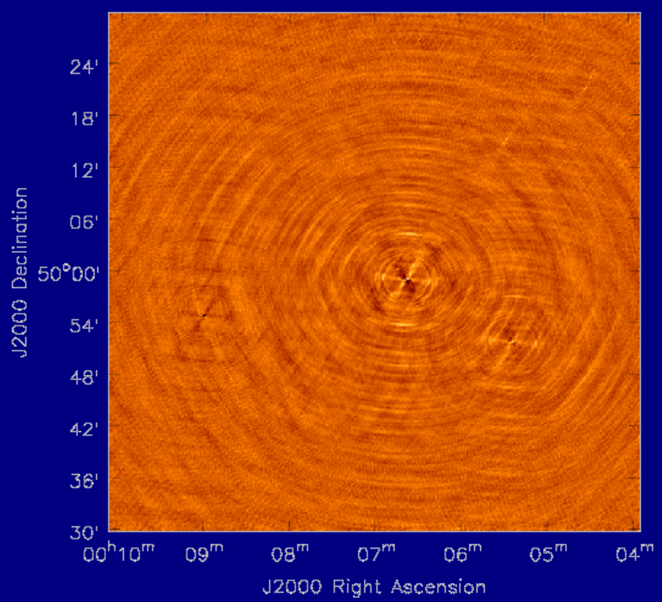
Direction dependent corrections



Pointing Correction
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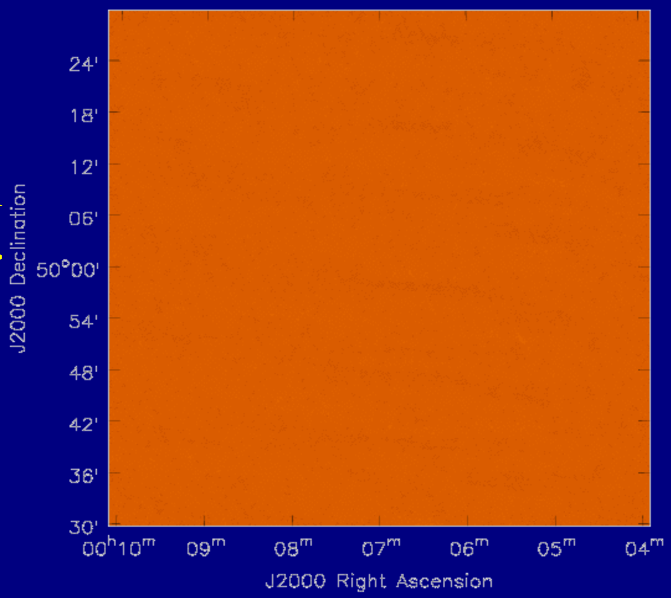

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The Measurement Equation

- Generic Measurement Equation:

$$V_{ij}^{Obs}(\nu) = J_{ij}(\nu, t) \int J_{ij}^{S}(\mathbf{S}, \nu, t)\, I^{o}(\mathbf{S})\, e^{\iota \mathbf{S.B}_{ij}}\, d\,\mathbf{S}$$

Data **Corruptions** **Sky**

***J_{ij}* *are the* direction independent corruptions.**

***J^{S}_{ij}* are the image plane errors (direction dependent).**

- $V_{ij}^{Obs} = J_{ij} . \left[E_{ij} * V_{ij}^{o} \right]$ where E_{ij} = FT[J^{S}_{ij}]. Unknowns: J_{ij}, J^{S}_{ij}, *and* I^{M}.

- SelfCal: Given I^{M} *and* knowing E_{ij} $\quad min : |V_{ij}^{Obs} - J_{ij} . V_{ij}^{M}|^2$ w.r.t. J_i

- Deconvolution: Given J_{ij} *and* E_{ij}, $\quad min : |J_{ij}^{-1} V_{ij}^{Obs} - V^{M}|^2$ w.r.t. I^{M}

Visibility correction

- In the absence of direction dependent errors $V_{ij}^{Obs} = J_{ij} V_{ij}^{o}$
- Observed data is corrected as $V_{ij}^{Corr} = J_{ij}^{-1} V_{ij}^{Obs}$
- Imaging is done using the corrected visibilities

Imaging and calibration are decoupled

- Antenna independent direction dependent corruptions (e.g. Primary beam effects) can be written as multiplicative terms in the image domain. $V_{ij}^{M} = FT[J^S I^M]_{ij}$

- This can be used in an iterative deconvolution scheme to make I^M which is corrected for J^S (VLA Poln. squint correction, mosaicking, etc.)

$$\vec{V}^{Obs} = E * \vec{V}^{M} \quad \text{and} \quad \Delta I^D \propto \Re \sum_{ij} [J_{ij}^{S^{*T}} \Delta V_{ij} e^{\iota S.B_{ij}}]$$

Visibility correction

- For effects which are separable as antenna based effects

$$J_{ij}^{S} = J_i^S \otimes J_j^{S*}$$

- When $J_i^S \neq J_j^S$, effects of J_{ij}^{S} cannot be reduced to the convolution of the visibility plane by a single function.

$$V_{ij}^{Obs} = E_{ij} * V_{ij}^{o}$$

- Each baseline is convolved by a different function.

Visibility inversion

$$V_{ij}^{Obs} = E_{ij} * [A \vec{I}^o]_{ij} \qquad \vec{V}^{Obs} = E[A \vec{I}^o]$$

V,I : The visibility and image vectors

A: The Measurement Matrix

E: The direction dependent effect in the Fourier plane

- To use FFT for Fourier transforming, re-sampling is done by convlutional gridding

$$V_{ij}^M = \left[C[A \vec{I}^M] \right](u_{ij}, v_{ij})$$

- Major cycle: $\Delta \vec{I}^D = A^T [C^T \vec{V}^R]$
- Minor cycle: $\vec{I}_i^M = \vec{I}_{i-1}^M + \alpha \Delta \vec{I}^D$
- Direction dependent effects can be incorporated in imaging by using *EA* as the transform operator

General approach for imaging

- Design filters to be used as Gridding Convolution Functions (GCF) for accurate forward transform (major cycle)

$$V^M(u_{ij}) = \left[E_{ij}(u) * [A I^M](n \Delta u) \right](u_{ij})$$

- Use an approximation for the inverse transform to compute the update direction (minor cycle)

- Major cycle: $\vec{I}^R = B[\vec{V}^{Obs} - A' \vec{I}^M] \quad where \quad B \approx A'^T$

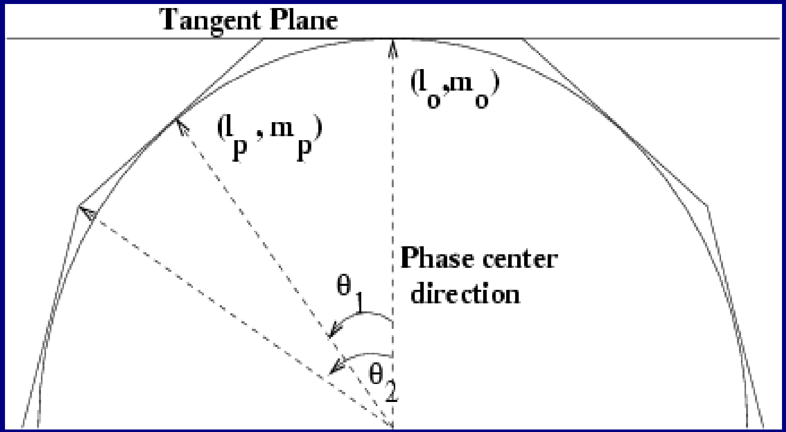
- Minor cycle: $\vec{V}^M = A' \vec{I}^M$

Known direction dependent effects

- Non-coplanar baselines

$$V(u,v,w)=\iint I(l,m)\,\mathrm{e}^{2\pi\iota\left(ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right)}\frac{dl\,dm}{\sqrt{1-l^2-m^2}}$$

- Traditional approach: Faceting



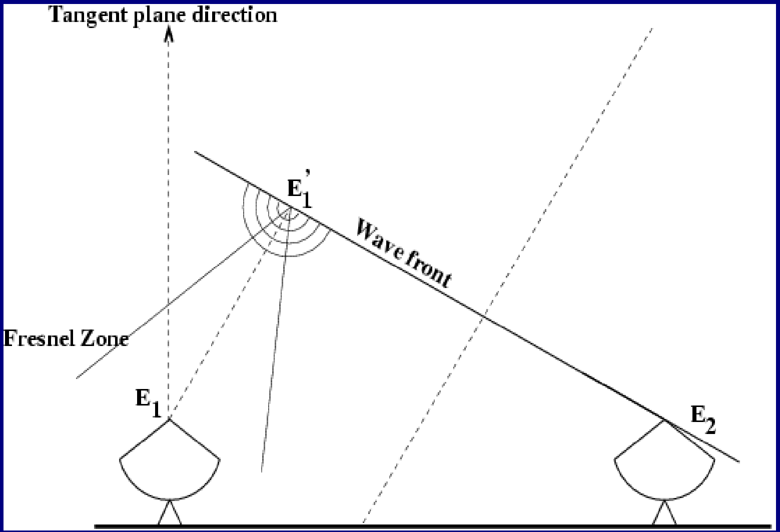
- W-projection: Visibility filtering (5-10 times faster)

$$V_{ij}^{Obs}(u,v,w)=E_{ij}*V_{ij}^{M}(u,v,w=0)\quad where\quad E_{ij}=\boldsymbol{FT}\left[e^{2\pi\iota w\left(\sqrt{1-l^2-m^2}\right)}\right]$$

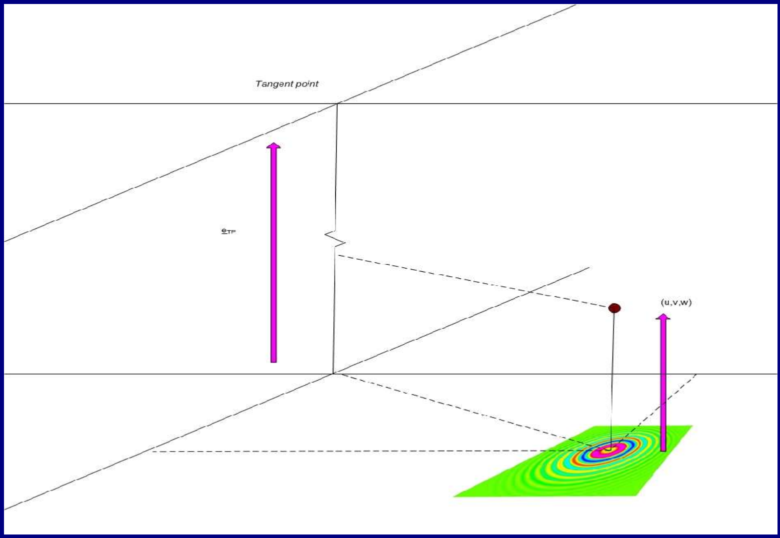
W-projection

- $\rho_{12}=\langle E_1(u,v,w=0)E_2^*(0,0,0)\rangle$

  $E_1=E'_1(u,v,w)$ propagated using Fresnel diffraction. The above convolution equation is reproduced with $r_F/\lambda\approx\sqrt{w}$



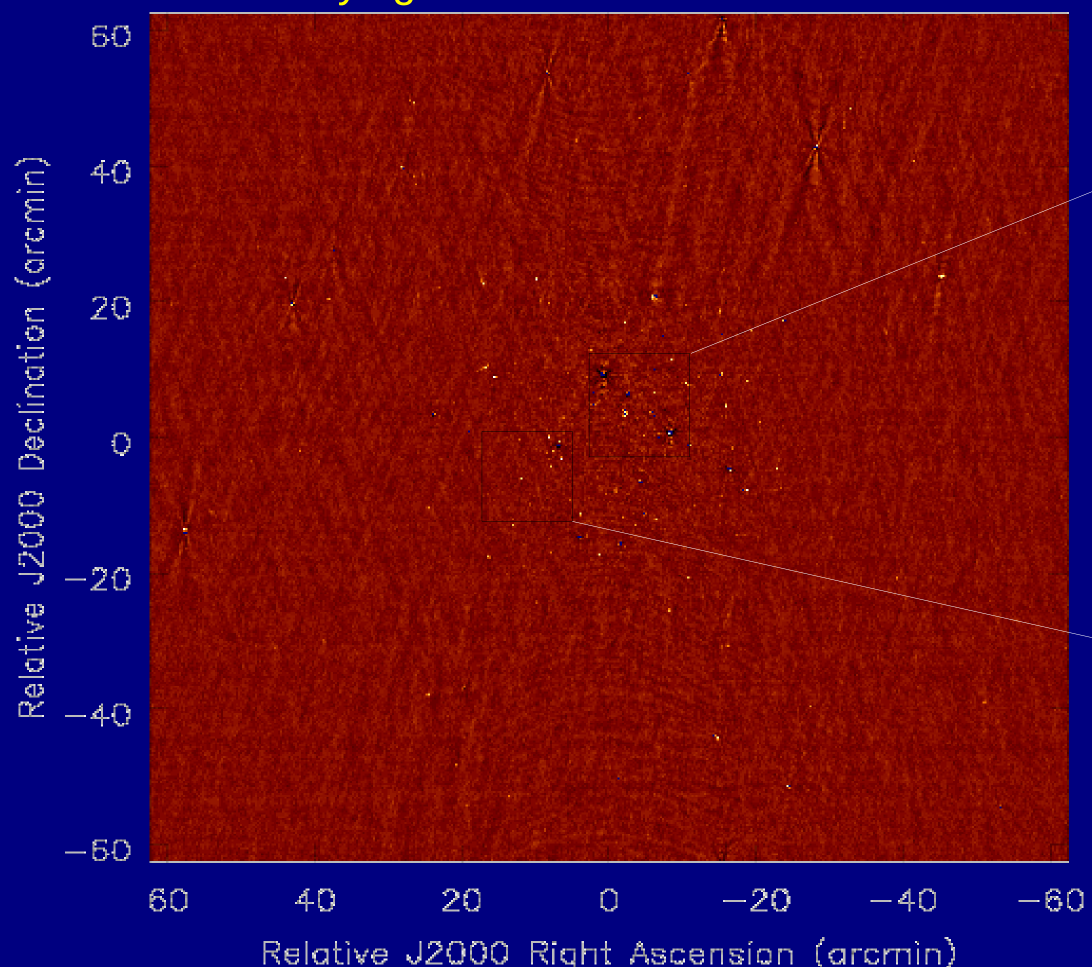
- A $w\neq 0$ interferometer is not a device to measure a single Fourier component.

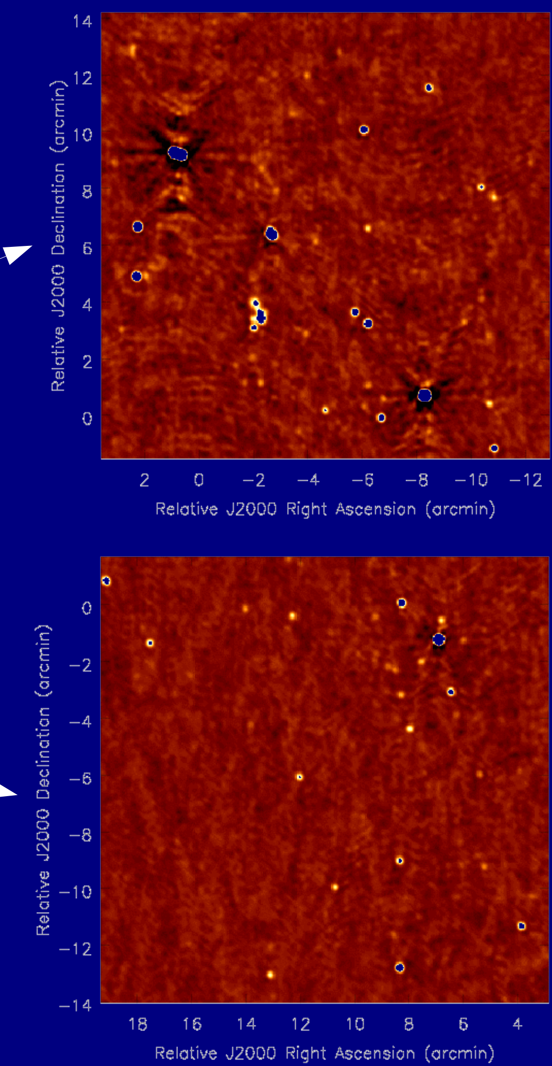


W-projection: Example

L-Band, VLA C-array, ~40hr. integration (Fomalont et al.)

- -RMS: ~15microJy, Peak: 40mJy
- -Errors vary across the image (pointing?).
- -Time varying first side-lobe not corrected.





Measured direction dependent effects

- E_{ij} as a function of direction is measured a priori

$$V^M_{ij} = [\boldsymbol{E}_{\boldsymbol{ij}} * [\boldsymbol{A}\,\boldsymbol{I}^M]_{ij}] \quad where \quad E_{ij}(l_i, l_j, u_{ij}; p_i, p_j)$$

- Beam squint: E_{ij} *separate* for each poln. product *pq*

$$E^{pq}_{ij} = E^{pq^o} \quad where \quad E^{pq^o}_i = \boldsymbol{FT}[Ideal\; PB_i]$$

- Pointing jitter: E_{ij} is not known a-priori

$$E^{pq}_{ij} = E^{pq^o} f(\phi_i - \phi_j) e^{\iota(\phi_i + \phi_j)}$$

Needs a solver: Pointing SelfCal

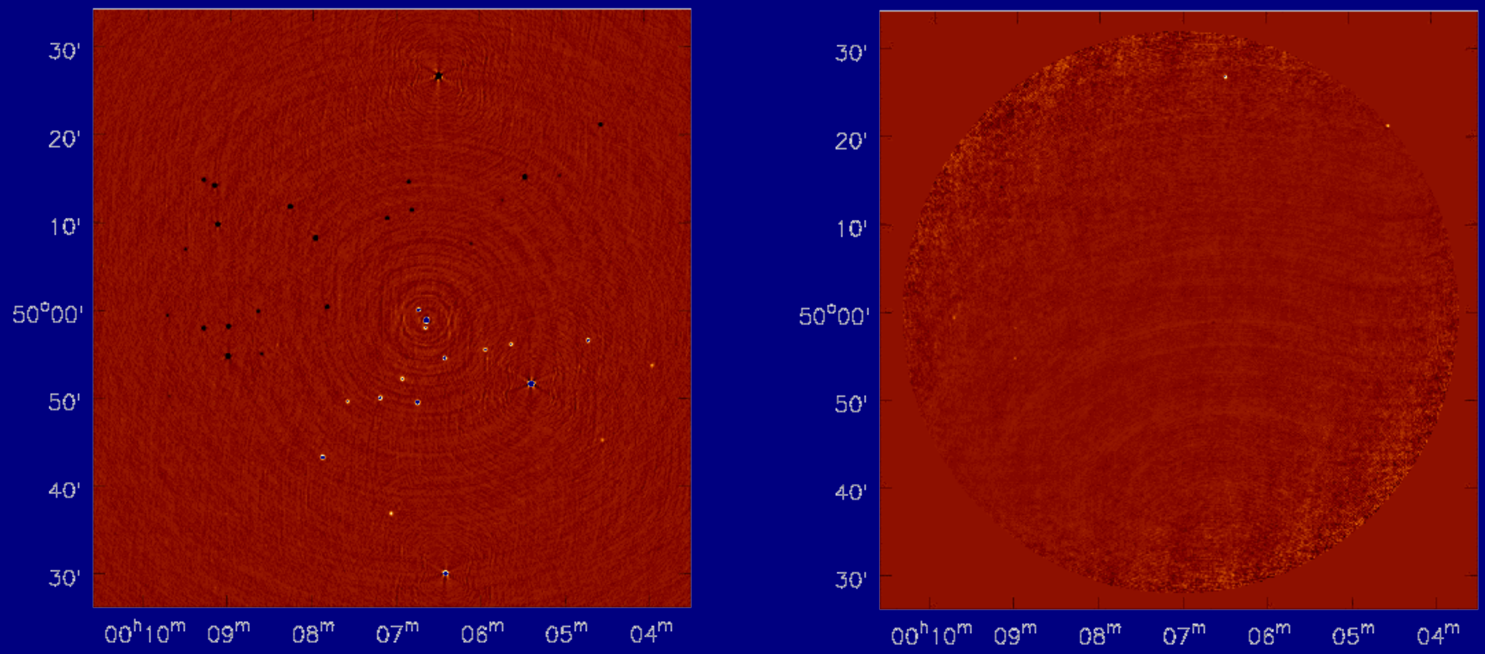
- Asymmetric Primary Beams:

$$E_{ij} = E^o_i * E^o_j \quad where \quad E^o_i = \boldsymbol{FT}[Measured\; PB_i]$$

PB-projection

Full beam polarimetry: Squint corrected Stokes-V imaging

Typical NVSS field: Peak 190mJy



Visibility filters computed for 15° PA increments.
Azimuthally symmetric Primary Beams.

Pointing Solver: Motivation

- Single pointing L-Band observations limited due to pointing ~10microJy/beam.
  - EVLA L-band sensitivity: 1microJy/beam
- Mosaicking dynamic range limited by pointing errors.
  - Significant fraction of ALMA observation will be mosaicking observations.
- Significantly increased computing:
  - Each iteration involves expensive visibility prediction

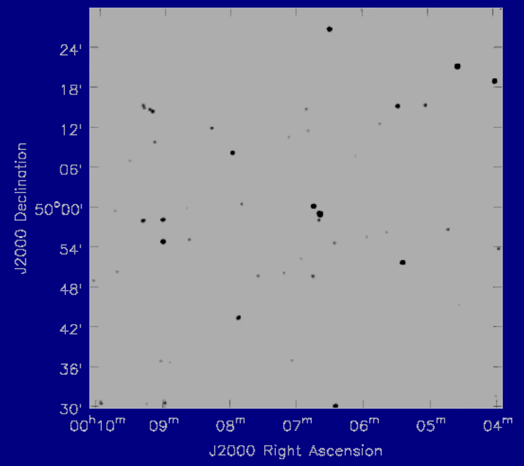
Pointing Solver

$$E_{ij}(l;l_i,l_j)=E^o(l)\,f(\phi_i-\phi_j)\,e^{-\pi\iota u_{ij}(\phi_i+\phi_j)}$$

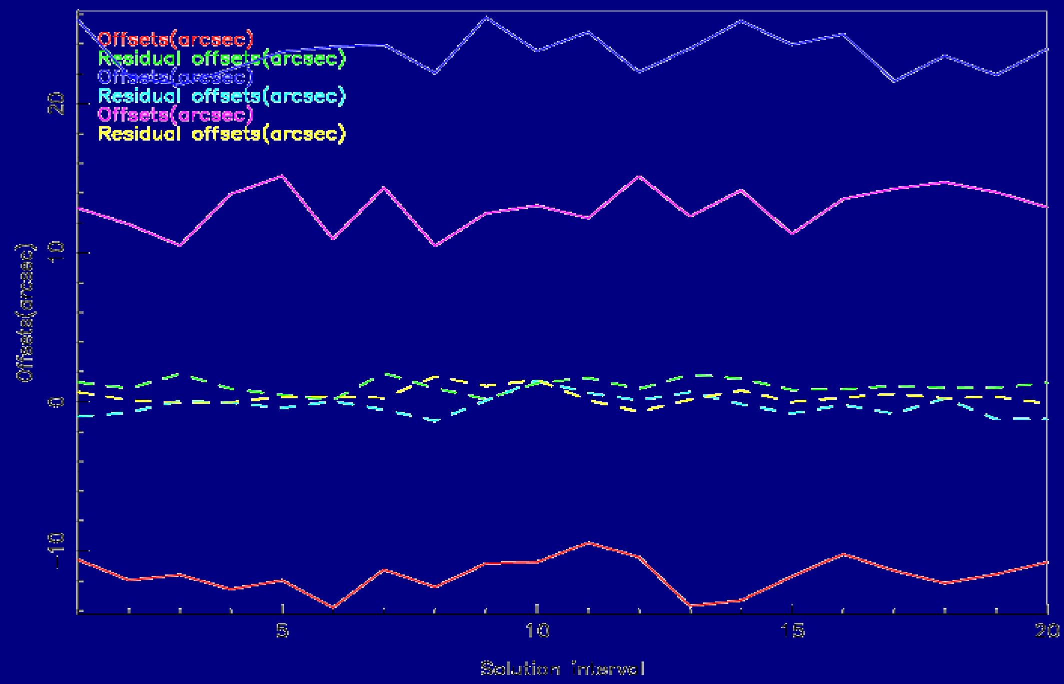
- Visibility prediction:
  - Gridded model $V^{M,Grid} = FFT[I^M]$
  - Re-sample on measured *(u,v)* using E_{ij} as the GCF for baseline *i-j:* $V_{ij}^M = E_{ij} * V^{M,Grid}$
- *GCF is different for each baseline!*
- Use lookup tables or approximation:

$$E_{ij}=E^o\left[1-(l_i-l_j)^2\sigma^2/2-\pi\iota u_{ij}(l_i+l_j)+\ldots\right]$$

Pointing correction



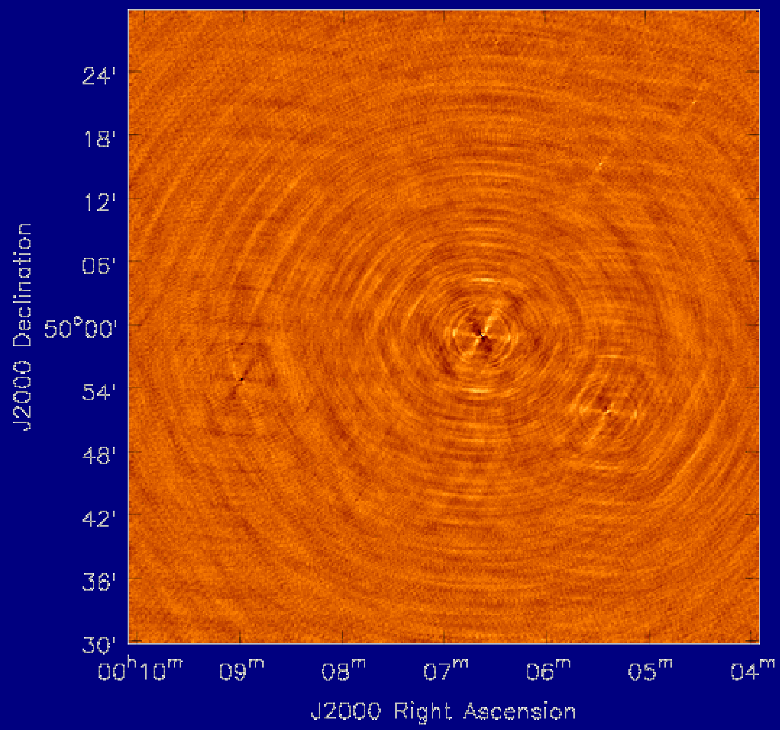
Model image using 59 sources from NVSS.
Flux range ~2-200 mJy

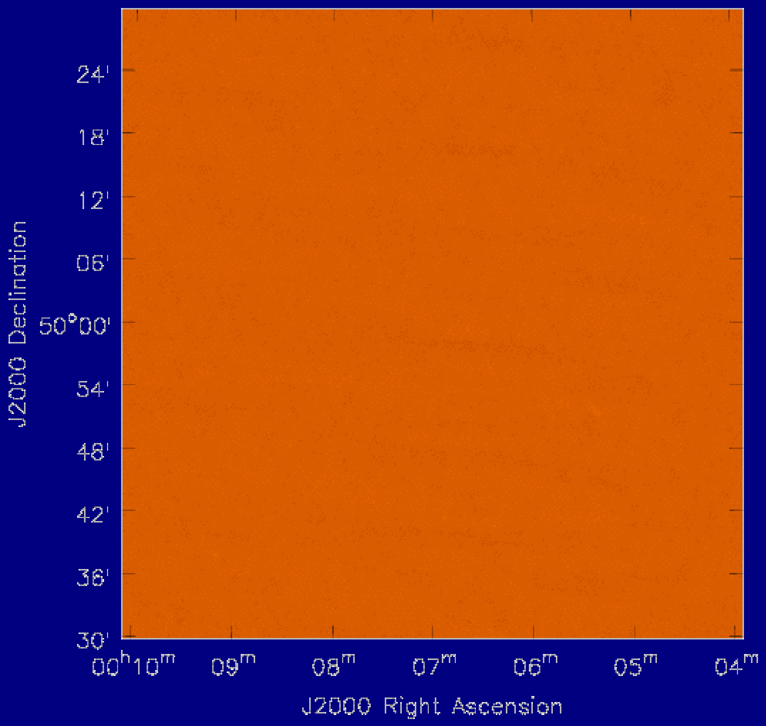


Continuous lines: Typical antenna pointing offsets for VLA as a function of time (Mean between +/-25" and RMS of 5").

Dashed lines: Residual pointing errors. RMS ~1".

Pointing correction



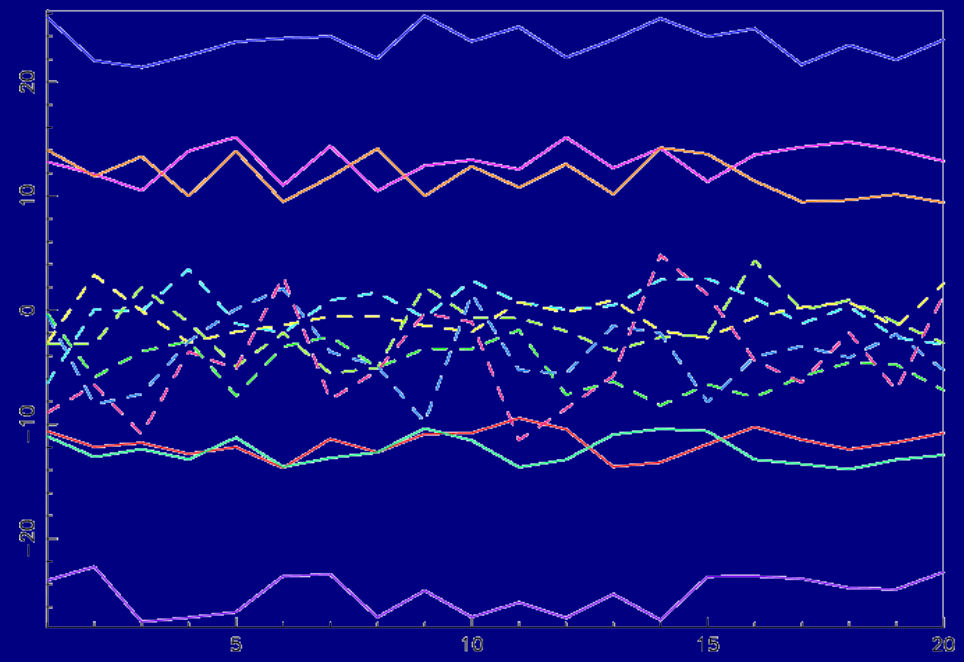


RMS: 15microJy/beam vs. 1microJy/beam
Peak: 250microJy vs. 5 microJy

Model image as components

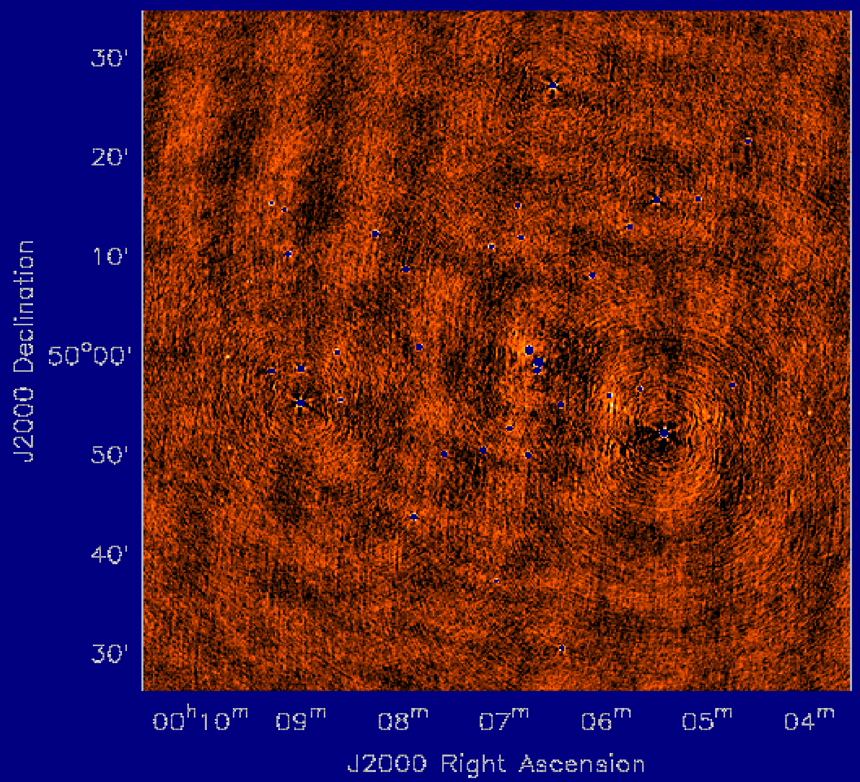
Pointing SelfCal

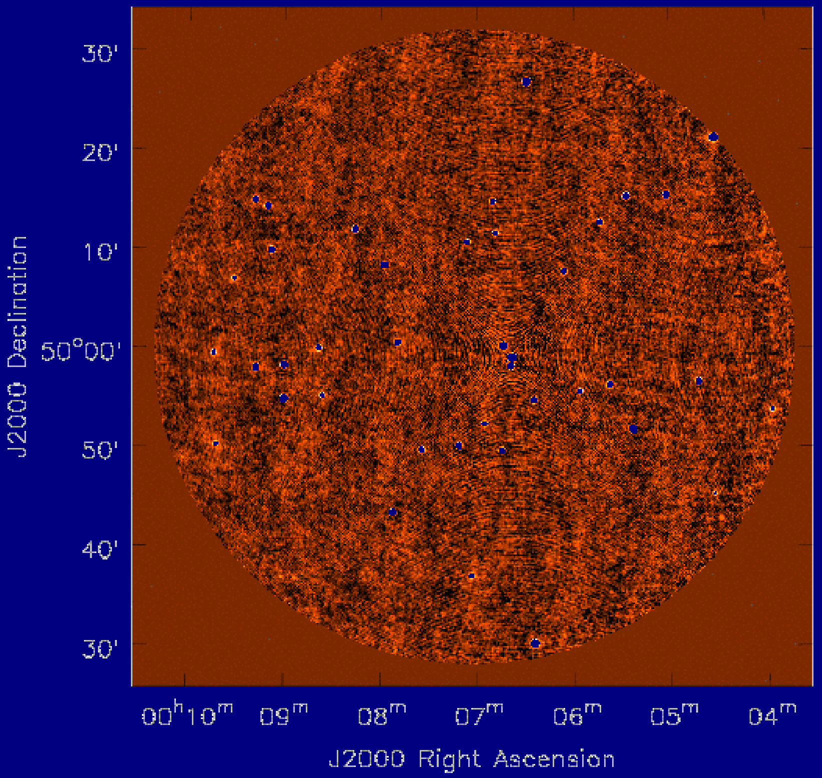
- Model image: deconvolved using entire data
- Pixelated model image



Pointing SelfCal

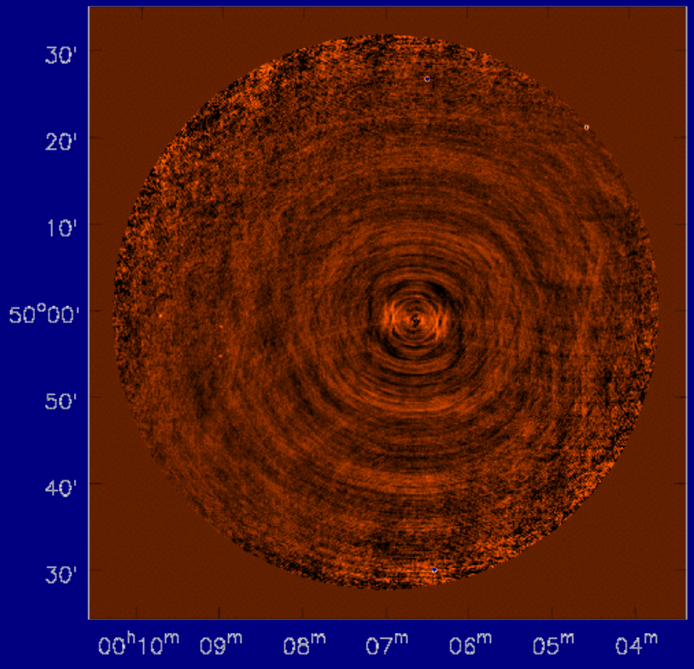
- Stokes-I imaging: Before and after pointing correction

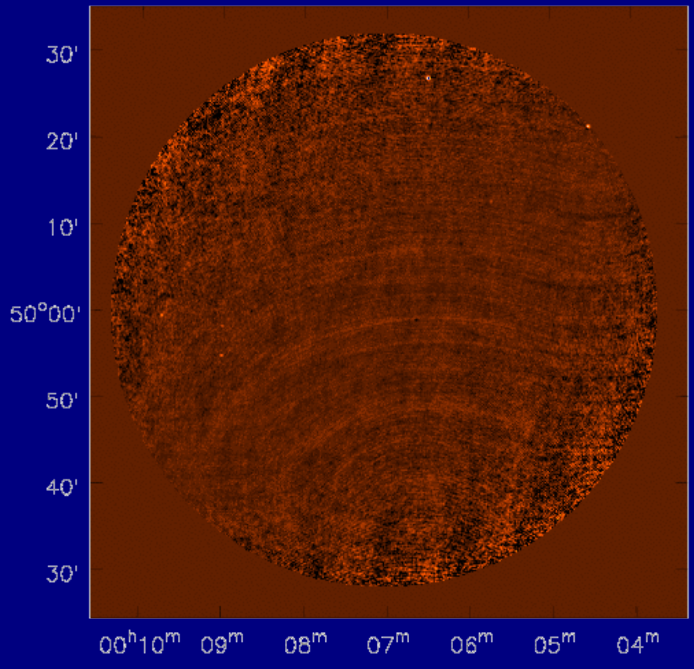




Pointing SelfCal

- Stokes-V imaging: Need to use component imaging?
- Use illumination patterns (work in progress)





Where to from here...

- SelfCal <-> imaging iterations to test the limits of this approach
- Component based imaging and prediction
- Is current deep L-band imaging pointing-error limited?

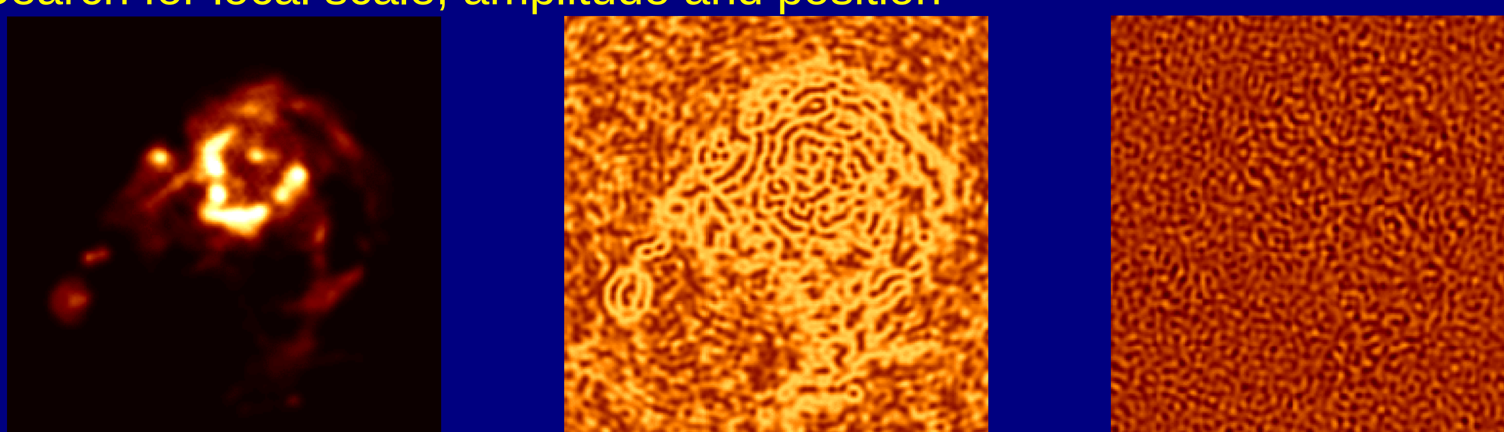
- Mosaicking dynamic range limited by pointing errors?
- Wide-band imaging
  - Use pb-projection to correct for PB scaling
  - MSF extensions: Freq. sensitive image plane modeling (Component based imaging?)
- Non-isoplanatic ionospheric calibration?

Scale sensitive imaging: Asp-Clean

- Pixel-to-pixel noise in the image is correlated

$$I^D = B * I^o + B * I^N \quad where\ B = PSF$$

- The scale of emission *fundamentally* separates signal *(I^o)* from the noise *(I^N).*
- Multi-Scale Clean (Cornwell & Holdaway, 2004, in prep.)
  - Decompose the sky in a set of components at few scales
- Asp-Clean (Bhatnagar & Cornwell, A&A,2004)
  - Search for local scale, amplitude and position



Wide-band continuum imaging

- EVLA bandwidth ratio of 2:1

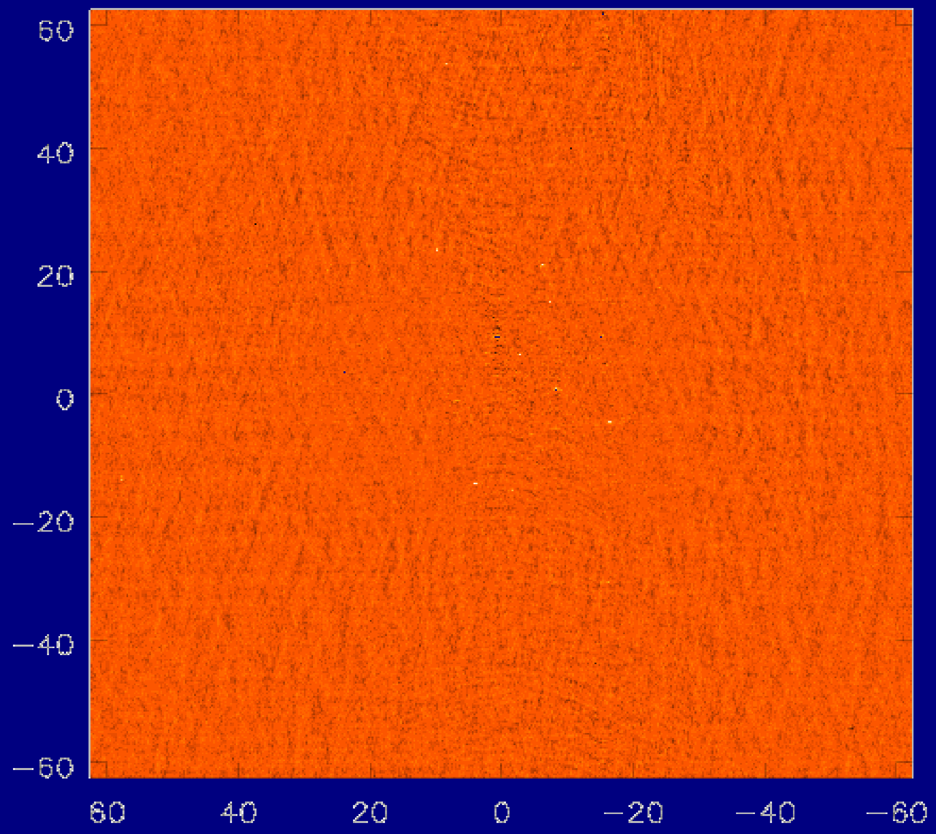
$$V(u_{ij}, v_{ij}) = \sum_{\nu_k} V(u_{ij}, v_{ij}; \nu_k) = \sum_{\nu_k} P_{ij}(\nu_k) FT[I^D(\nu_k)]$$

Sky emission, the Primary Beams, etc. are a function of frequency.
Ideas: Apply PB effects during predict using pb-projection.
Parameterized sky model for prediction.

- Combining Scale sensitive + frequency sensitive deconvolution with image plane corrections.

Wide-band imaging: The problem

- IF1 – IF2: 50MHz apart
  - Spectral index? Frequency dependence of the PBs?



- EVLA bandwidth ratio of 2:1

Ionospheric/atmospheric calibration

$$V_{ij}^{Obs}(\nu,t)=\int\int K_{ij}(S;\nu,t)\,I^M(S)\,e^{2\pi\iota S.B_{ij}}\,dl\,dm$$

where K_{ij} is the ionospheric, direction dependent phase

- General form for residuals: $V_{ij}^{R}=V_{ij}^{Obs}-X_{ij}*FT[I^M]$
  where $X_{ij}=W_{ij}*E_{ij}*K_{ij}$
- Express ionospheric phase in the antenna beam using ionospheric physics/geometrical effects/primary beam effects (de-focusing, time varying illumination, etc.)

Computing and I/O costs

- Significant increase in run-time due to more sophisticated parameterization
  - Deconvolution: Fast evaluation of $B * \sum_k A_k P(Scale_k, Pos_k)$
  - Calibration: Fast evaluation of $E_{ij} * V_{ij}$
- Cost of computing residual visibilities is dominated by I/O costs for large datasets (~200GB for EVLA)
  - Deconvolution: Approx. 20 access of the entire dataset
  - Calibration: Each trial step in the search accesses the entire dataset
- Solutions: Analytical approximations, caching, Parallel computing and I/O,...