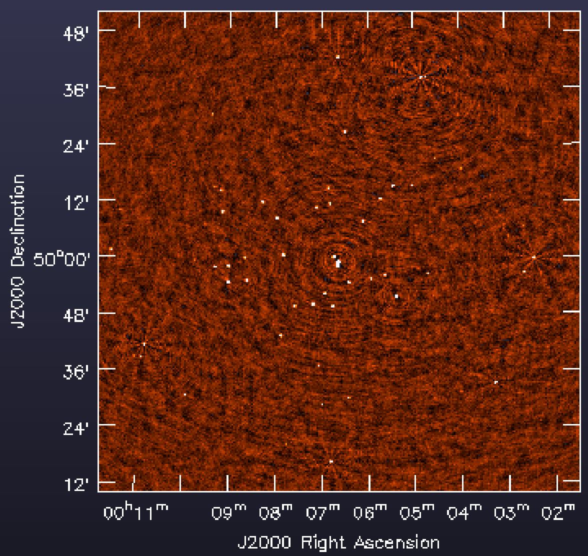
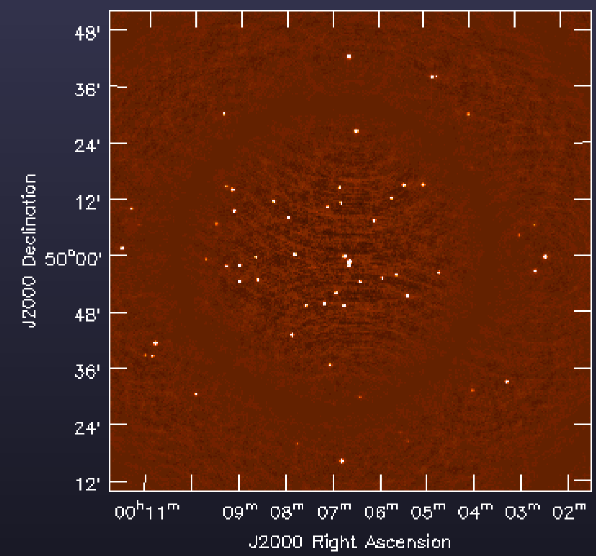
Parametrization of the Measurement Equation





S. Bhatnagar

NRAO, Socorro

The Measurement Equation

- Generic Measurement Equation:

$$V_{ij}^{Obs}(\nu) = J_{ij}(\nu, t) \int J_{ij}^{S}(S, \nu, t) \; I(S) \; e^{\iota S.B_{ij}} \; dS$$

Data **Corruptions** **Sky**

$J_{ij} = J_i \otimes J_j^*$:direction independent corruptions.

$J_{ij}^s = J_i^s \otimes J_j^{s*}$:direction dependent (image plane) corruptions.

- $V_{ij}^{Obs} = J_{ij} W_{ij} E_{ij} V^o$ where $E_{ij} = F J_{ij}^s F^T$
  - J_{ij} is multiplicative in the Fourier domain
  - J^s_{ij} is multiplicative in the Image domain *only if* $J^s_i = J^s_j$
  - $WF \equiv A$: The Measurement Matrix

Generalized Data Reduction

- Calibration:
  - ➔ Keep sky fixed: $min : |V_{ij}^{Obs} - J_{ij} . V_{ij}^{M}|^2$ w.r.t. J_i
  - ➔ V^M = Data on a source with known structure (Primary Cal.)

- Imaging: A^{-1} does not exist. Non-linear inversion required
  - ➔ Keep J_{ij} fixed $min : |J_{ij}^{-1} V_{ij}^{Obs} - A I^M|^2$ w.r.t. I^M
  - ➔ Final image: $I = I^M / PB$

- Self-Calibration: Treat J_{ij} and V^M (or I^M) as orthogonal
  - ➔ Iteratively, but independently, improve J_{ij} and V^M

Parameters of the Measurement Equation

- Unknowns of the ME
  - Sky: Position, Flux, scale
    - Frequency, polarization & time dependence
  - Known effects
    - W-term, PB, dependence of PB on poln. and frequency
  - Unknown instrumental & atmospheric effects
    - Complex gains, poln. leakage (direction indep.)
    - Pointing, PB deformation, ionosphere

Existing Parametrization

- Parametrization of the ME in existing algorithms :

$$V_{ij}^{Obs}(\nu)=J_{ij}(\nu,t)\int J_{ij}^{S}(\boldsymbol{S},\nu,t)\; I(\boldsymbol{S})\; e^{\iota \boldsymbol{S}.\boldsymbol{B}_{ij}}\; d\boldsymbol{S}$$

$$J_{ij}^{S}(\boldsymbol{S},\nu,t)=PB: \text{Indep. of } t,\nu,\boldsymbol{S}\,\&\, i-j$$

$$I(\boldsymbol{S})=\sum_{k} P(x_k,y_k): \text{Pixel Basis}$$

$$J_{ij}(\nu,t)=J_{i}(\nu,t)\otimes J_{j}^{*}(\nu,t): \text{Direction indep.}$$

- More sophisticated parametrization required for imaging dynamic range (DR) >50dB
  - Correct for PB effects, DD atmospheric effects
  - Scale sensitive image decomposition

Motivation

- Use of pixel basis for image representation: deconvolution errors limit: DR ~ 10^4-10^5

- Single pointing L-Band observations limited due to pointing/PB asymmetries: ~DR ~ few X 10^5.
  - Next generation telescopes hope to do >10x better
- Mosaicking dynamic range limited by pointing errors and azimuthally asymmetric PB/sidelobes.

- Frequency dependence of the sky & the instrument: DR ~ 10^4-10^5

Pieces of the Puzzle

- Decomposition of the sky in a more appropriate basis
  - ➔ Frequency sensitive
- Efficient algorithms to correct for image plane effects
  - ➔ Approximate inverse transform (Vis -> Image)
  - ➔ Forward transform (accurate)
- Solvers for the "unknown" image plane effects
  - ➔ As expensive as imaging!
- Larger computers! (More memory, CPU power, fast I/O)
  - ➔ Parallel computing and I/O, GPU computing?

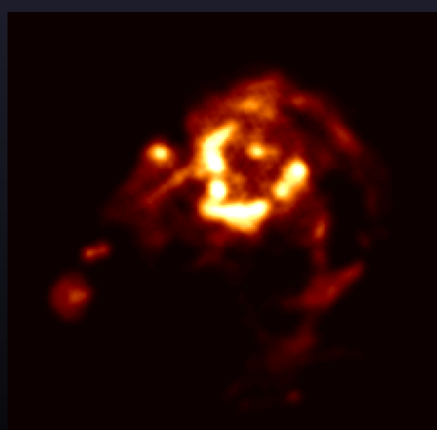
Scale Sensitive Imaging: Asp-Clean

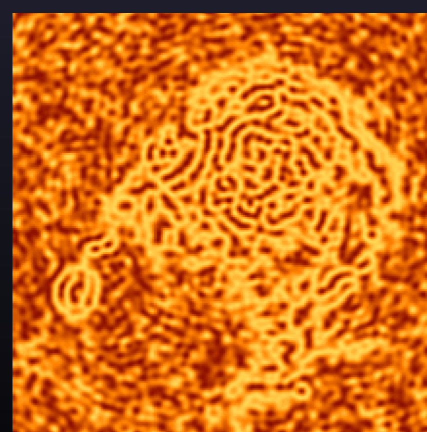
- Pixel-to-pixel noise in the image is correlated

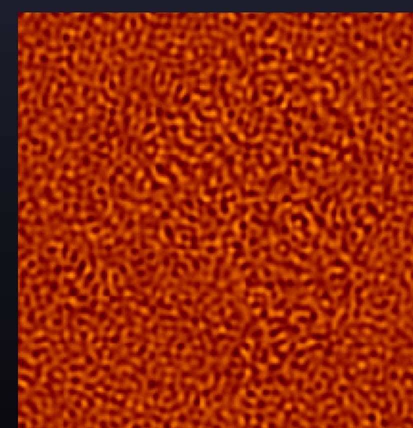
$$I^D = PI^o + PI^N \quad \text{where} \quad P = \text{Beam Matrix}$$

The scale of emission *fundamentally* separates signal *(I^o)* from the noise *(I^N).*

- Asp-Clean (Bhatnagar & Cornwell, A&A,2004)
  - Search for local scale, amplitude and position







General Structure of the Imaging Algorithms

- Solve the normal equation $A^T[\vec{V} - A\vec{I}] = 0$
  - Compute the approx. update direction: $I^R = A^T[\Delta V^R]$
  - Update the model: (Minor Cycle) $I_i^M = T(I_{i-1}^M, I^R)$
    (Steepest Descent minimization: Clean algorithm)
  - Compute residuals: (Major Cycle) $Forward: \overrightarrow{V^M} = A\overrightarrow{I^M}$
    $Backward: \overrightarrow{I^R} = [A^T A]^{-1} A^T \overrightarrow{V^R}$
- Transform implemented using FFT:
  - Forward transform: *CA* $V^M = C[A\vec{I}^M]$
  - Backward transform: $[CA]^{-1} = A^T C^T$
- Approximate methods:
  - Accurate forward and approximate backward transform is sufficient

Hierarchy of Algorithms

- Unknowns of the problem: J_{ij}, J^s_{ij}, *and* I^M.
- $J^s_i = J^s_j$ and *independent* of time

Imaging and calibration as orthogonal operations

- $J^s_i(t) = J^s_j(t)$ (Poln. squint, PB correction, etc.)
  (Cornwell,EVLA Memo 62)

  - ➔ J^s_{ij} is multiplicative in the image plane for appropriate ∇T

$$\Delta I^D \propto \Re\left[\sum_n J^{s^T}(n\nabla T)\sum_{ij}\left[\Delta V_{ij}(\nabla T)e^{\iota S.B_{ij}}\right]\right]$$

Hierarchy of Algorithms

- $J_i^s(t) \neq J_j^s(t)$ (Pointing offsets, PB variations, etc.)
  - Image plane effects not known a-priori
    - Pointing selfcal (EVLA Memo 84)
  - Correct for J^s_{ij} during image deconvolution
    - W-Projection, PB-Projection
      (EVLA Memo 67) (EVLA Memo 100)
- Simultaneous solver for J_{ij}, J^s_{ij}, *and* I^M*!!*

Measured direction dependent effects

- E_{ij} as a function of direction is measured a-priori (nominal full beam polarimetric imaging)

$$V_{ij}^{M} = \boldsymbol{E}_{\boldsymbol{ij}} [\boldsymbol{A}\,\boldsymbol{I}^{M}]_{ij} \quad where \quad E_{ij}(u_i, u_j; p_i, p_j) = E^{o} f(p_i, p_j)$$

- Aperture Function: E_i *different* for each poln. product *pq and baseline* (pointing offsets correction)

    Needs a solver: Pointing SelfCal

- Asymmetric Primary Beams:

    Parameterize E^o

Full-beam Full-Stokes Imaging

- Full Stokes imaging requires full Sky-Muller matrix

$$\mathrm{J}_i(S) = \begin{bmatrix} \mathrm{J}_i^{p} & \mathrm{J}_i^{pq} \\ \mathrm{J}_i^{qp} & \mathrm{J}_i^{q} \end{bmatrix}$$

$$\begin{bmatrix} OUT_{pp} \\ OUT_{pq} \\ OUT_{qp} \\ OUT_{qq} \end{bmatrix} = \begin{bmatrix} \mathrm{J}_i^{p}\mathrm{J}_j^{p^*} & \mathrm{J}_i^{p}\mathrm{J}_j^{pq^*} & \mathrm{J}_i^{pq}\mathrm{J}_j^{p^*} & \mathrm{J}_i^{pq}\mathrm{J}_j^{pq^*} \\ \mathrm{J}_i^{p}\mathrm{J}_j^{qp^*} & \mathrm{J}_i^{p}\mathrm{J}_j^{q^*} & \mathrm{J}_i^{pq}\mathrm{J}_j^{qp^*} & \mathrm{J}_i^{pq}\mathrm{J}_j^{q^*} \\ \mathrm{J}_i^{qp}\mathrm{J}_j^{p^*} & \mathrm{J}_i^{qp}\mathrm{J}_j^{pq^*} & \mathrm{J}_i^{q}\mathrm{J}_j^{p^*} & \mathrm{J}_i^{q}\mathrm{J}_j^{pq^*} \\ \mathrm{J}_i^{qp}\mathrm{J}_j^{qp^*} & \mathrm{J}_i^{qp}\mathrm{J}_j^{q^*} & \mathrm{J}_i^{q}\mathrm{J}_j^{qp^*} & \mathrm{J}_i^{q}\mathrm{J}_j^{q^*} \end{bmatrix} \begin{bmatrix} IN_{pp} \\ IN_{pq} \\ IN_{qp} \\ IN_{qq} \end{bmatrix}$$

$$\mathrm{J}_{ij}(S) = \mathrm{J}_i(S) \otimes \mathrm{J}_j^{*}(S)$$

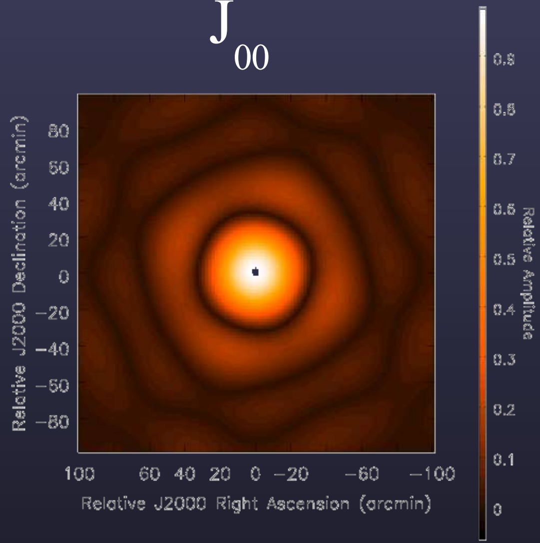
- $J_i^{p}(S) \equiv$ Antenna voltage patterns

- $J_i^{pq}(S) \equiv$ Polarization leakage

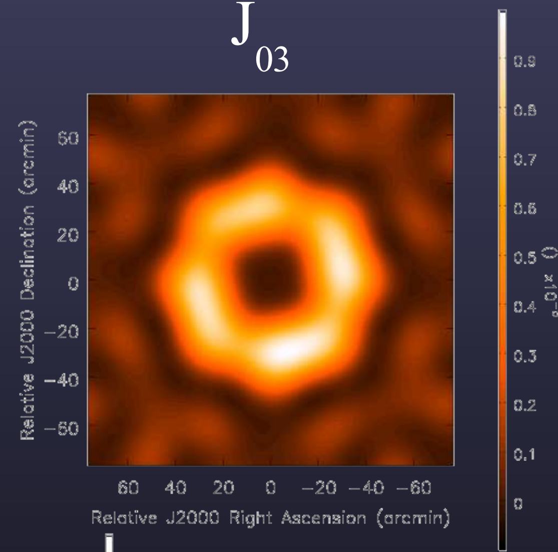
- $J_{ij}(S)$ is not identity or even diagonal matrix for DR $> 10^4$

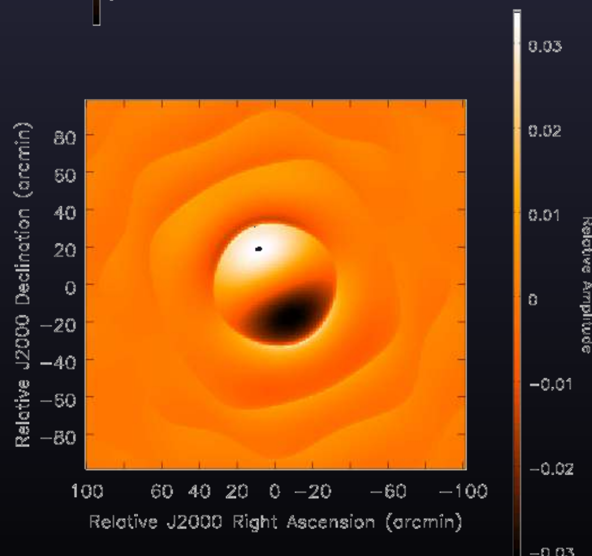
Structure of the Sky-Muller Matrix

J_{00}



J_{03}





$J_{00}-J_{11}$

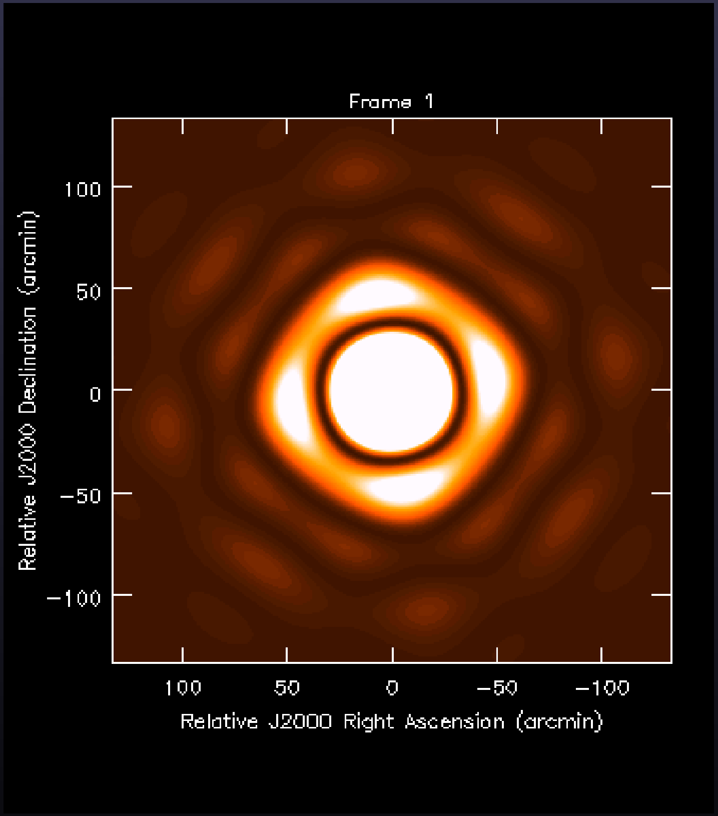
Correction of DD effects during imaging

- A class of Direction Dependent effects can be corrected during imaging
  - The effect should be “band limited”
  - Incorporate DD but “band limited” effects in the operator C
  - The operator C should be unitary (or approximately so)
- Devise an approximate backward and accurate forward transform
- Forward transform:
  - Replace the operator C by: $E^{P}_{ij} = E^{P^o} f(\phi_i - \phi_j) e^{\iota(\phi_i - \phi_j)}$
- Backward transform:
  - $\boldsymbol{V}^{\boldsymbol{P},\boldsymbol{G}}(n\Delta u, m\Delta v) = (\boldsymbol{E}^{\boldsymbol{P}^T}_{\boldsymbol{ij}} \boldsymbol{V}^{\boldsymbol{P}}(u_{ij}, v_{ij}))(n\Delta u, m\Delta v)$

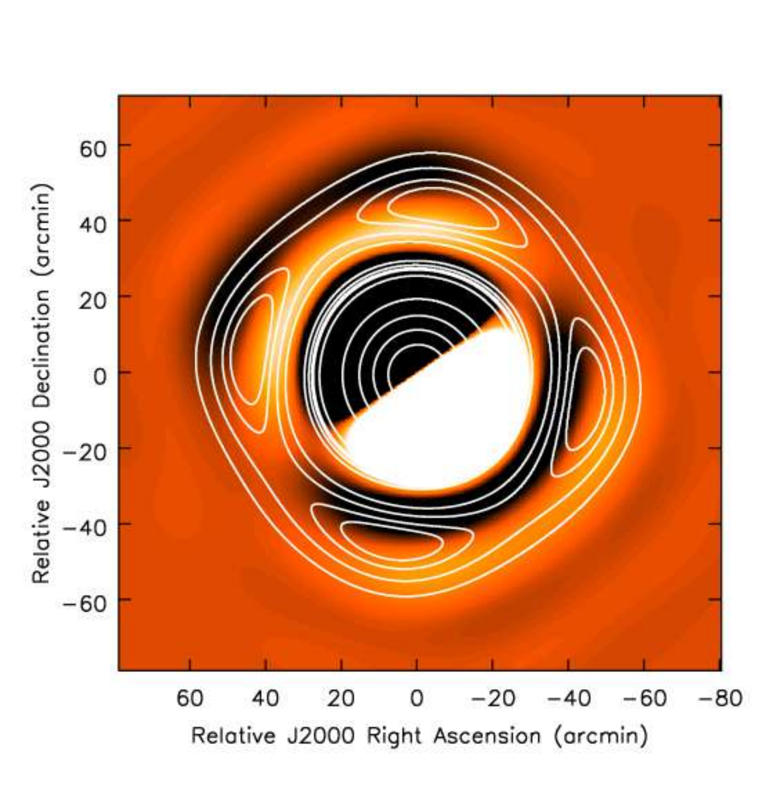
    $\boldsymbol{I}^d = det(\boldsymbol{F}^T[\boldsymbol{E}^{\boldsymbol{P}^T}])^{-1} \boldsymbol{F}^T \boldsymbol{V}^{\boldsymbol{P},\boldsymbol{G}}$

Power Patterns

Model for VLA antenna power patterns at L-band (modeling code courtesy W.Brisken)

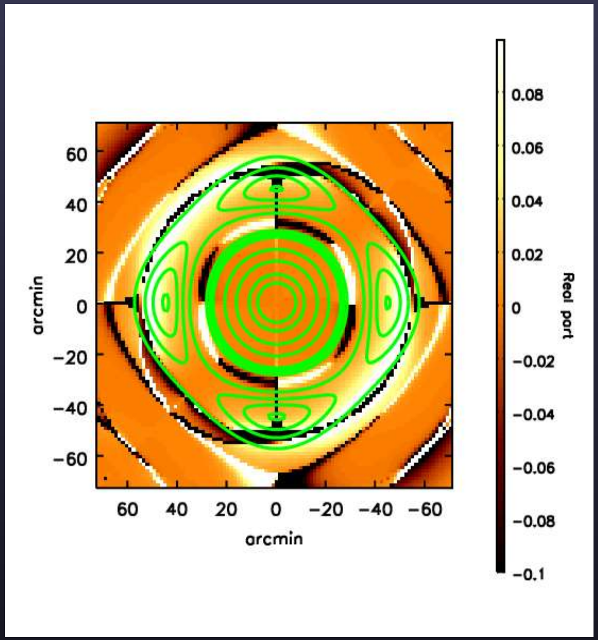


Stokes-I

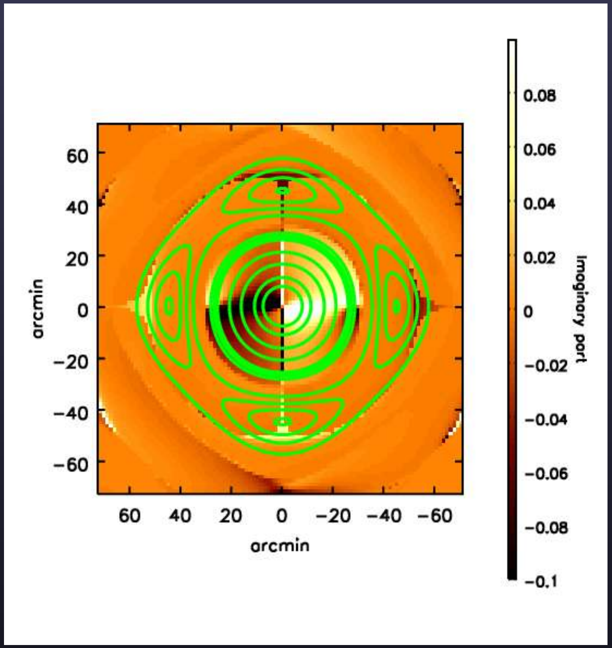


Stokes-V

Approximately Unitary Sky-Jones Matrix



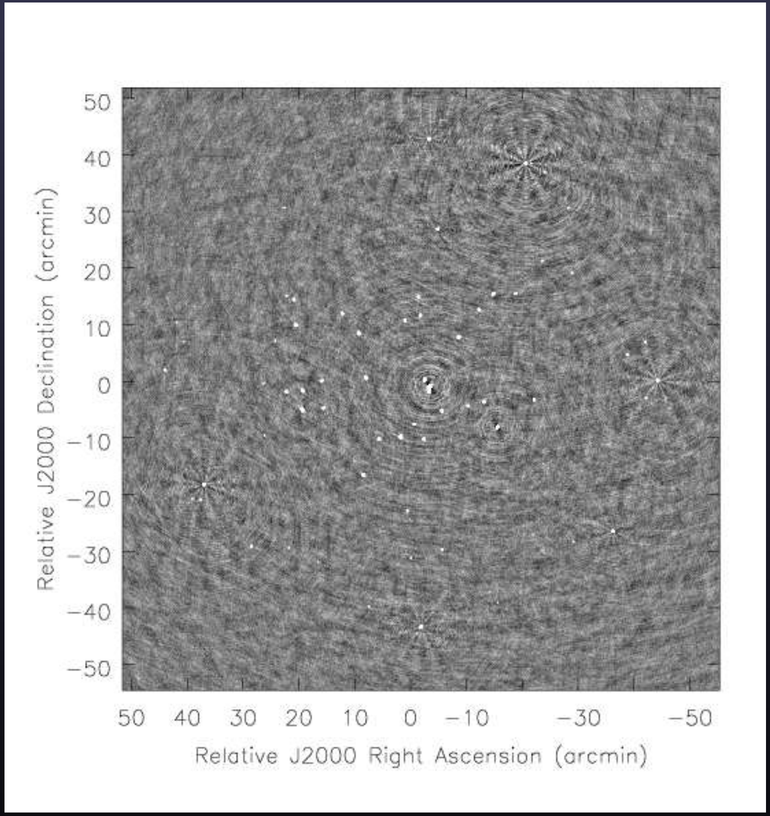
Real part of $J_i J^T_i(0,1)$



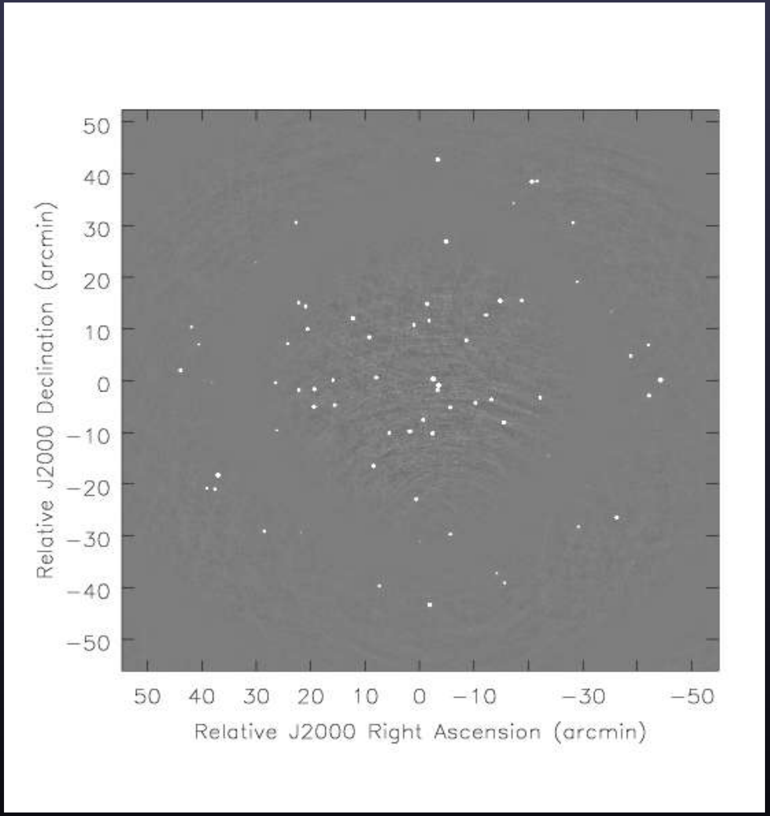
Imag. part of $J_i J^T_i(0,1)$

Simulations: Stokes-I

Stokes-I imaging with and without PB effects (Polarization squint, Pointing offsets, PB rotation)



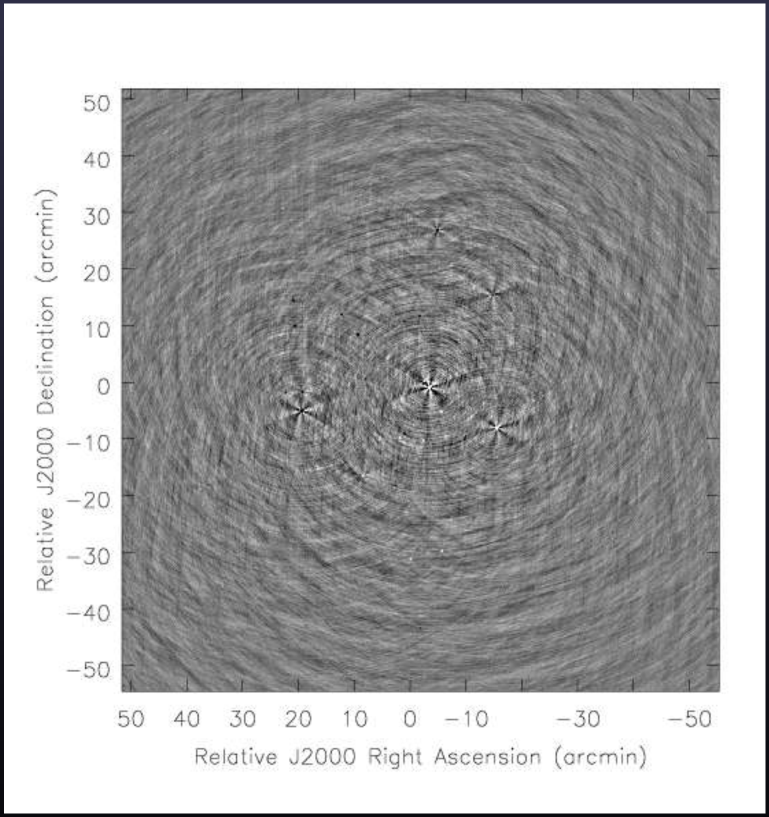
RMS ~15μJy/beam

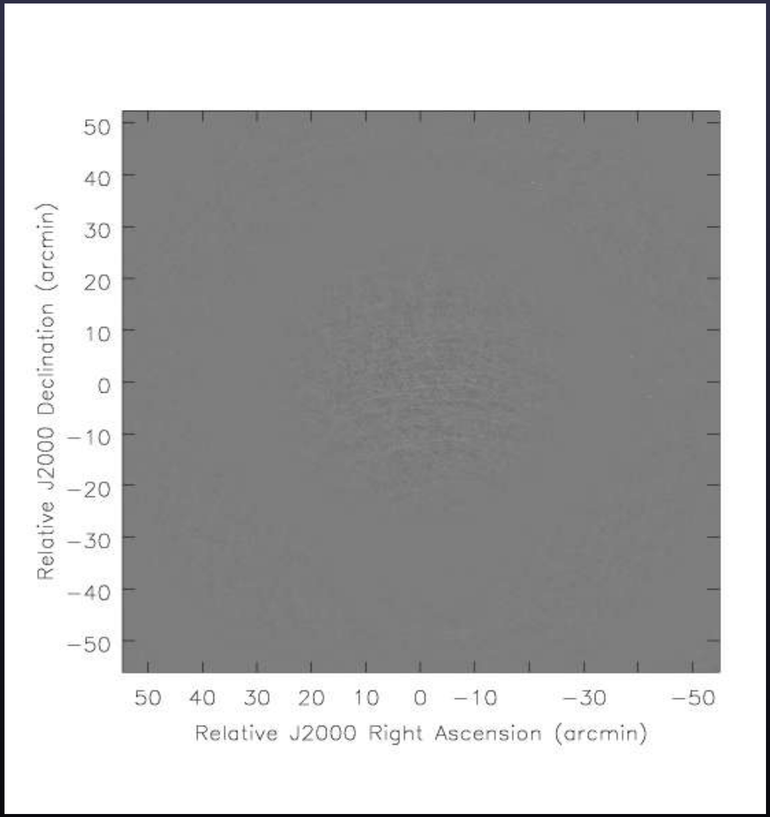


RMS ~1μJy/beam

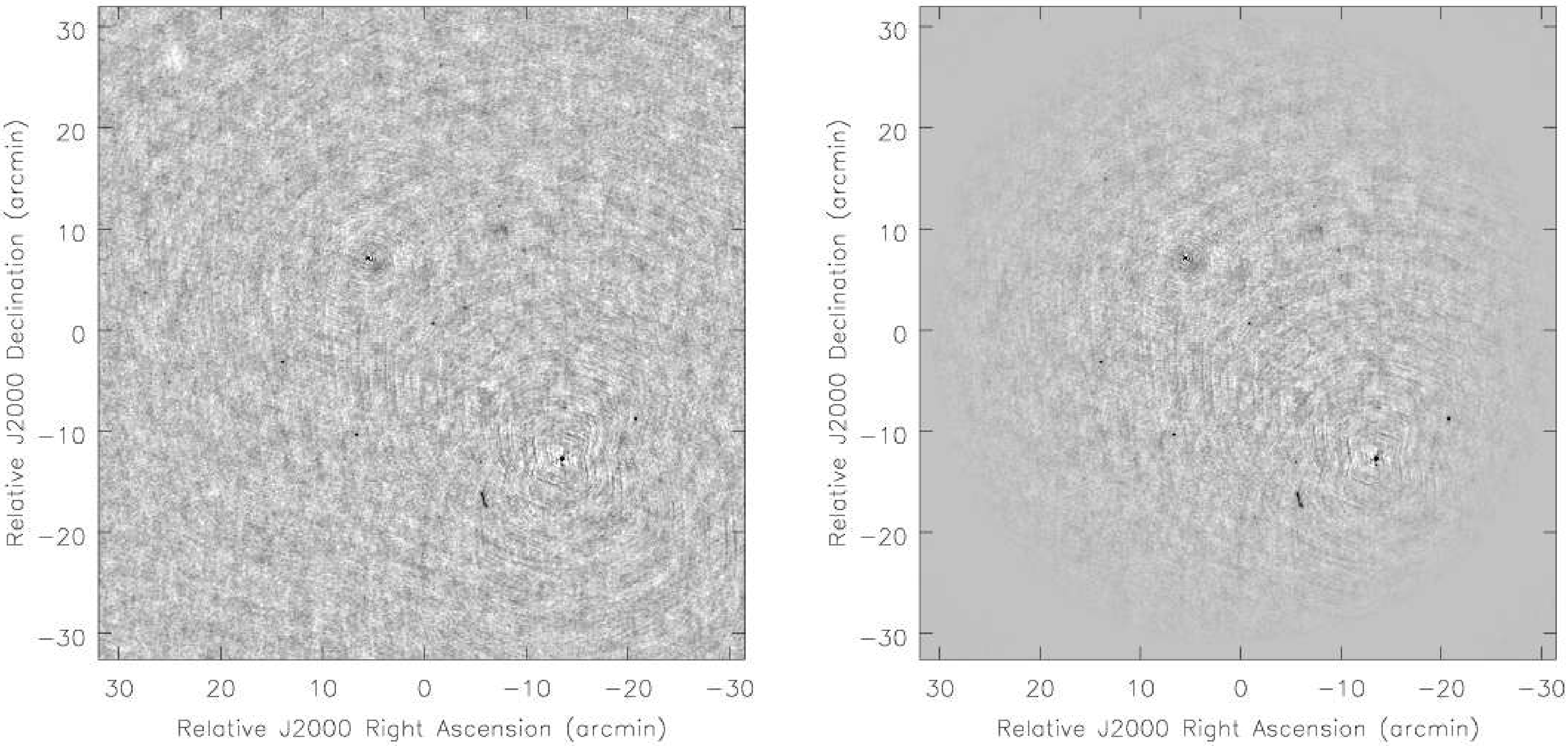
Simulations: Stokes-V

Stokes-V imaging with and without PB effects (Polarization squint, Pointing offsets, PB rotation)

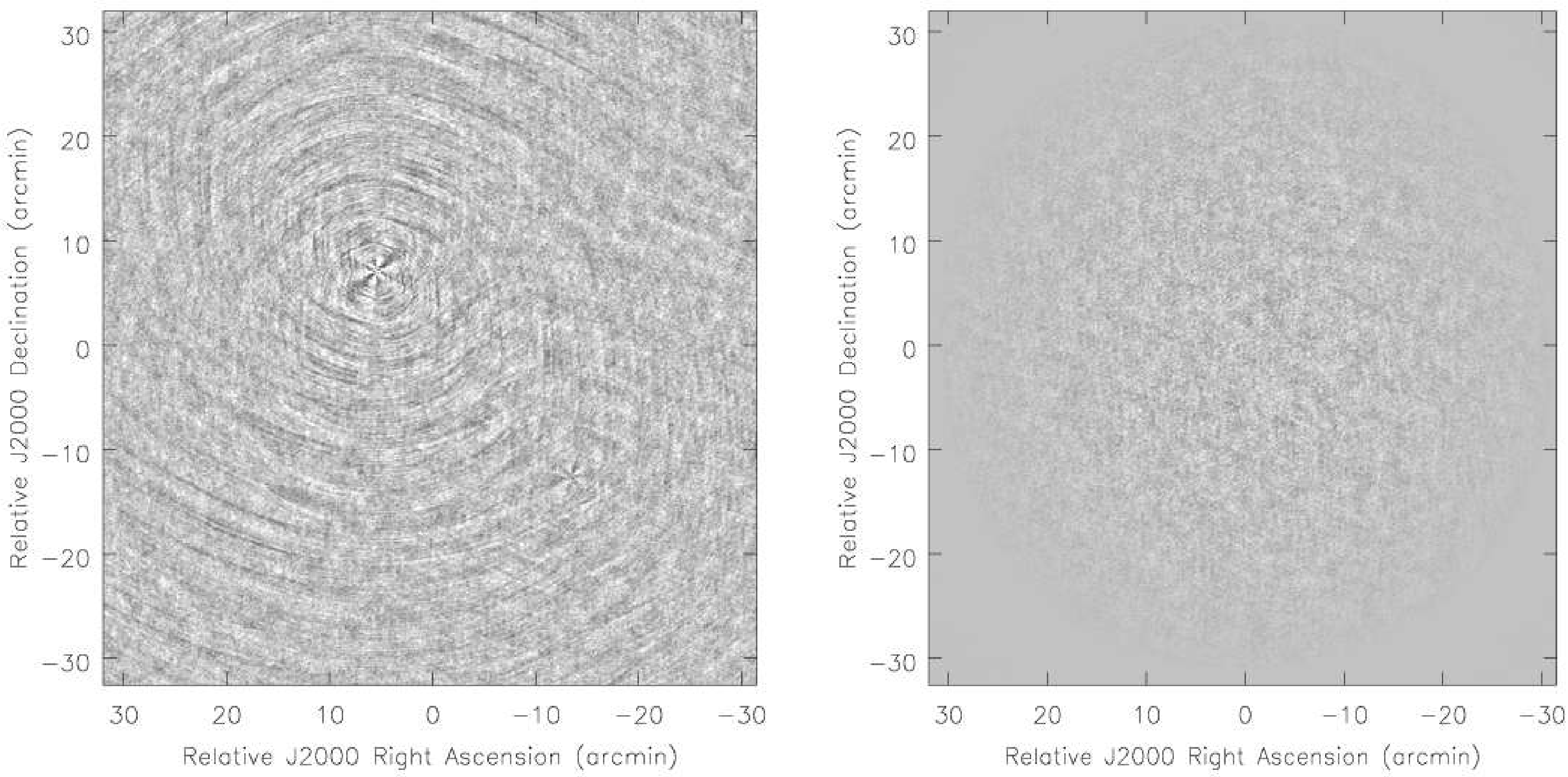




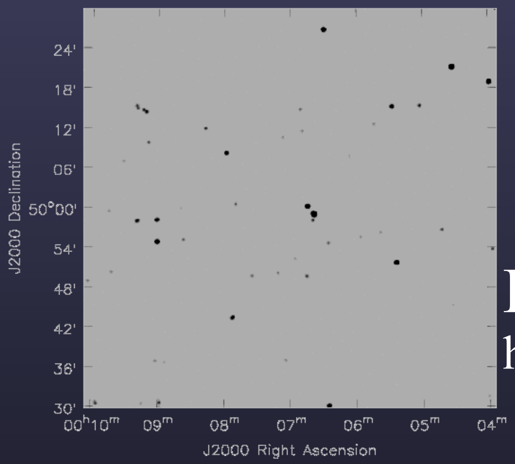
VLA L-Band, C-array, Stokes-I



VLA L-Band, C-array, Stokes-V

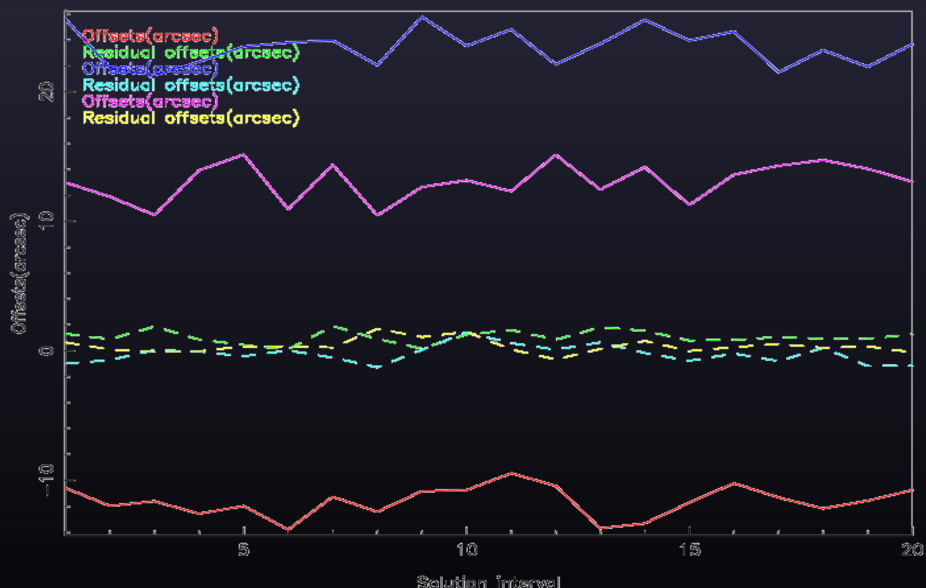


Pointing correction



Model image using 59 sources from NVSS.
Flux range ~2-200 mJy

Details in EVLA Memo 84 (2004)
http://www.aoc.nrao.edu/evla/geninfo/memoseries/evlamemo84.pdf

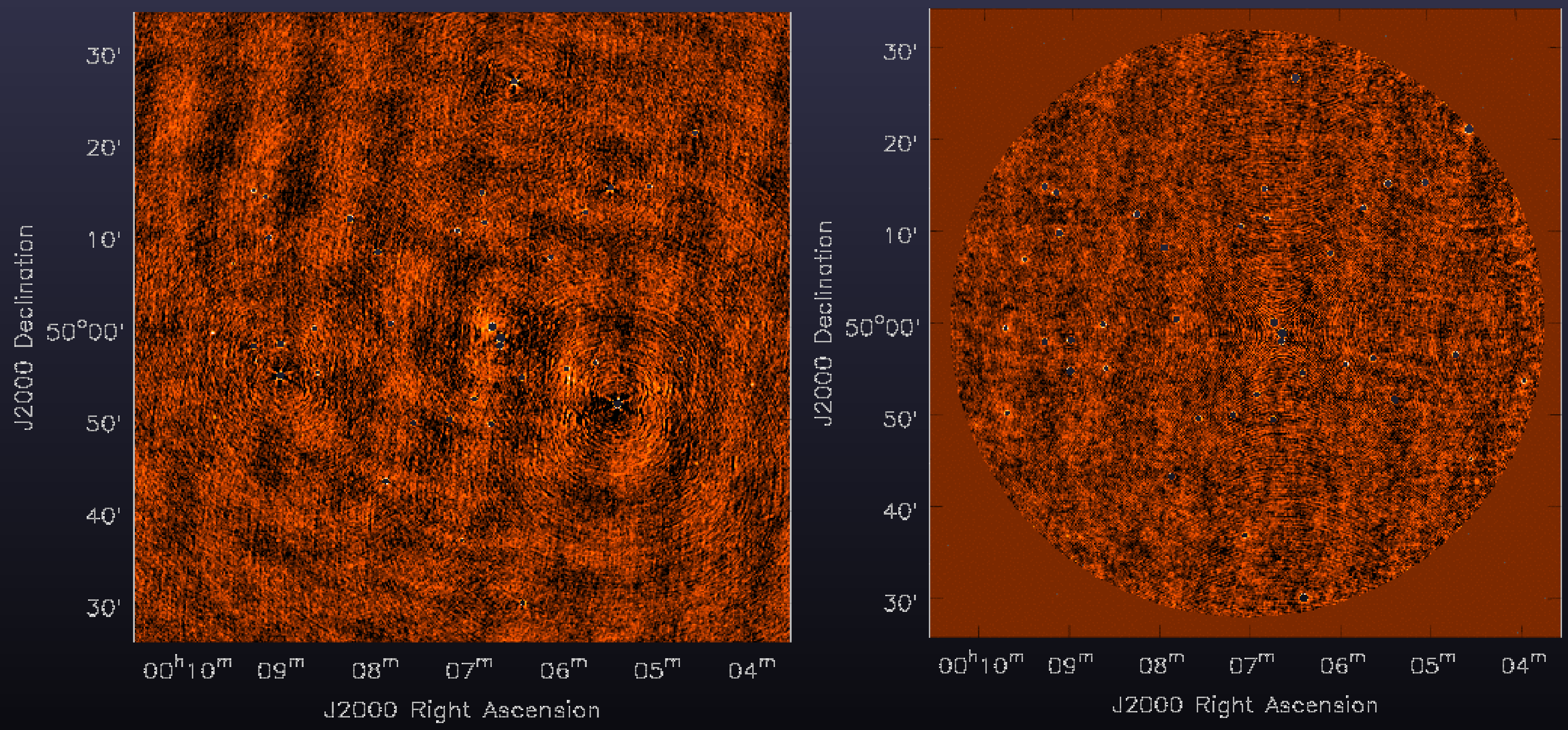


Continuous lines: Typical antenna pointing offsets for VLA as a function of time (Mean between +/- 25” and RMS of 5”).

Dashed lines: Residual pointing errors. RMS ~1”.

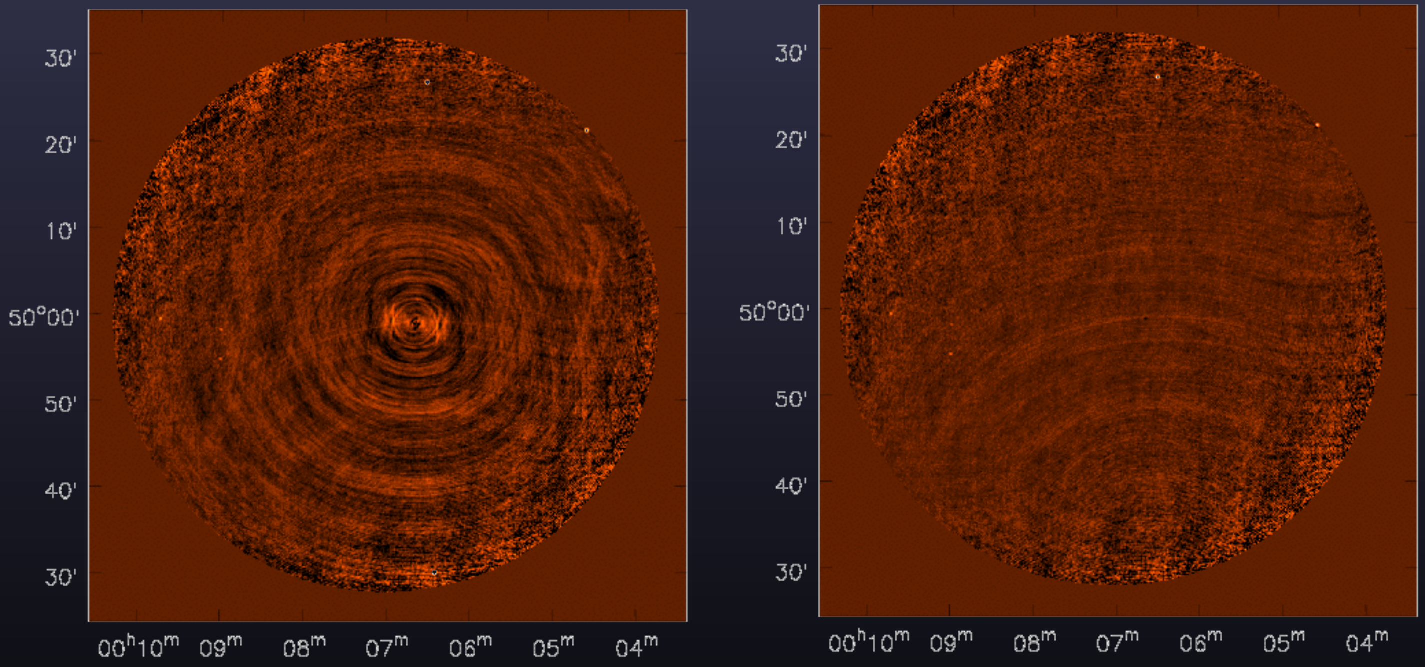
Pointing Selfcal

- Stokes-I imaging: Before and after pointing correction



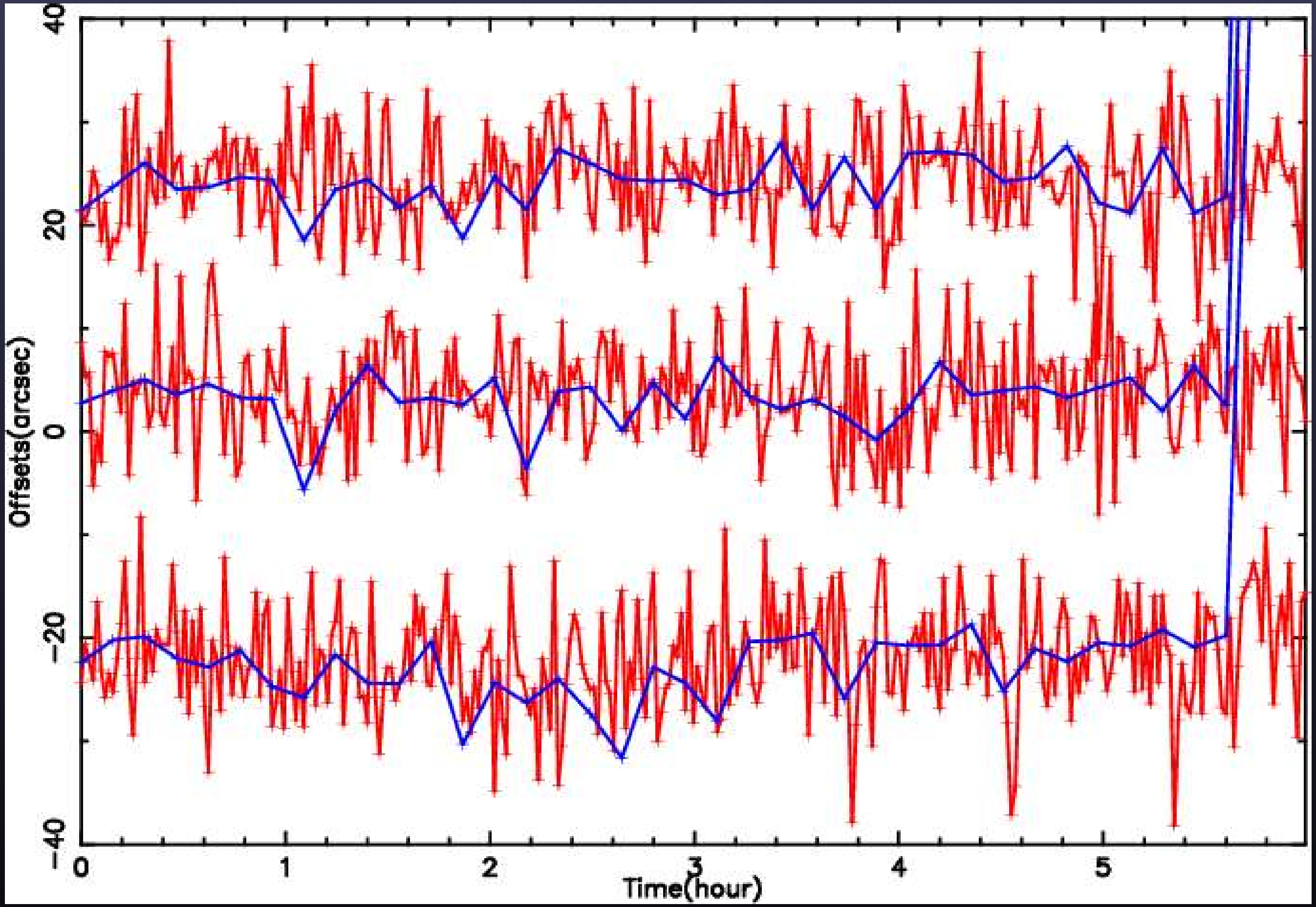
Pointing Selfcal

- Stokes-V imaging: Need to use component imaging?



Pointing selfcal: Unit test

Test for the solver using simulated data



Red: Simulated pointing offsets τ=60sec

Blue: Solutions τ=600sec

Time lines: Development

- Code development time much longer
  - Complexity – code base
    - Partly unavoidable
    - Improvements: Possible & Necessary
    - Use of simpler UI (UNIX command-line, inp/set/save/go)
      - Currently usable on real data (minus data selection)
  - Complexity – algorithm
    - Very difficult to predict (more difficult in evolving code-base)
    - Can run into dead-ends
- Optimization and stability/robustness
  - From “working algorithm” to “usable implementation”
  - Stability/robustness/numerical testing: Time consuming & related to the code-base complexity/stability/evolution.

Progress so far

- Use aperture function / eliminate re-gridding [Done]
- Write the imaging and solver code [Done]
- SelfCal <-> imaging iterations [Testing]
- Component image model (Asp-Clean + PB-Projection + W-Projection) [Next!]

- Is current deep L-band imaging pointing-error limited?
- Mosaicking dynamic range limited by pointing errors?
- Wide-band imaging
  - Use PB-projection to correct for PB scaling
  - MSF extensions: Freq. sensitive image plane modeling (Component based imaging)

Computing and I/O costs

- Significant increase in run-time due to more sophisticated parameterization
  - Deconvolution: Fast transform (both ways)
    - E.g. limits the use of MCMC approach
  - Calibration: Fast prediction

- Cost of computing residual visibilities is dominated by I/O costs for large datasets (~500GB for EVLA)
  - Deconvolution: Approx. 20 access of the entire dataset
  - Calibration: Each trial step in the search accesses the entire dataset

- 10 -100 Gflops + multi Terabyte I/O load

References

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2. Cornwell, 1995, The Generic Interferometer: II Image Solvers, AIPS++ Note 184
3. Bhatnagar & Cornwell, 2004, Scale sensitive deconvolution of interferometric images, A&A, 426, 747-754, 2004 (astro-ph/0407225)
4. Cornwell, Golap & Bhatnagar, 2003, W-Projection: A new algorithm for non-coplanar baselines, Tech. rep., EVLA Memo 67
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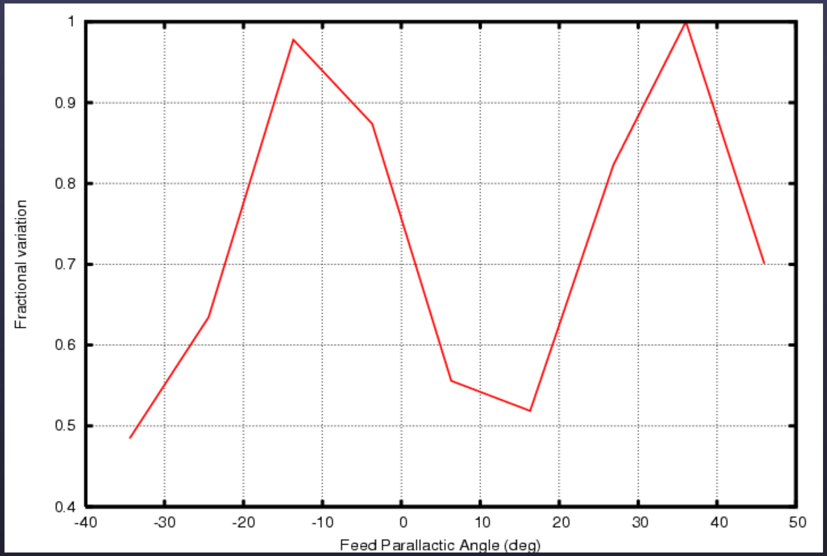
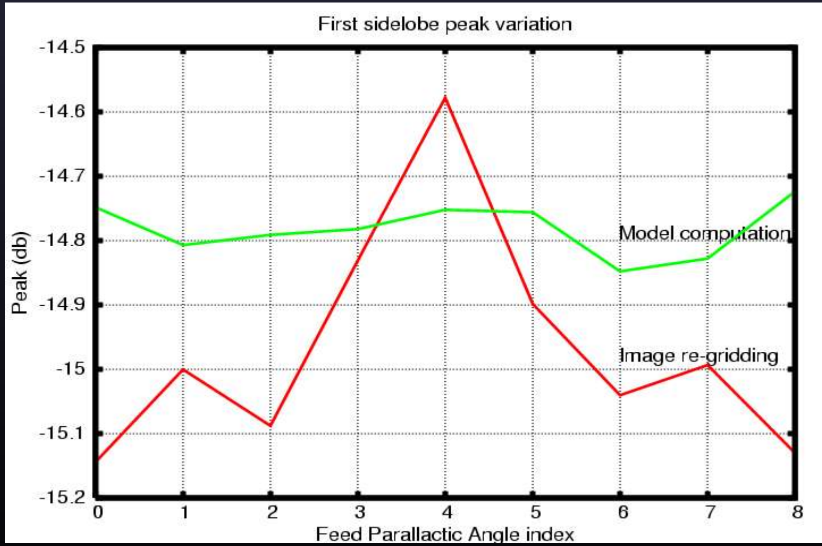
6. Bhatnagar, Cornwell & Golap, 2004, Solving for the antenna based pointing errors, Tech. rep., EVLA Memo 84
7. Bhatnagar, Cornwell & Golap, 2006., Image plane corrections, EVLA Memo 100

http://www.aoc.nrao.edu/~sbhatnag/talks.html

Variable sidelobe gains



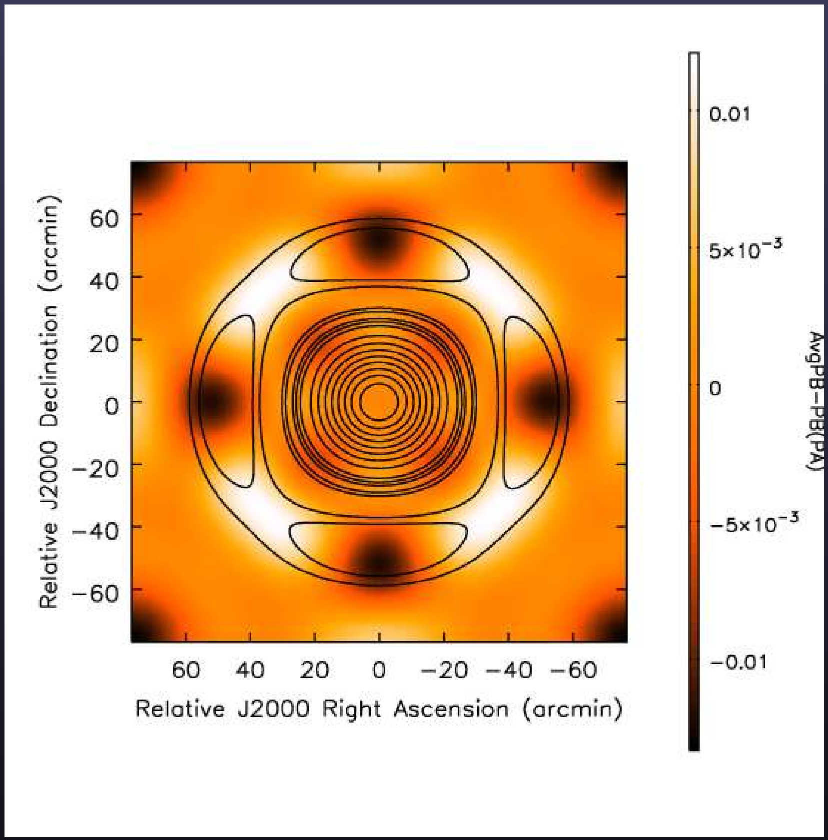
Maximum gain variations at the location of the sidelobes



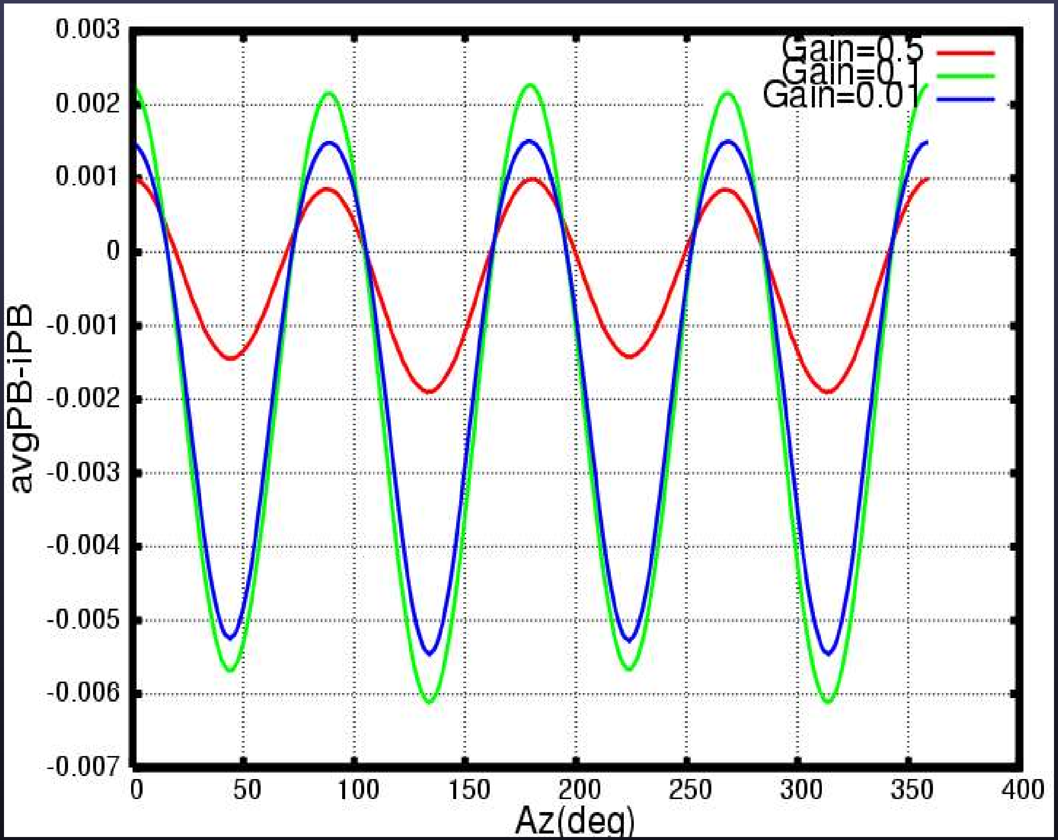
Numerical errors: Variations in the peak of the sidelobe.

Image re-gridding vs. direct evaluation

Error Patterns



AvgPB - PB(t_o)



Azimuthal cuts at 50%, 10% and 1% of the Stokes-I error pattern AvgPB - PB(t_o)