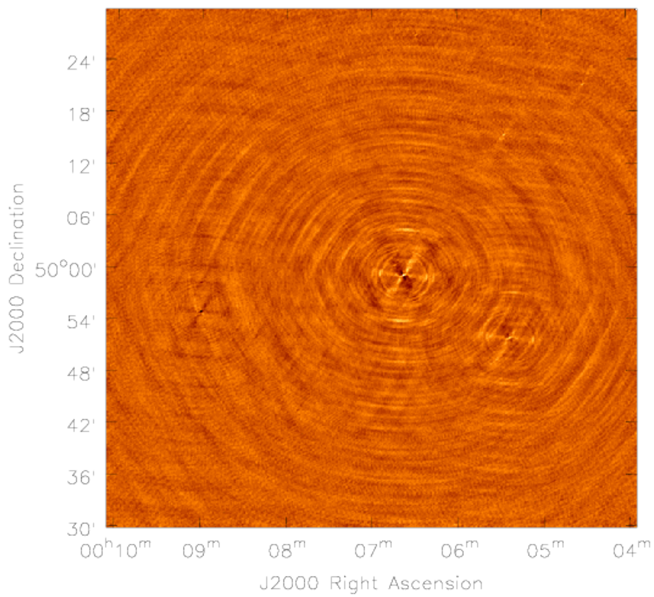
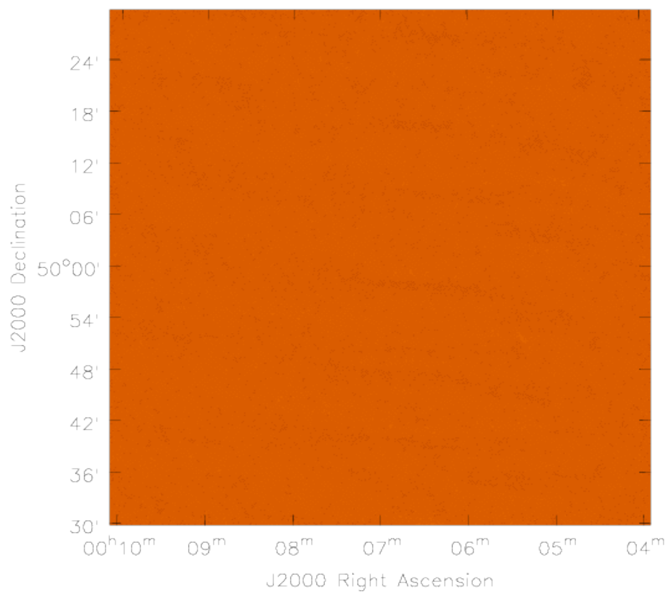
Steps towards correction of image plane effects





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Basic Interferometry

- Interferometers measure the source coherence function (the Visibility function) $V(u_{ij}, v_{ij}, w_{ij}) = \langle E_i E_j^* \rangle$

  E_i is the electric field measured at antenna *i*

  u,v,w are the projected separation between the antennas *i* and *j*

- In terms of the sky brightness distribution ($I^o(l,m)$)

$$V(u_{ij}, v_{ij}, w_{ij}) = \int\int I^o(l,m) e^{-2\pi\iota(u_{ij}l + v_{ij}m + w_{ij}n)} \frac{dl\,dm}{n}$$

- In the small angle approximation, sky is the 2D Fourier transform of the Visibility function (van Cittert-Zernike Theorem)

$$V(u_{ij}, v_{ij}) = \int\int I^o(l,m) e^{-2\pi\iota(u_{ij}l + v_{ij}m)} dl\,dm$$

$$I^o = FT[V]$$

Imaging and calibration errors

- Generic Measurement Equation:

$$V_{ij}^{Obs}(\nu,t)=G_{ij}(\nu,t)[\int\int X_{ij}(\nu,t)I^{M}(l,m)e^{2\pi\iota(lu_{ij}+mv_{ij})}dl\,dm]$$

Data — Corruptions — Sky

$G_{ij}=G_iG_j^*$ where G_i are antenna based gains (direction independent)

$X_{ij}=X_i(l,m)X_j(l,m)$ where X_i are image plane errors (direction dependent).

- $V_{ij}^{Obs}=G_{ij}.[E_{ij}*V_{ij}^{M}]$ Unknowns are: G_{ij}, E_{ij}, *and* I^M.
- SelfCal: Given I^M *and* knowing E_{ij}, G_{ij} is the dominant error term.
- Imaging/deconvolution: Knowing G_{ij} *and* E_{ij}, solve for I^M

Visibility inversion

- Visibility inversion is done using the FFT algorithm which operates on a regular grid.
- V^{Obs} does not regularly sample the (u,v) plane. $V^{M}(u,v)$ is computed by re-sampling the grid using a Gridding Convolution Function (GCF)

$$V^M(u_{ij},v_{ij}) = \left(GCF(u,v) * \boldsymbol{FFT}\left[I^M\right]\right)(u_{ij},v_{ij})$$

- Image plane effects can be efficiently applied by incorporating them in the GCF.

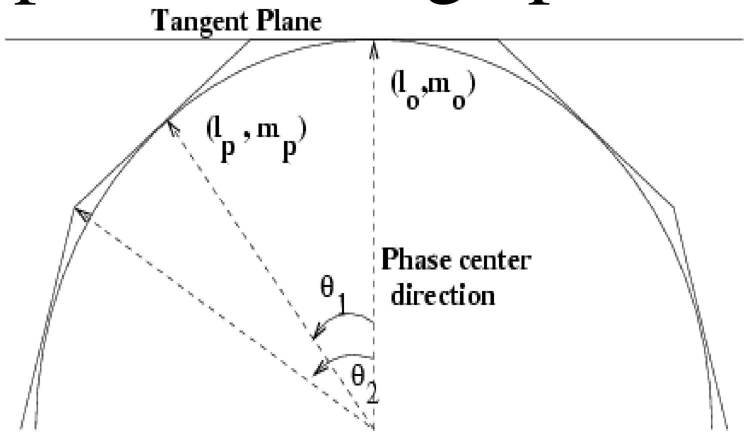
- Challenge: Fast evaluation of the modified GCF.

Known direction dependent errors

- Non co-planar baselines

$$V(u,v,w)=\iint I(l,m)\, e^{2\pi\iota\left(ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right)}\frac{dl\,dm}{\sqrt{1-l^2-m^2}}$$

- Traditional approach: Image plane faceting



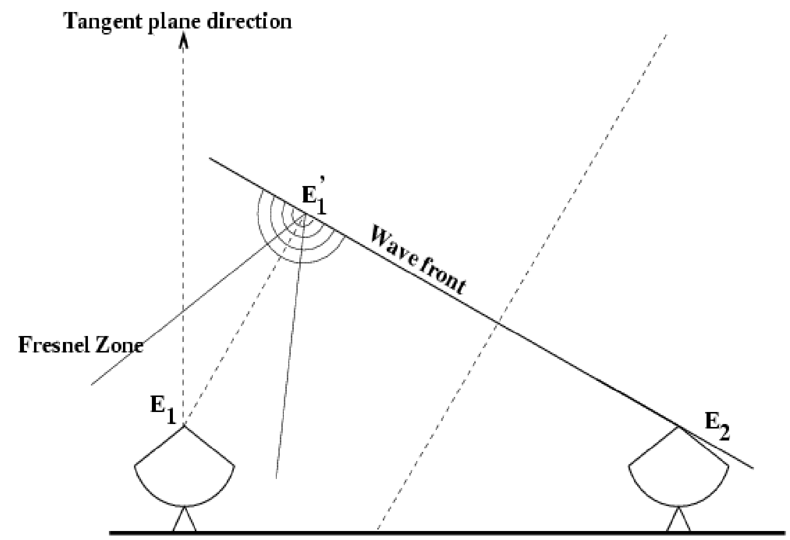
- W-projection

$$V_{ij}^{Obs}(u,v,w)=E_{ij}*V_{ij}^{M}(u,v,w=0) \quad where \quad E_{ij}=\mathbf{FT}\left[e^{2\pi\iota w\left(\sqrt{1-l^2-m^2}\right)}\right]$$

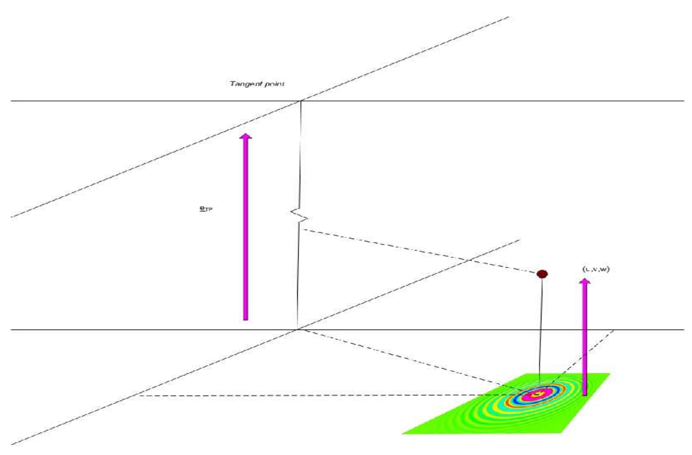
W-projection

- $\rho_{12}=\langle \boldsymbol{E}_1(u,v,w=0)\boldsymbol{E}_2^*(0,0,0)\rangle$

  $E_1=E'_1(u,v,w)$ propagated using Fresnel diffraction theory. The above convolution equation is reproduced with $r_F/\lambda\approx\sqrt{w}$

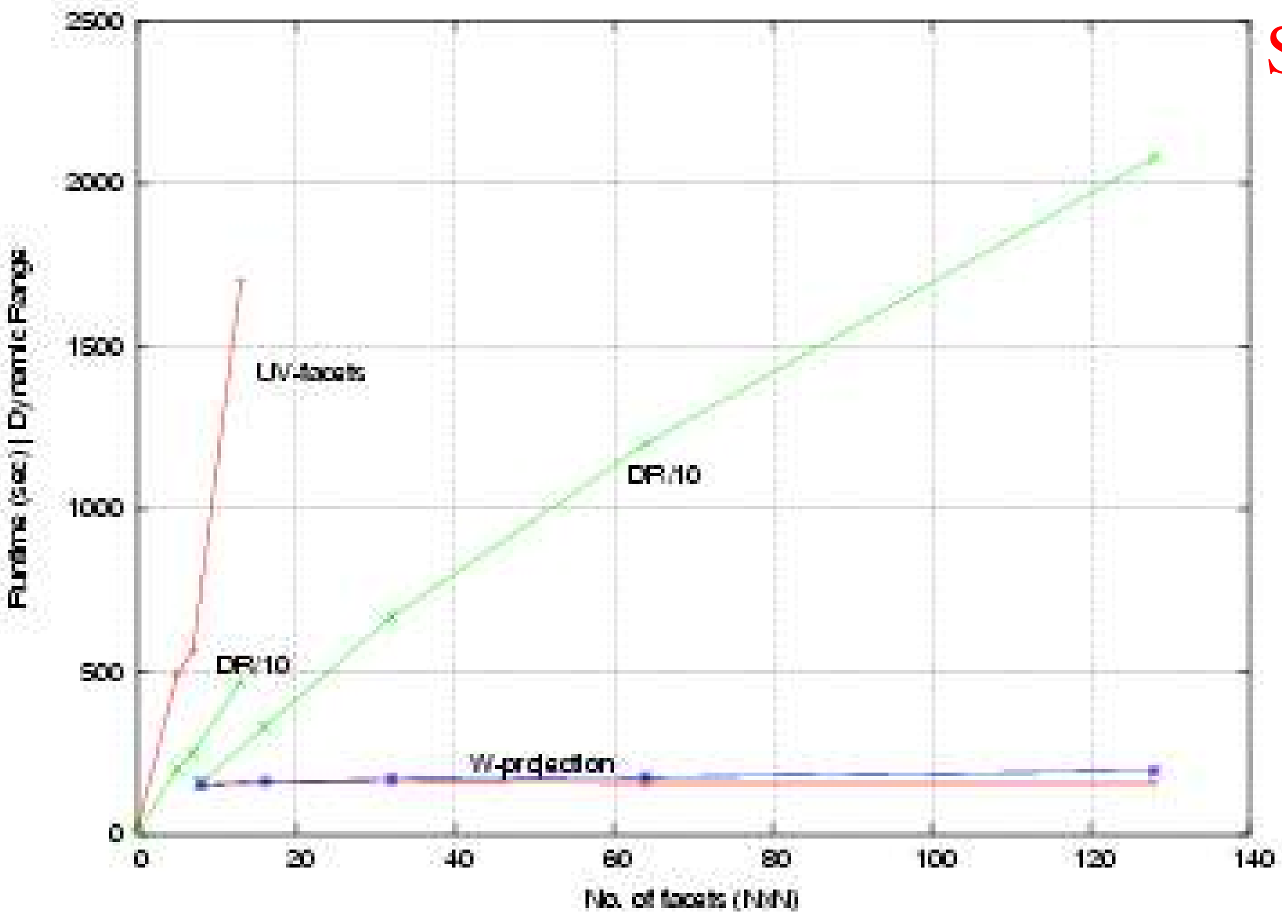


- A $w\neq 0$ interferometer is not a device to measure a single Fourier component.



W-projection algorithm
(CGB, EVLA Memo #67)

- Pre-compute *GCFs* with uniform sampling in $\sqrt{w}$ such that aliasing effects are less than the required dynamic range.



Scaling laws:

W-projection:

$$(N^2_{wproj} + N^2_{GCF})\, N_{vis}$$

UV-facet:

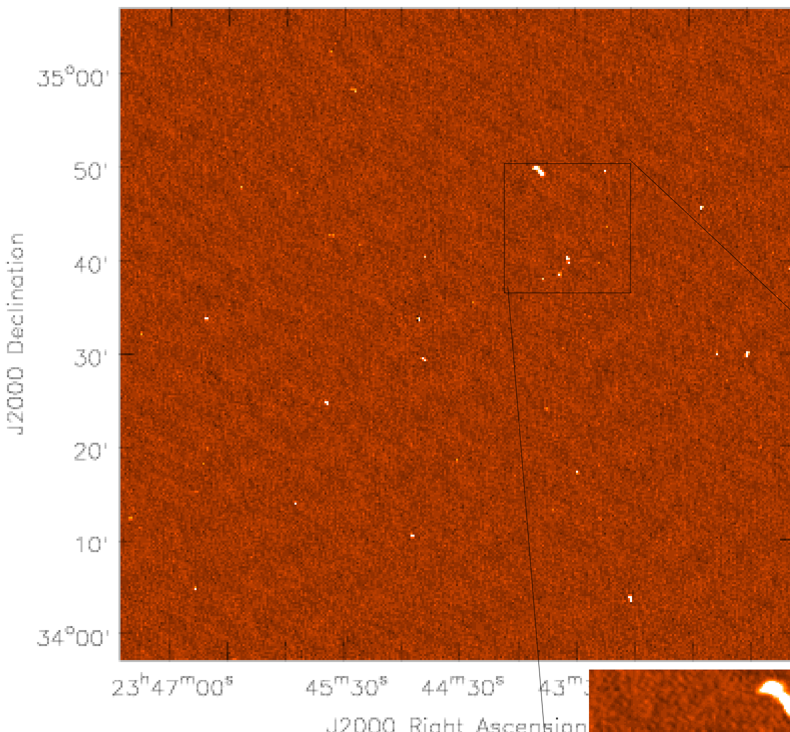
$$N^2_{facets}\, N^2_{GCF}\, N_{vis}$$

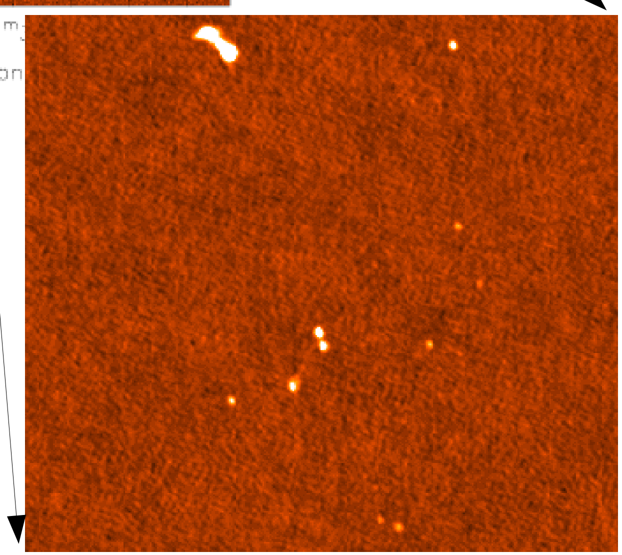
Ratio:

$$\approx N^2_{GCF} \text{ for large no. of facets}$$

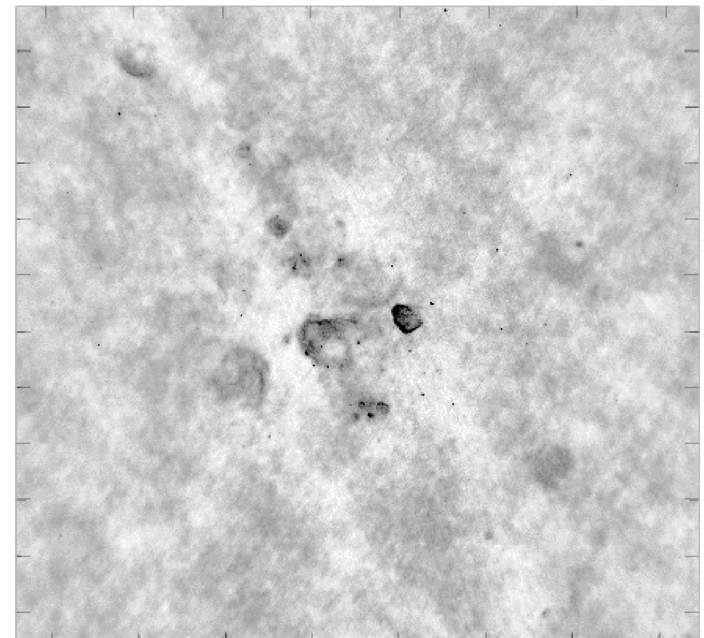
W-projection: Examples

GMRT P-band image of TXS2342+342 (Kanekar et al.)





Galactic Plane at P-band – VLA B,C,D (Brogan et al.)



Measured direction dependent effects

- E_{ij} as a function of direction is measured a priori

$$V_{ij}^{Obs} = G_{ij} . [E_{ij} * V^M] \quad where \quad E_{ij}(l_i, l_j, u_{ij}; p_i, p_j)$$

- Primary beam effects
  - Asymmetric Primary Beams:

$$E_{ij} = E_i^o * E_j^o \quad where \quad E_i^o = \boldsymbol{FT}[Measured\, PB_i]$$

  - Polarized primary beam: Beam squint
    - E_i separately measured for the two polarizations
    - Applied with the known squint

Unknown direction dependent effects

- E_{ij} as function of direction is not known a priori

$$V_{ij}^{Obs} = G_{ij}.\left[E_{ij} * \mathbf{FFT}\left[I^M\right]\right] \quad where \quad E_{ij}\left(l_i, l_j, u_{ij}; p_i, p_j\right)$$

- E_{ij} is parameterized *(p_i)*. *Needs a solver*

- Primary beam effects

  - <u>Pointing offsets:</u>

$$E_{ij} = E^o\left[1 + \Delta E_{ij}\left(l_i, l_j; t\right)\right] \quad l_i \, is\, the\, pointing\, offset$$

- Ionospheric/atmospheric effects: <u>non-isoplanatic phases</u>

- Source structure as a function of polarization and frequency: I^M is also parameterized <u>(spectral index effects)</u>

$$I^M = \sum_k Components\left(\nu, Poln, Position, Scale\right)$$

Pointing offset calibration

$$E_{ij}(l;l_i,l_j)=E^0_{ij}(l)\,e^{-\left(\frac{l_i-l_j}{2}\right)^2}\,e^{-\pi\iota u_{ij}(l_i+l_j)}\quad (l_i \textit{ is the pointing offset})$$

- Visibility prediction:
  - Gridded model $V^{M,Grid}$ = ***FFT[I^M]***
  - Re-sample on measured *(u,v)* using E_{ij} as the GCF for baseline *i-j*: $V^M_{ij}=E_{ij}*V^{M,Grid}$
- *GCF is different for each baseline!*
- Use lookup tables

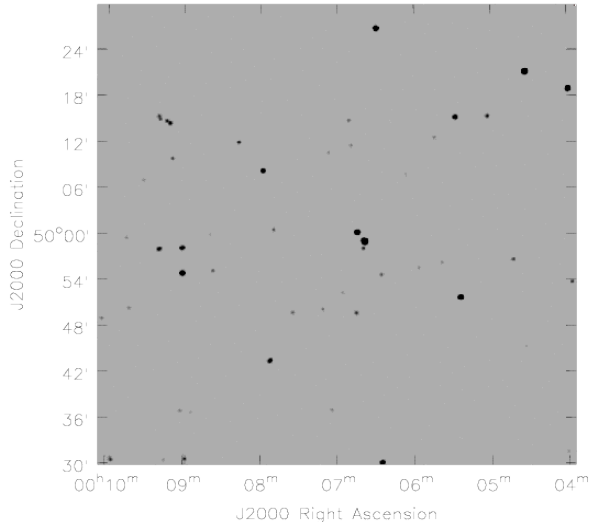
OR

- Approximation: $E_{ij}=E^0_{ij}\left[1-(l_i-l_j)^2\sigma^2/2-\pi\iota u_{ij}(l_i+l_j)+...\right]$

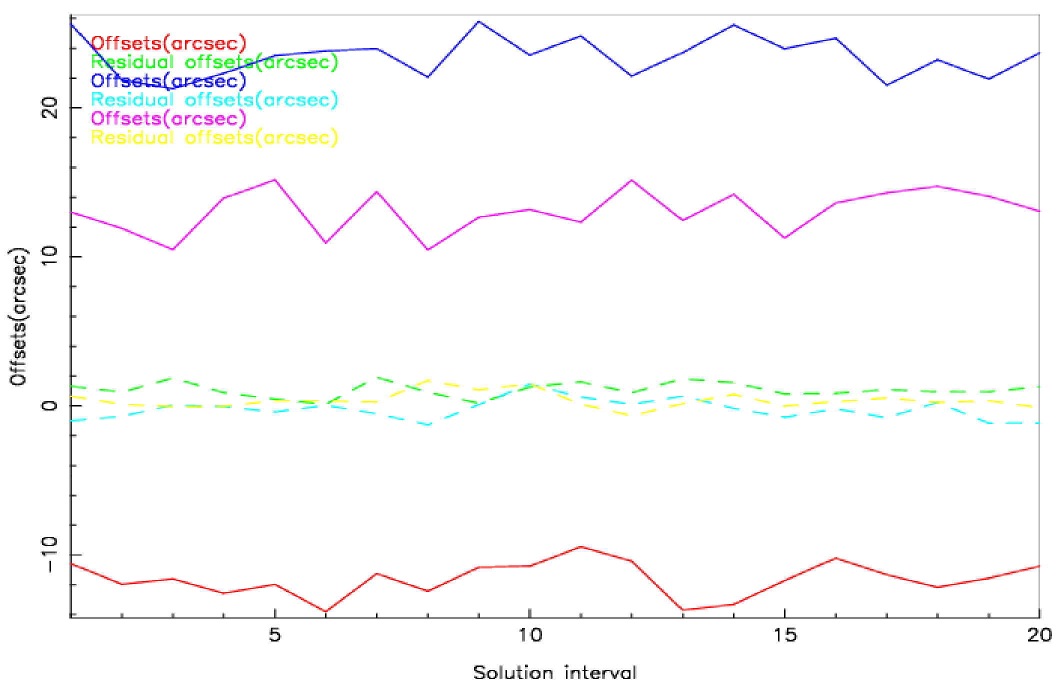
Pointing SelfCal

- Pointing errors for single pointing observation of typical L-Band field limits the RMS noise to ~10microJy.
    - EVLA L-band sensitivity: 1microJy/beam
- Mosacing dynamic range limited by pointing errors.
    - Significant fraction of ALMA observation will be mosacing observations.
- This implies significantly increased computing:
    - Each iteration involves expensive visibility prediction

Pointing SelfCal: Example



Model image using 59 sources from NVSS. Flux range ~2-200 mJy/beam

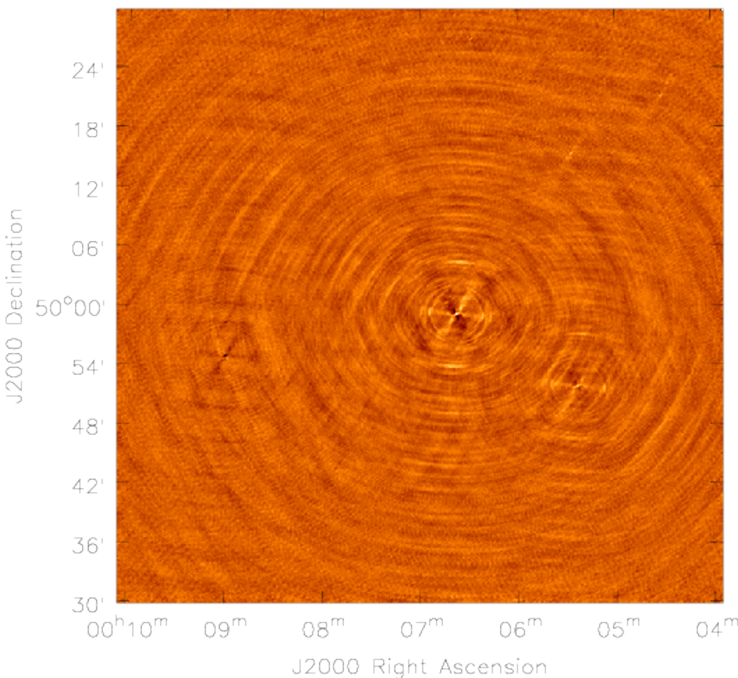


Continuous lines: Typical antenna pointing offsets for VLA as a function of time (Mean between +/- 20arcsec and RMS of 5arcsec).

Dashed lines: Residual pointing errors. RMS ~ 1arcsec.

Pointing SelfCal: Example

Ref: BCG, EVLA Memo #84

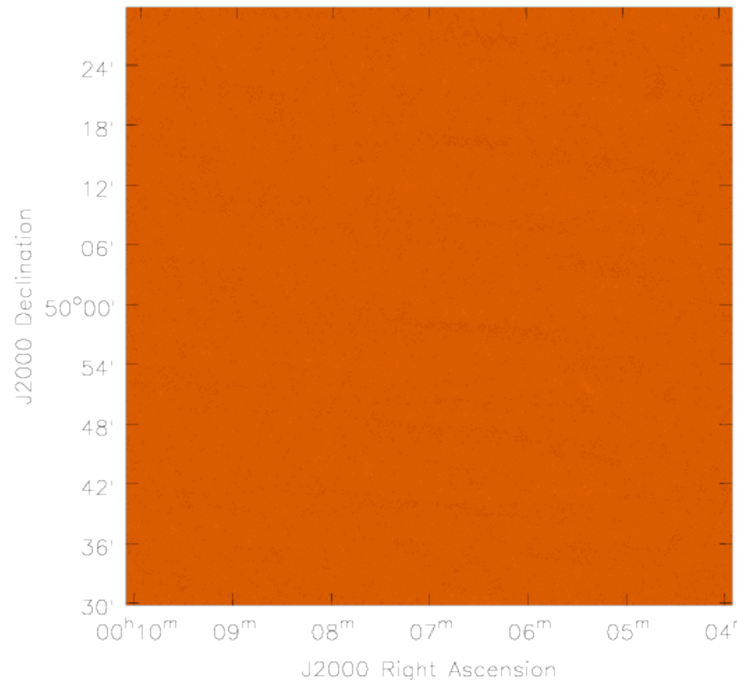


Residual image without pointing correction:

FT$[V^{Obs}-V^{M}]$

Peak flux ~250 microJy/beam

RMS ~ 15microJy



Residual image after pointing correction:

FT$[V^{Obs}-E_{ij}*V^{M}]$

Peak flux ~5microJy/beam

RMS ~ 1microJy

Wide band continuum imaging

- EVLA bandwidth ratio of 2:1
  - $V(u_{ij}, v_{ij}) = \sum_{\nu_k} V(u_{ij}, v_{ij}; \nu_k) = \sum_{\nu_k} P_{ij}(\nu_k) FT\left[I^D(\nu_k)\right]$

  Sky emission, the Primary Beams, etc. become a function of frequency.

  Ideas: Apply PB effects during predict. Sky model parameterized in frequency as well.

- Combining Scale sensitive + frequency sensitive deconvolution with image plane corrections.
- Work in progress: Extension of MSF and Scale-sensitive Clean (Bhatnagar&Cornwell, A&A, 2004, in press)

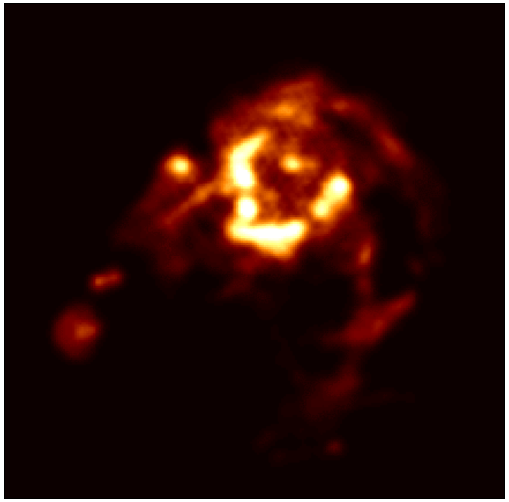
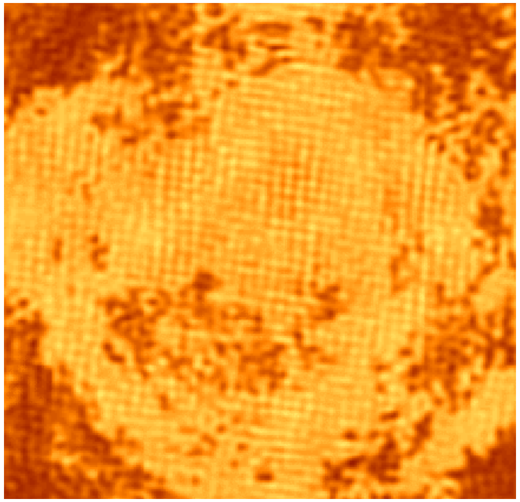
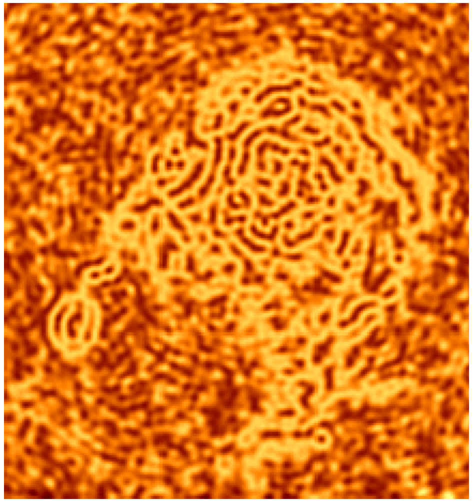
Scale sensitive deconvolution-I

- Pixel-to-pixel noise in the image is correlated at the scale of the resolution element

$$I^D = B * I^o + B * I^N \quad where\ I^N = FT[Visibility\ Noise];\ B = PSF$$

- The scale of emission *fundamentally* separates signal (I^o) from the noise (I^N).

- Multi-Scale Clean (Cornwell & Holdaway, 2004, in prep)
    - Decompose the sky in a set of components at few scales $I^M = \sum_{k=1}^{N} A_k P(Scale_k)$

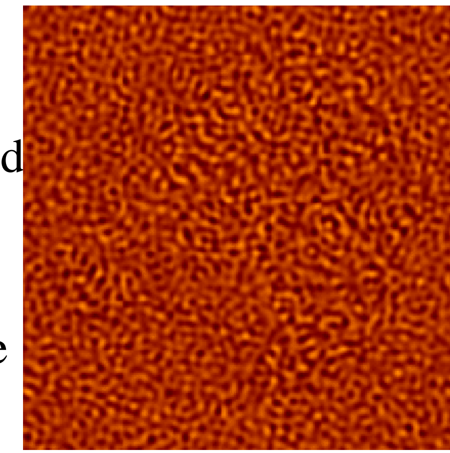
  

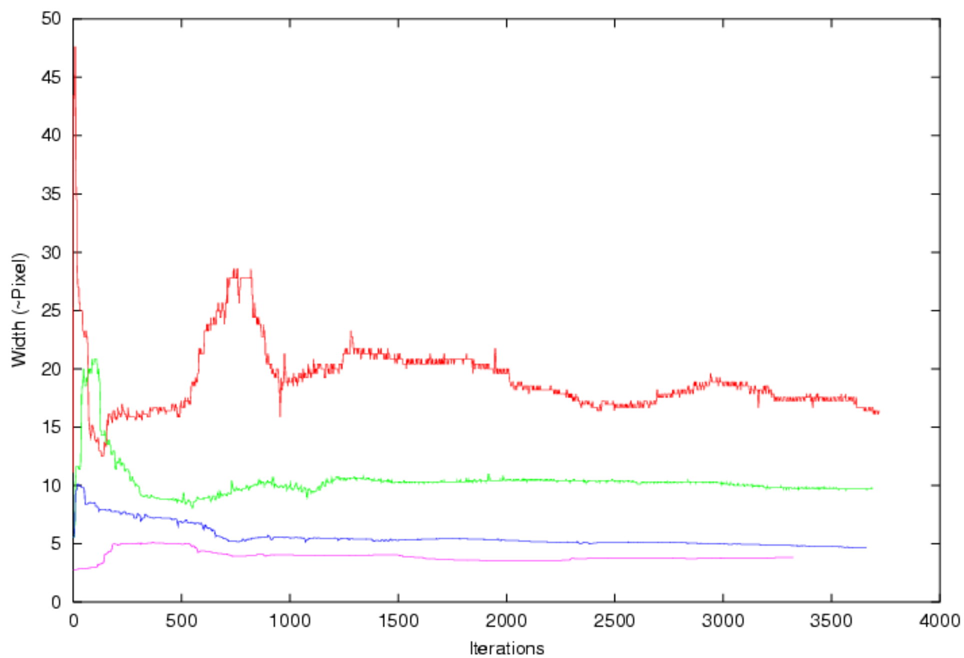
Scale sensitive deconvolution-II

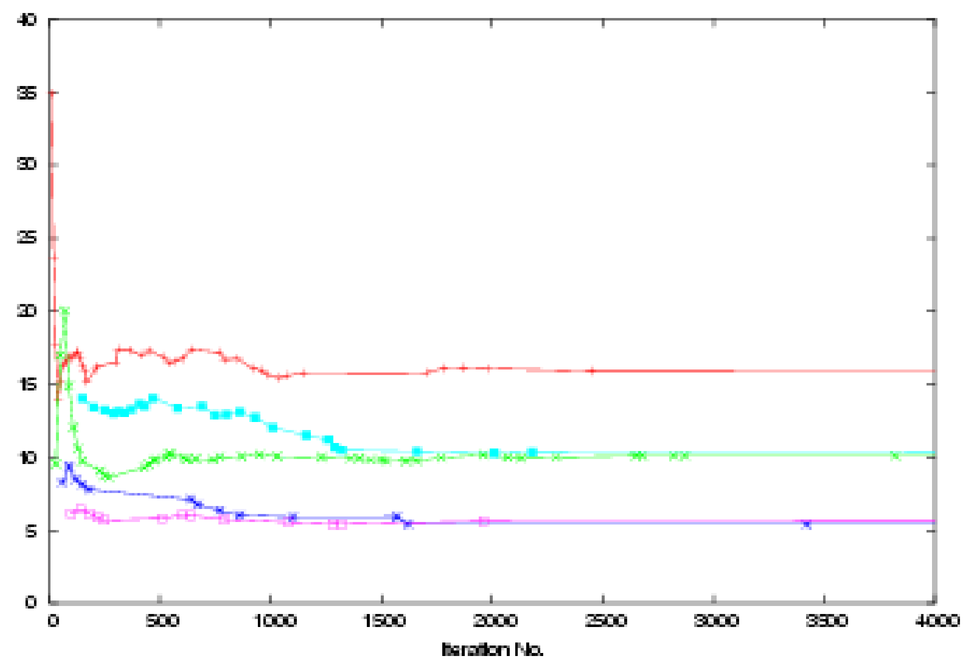
- Asp-Clean (Bhatnagar & Cornwell, A&A, in press)
  - Explicitly solve for the local scale, position and amplitude of the pixel model

$$I^M = \sum_k A_k P(Scale_k, Pos_k)$$

  - Large scale emission and asymmetric structures are better reconstructed
  - Computationally expensive: cost increases with the no. of components
  - Acceleration: Solve in a sub-space; adaptively determine the sub-space







Ionospheric phase correction

- $V_{ij}^{Obs}(\nu,t)=\int\int K_{ij}(l,m;\nu,t)I^M(l,m)e^{2\pi\iota(u_{ij}l+v_{ij}m)}dl\,dm$

  where K_{ij} is the ionospheric, direction dependent phase

- The general form for residuals: $V_{ij}^R=V_{ij}^{Obs}-X_{ij}*FT[I^M]$

  where $X_{ij}=W_{ij}.E_{ij}.K_{ij}$ (and other direction dependent terms)

- Express ionospheric phase in the antenna beam using ionospheric physics/geometrical effects/primary beam effects (de-focusing, time varying illumination, etc.)

- X_{ij} is separable into antenna based quantities $X(p_i).X^*(p_j)$
  Solve for p_i's

Computing and I/O costs

- Increase in computing due to more sophisticated parameterization
  - Deconvolution: Fast evaluation of $B * \sum_k A_k P(Scale_k, Pos_k)$
  - Calibration: Fast evaluation of $E_{ij} * V^M$
- Cost of computing residual visibilities is dominated by I/O costs for large datasets (~200GB for EVLA)
  - Deconvolution: Approx. 20 access of the entire dataset
  - Calibration: Each trial step in the search accesses the entire dataset
- Solutions: Analytical approximations, caching, Parallel computing and I/O,...