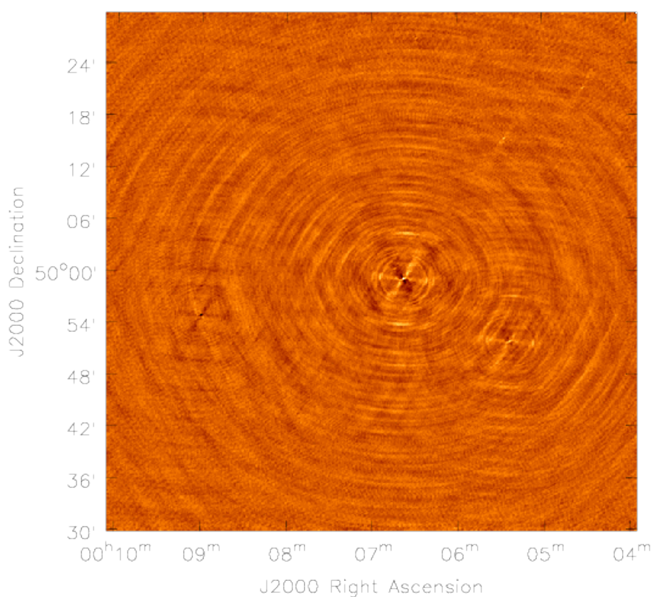
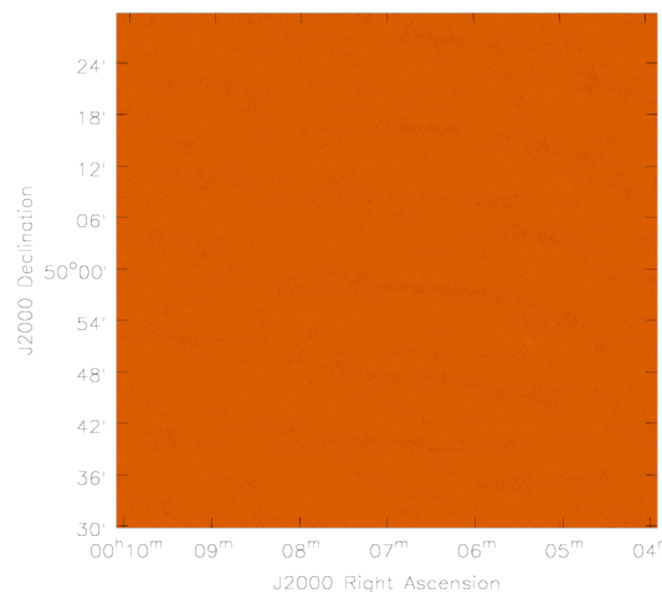
Synthesis imaging in the presence of image plane effects





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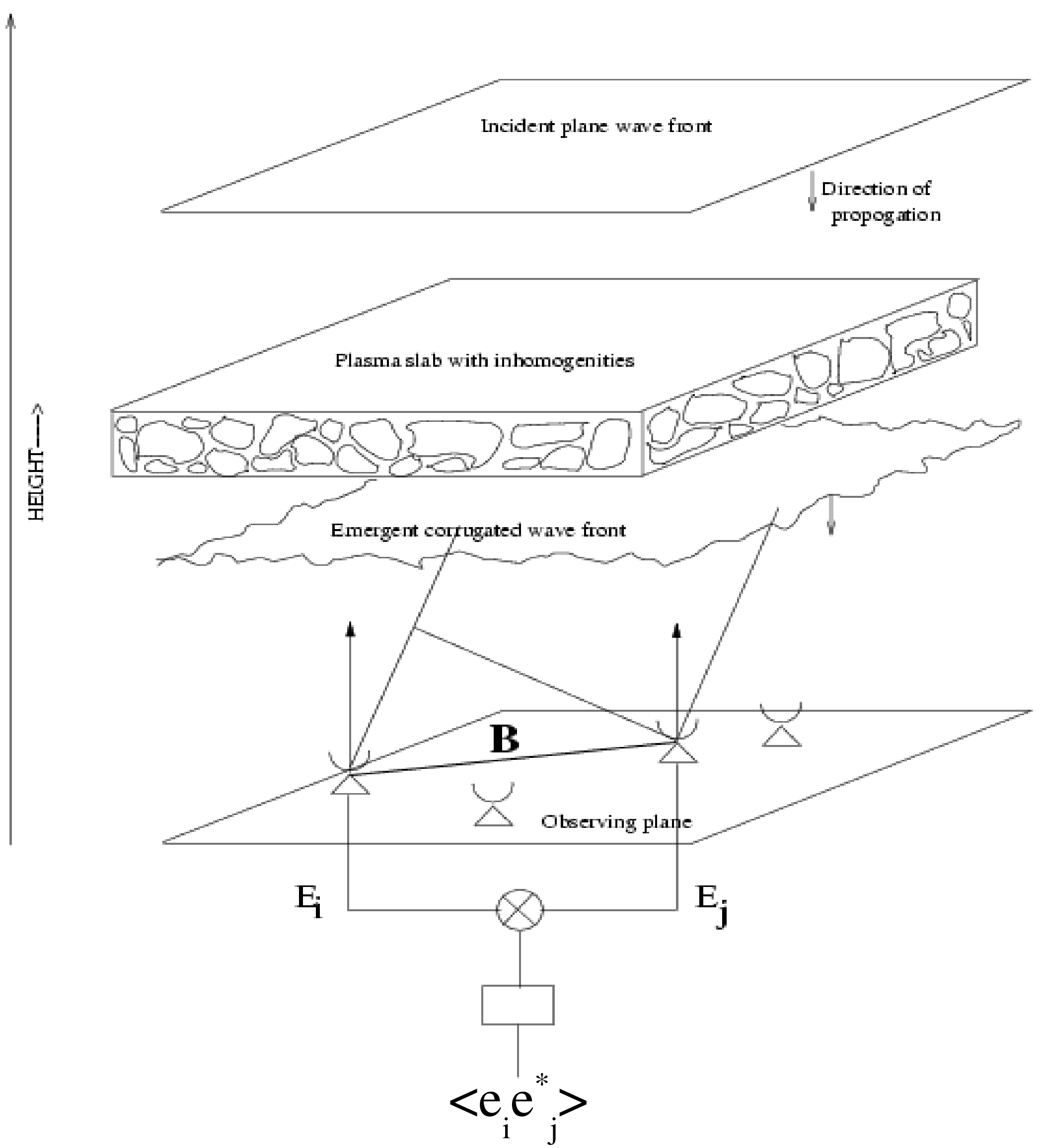
Basic Interferometry



Terminology:

- Visibility: $V_{ij}=<e_i e^*_j>$,
  e_i is the electric field
- Baseline: Length of the projected separation between the antennas (**B**).
  - Only relative separation matters for incoherent emission
  - Co-ordinates: (u_{ij},v_{ij},w_{ij})
    $u_{ij}=u_i-u_j$
- An *N*-element array
  instantaneously measures *N(N-1)/2* baselines (complex values)

Basic Imaging

- In terms of the sky brightness distribution (*$I^o(l,m)$*)

$$V(u_{ij}, v_{ij}, w_{ij}) = \int\int I^o(l,m) S(u,v,w) e^{-2\pi\iota(u_{ij}l + v_{ij}m + w_{ij}(n-1))} \frac{dl\, dm}{n} + N_{ij}$$

where S : uv-sampling function, *(l,m):* direction in the sky

- $n = \sqrt{(1 - l^2 - m^2)}$ For $l^2 + m^2 << 1$ or *small* w_{ij}, sky is the 2D Fourier transform of the Visibility function (van Cittert-Zernike Theorem) $I^o = FT[V^o]$ and $V^{Obs} = S V^o$

$$I^d = PSF * I^o \qquad \text{and} \qquad PSF = FT[S]$$

The Dirty Image — The Dirty Beam

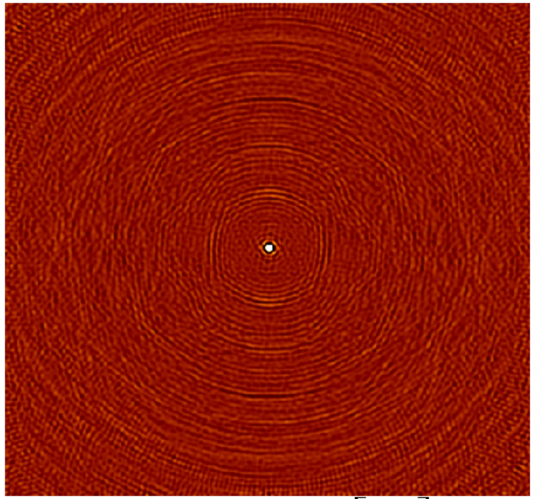
Basic Imaging

- The Measurement Eq.: $V = AI^o + AN$ where ***A*** is the measurement matrix, ***V, I*** and ***N*** are the Visibility, Image and noise vectors.
- ***A=FS*** where ***F*** is the Fourier Transform operator and ***S*** is a diagonal matrix of weights.
  - Due to finite Fourier plane sampling, ***A*** is singular and in general rectangular
- Image reconstruction: Solve for I^o

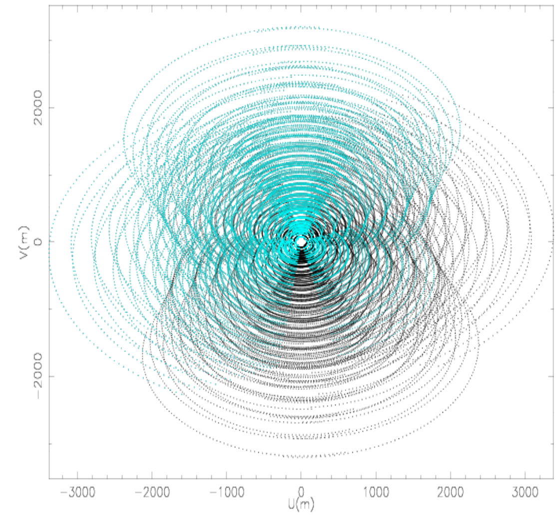
$$A^T V = BI^o + BN \quad where \quad B = A^T A$$

- ***B***: The Toeplitz Beam Matrix
- $A^T V = I^d$: The Dirty Image vector

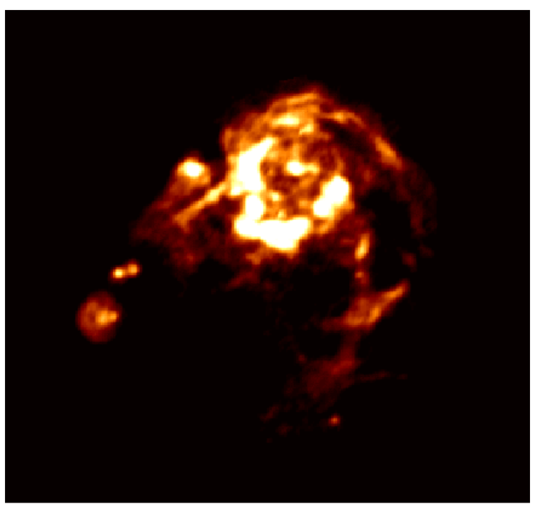
The image plane



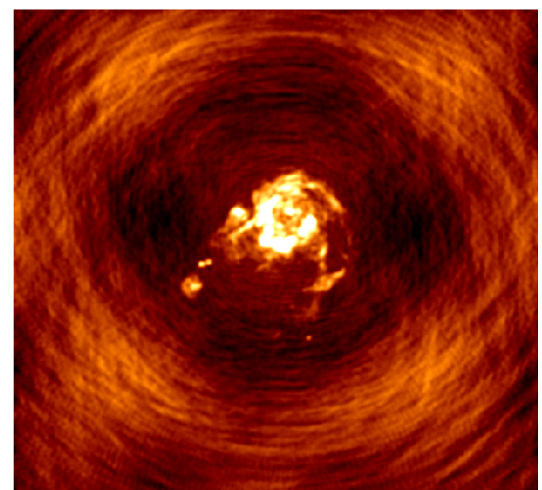
$$PSF = FT[S]$$



Sampling function



$$I^o = FT[V^o]$$



$$I^d = FT[V^{Obs}] = PSF * I^o$$

Data corruptions

- The full Measurement Equation:

$$V_{ij}^{Obs}(\nu,t)=G_{ij}(\nu,t)\left[\int\int X_{ij}(\nu,t) I^M(l,m) e^{2\pi\iota(lu_{ij}+mv_{ij})} dl\, dm\right]$$

Data Corruptions Sky

  - G_{ij}: direction independent corruptions (e.g., multiplicative complex gains, etc.)
  - X_{ij}: direction dependent corruptions (e.g., Primary beam effects, etc.)
  - Often G_{ij}, X_{ij} are separable into antenna based quantities as $G_{ij}=G_i G_j^*$

    ==> *N unknowns; O(N*^{2}*) measurements*

$$V_{ij}^{Obs}=G_{ij}.\left[E_{ij}*V_{ij}^{o}\right]$$

$$\text{where } E_{ij}=E_i*E_j^* \text{ and } E_i=FT\left[X_i\right]$$

General Imaging approach

- Inverse transform strictly does not exist.
  - Use accurate transform for Image->Data and approximate for Data->Image

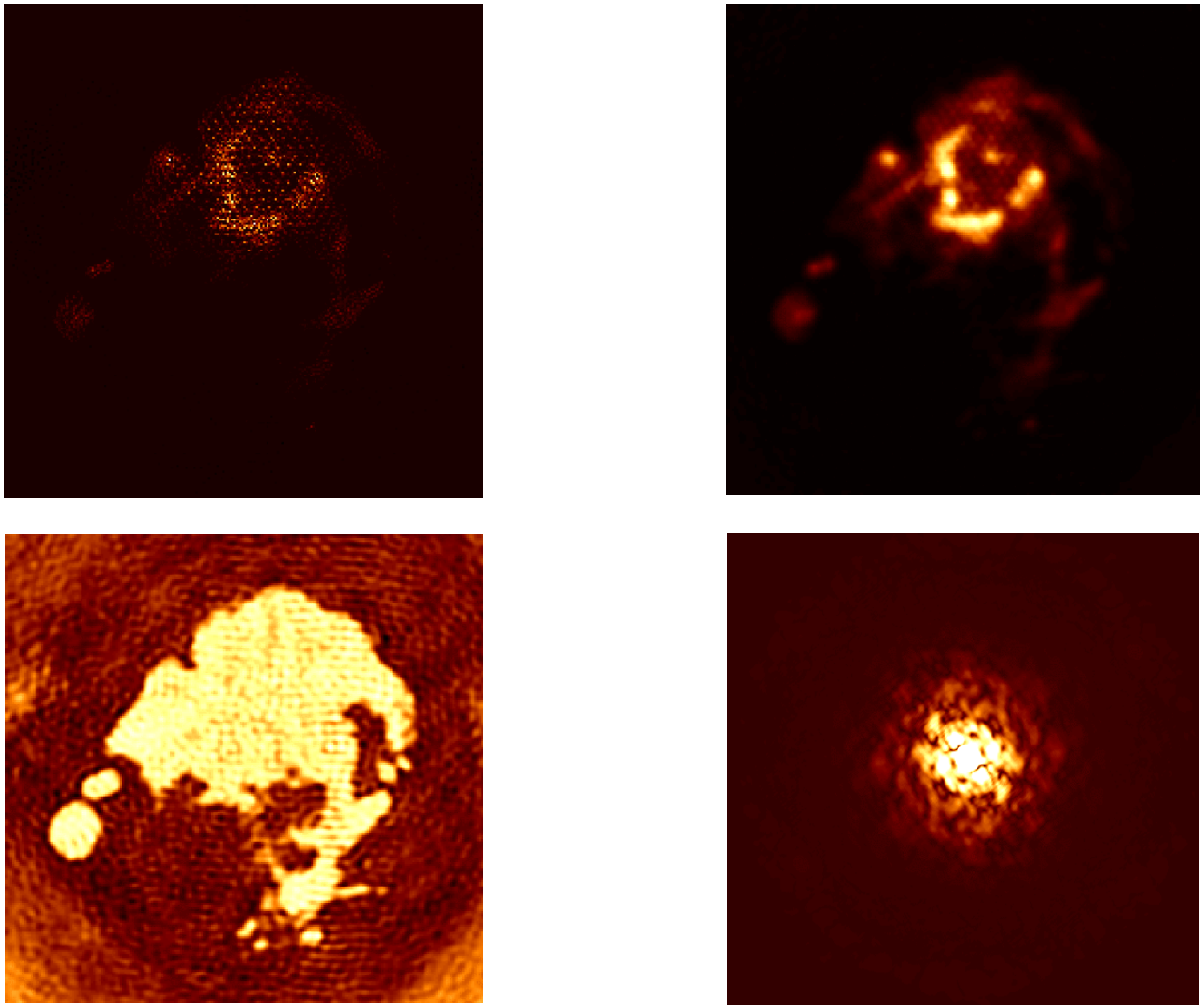
$$\mathbf{V}^{\mathbf{M}} = \mathbf{A}\mathbf{I}^{\mathbf{M}} \quad \text{and} \quad \mathbf{I}^{\mathbf{R}} = \mathbf{D}[\mathbf{V} - \mathbf{A}\mathbf{I}^{\mathbf{M}}] \quad where \quad \mathbf{D} \approx \mathbf{A}^{\mathbf{T}}$$

  - Use **D** to compute approximate update direction to be used in a non-linear iterative scheme to estimate $\boldsymbol{I^M}$ (deconvolution).
- Use $\boldsymbol{I^M}$ to solve for corruptions (calibration):

$$\text{minimize} : [\mathbf{V}^{Obs} - G_{ij} \mathbf{V}^{M}] \, w.r.t. \, G_i$$

- Use $G_{ij}^{-1} \mathbf{V}^{Obs}$ to improve the estimate for $\boldsymbol{I^M}$
- Iterate between deconvolution and calibration.

Example: The CLEAN algorithm



Direction dependent errors

- Antenna based, direction dependent errors needs correction in the data from each baseline by a different function: $V_{ij}^{Obs} = [E_{ij} * V_{ij}^{o}] \text{ where } E_{ij} = E_i * E_j^*$

- Such errors need to be corrected during deconvolution.

- General structure of deconvolution algorithms:
  - *Major cycle:* Compute the update direction:
  $$\Delta \chi^2 \propto \boldsymbol{I}^{\boldsymbol{R}} = \boldsymbol{D}[\boldsymbol{V} - \boldsymbol{A}\boldsymbol{I}^{\boldsymbol{M}}]$$
  - *Minor cycle:* Update the model image
  $$\boldsymbol{I}_{\boldsymbol{i}}^{\boldsymbol{M}} = \boldsymbol{I}_{\boldsymbol{i-1}}^{\boldsymbol{M}} + \alpha \, max[\Delta \chi^2]$$

Direction dependent error correction

- Use FFT for ***A*** to compute ***V^o^***. Needs data re-sampling.

$$\boldsymbol{V}^{\boldsymbol{G}}(k\,\Delta u)=(GCF(u)*\boldsymbol{V}(u))(k\,\Delta u)$$

- Use E_{ij} for GCF when predicting model visibilities.

$$\boldsymbol{V}^{\boldsymbol{M}}(u)=(E_{ij}(u)*\boldsymbol{A}\boldsymbol{I}^{\boldsymbol{M}})(u)$$

- Compute the update direction ($\boldsymbol{I}^{\boldsymbol{R}}$) as:

$$\frac{\boldsymbol{A}^{\boldsymbol{T}}[V^{Obs}-E_{ij}^{T}(u)*\boldsymbol{V}^{\boldsymbol{M}}]}{\overline{\boldsymbol{A}^{\boldsymbol{T}}E^{2}}}$$

Known direction dependent effects

- Non co-planar baselines

$$V(u,v,w)=\iint I(l,m)\,e^{2\pi\iota\left(ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right)}\frac{dl\,dm}{\sqrt{1-l^2-m^2}}$$

- W-projection algorithm

$$V_{ij}^{Obs}(u,v,w)=E_{ij}*V_{ij}^{M}(u,v,w=0)\quad where\quad E_{ij}=\boldsymbol{FT}\left[e^{2\pi\iota w\left(\sqrt{1-l^2-m^2}\right)}\right]$$

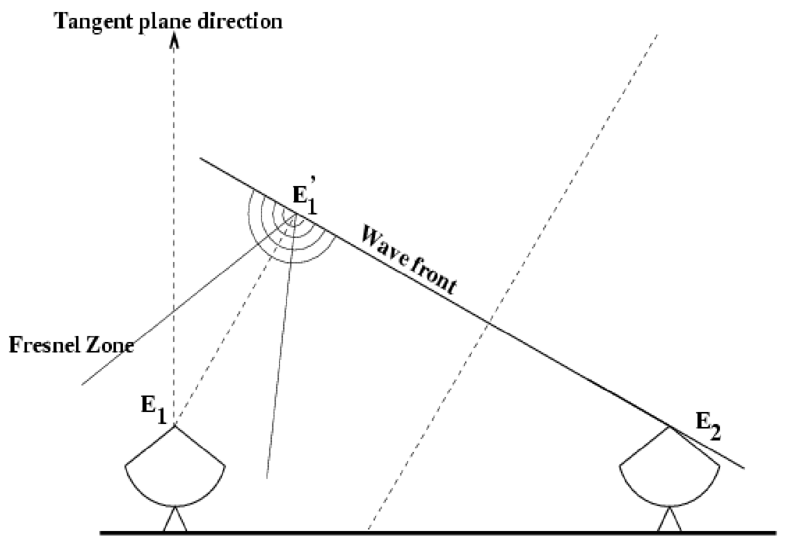
- Pre-compute *GCFs* with uniform sampling in $\sqrt{w}$ such that aliasing effects are less than the required dynamic range.

(Cornwell,Golap,Bhatnagar: EVLA Memo#67;A&A,in press)

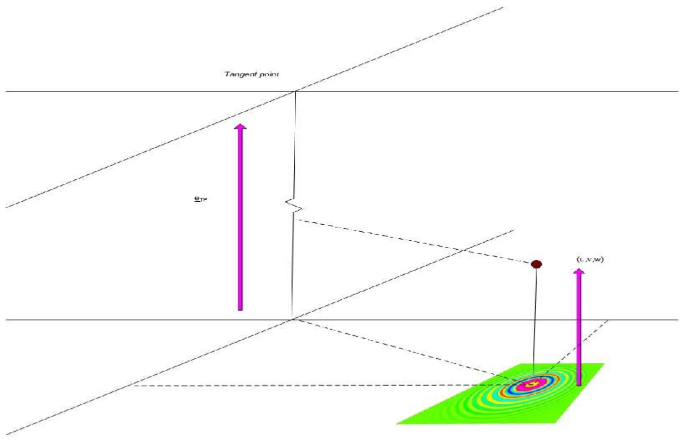
W-projection

- $\rho_{12}=\langle \boldsymbol{e}_1(u,v,w=0)\,\boldsymbol{e}_2^*(0,0,0)\rangle$

  $e_1 = e'_1(u,v,w)$ propagated using Fresnel diffraction theory. The above convolution equation is reproduced with $r_F/\lambda \approx \sqrt{w}$

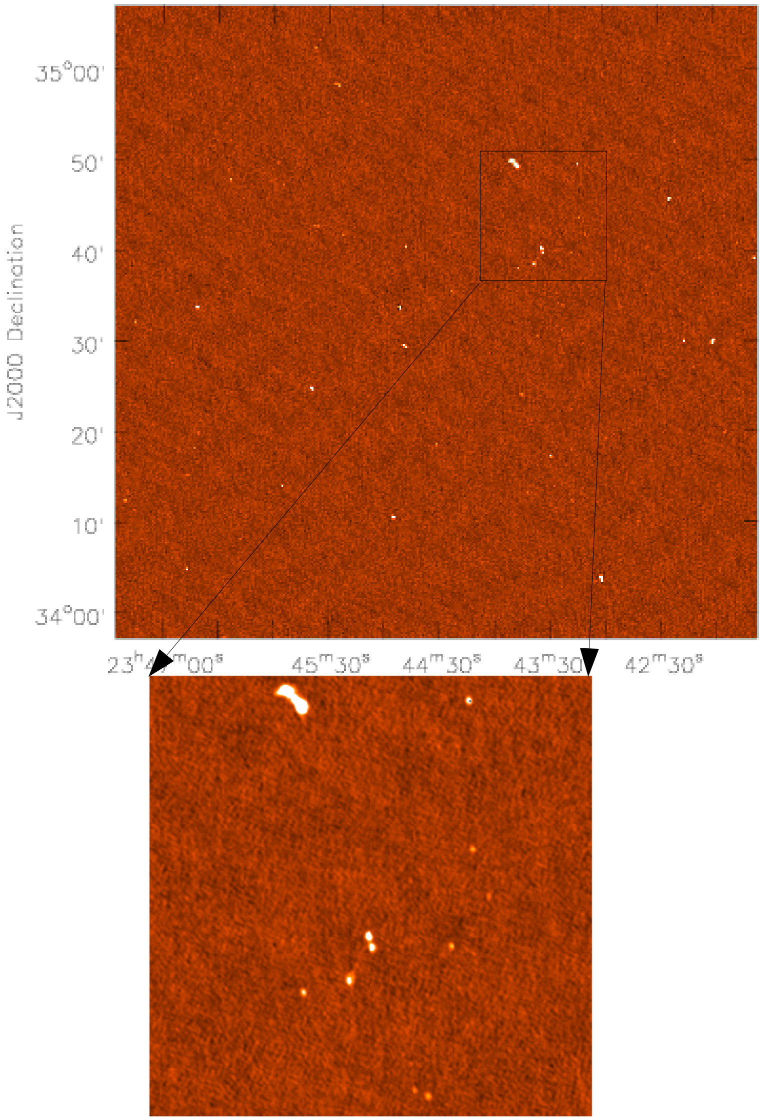


- A $w \neq 0$ interferometer is not a device to measure a single Fourier component.

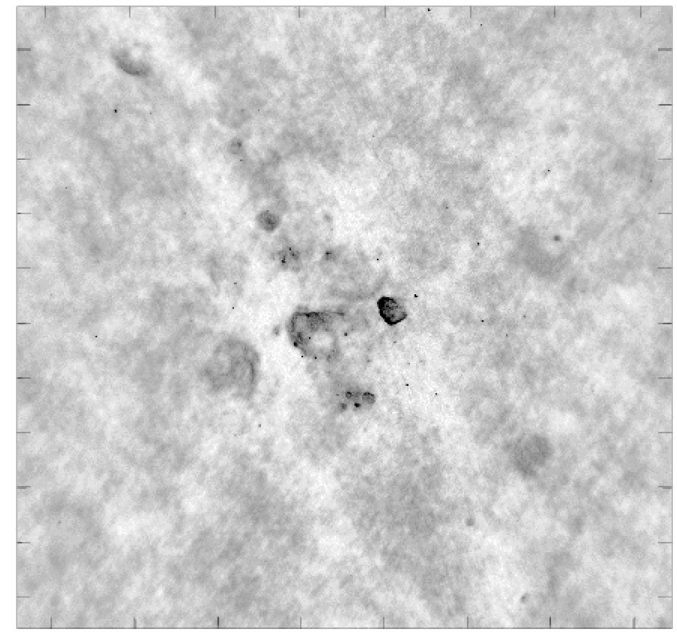


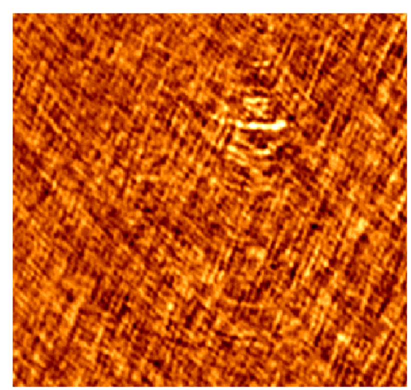
W-projection

GMRT P-band image of TXS2342+342 (Kanekar et al.)

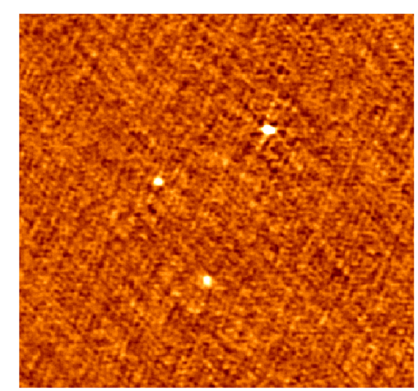


Galactic Plane at P-band – VLA B,C,D (Brogan et al.)



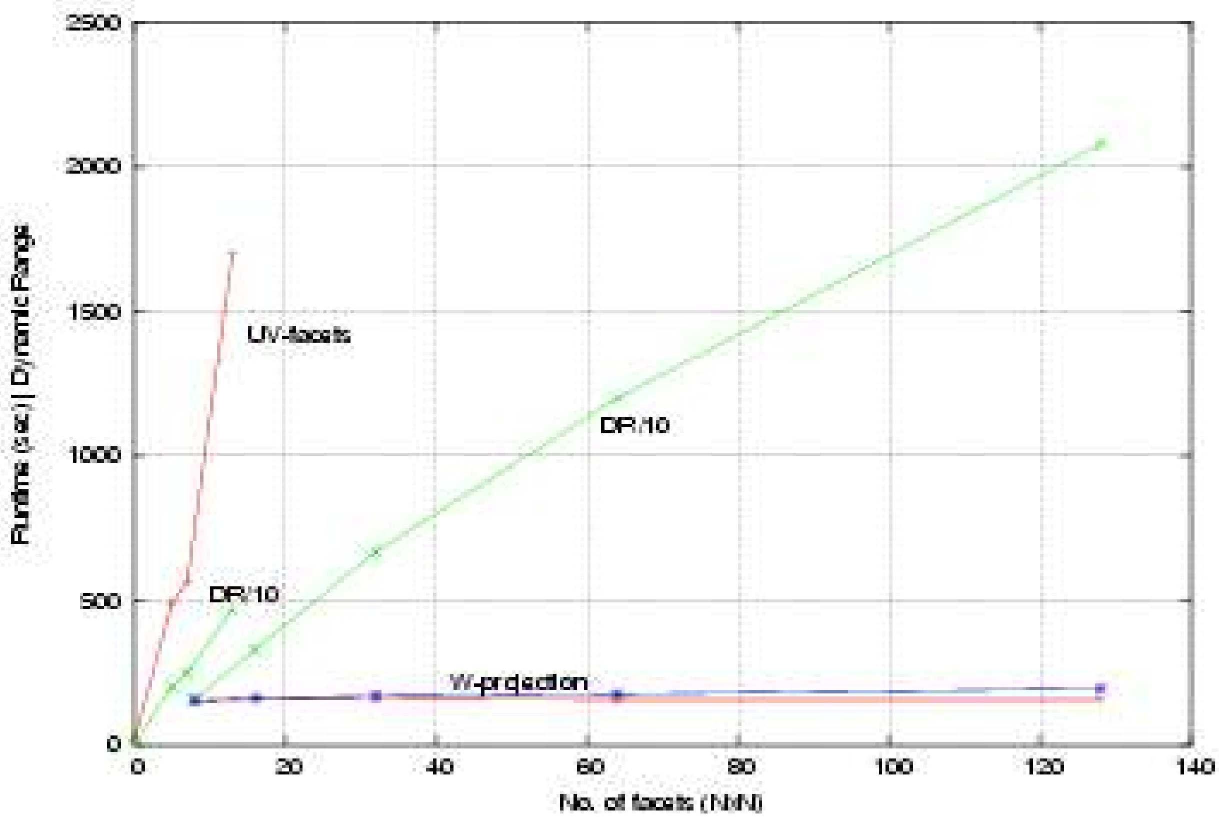


Without correction



With correction

W-projection scaling laws



Scaling laws:

W-projection:

$$(N^2_{wproj} + N^2_{GCF})\,N_{vis}$$

UV-facet:

$$N^2_{facets}\,N^2_{GCF}\,N_{vis}$$

Ratio:

$$\approx N^2_{GCF}\ \textit{for large no. of facets}$$

Measured direction dependent effects

- E_{ij} as function of direction is measured a priori

$$V_{ij}^{Obs} = G_{ij} . [E_{ij} * V^M] \quad where \quad E_{ij}(l_i, l_j, u_{ij}; p_i, p_j)$$

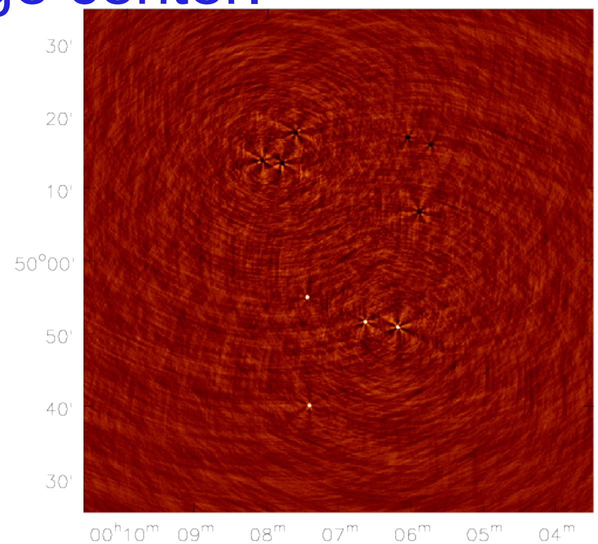
- Primary beam effects
  - Non-identical Primary Beams:

$$E_{ij} = E_i^o * E_j^o \quad where \quad E_i^o = \boldsymbol{FT}[Measured\ PB_i]$$

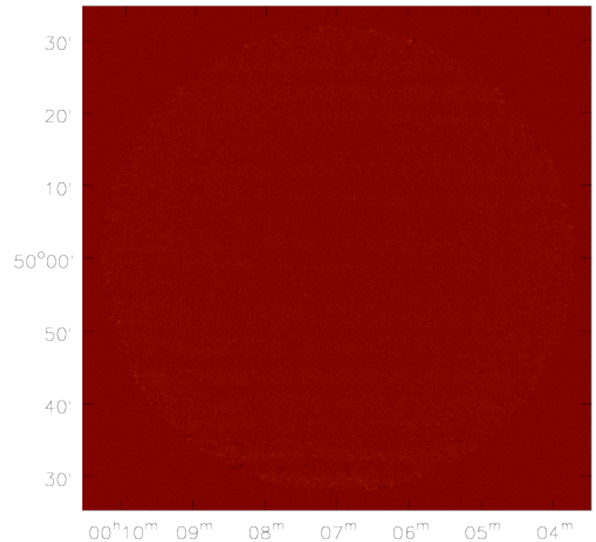
  - Polarized primary beam: Beam squint
    - E_i separately measured for the two polarizations
    - Applied with the known squint

Polarization Beam Squint: Example

- VLA: The two orthogonal polarization power patterns have a squint w.r.t. to the pointing direction (6% of FWHM).
- If not corrected, polarization properties are correct only at the image center.



Stokes-V image without squint correction



Stokes-V image with squint correction

Dynamic range improvement: ~15x Fidelity improvement: ~50x

Unknown direction dependent effects

- E_{ij} as a function of direction is not known a priori

$$V_{ij}^{Obs} = G_{ij}.[E_{ij} * \mathbf{FFT}[I^M]] \quad where \quad E_{ij}(l_i, l_j, u_{ij}; p_i, p_j)$$

- E_{ij} is parameterized *(p_i).* Needs a solver

- Primary beam effects (Pointing offsets):

$$E_{ij} = E^o [1 + \Delta E_{ij}(l_i, l_j; t)] \quad l_i \text{ is the pointing offset}$$

- Source structure as a function of polarization and frequency: I^M is also parameterized $I(\nu) = I(\nu_o)(\nu / \nu_o)^{-\alpha}$

$$I^M = \sum_k Components(\nu, Poln, Position, Scale)$$

Pointing offset calibration

$$E_{ij}(l;l_i,l_j)=E^o(l)\left[e^{-\left(\frac{l_i-l_j}{2}\right)^2} e^{-\pi\iota u_{ij}(l_i+l_j)}\right]$$

$$(l_i \text{ is the pointing offset})$$

- Separates into constant E^o and antenna dependent terms
- *E_{ij} (the GCF) is different for each baseline!*
- Use lookup tables or approximation:

$$E_{ij}=E^o\left[1-(l_i-l_j)^2\sigma^2/2-\pi\iota u_{ij}(l_i+l_j)+\ldots\right]$$

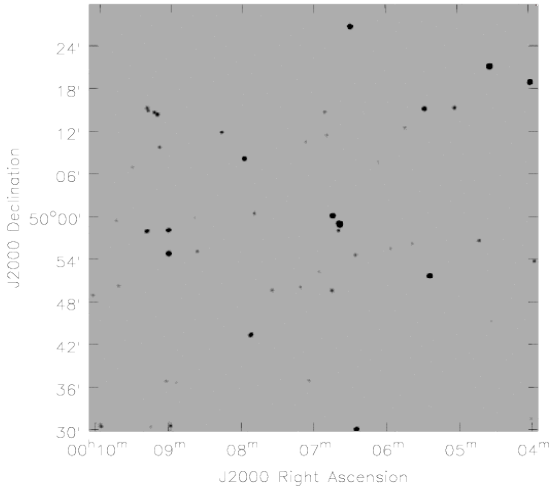
Pointing SelfCal

- Pointing errors for single pointing observation of typical L-Band field limits the RMS noise to ~10microJy.
    - EVLA L-band sensitivity: 1microJy/beam

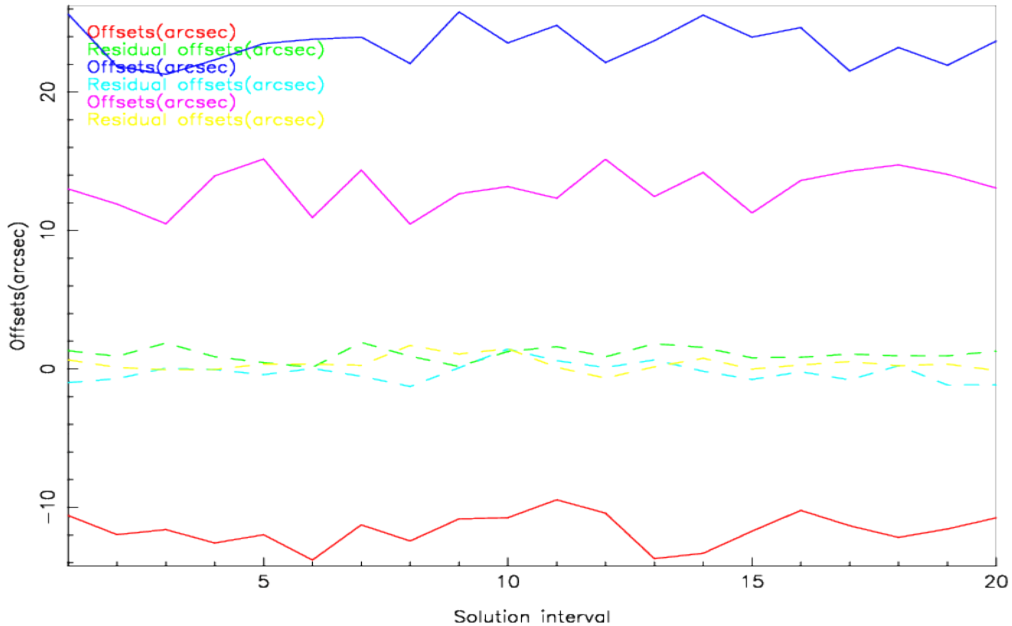
- Mosacing dynamic range limited by pointing errors.
    - Significant fraction of ALMA observation will be mosacing observations.

- This implies significantly increased computing:
    - Each iteration involves expensive visibility prediction

Pointing SelfCal: Example



Model image: 59 sources from NRAO-VLA All Sky Survey(NVSS). Flux range ~2-200 mJy/beam

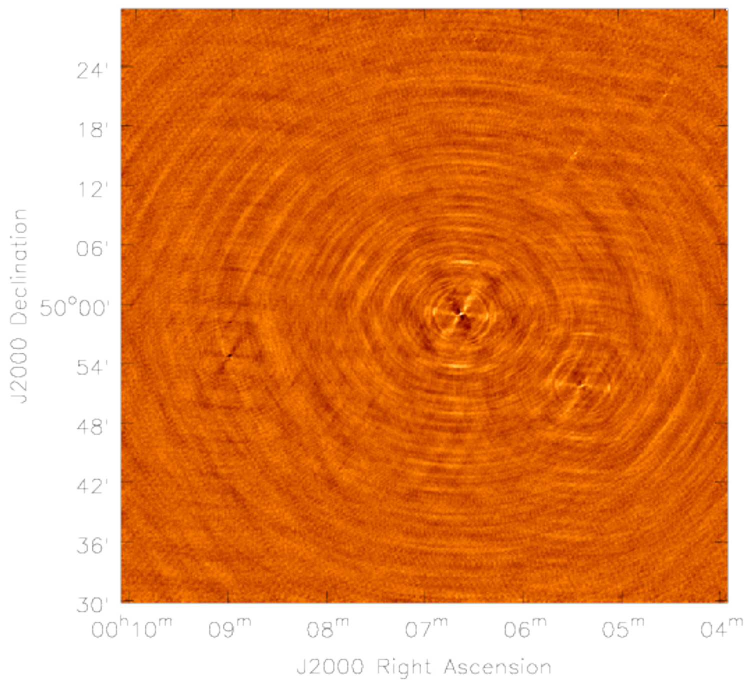


Continuous lines: Typical antenna pointing offsets for VLA as a function of time (Mean between +/- 20arcsec and RMS of 5arcsec).

Dashed lines: Residual pointing errors. RMS ~ 1arcsec.

Pointing SelfCal: Example

Ref: Bhatnagar,Cornwell,Golap, EVLA Memo #84

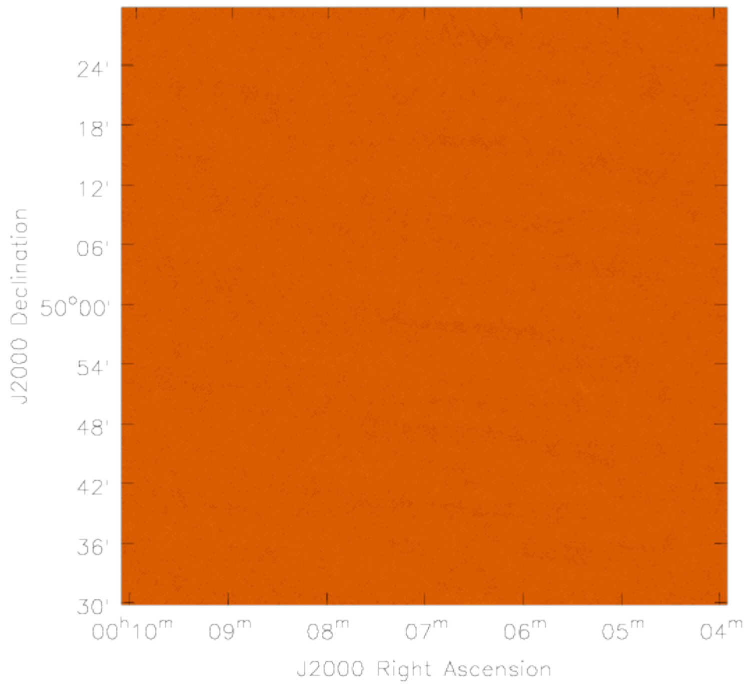


Residual image without pointing correction:

FT[V^{Obs}-V^{M}]

Peak flux ~250 microJy/beam

RMS ~ 15microJy



Residual image after pointing correction:

FT[V^{Obs}-E_{ij}*V^{M}]

Peak flux ~5microJy/beam

RMS ~ 1microJy

Wide-band continuum imaging

- EVLA bandwidth ratio of 2:1
  - $V(u_{ij}, v_{ij}) = \sum_{\nu_k} V(u_{ij}, v_{ij}; \nu_k) = \sum_{\nu_k} P_{ij}(\nu_k) FT[I^D(\nu_k)]$

    Sky emission, the Primary Beams, etc. become a function of frequency.

    Ideas: Apply PB effects during predict. Sky model parameterized in frequency as well.

- Combining Scale sensitive + frequency sensitive deconvolution with image plane corrections.
- Work in progress: Extension of Multi-Frequency Synthesis and Scale-sensitive Clean.

Scale sensitive deconvolution

- Pixel-to-pixel noise in the image is correlated at the scale of the resolution element

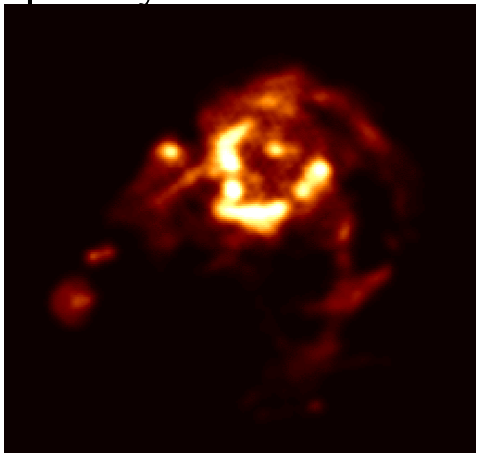
$$I^D = B * I^o + B * I^N \quad where\ I^N = FT[Visibility\ Noise];\ B = PSF$$

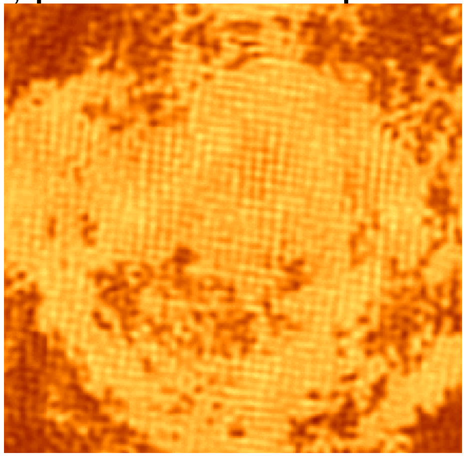
- The scale of emission *fundamentally* separates signal (I^o) from the noise (I^N).

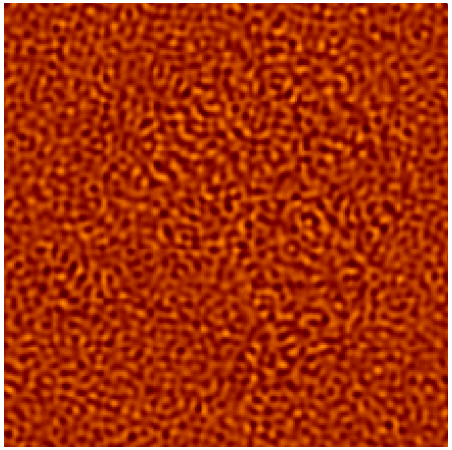
- Asp-Clean (Bhatnagar&Cornwell, 2004, A&A)

$$I^M = \sum_{k=1}^{N} A_k P(Scale_k)$$

  - Explicitly solve for local scale, position and amplitude

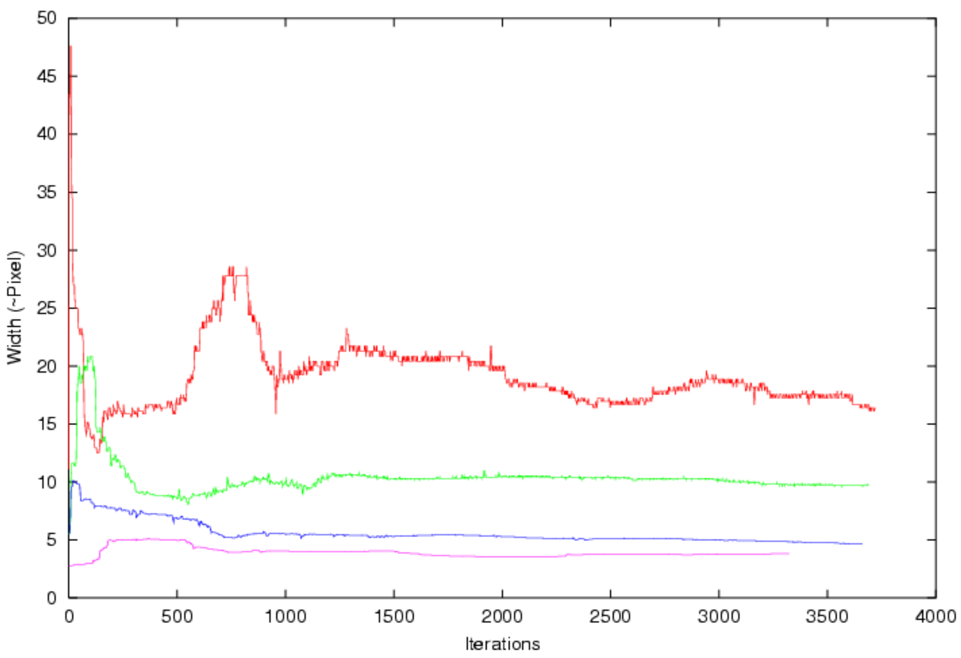


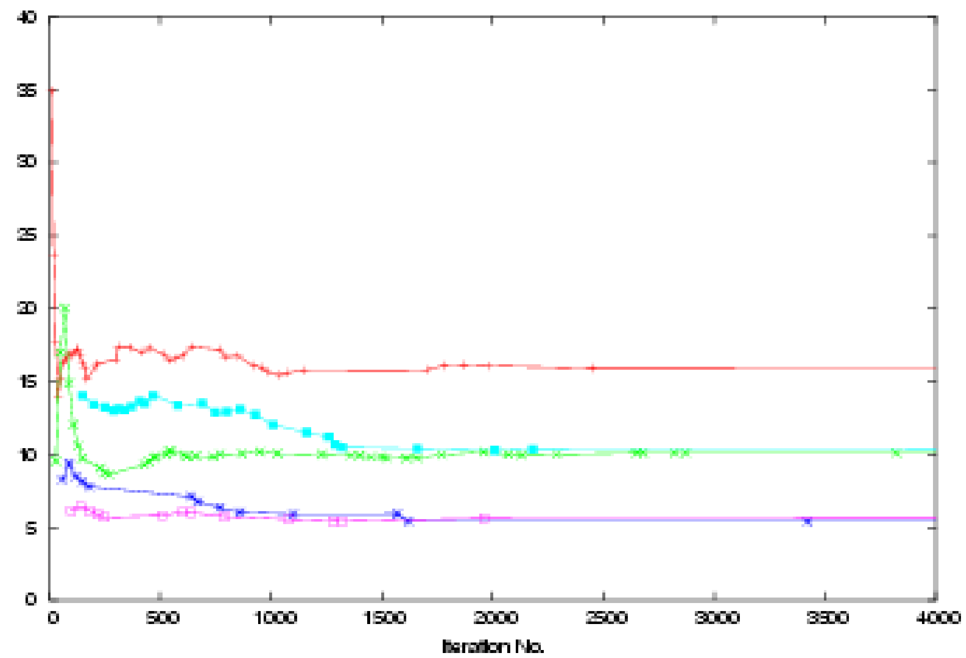




Asp-Clean acceleration

- Asp-Clean (Bhatnagar & Cornwell, A&A, in press)
  - Large scale emission and asymmetric structures are better reconstructed
  - Computationally expensive: cost increases with the no. of components
  - Acceleration: Solve in a sub-space; adaptively determine the sub-space





Ionospheric phase correction

- $$V_{ij}^{Obs}(\nu,t)=\int\int K_{ij}(l,m;\nu,t) I^M(l,m) e^{2\pi\iota(u_{ij}l+v_{ij}m)} dl\,dm$$

  where K_{ij} is the ionospheric, direction dependent phase

- The general form for residuals: $V_{ij}^R = V_{ij}^{Obs} - X_{ij} * FT[I^M]$

  where $X_{ij}=W_{ij}.E_{ij}.K_{ij}$ (and other direction dependent terms)

- Express ionospheric phase in the antenna beam using ionospheric physics/geometrical effects/primary beam effects (de-focusing, time varying illumination, etc.)

- X_{ij} is separable into antenna based quantities $X(p_i).X^*(p_j)$ Solve for p_i's

Computing and I/O costs

- Increase in computing due to more sophisticated parameterization
  - Deconvolution: Fast evaluation of $B * \sum_k A_k P(Scale_k, Pos_k)$
  - Calibration: Fast evaluation of $E_{ij} * V^M$
- Cost of computing residual visibilities is dominated by I/O costs for large datasets (~200GB for EVLA)
  - Deconvolution: Approx. 20 access of the entire dataset
  - Calibration: Each trial step in the search accesses the entire dataset
- Solutions: Analytical approximations, caching, Parallel computing and I/O,...