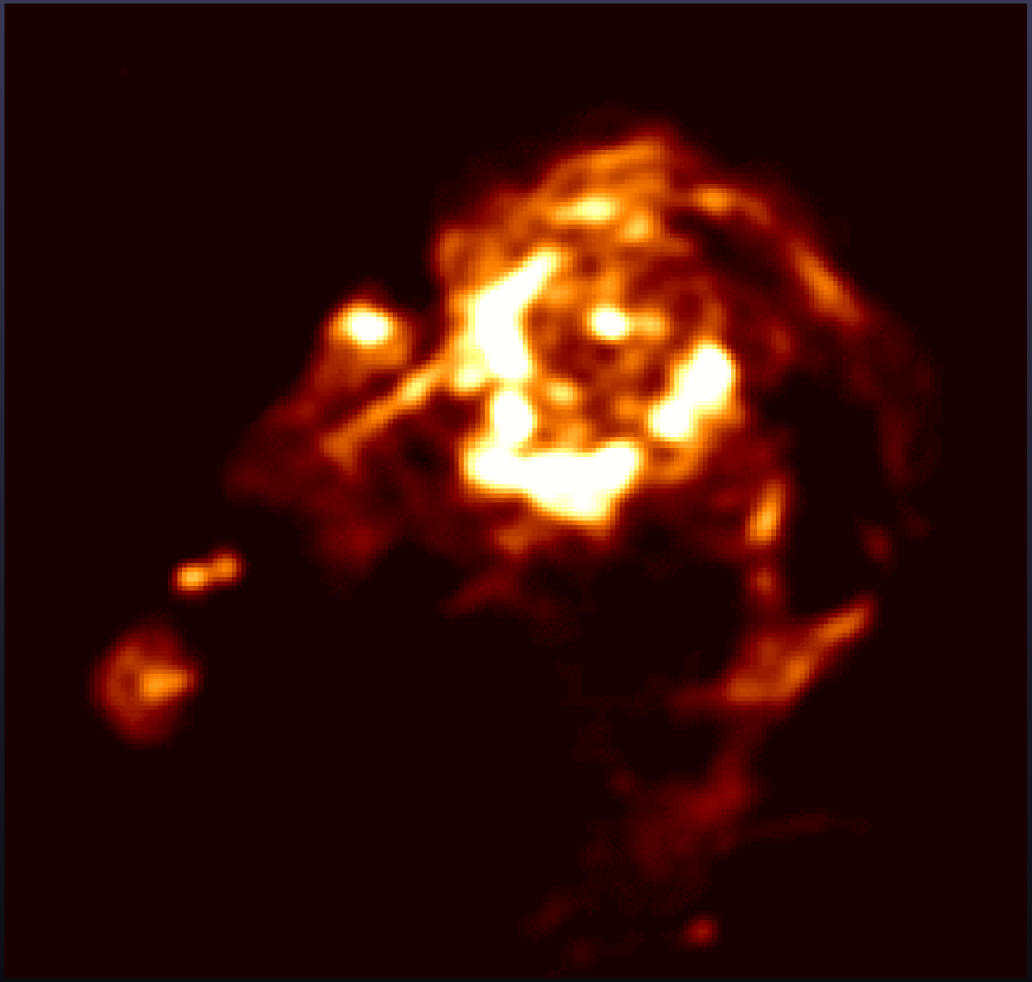
m_k is the prior image – usually a flat default image.

- Default image is a very useful in incorporating model images from other algorithms etc.
- Naturally useful when final image is some combination of images (like mosaic images).

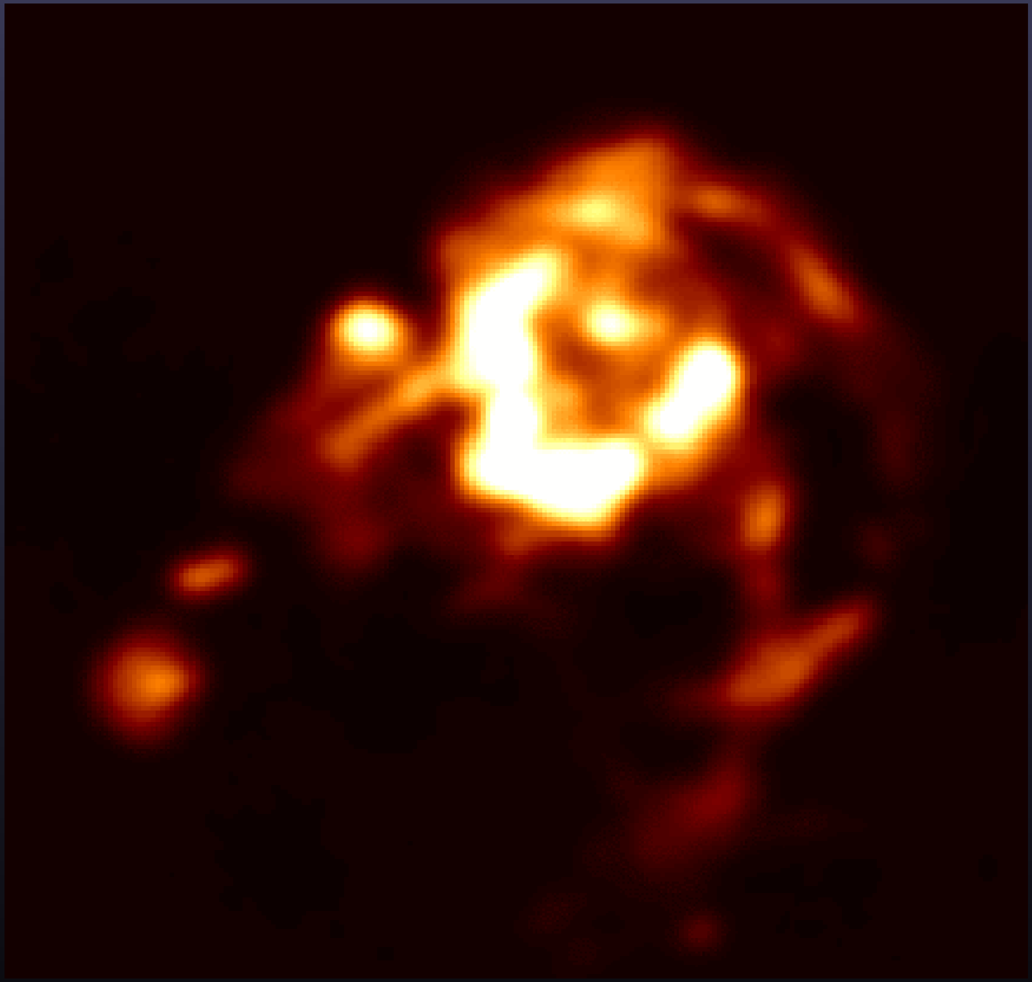
MEM: Some points

- **Works better than Clean for extended emission.**
- **Every pixel is treated as a potential degree of freedom – a scale insensitive algorithm.**
- **Point sources are a problem, particularly along with large scale background emission – but can be removed with, say, Clean before hand.**
- **Easier to analyze and understand.**

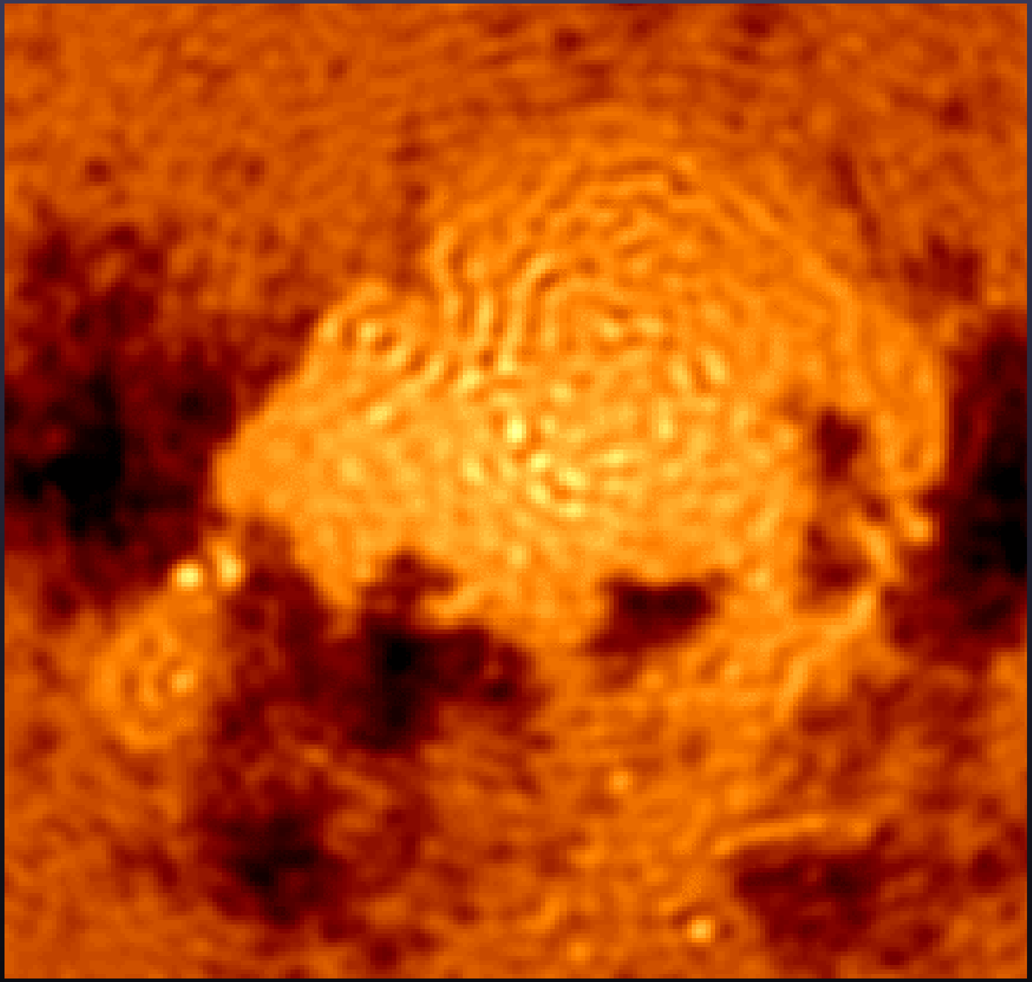
MEM: Model



MEM: Restored

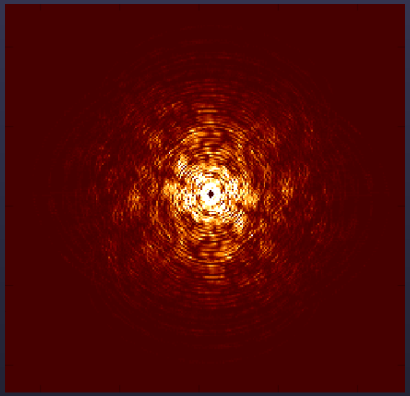


MEM: Residual

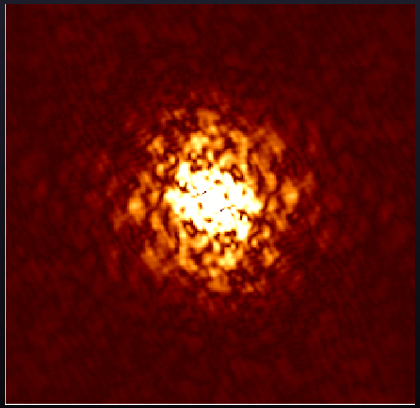


MEM: Model visibilities

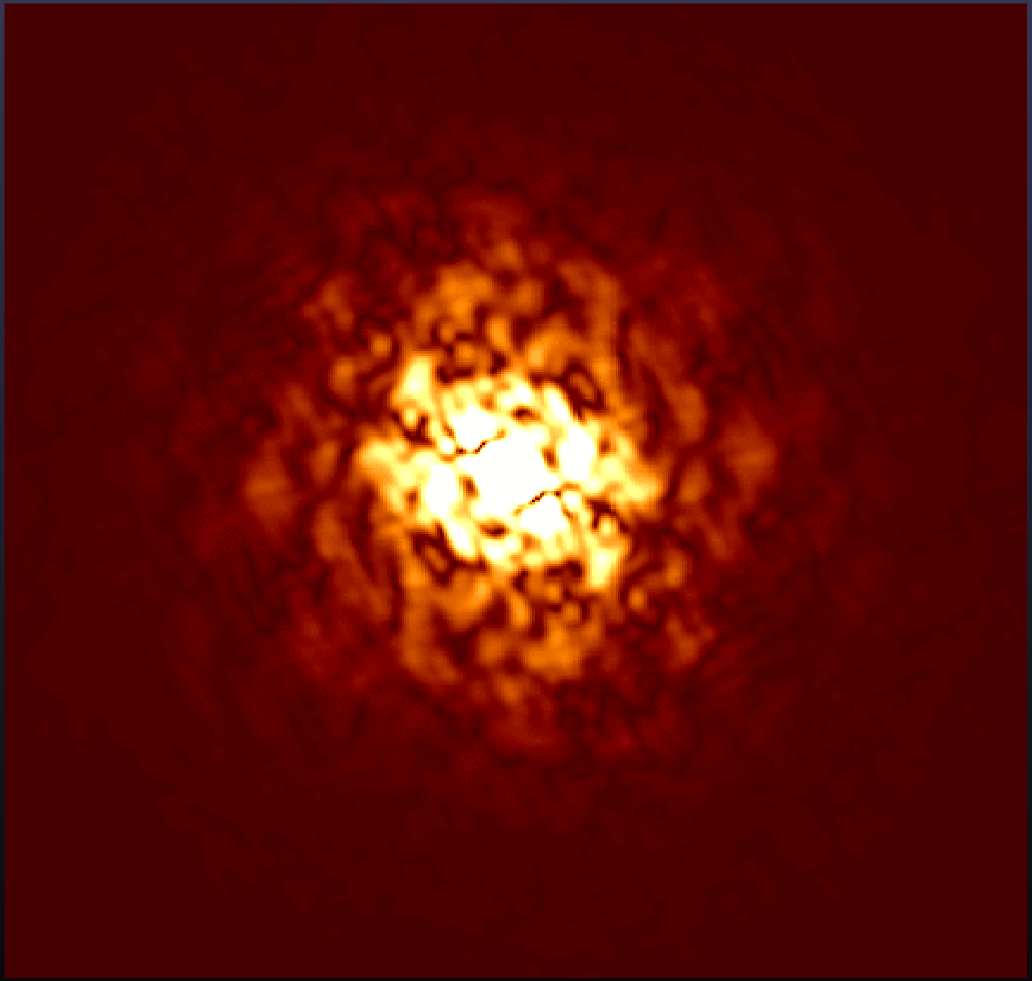
Sampled Vis



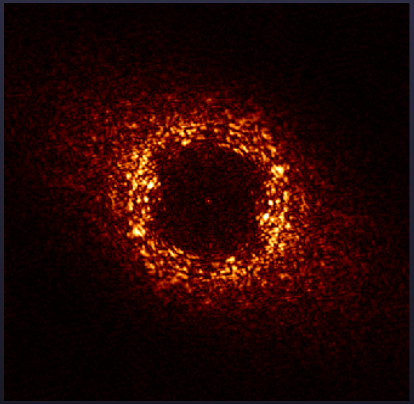
True Vis.



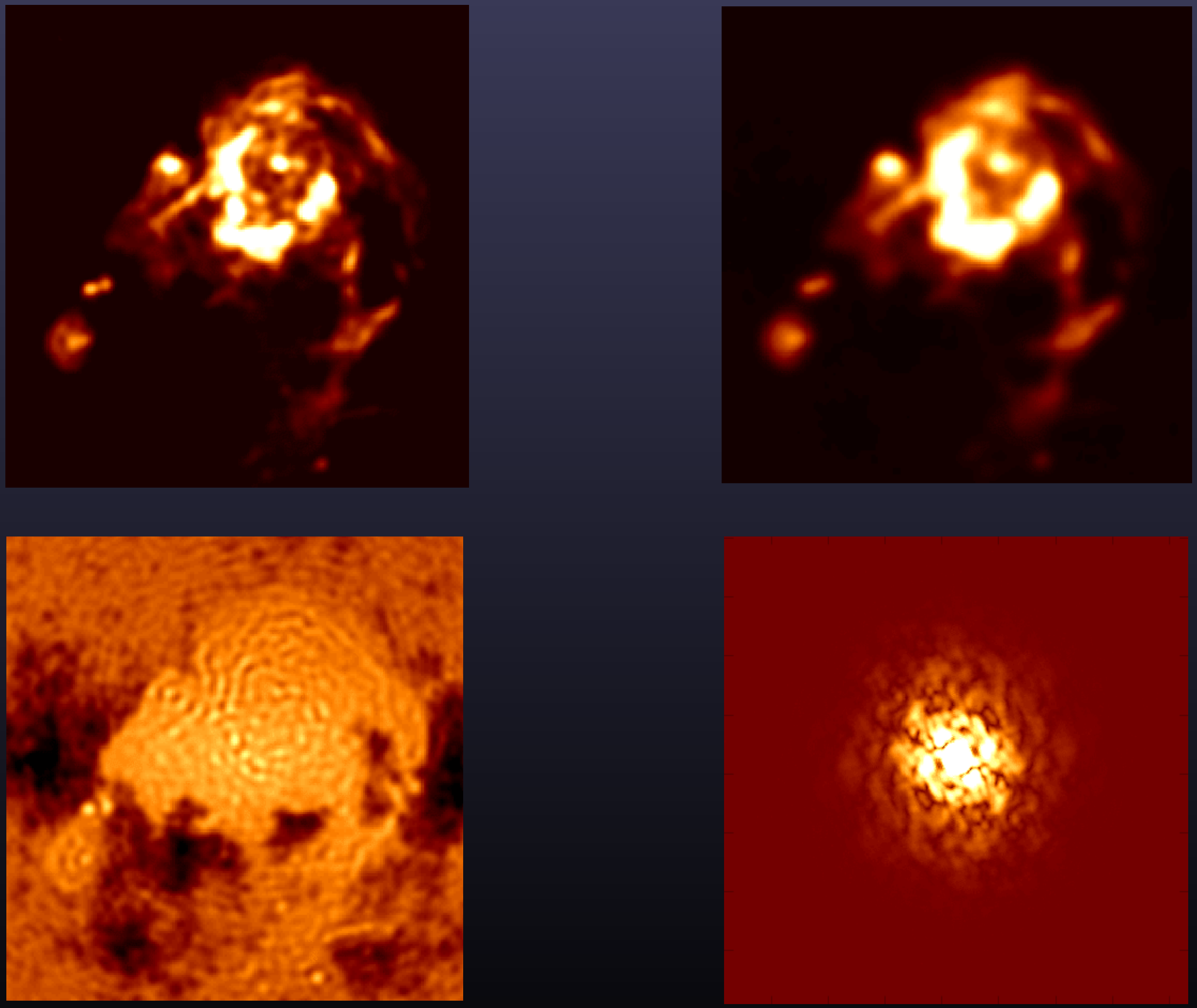
Model Vis.



Residual Vis.



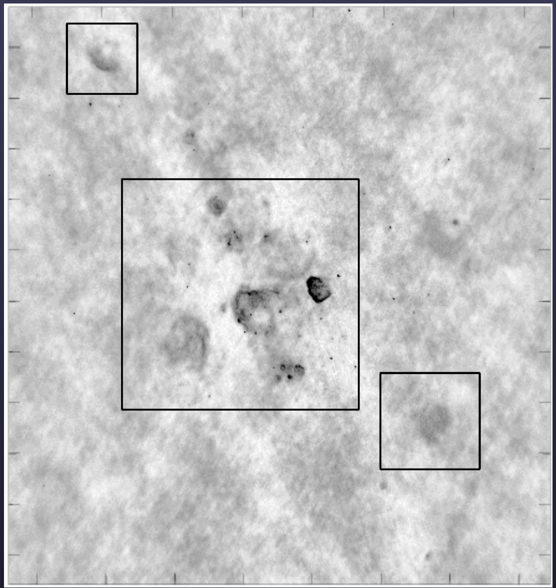
MEM: Example



Role of boxes

- **Limit the search for components to only parts of the image.**

  A way to regularize the deconvolution process.



- **Useful when small no. of visibilities (e.g. VLBI/snapshots).**
- **Do not over-Clean within the boxes (over-fitting).**
- **Deeper Clean with no/loose boxes and lower loop gain can achieve similar (more objective) results.**
- **Stop when Cleaning within the boxes has no global effect (insignificant coupling of pixels due to the PSF).**

Fundamenal problem with scale-less decomposition

- **Each pixel is not an independent degree of freedom (DOF).**

E.g., a gaussian shaped source covering 100 pixels can be represented by 5 parameters.

- **Clean/MEM treats each pixel within a clean-box as an independent degree of freedom.**
- **Scale fundamentally separates noise and signal.**

- *Largest* coherent scale in *BI^N ~ the size of the resolution element.*

- Physically plausible I^M is composed of scales >= the resolution element (smallest scale is of the size of the resolution element).

- ***Scale-sensitive reconstruction therefore leaves more noise-like (uncorrelated) residuals.***

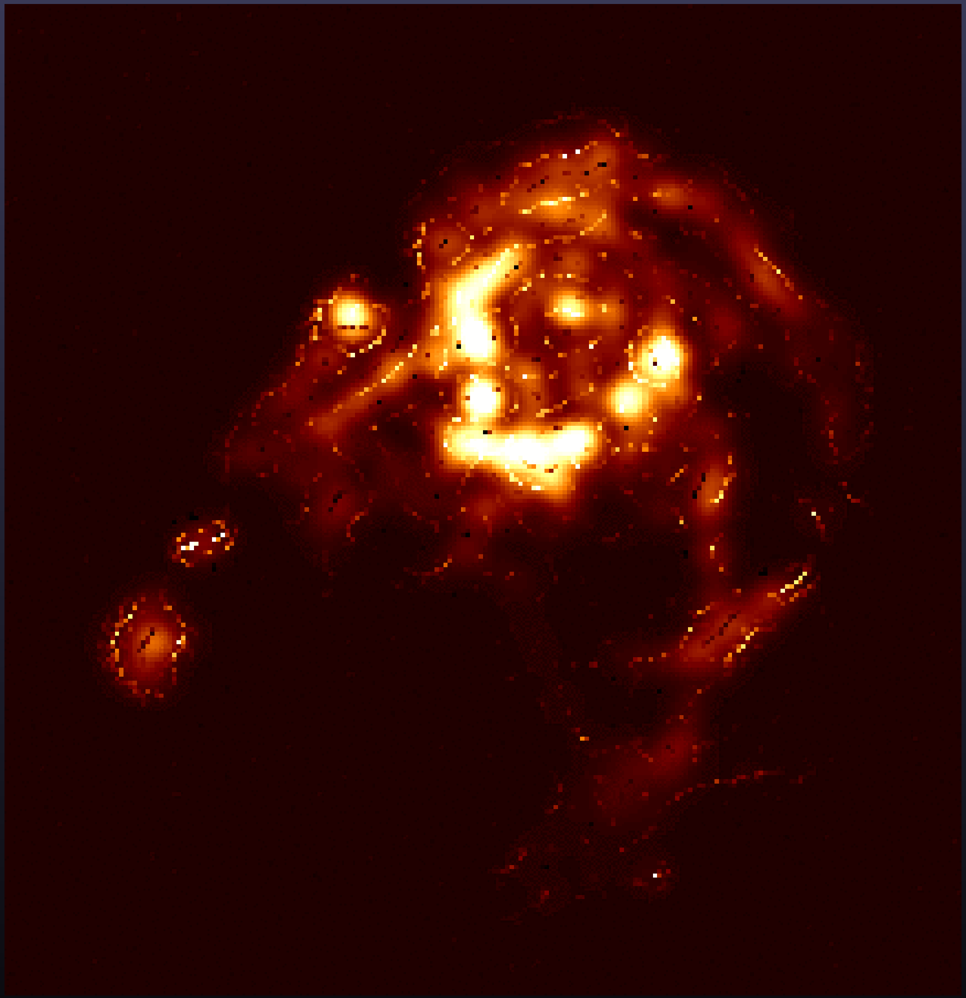
Scale Sensitive Deconvolution: MS-Clean

- **Inspired by the Clean algorithm (Cornwell & Holdaway).**
- **Decompose the image into a pre-computed set of symmetric "blobs" at a few scales (e.g. Gaussians).**
- **Algorithm**
  1. **Make residual images smoothed to a few scales.**
  2. **Find the peak among these residual images.**
  3. **Subtract from all residual images a blob of scale corresponding to the scale of the residual image which had the peak.**
  4. **Add the blob to the model image.**
  5. **If more peaks in the residual images, goto 1.**
  6. **Smooth the model image by the "clean beam" and add the residuals.**

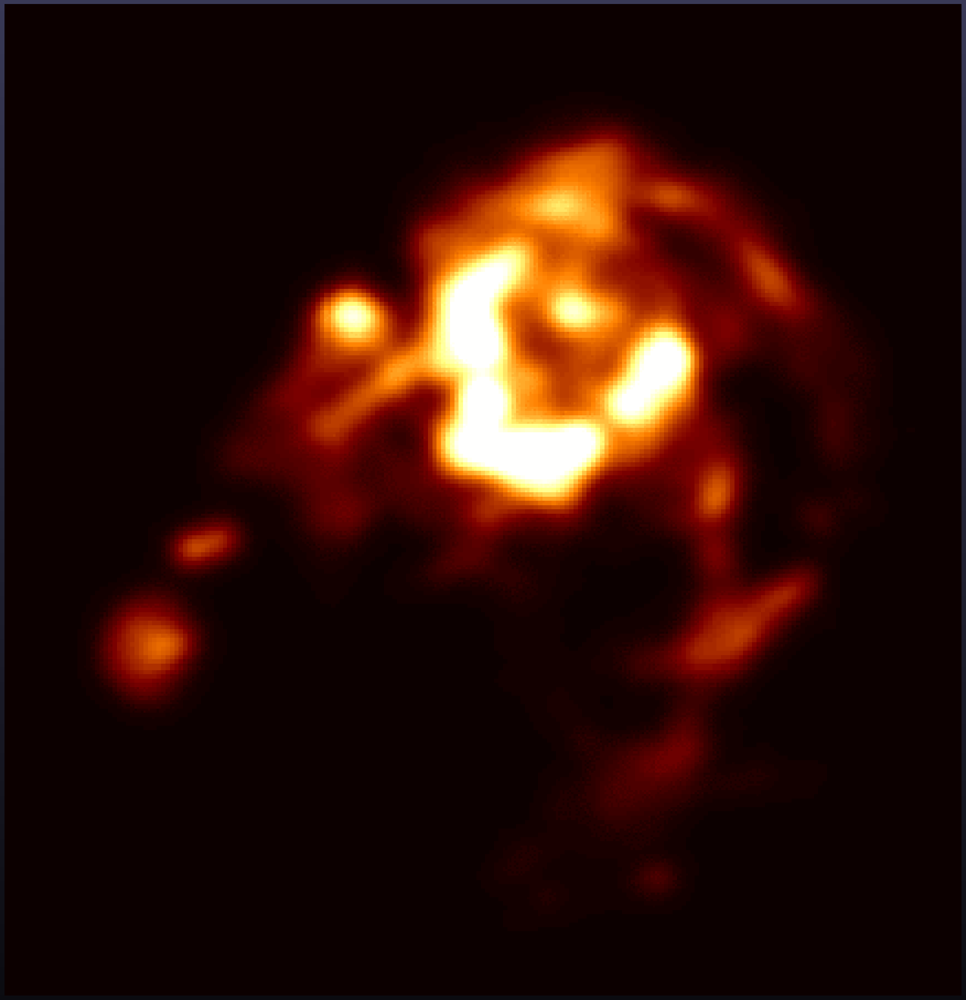
MS-Clean details

- ✓ Deals with compact as well as extended emission better (need to include a blob of zero scale).
- ✓ Retains the scale-shift-n-subtract nature of Clean – easy to implement.
- ✓ Reasonably fast (for what it does!)
- ✗ Breaks up non-symmetric structures (as in Clean – but the errors are at larger scales than in Clean).
- ✗ Ignores coupling between blobs.
  - Assumes an orthogonal space and steepest descent minimization.

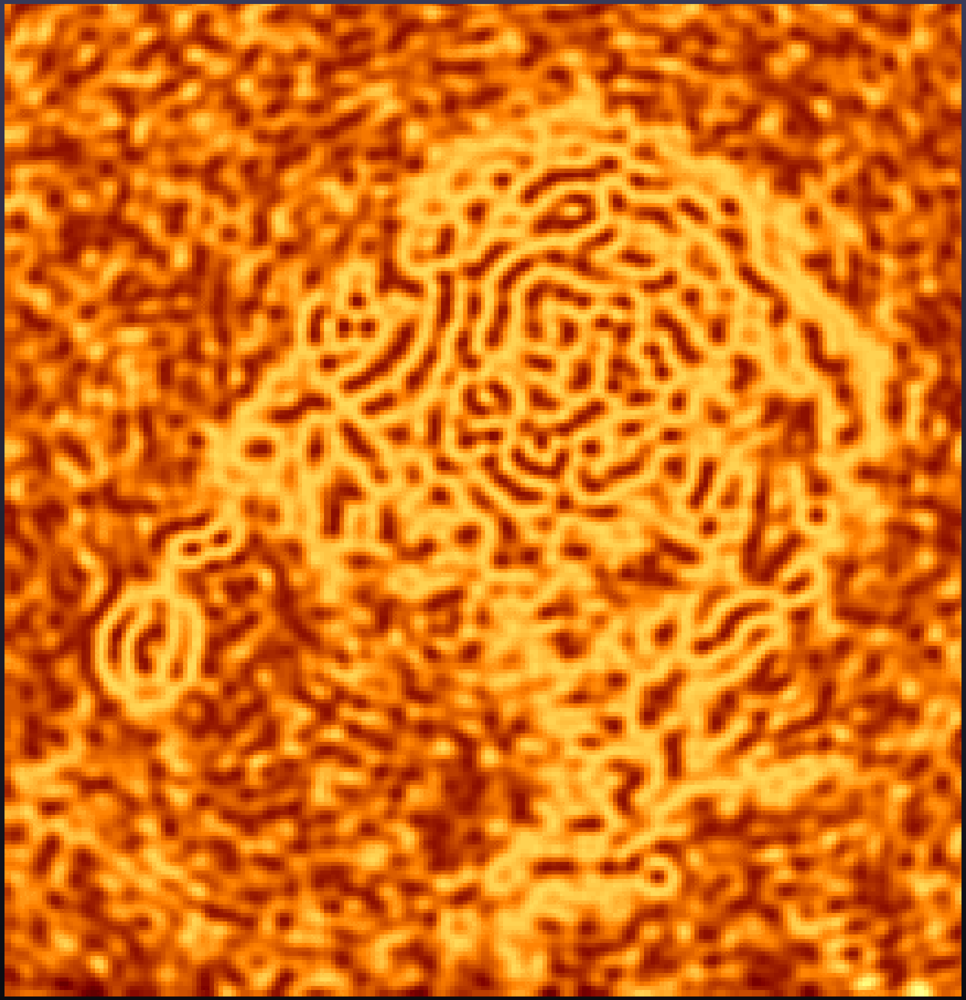
MS-Clean: Model



MS-Clean: Restored

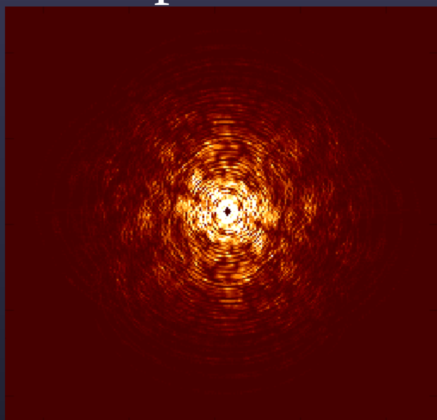


MS-Clean: Residual

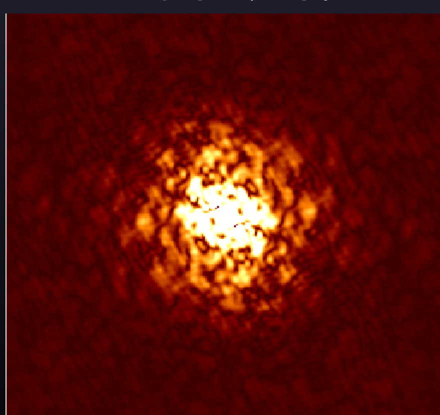


MS-Clean: Model visibilities

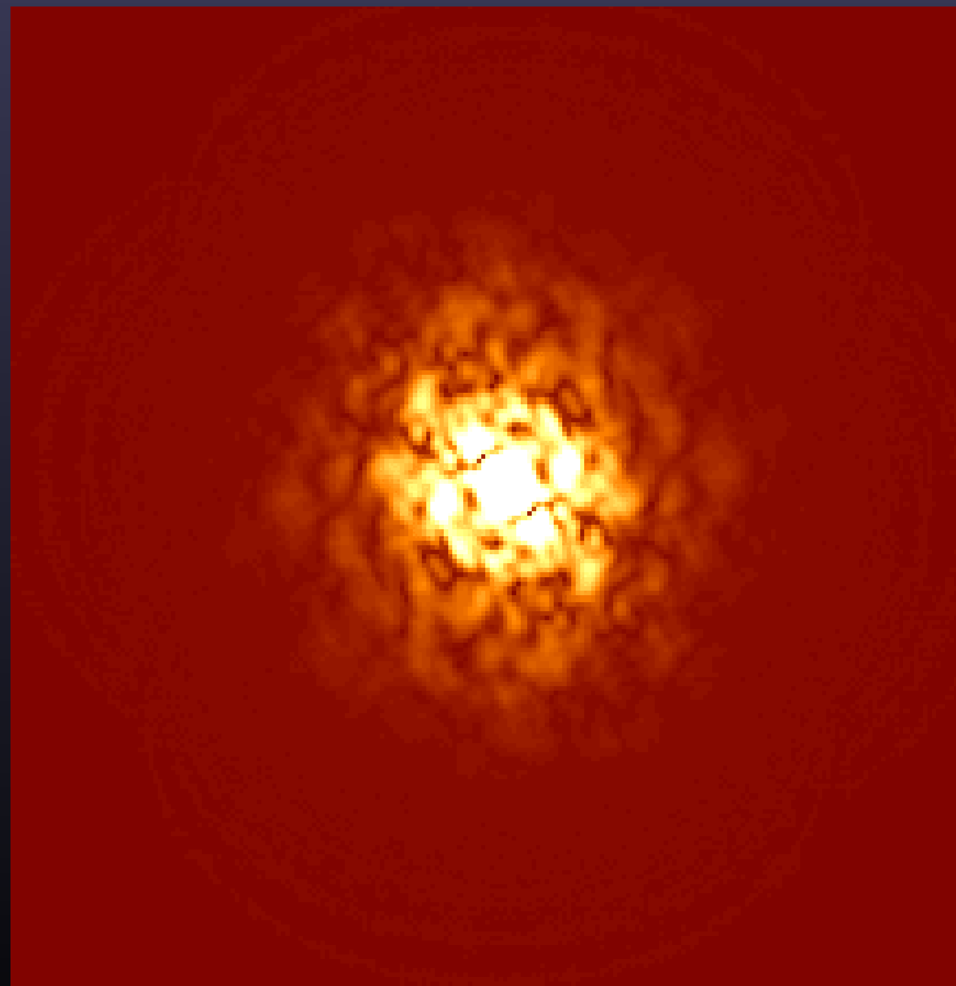
Sampled Vis



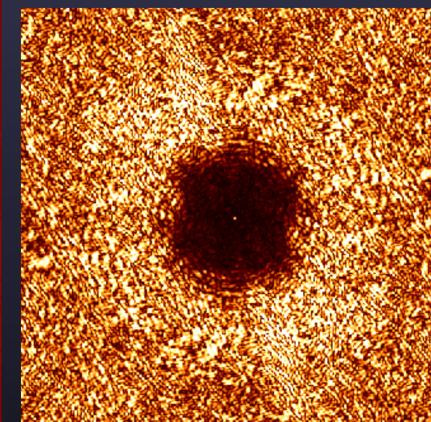
True Vis.



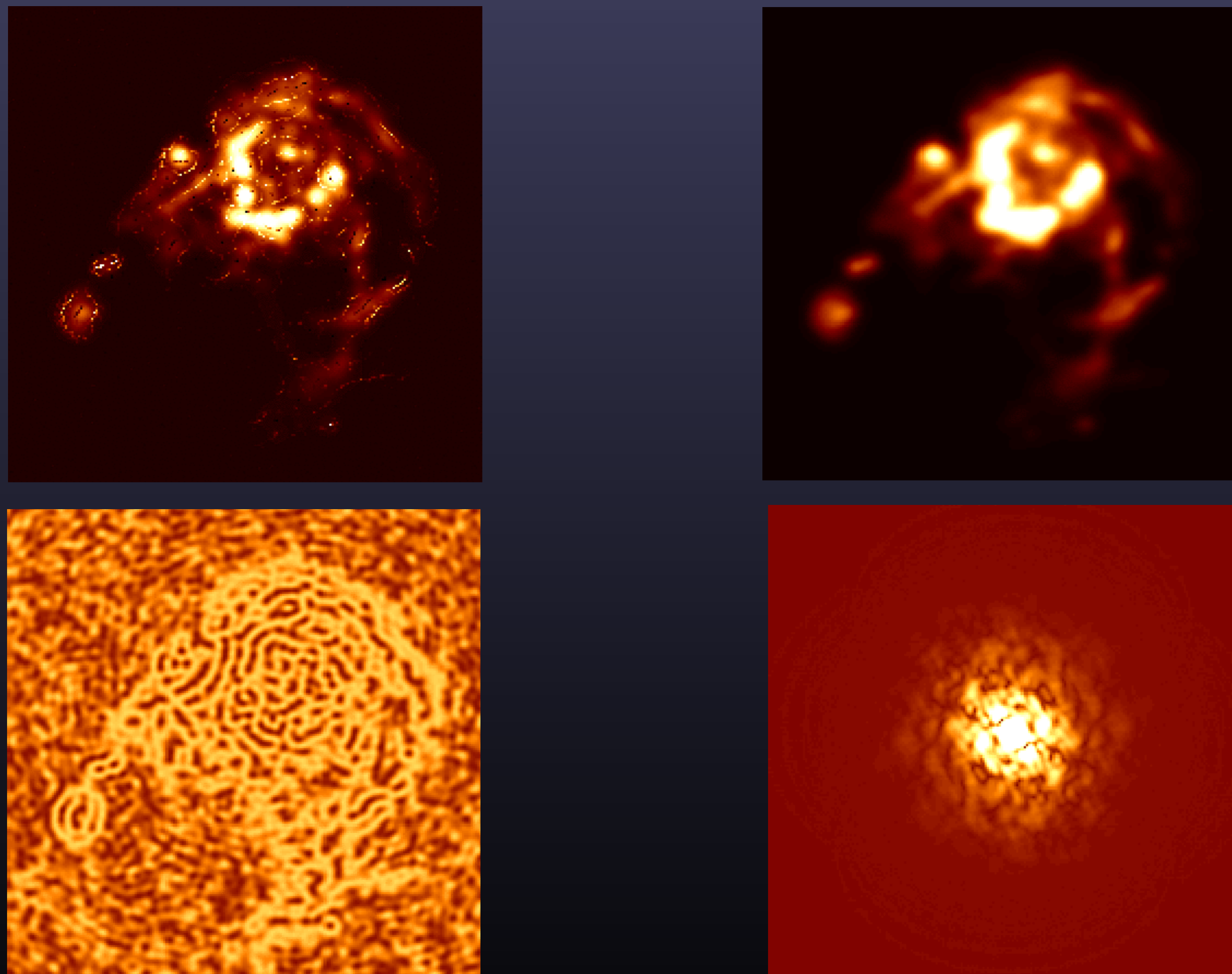
Model Vis.



Residual Vis.



MS-Clean: Example



Multi-resolution vs. Multi-scale Clean

- **Subtle difference between AIPS and AIPS++ implementations of scale sensitive**

  AIPS++: Each *iteration of the minor cycle* removes the optimal scale (one which reduces the residuals globally). Effectively, this achieves a "simultaneous" deconvolution at various scales [Multi-Scale Clean]

  AIPS: A decision, based on a user defined parameter, is made at the *start of each minor cycle* about the optimal scale to deconvolve [Multi-resolution Clean].

- **MS-Clean naturally detects and removes the scale with maximum power**
- **Removal of the optimal scale in MR-Clean strongly depends on the value of the user defined parameter.**

Scale Sensitive Deconvolution: Asp-Clean

- **Inspired by Pixon reconstruction (Puetter & Pina, 1994).**
- **Decompose the image into a set of Adaptive Scale Pixel (Asp) model (Bhatnagar & Cornwell, 2004).**
- **Algorithm:**
  1. **Find the peak at a few scales, and use the scale with the highest peak as an initial guess for the optimal dominant scale.**
  2. **Make a set of active-aspen containing Aspen found in earlier iterations and which are likely to have a significant impact on convergence.**
  3. **Find the best fit set of active-Aspen (expensive step).**
  4. **If termination criterion not met, goto 1.**
  5. **Smooth with the clean-beam. Add residuals if it has systematics.**

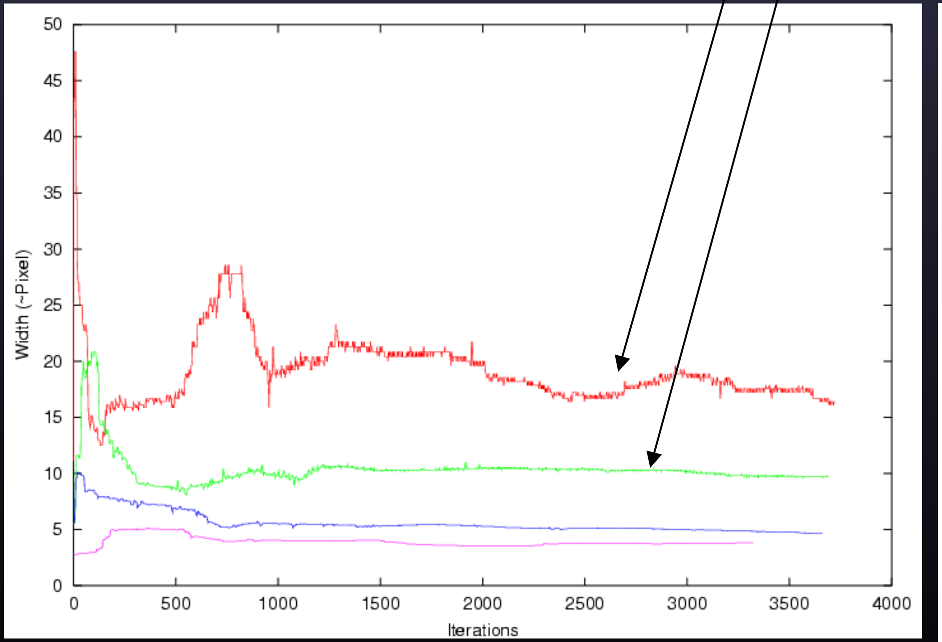
Asp-Clean details

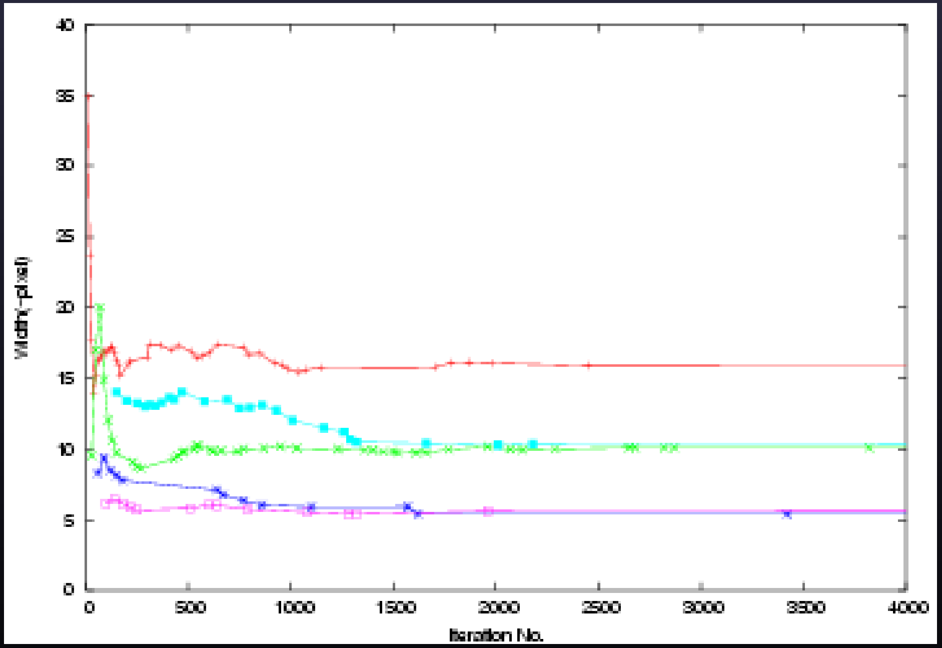
- ✓ **Deals with non-symmetric structures better.**
- ✓ **Incorporates the fact that scale changes across the image. Residuals are more noise-like.**
- ✓ **Incorporates the fact that search space is potentially non-orthogonal (inherent coupling between Aspen).**
- ✓ **Aspen found in earlier iterations are not frozen.**
- ✓ **Scales well with computing power.**
- ✗ **Slower in execution speed.**

Asp-Clean details: acceleration

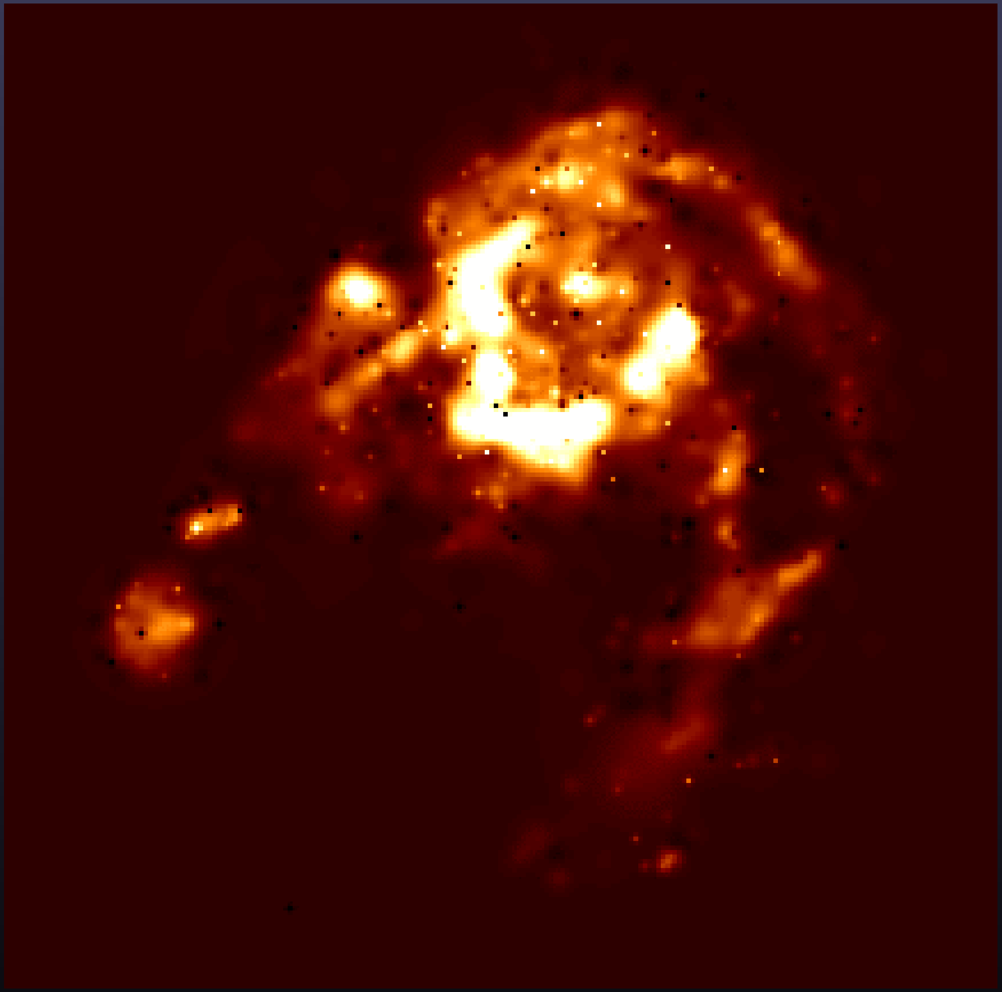
Fig 1: All Aspen are kept in the problem for all iterations. Scales all Asp scales evolve as a function of iterations.
Not all Aspen evolve significantly for at all iterations.

Fig 2: The active-set is determined by thresholding the first derivative. Only those Aspen, shown by symbols, are kept in the problem which are likely to evolve significantly at each iteration.

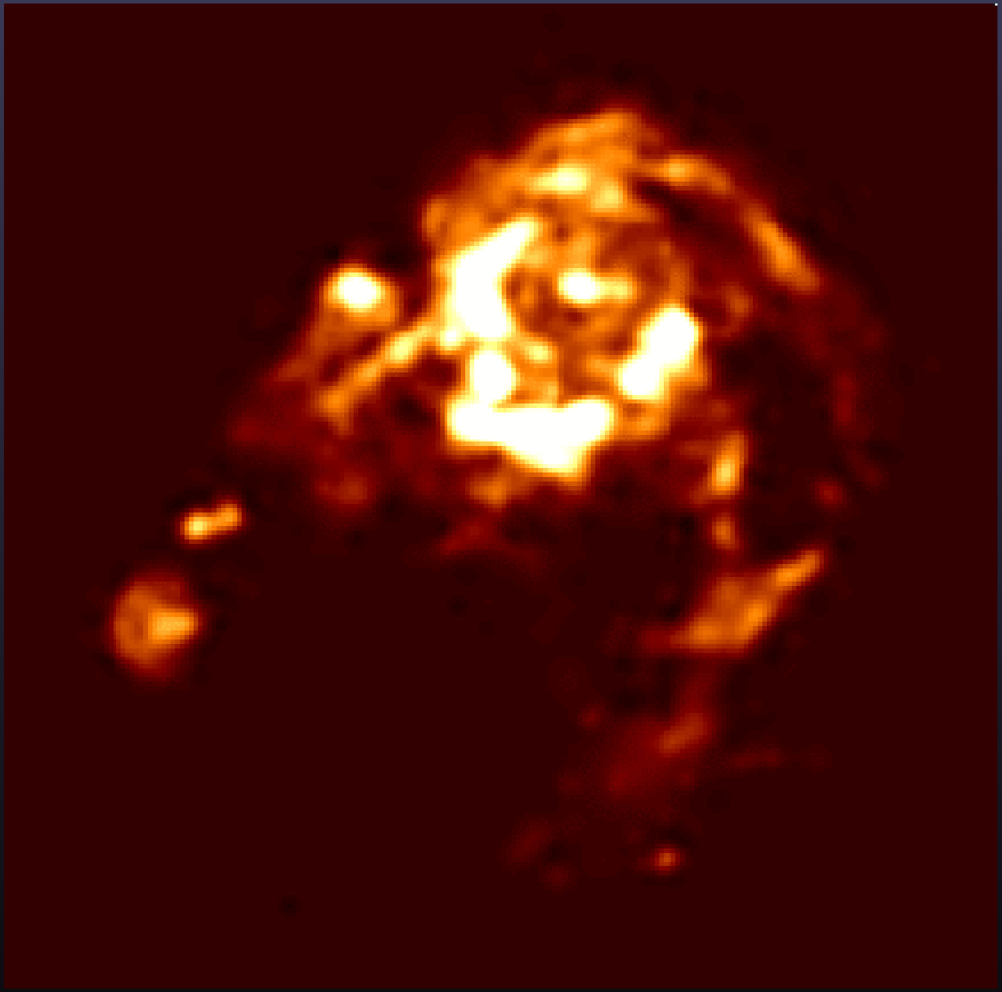




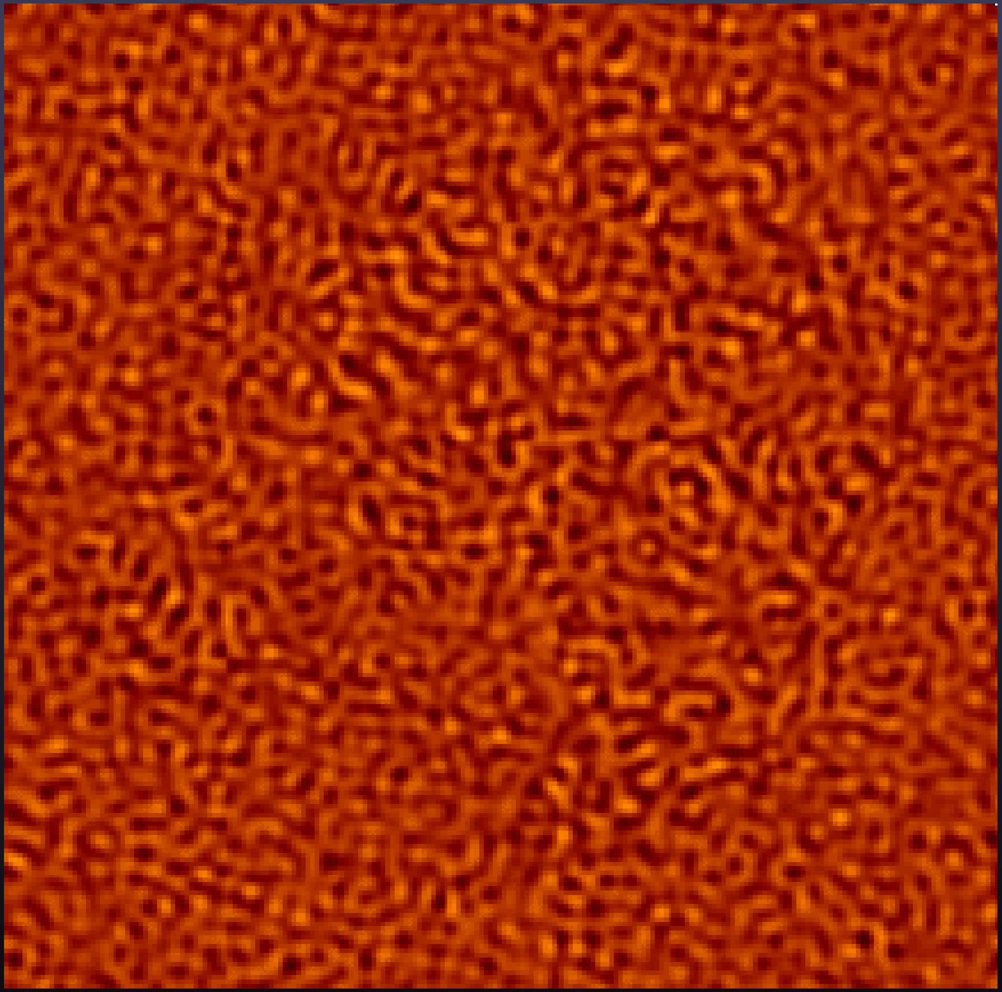
Asp-Clean: Model



Asp-Clean: Restored

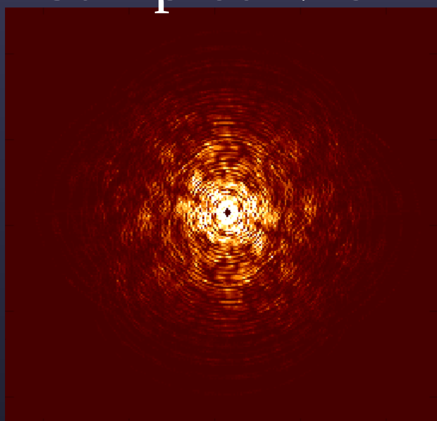


Asp-Clean: Residual

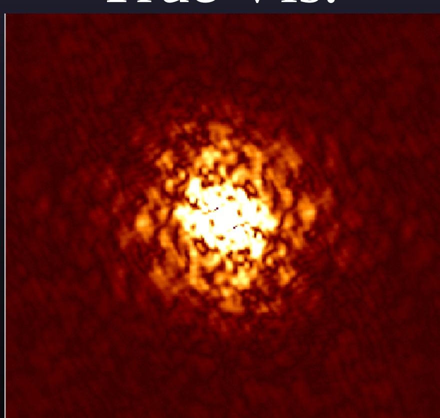


Asp-Clean: Model visibilities

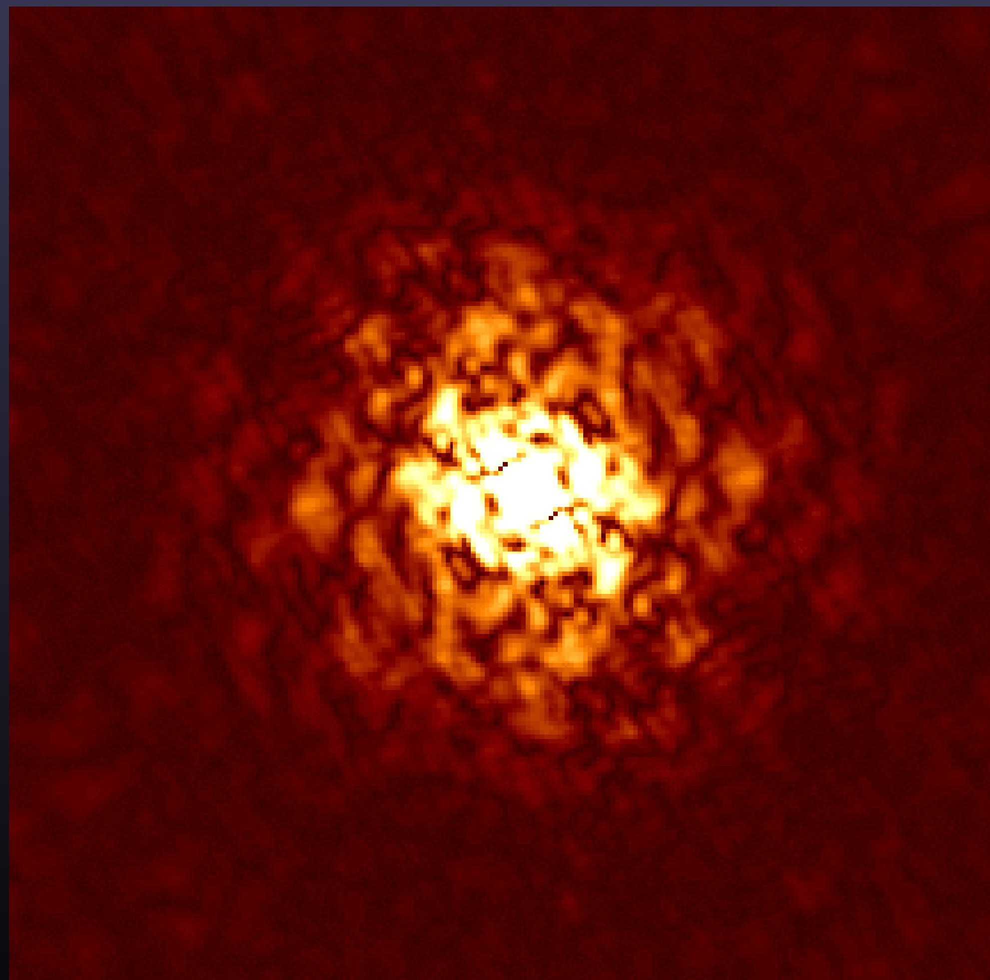
Sampled Vis



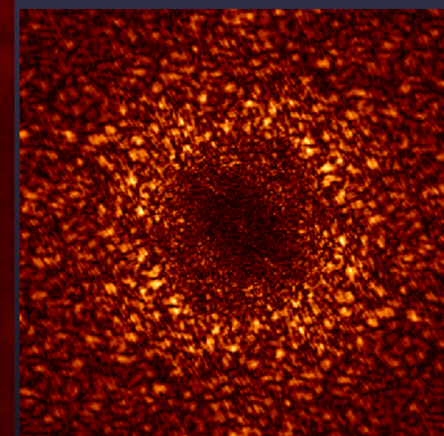
True Vis.



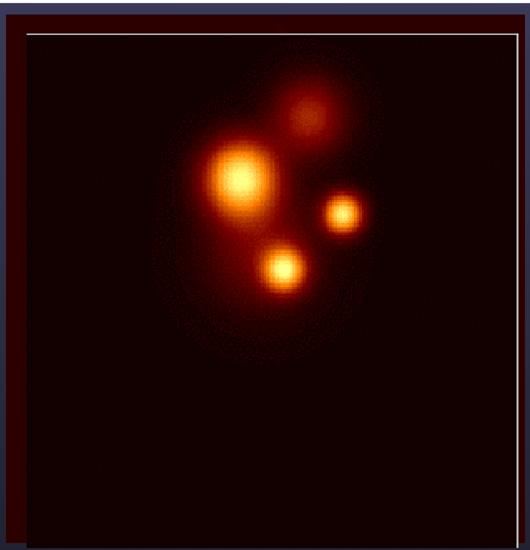
Model Vis.

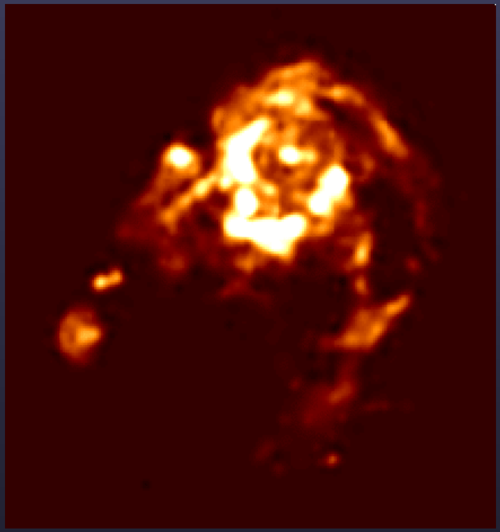


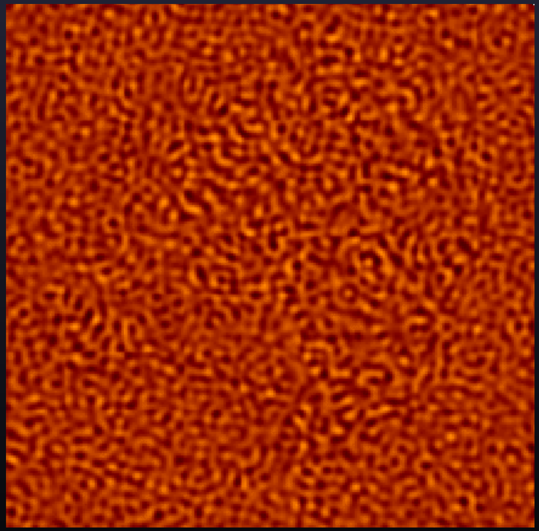
Residual Vis.

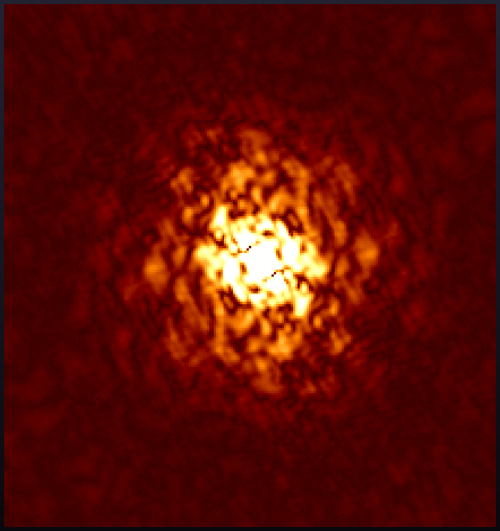


Asp-Clean: Example

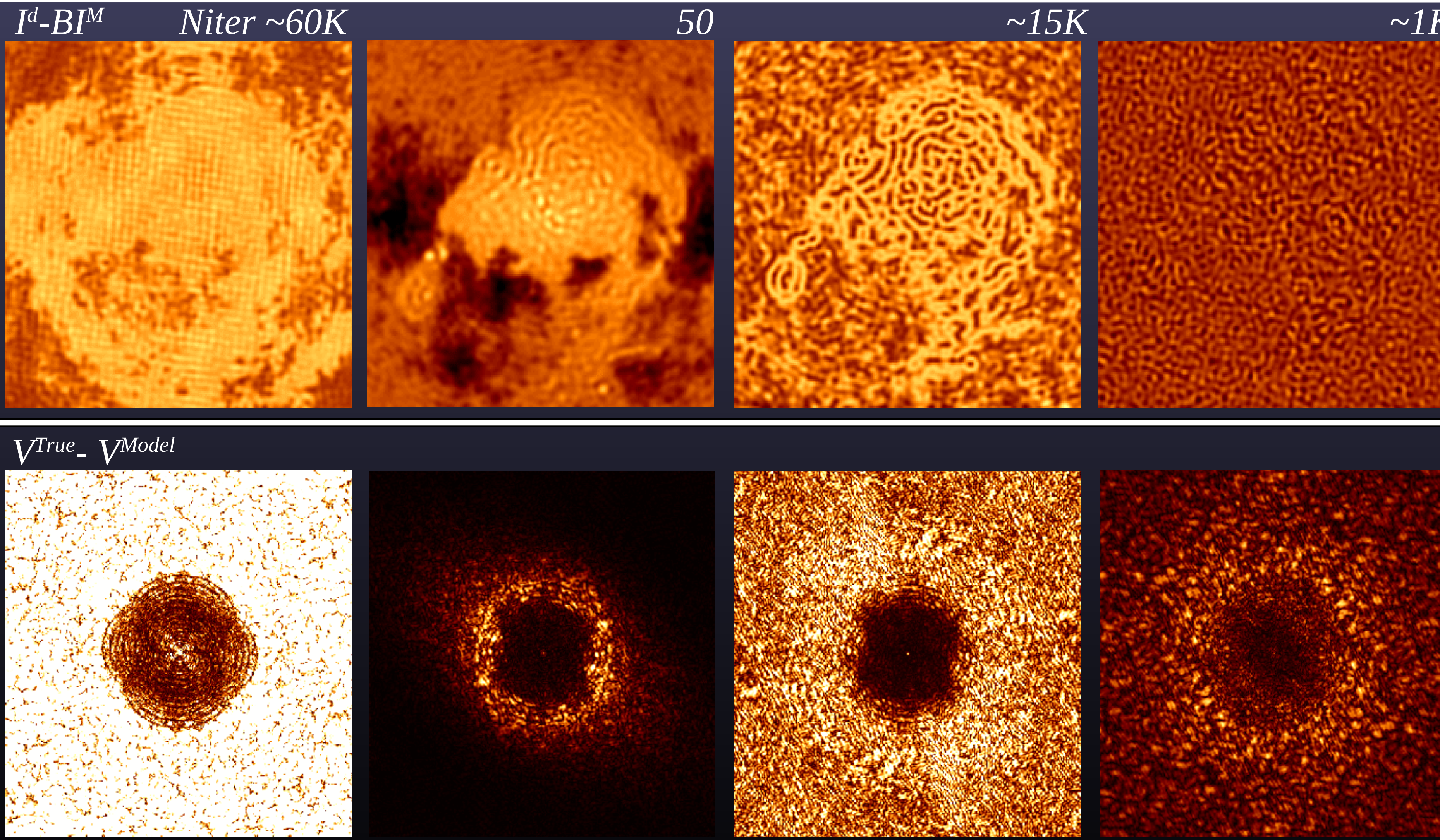








Clean, MEM, MS-Clean, Asp-Clean



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1. Synthesis Imaging in Radio Astronomy II; Imaging and deconvolution.
2. "High Fidelity Imaging of Moderately Resolves Sources"; Briggs, D. S., PhD Thesis, New Mexico Tech., 1995
3. Astronomers at AOC in general