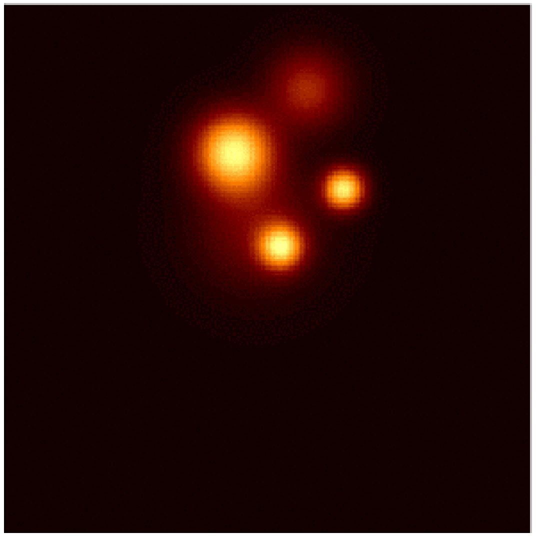
Challenges in advanced imaging with radio interferometric telescopes



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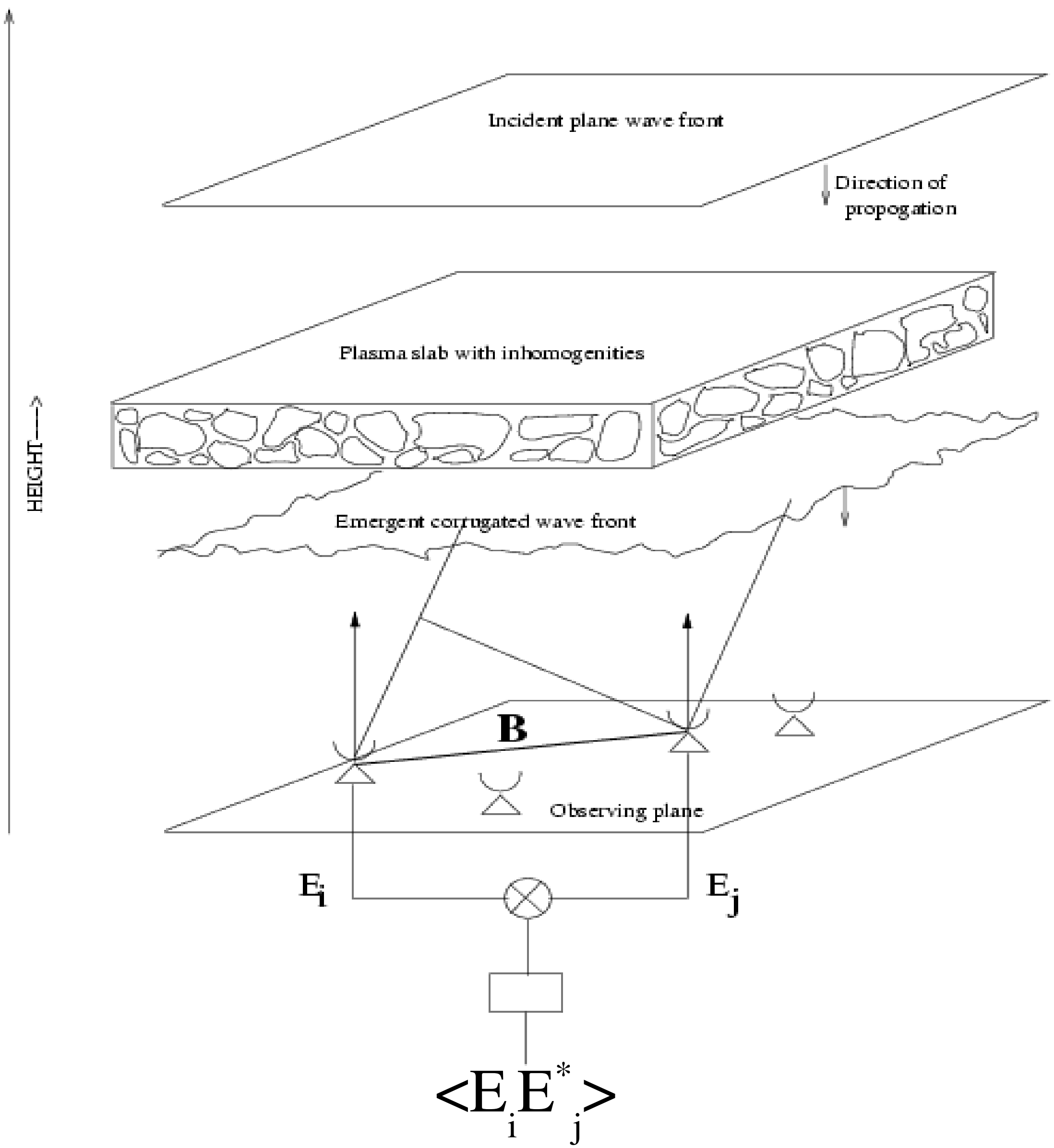
Basic Interferometry



<u>Terminology:</u>

- Visibility: $V_{ij}=<E_iE^*_j>$, E_i is the electric field
- Baseline: Length of the projected separation between the antennas (**B**).
  - Only relative separation matters for incoherent emission
  - Co-ordinates: (u_{ij},v_{ij},w_{ij})

    $u_{ij}=u_i-u_j$
- An *N*-element array instantaneously measures $N(N-1)/2$ baselines (complex values)

Basic Interferometry

- In terms of the sky brightness distribution ($I^o(l,m)$)

$$V(u_{ij}, v_{ij}, w_{ij}) = \int \int I^o(l,m) S(u,v,w) e^{-2\pi\iota(u_{ij}l + v_{ij}m + w_{ij}(n-1))} \frac{dl\,dm}{n} + N_{ij}$$

- S : uv-sampling function, *(l,m):* direction in the sky

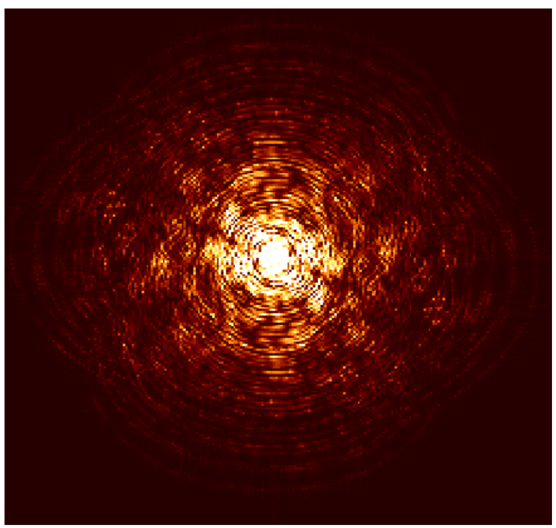
- $n=\sqrt{(1-l^2-m^2)}$ For $l^2+m^2<<1$ or *small* w_{ij}, sky is the 2D Fourier transform of the Visibility function (van Cittert-Zernike Theorem)

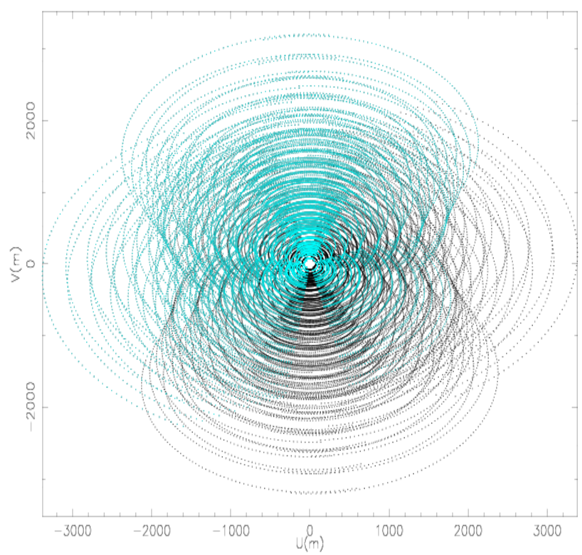
$$I^o = FT[V^o] \text{ and } V^{Obs} = S\,V^o$$

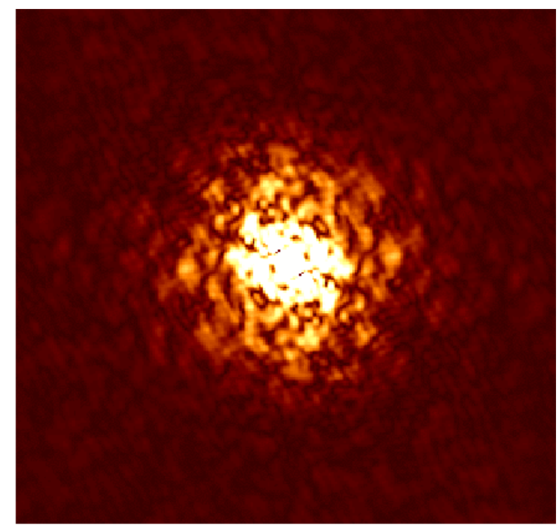
$$I^d = PSF * I^o \qquad PSF = FT[S]: \text{ The Dirty Beam}$$

Sampling in the Fourier plane

- N-element array instantaneously measures $O(N^2)$ Fourier components.
- As earth rotates the projected baseline changes ==> each baseline measures a new Fourier component

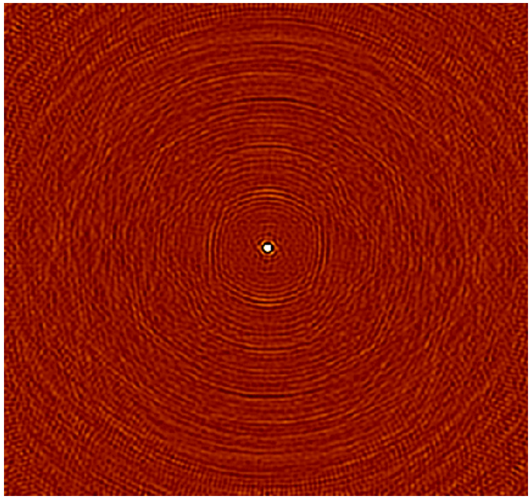




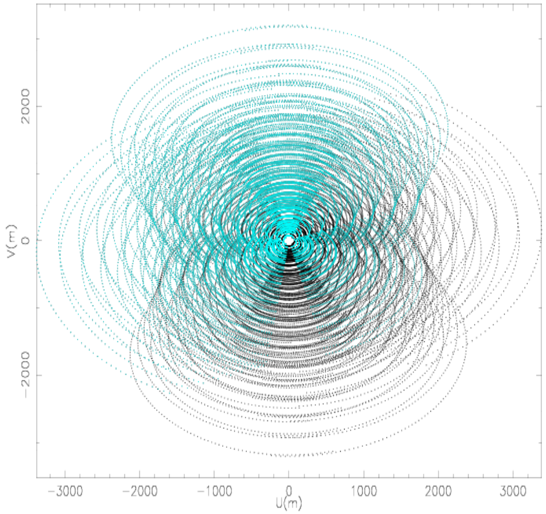


$$V^{Obs} \quad = \quad S \quad . \quad V^{o}$$

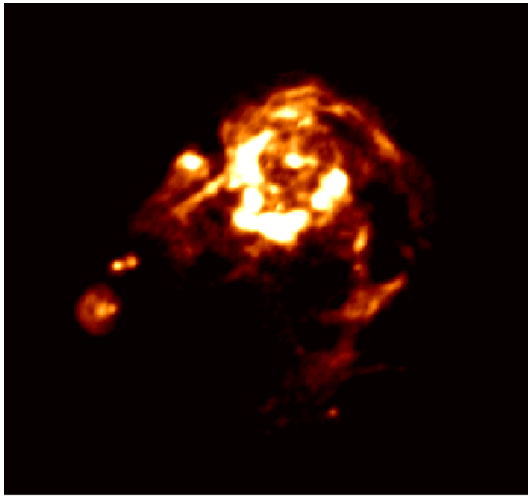
Image plane



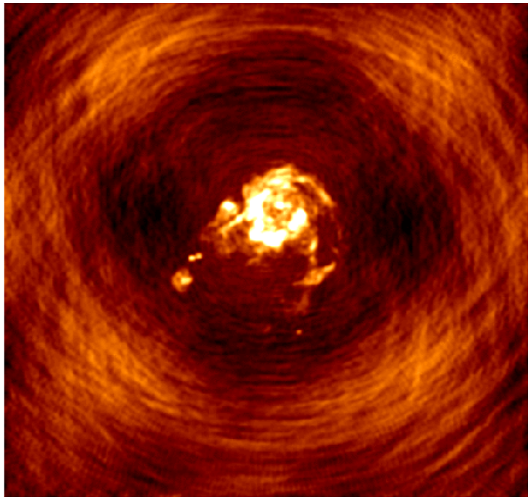
$$PSF = FT[S]$$



Sampling function



$$I^o = FT[V^o]$$



$$I^d = FT[V^{Obs}] = PSF * I^o$$

Imaging

- The Measurement Eq.: $V = AI^o + AN$ where A is the measurement matrix, V, I and N are the Visibility, Image and noise vectors.
- $A=FS$ where F is the Fourier Transform operator and S is a diagonal matrix of weights (w_{ij}).
  - Due to finite Fourier plane sampling, A is singular and in general rectangular
- Image reconstruction: Solve for I^o

$$A^T V = BI^o + BN \quad where \quad B = A^T A$$

- B: The Toeplitz Beam Matrix
- $A^TA=I^d$: The Dirty Image vector

Data corruption

- The full Measurement Eq.:

$$V_{ij}^{Obs}(\nu,t)=G_{ij}(\nu,t)\left[\int\int X_{ij}(\nu,t)\,I^{M}(l,m)\,e^{2\pi\iota(lu_{ij}+mv_{ij})}\,dl\,dm\right]$$

Data — Corruptions — Sky

- G_{ij} are the direction independent corruptions (e.g., multiplicative complex gains, etc.)
- X_{ij} are the direction dependent corruptions (e.g., ionospheric/atmospheric effects, Primary beam effects, etc.)
- Often G_{ij}, X_{ij} are separable into antenna based quantities as $G_{ij}=G_i G_j^*$

$$V_{ij}^{Obs}=G_{ij}.\left[E_{ij}*V_{ij}^{o}\right]$$

$$\text{where } E_{ij}=E_i*E_j^* \text{ and } E_i=FT\left[X_i\right]$$

Data analysis operations

$$V_{ij}^{Obs} = J_{ij}\left(\boldsymbol{A}\boldsymbol{I}^{M}\right)_{ij}$$

- Calibration: Keeping I^M fixed, determine J_i's.
- Calibrate the data:
  - For direction independent corrections: $V_{ij}^{Corrected} = J_{ij}^{-1} V_{ij}^{Obs}$
  - Incorporate direction dependent corrections in imaging

- Imaging: Keeping J_{ij} fixed
  - Estimate I^M such that $V^{Corrected} - \boldsymbol{A}\boldsymbol{I}^{M}$ is noise like

Calibration

- For full polarization treatment: $V_{ij}^{Obs} = J_{ij} . V_{ij}^{o}$

$V = [V_{RR}, V_{RL}, V_{LR}, V_{LL}]$

$J_{ij} = J_i \otimes J_j$

$$J_i = \begin{pmatrix} g_i^R & d_i^L \\ -d_i^R & g_i^L \end{pmatrix}$$

- Given I^M, solve for $J_i s$ *such that* $V_{ij}^{Obs} - J_{ij} . V_{ij}^{M}$ is noise like
  - Image using $V_{ij}^{Corrected} = J_{ij}^{-1} V_{ij}^{Obs}$

- For single polarization: min: $\{ \frac{V_{ij}^{Obs}}{V_{ij}^{M}} - g_i g_j^* \}$

Imaging

- Use of FFT requires re-sampling the Vis. (V(u,v) is not measured on a regular grid)
- Convolutional gridding: $V^G = (GCF * V^{corrected})(k\,\Delta u)$
- Make the Dirty Image as: $I^d = FFT[V^G]$
- Deconvolve the PSF to estimate I^o
- General structure of deconvolution algorithms:
  - *Major cycle:* Compute the model and the residual vis as $\boldsymbol{AI}^M$ and $V - \boldsymbol{AI}^M$
  - *Minor cycle:* Update the model image

Deconvolution: 2D geometry

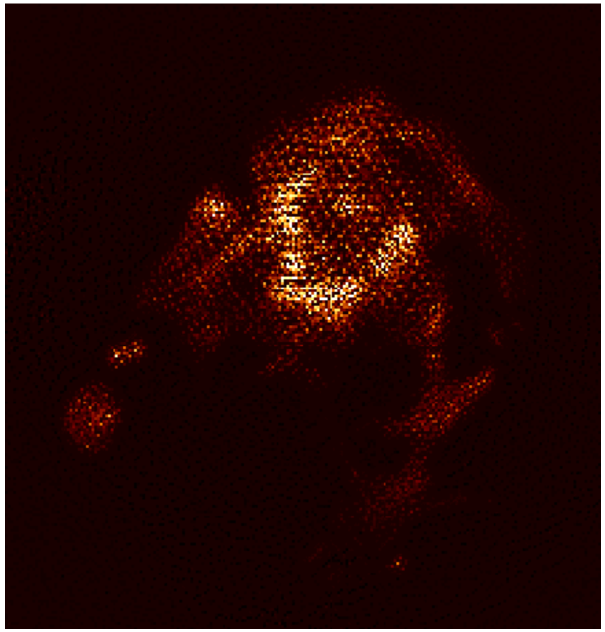
- General formulation
- $\chi^2 = [\boldsymbol{V} - \boldsymbol{A}\boldsymbol{I}^M]^T \boldsymbol{W} [\boldsymbol{V} - \boldsymbol{A}\boldsymbol{I}^M]$ $\quad \boldsymbol{I}^M = \sum_k a_k \boldsymbol{P}(\boldsymbol{p_k})$

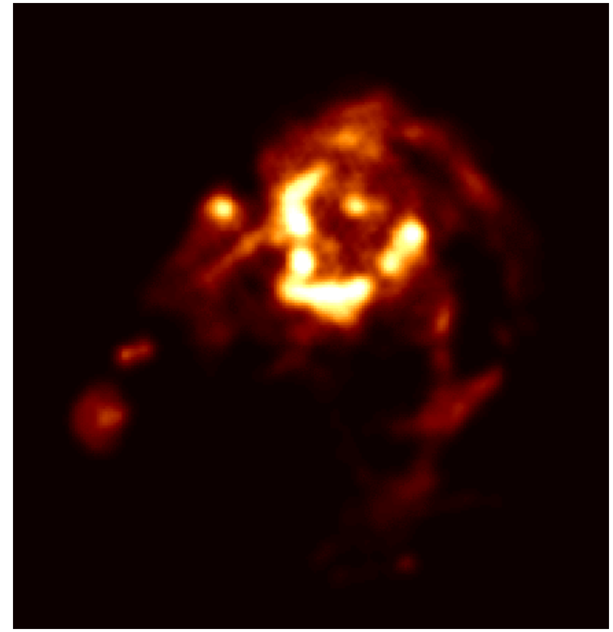
$$\nabla \chi^2 = -\boldsymbol{I}^R \nabla P$$

- The Clean algorithm: $I^M = \sum_k a_k \delta(x - x_k)$
  - Image representation in the pixel basis. $\nabla \chi^2 = -\boldsymbol{I}^R$
  - Steepest descent
    - ➔ Minor cycle: Step size: s=max$\{\nabla \chi^2\}$
    - ➔ $\boldsymbol{I}_i^M = \boldsymbol{I}_{i-1}^M + \alpha s$
    - ➔ Major cycle: Update using FFT: $\boldsymbol{V}_i^R = \boldsymbol{V} - \boldsymbol{A}\boldsymbol{I}_i^M$

Example

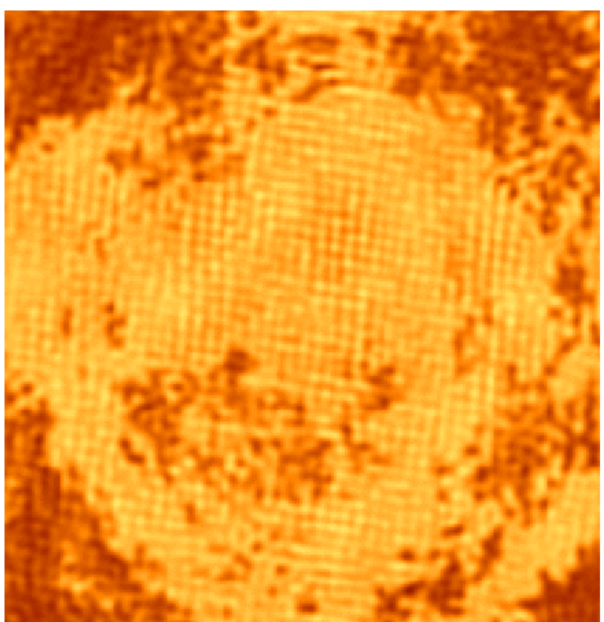
I^M

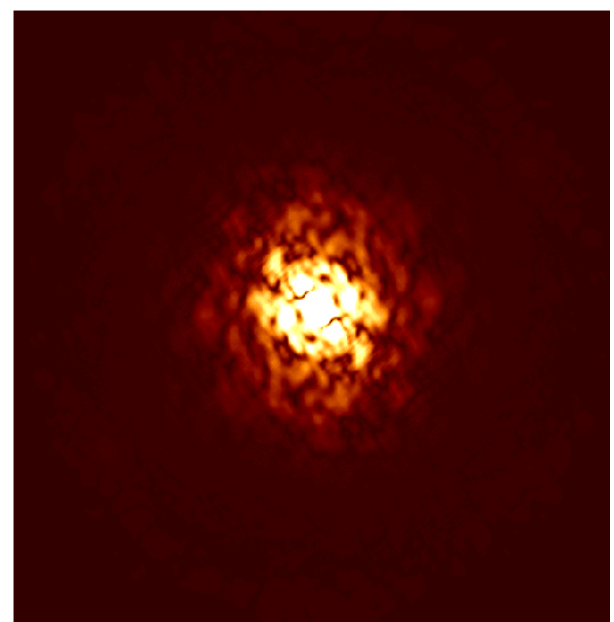




I^M

$A^T(V-AI^M)$





$V-AI^M$

Some observations

- Expensive function evaluation is simplified: Convolution becomes a shift-and-add operation.
- Clean (and its variants) ignore inherent pixel level correlation in I^o (each pixel is a DOF) – errors are correlated with extended emission.
    - Search space is assumed to be orthogonal.
- Too many DOFs for extended emission (no. of pixels).
- Search space constrained by user defined boxes.

Image plane corrections

$$V_{ij}^{Obs}(u,v,w)=\int\int X_{ij}(l,m,w)\, I^M(l,m)\, e^{2\pi\iota(lu_{ij}+mv_{ij})}\, dl\, dm$$

$$V_{ij}^{Obs}(u,v,w)=E_{ij}(u,v,w)*V^{Obs}(u,v)$$

$\boldsymbol{A}$ is more complicated (not a FT operator)

General approach:

- Major cycle involves: $\boldsymbol{V}-\boldsymbol{A}\boldsymbol{I}^{\boldsymbol{M}}$ and $\boldsymbol{A}^{\boldsymbol{T}}(\boldsymbol{V}-\boldsymbol{A}\boldsymbol{I}^{\boldsymbol{M}})$
  - Use E_{ij} as the GCF to predict the model data ($\boldsymbol{A}\boldsymbol{I}^{\boldsymbol{M}}$)
  - Compute $\boldsymbol{V}^{\boldsymbol{R}}$ at high accuracy.
  - Use an approximation for $\boldsymbol{A}^{\boldsymbol{T}}$: Use E^T_{ij} as the GCF

Primary Beam Effects

- E_{ij} as function of direction is measured a priori

$$V_{ij}^{Obs} = G_{ij}.[E_{ij} * V^M] \quad where \quad E_{ij}(l_i, l_j, u_{ij}; p_i, p_j)$$

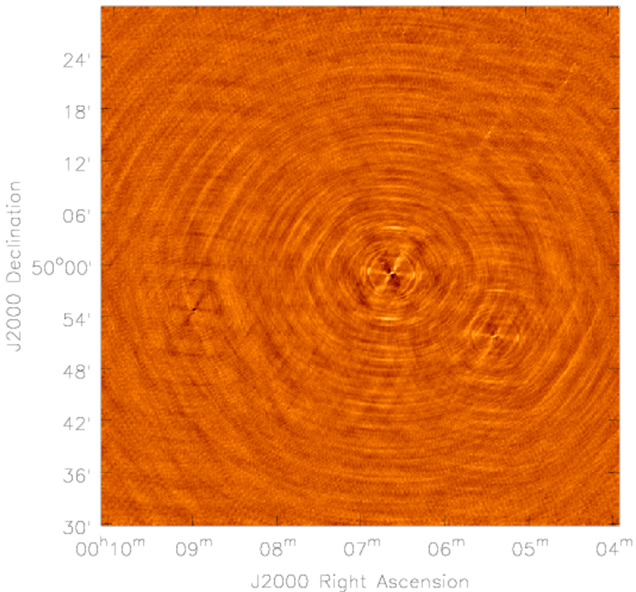
- Primary beam effects

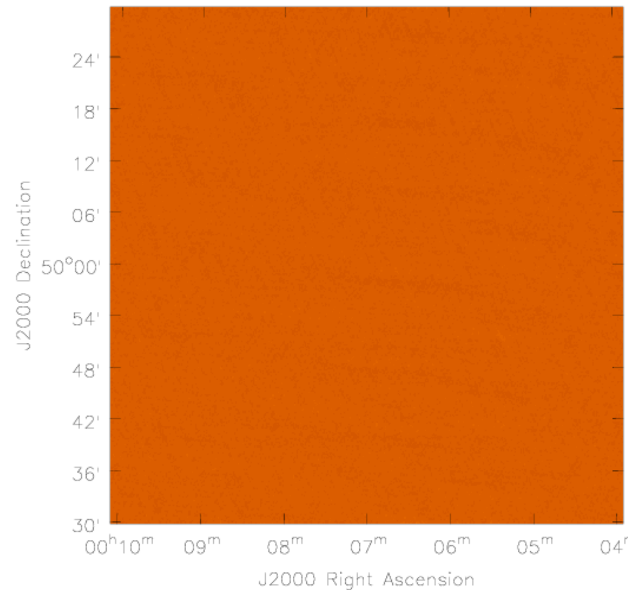
$$E_{ij} = E_i^o * E_j^o \quad where \quad E_i^o = \boldsymbol{FT}[Measured\ PB_i]$$

  - Polarized primary beam: Beam squint
    - For full beam polarimetry (EVLA)
  - Pointing offset calibration
    - For mosaicing (EVLA, ALMA)
    - GCF different for each visibility!

Example

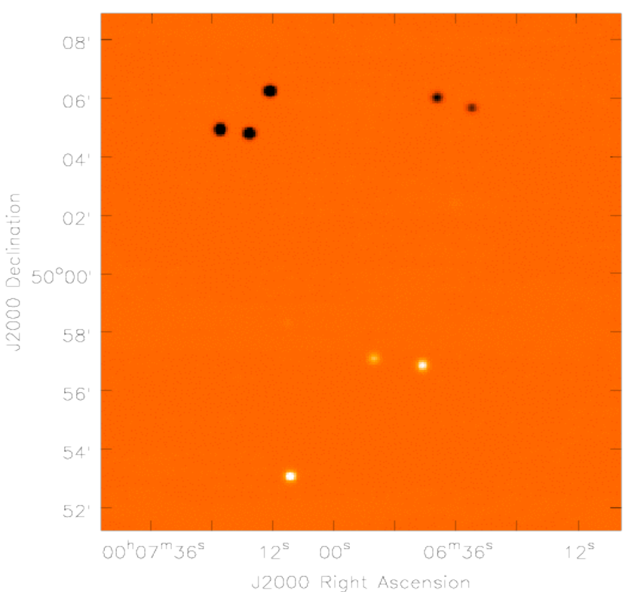
Residual image before and after pointing correction.

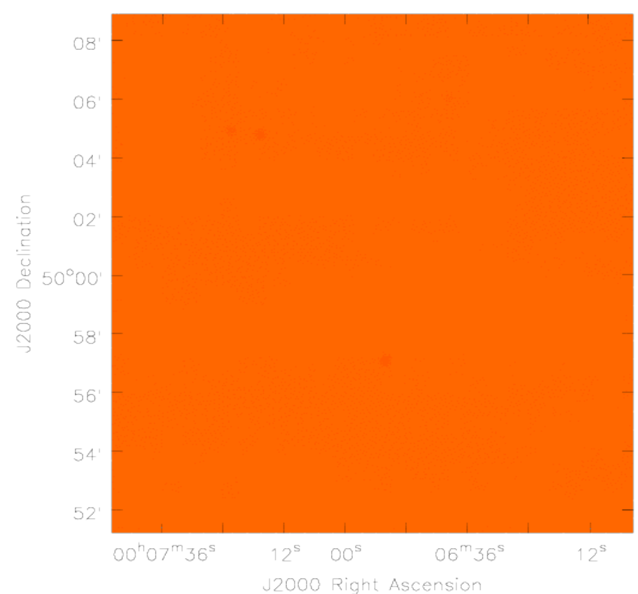




$I^V = PB(I^{RR} - I^{LL})$

Peak ~4%





$I^V = PB^R I^{RR} - PB^L I^{LL}$

Peak ~0.2%

Scale-sensitive deconvolution

- Pixel-to-pixel noise is correlated at the scale of the PSF support $\boldsymbol{I^d = BI^o + BI^N}$ (Pixon methods not applicable – they are patented anyway!).
- The scale of the emission fundamentally separates the signal (I^o) from noise (I^N).

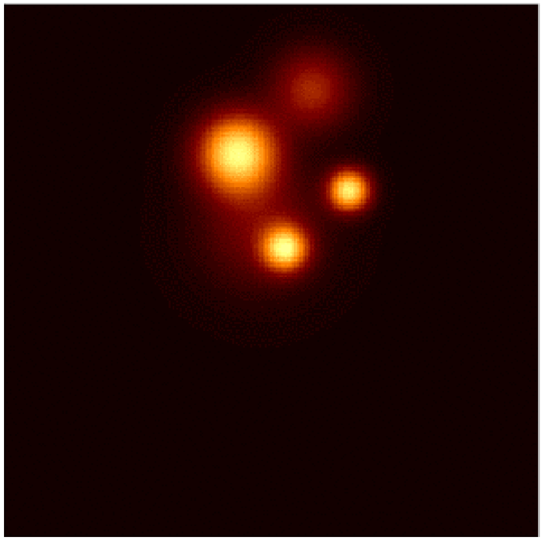
- Use a pixel-model with finite support
  - Convolution is no more a shift-n-add operation. Function evaluation becomes very expensive.

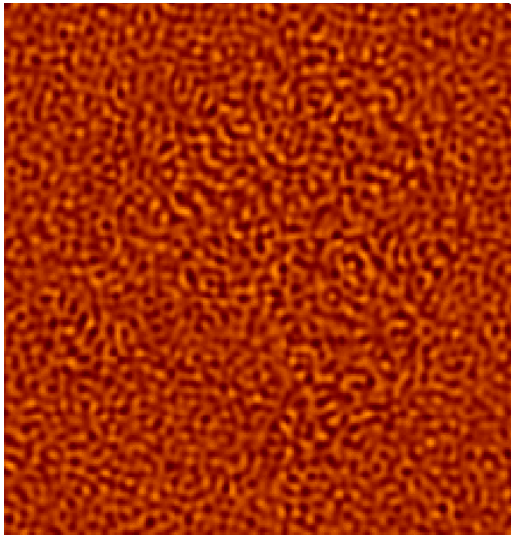
$$V^R = FT[I^d - PSF * I^M]$$

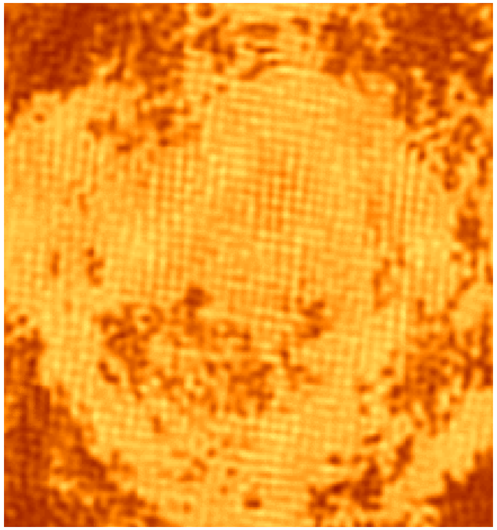
Scale sensitive deconvolution

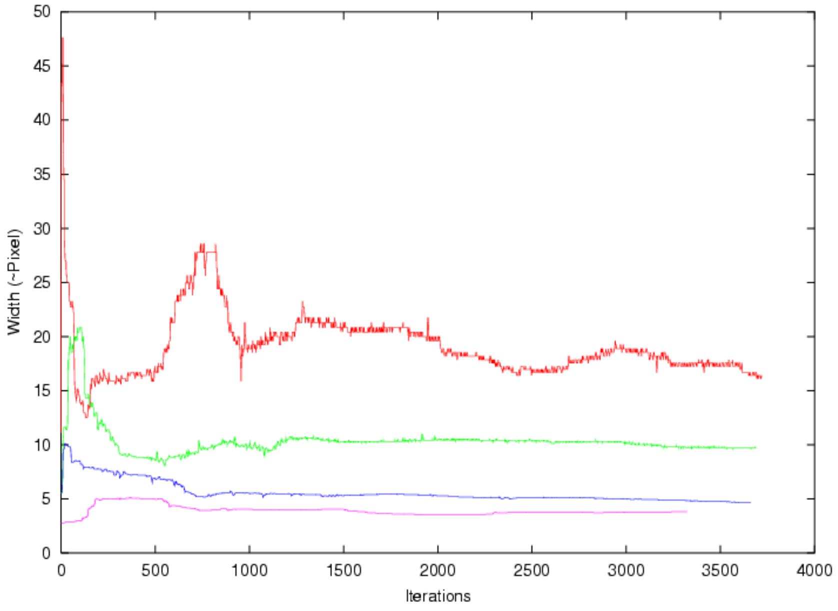
- Search space is non-orthogonal
- The total DOFs decrease, but computing cost is a strong function of the number of parameters.
- Asp-Clean (Adaptive Scale Pixel model) [A&A, 2004]
  - ➔ Use $I^M = \sum_k a_k P(p_k)$ e.g., where $P(p_k) = Gaussian$
  - ➔ Find the local scale by fitting
- For emission covering 100,000 pixels, required 1000 $P_k s$
- Computing cost prohibitive for more ~100 active parameters.
  - ➔ Sub-spaces

Example








Width (~Pixel)
Iterations
0
5
10
15
20
25
30
35
40
45
50
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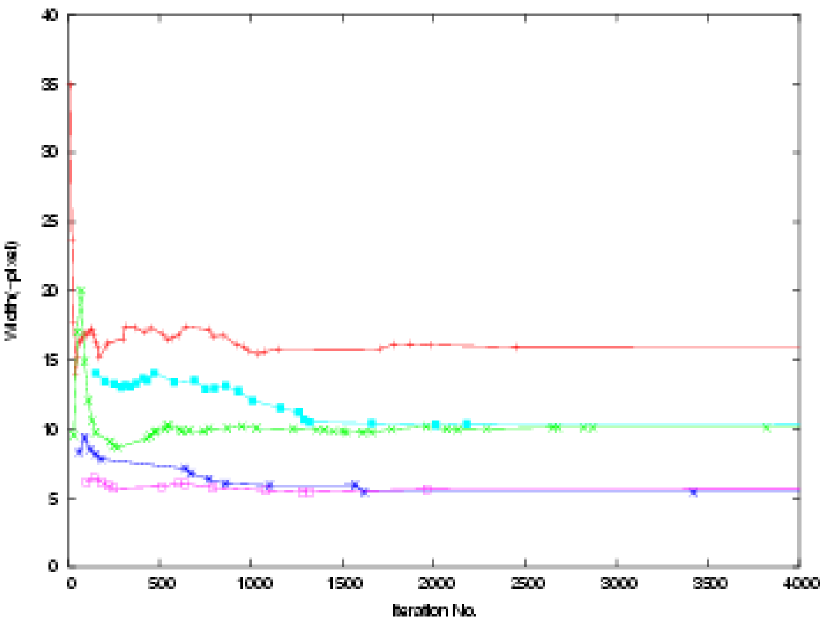
500
1000
1500
2000
2500
3000
3500
4000


Width (~pixel)
Iteration No.
0
5
10
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25
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40
0
500
1000
1500
2000
2500
3000
3500
4000

Challenges

- Fast and accurate forward and backward transform in the presence of image plane effects

$$min: V_{ij}^{Obs} - E_{ij} * [AI^M]_{ij}$$

- Solve for direction dependent effects:
  - ➔ Primary beam effects (pointing, squint, variable beam shape,...) [EVLA, ALMA]
  - ➔ Wide-band imaging: $E_{ij} = E_{ij}(\nu)$ [EVLA]
  - ➔ Need more sophisticated parametrization of E_{ij} [EVLA,ALMA,LWA,SKA...] $E_{ij} = E_{ij}(\nu, t)$
- Ionospheric/atmospheric calibration [LWA, SKA]
  - ➔ Simple functional representation may be difficult

Challenges

- Component based image model
  - ➔ Gridding errors limit the highest dynamic range achievable.
  - ➔ FFT based inversion of the ME may not be usable.

- Image sizes and visibility data sets will be large
  - ➔ 100s of GB – 1TB of data, 15K X 15K images
  - ➔ 10-100 transforms between image and data
  - ➔ Parallelization of computing as well as I/O

Challenges

- Scale sensitive deconvolution
  - Pixel model parameterized by the amplitude, (local) scale, position, frequency, and perhaps polarization.
  - Total no. of parameters: ~10K
  - Fast algorithm for parameter estimation
  - Handle the coupling between pixels (non-orthogonal search space)
  - Adaptively control on the dimensionality of the search space.

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