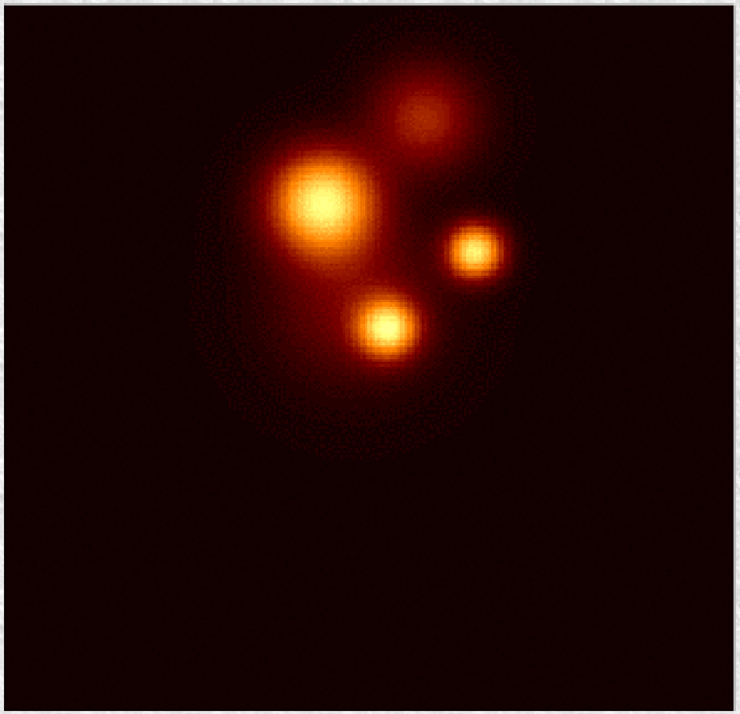
Scale Sensitive Deconvolution



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Deconvolution: Problem definition

- Interferometers measure the data in Fourier space
- Final product needed is the image

Measurement Equation: $V = F\, I^D + noise$

F: Fourier Transform operator $I^D = B * I^M$

The goal is to estimate the Model Image (I^M), given the measurement (V) and an estimate of the PSF (B).

- Inverse of the Beam Matrix does not exist. Direct methods for deconvolution are not practical
- Represent $I^M = \sum_k P(p_k)$: *P* is the Pixel Model and p_k are the Degrees of Freedom (DOFs)

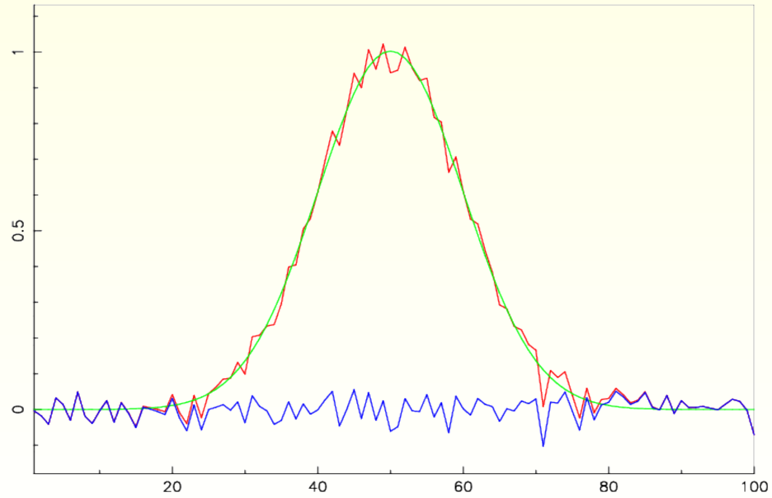
Deconvolution: As an optimization problem

- χ^2 is an optimal estimator for a gaussian random process (the noise). $\chi^2 = \sum [V^{Obs} - F\, I^M]^T W [V^{Obs} - F\, I^M]$

Minimize: χ^2 w.r.t. I^M

- Step size $\propto \frac{\partial \chi^2}{\partial p_k} = -2 \sum_k [I^D - B * I^M][\frac{\partial P}{\partial p_k}]$ Residual image provides the update direction

- χ^2 is an optimal estimator <u>provided the model for the data fundamentally separates signal from noise</u>



Fitting individual pixels will result into over-fitting

Scale-less deconvolution:

- Scale (correlation length) fundamentally separates the signal from the noise.
- $I^M = \sum_k A_k \delta(x - x_k)$

  The image is decomposed into delta functions at discrete pixel locations (quantized). A_k is the only parameter.

- Clean Iterations: $I_i^M = I_{i-1}^M + \alpha\, Max(I_i^R)$
- Each pixel is an independent DOF
  - Dimensionality of the search space: No. of pixels in the Box
  - Minimize along the axis of maximum derivative

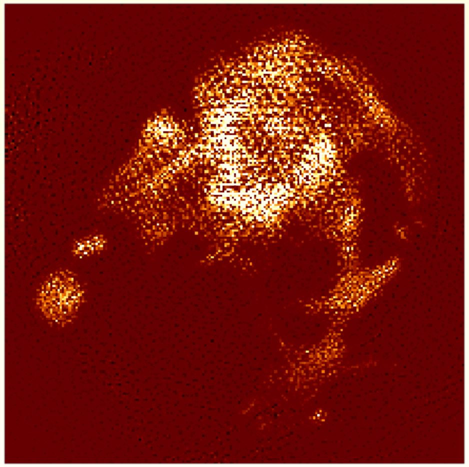
Scale-less deconvolution:

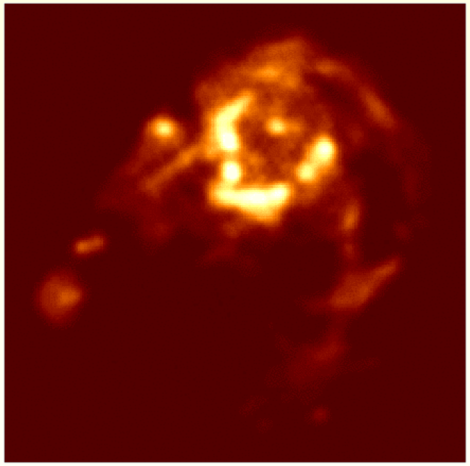
- Regularization:

  - Box (limit the search space)

  - Maximum number of components (limit over-fitting)

- MEM: Constrained minimization: $\chi^2 + \lambda\, Entropy$
- Example: Clean (50K components)

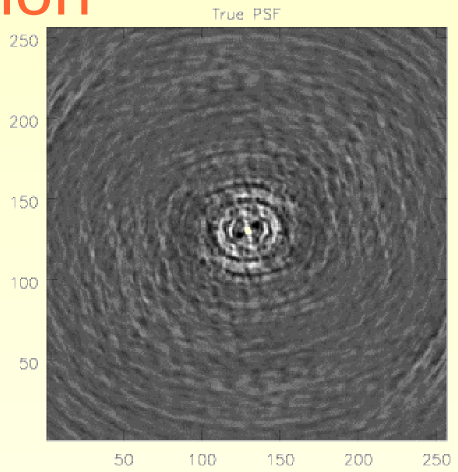




- Diagonal approximation: Hessian: $H \sim H_{ii}$

Adaptive Scale Pixel (ASP) Model:

- Interferometric PSF has widespread side lobes
  - Diagonal or band-diagonal approximation of the Hessian is not sufficient
- Decompose the image into a scale sensitive basis (Aspen).

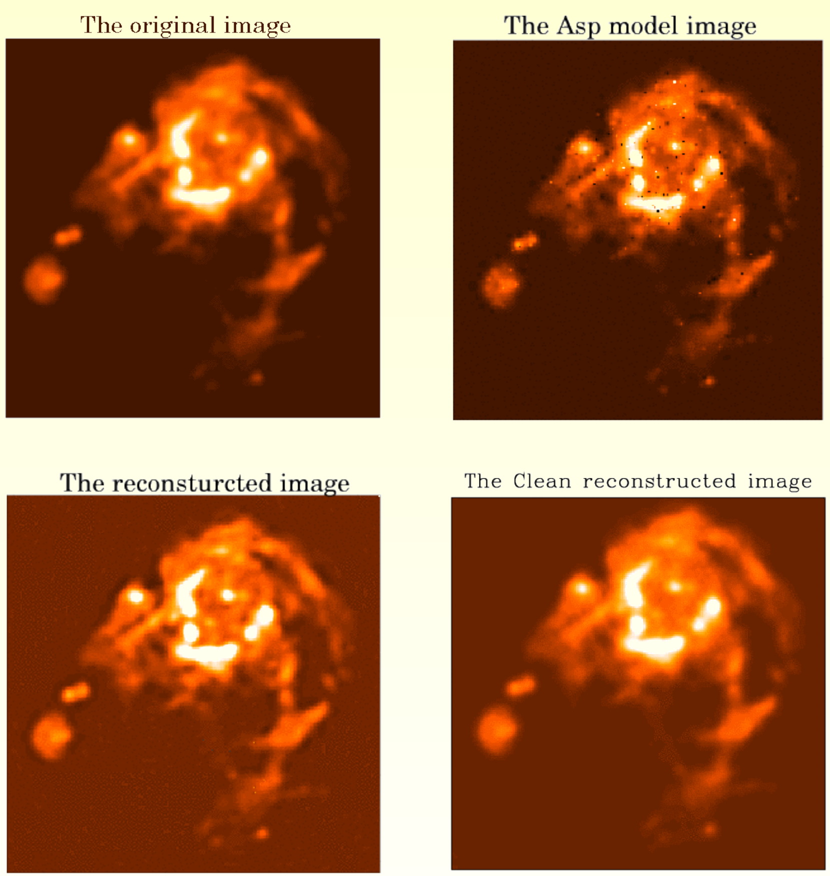


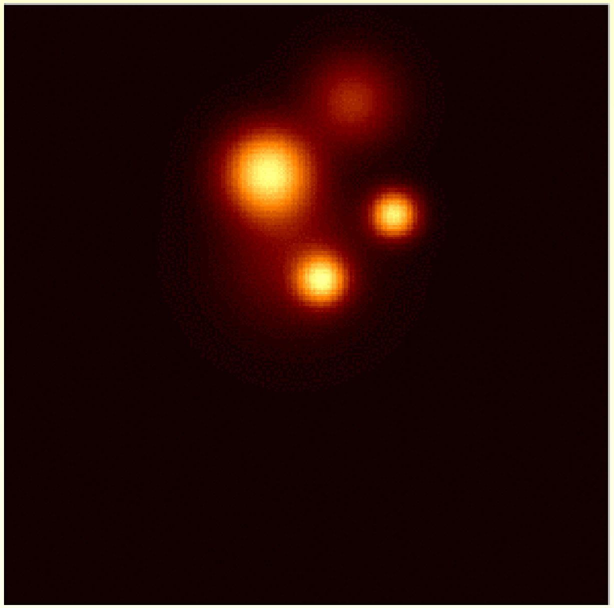
- Minor cycle:
  - Find a set of 'active' Aspen {A}
  - Solve for the best-fit set {A + New Asp at Max:I^R}=$\{A_i\}$
  - Compute: $I^R = I^D - B*\{A_i\}$
- Major cycle:

  Compute: $V_i^R = V^O - FI_i^M$ and $I_i^R = FV_i^R$

Asp deconvolution: Example

- Minimizes the number of DOFs used
- Iterations are not independent



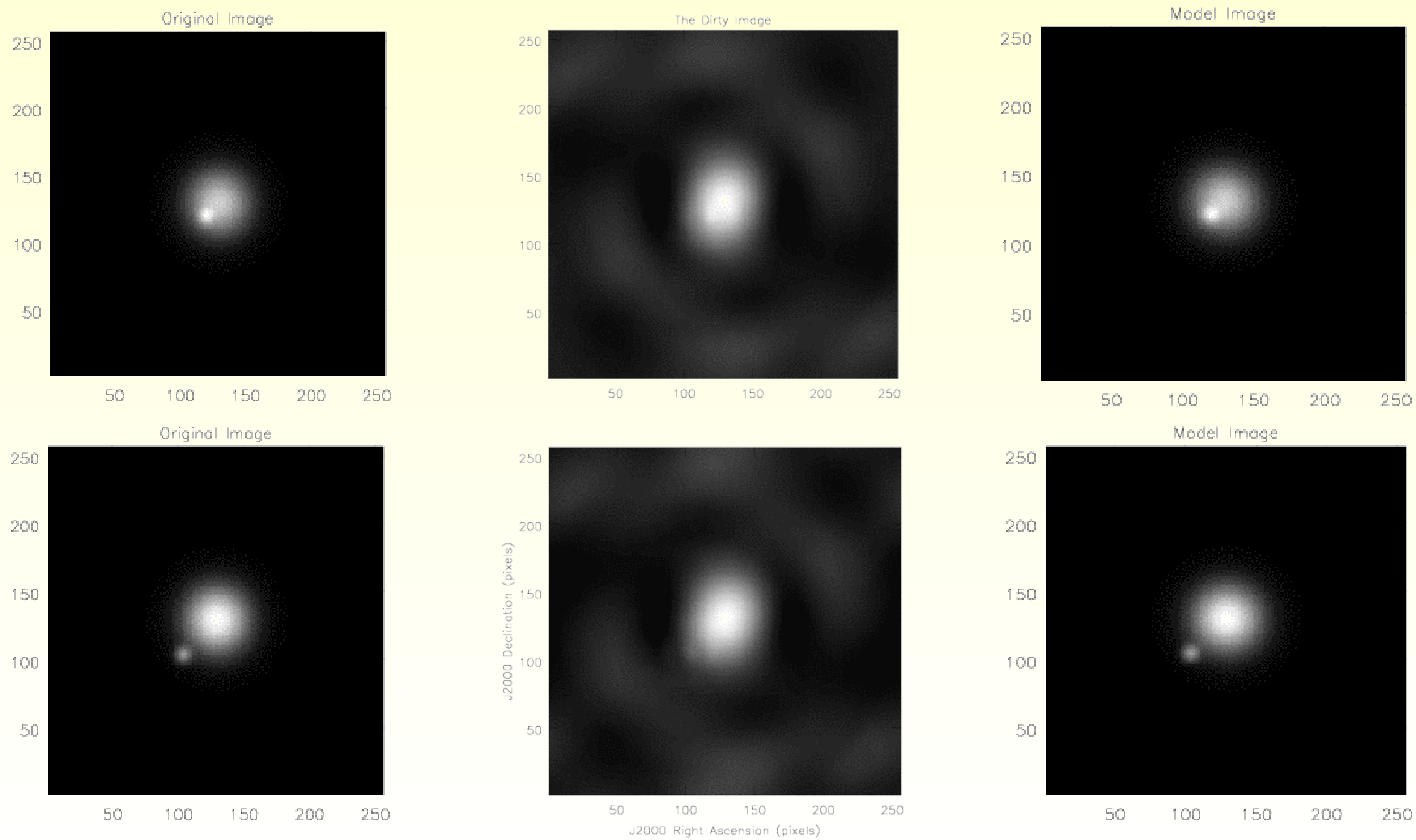


500 Asp reconstruction

✗ Slower: Step size computation needs convolution

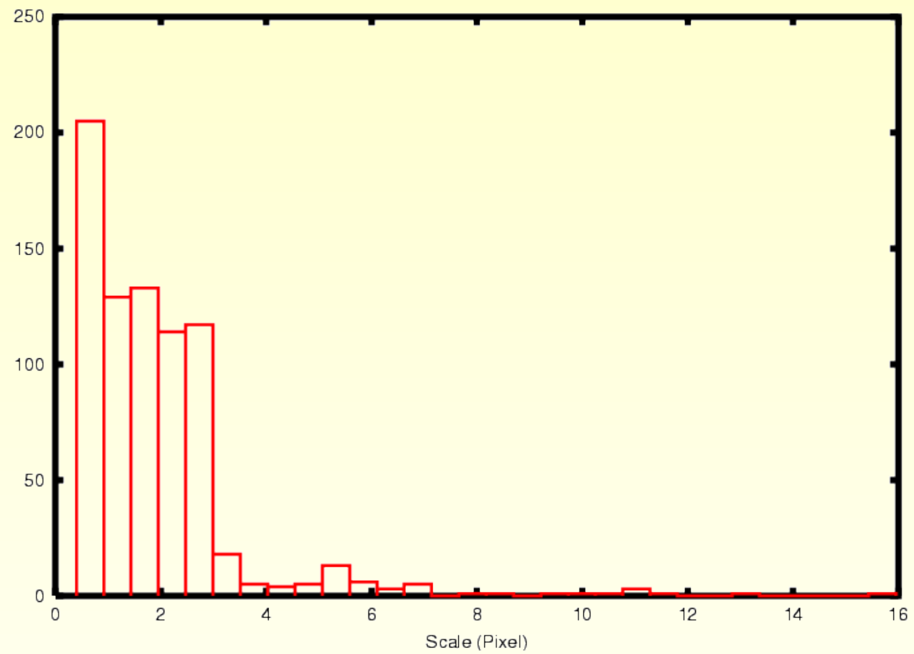
Asp deconvolution: Features

- Sensitive to the local scale and SNR
  - Detects overlapping and well separated scales equally well
  - Uses a continuous range of scales and positions



Asp deconvolution: ...Features

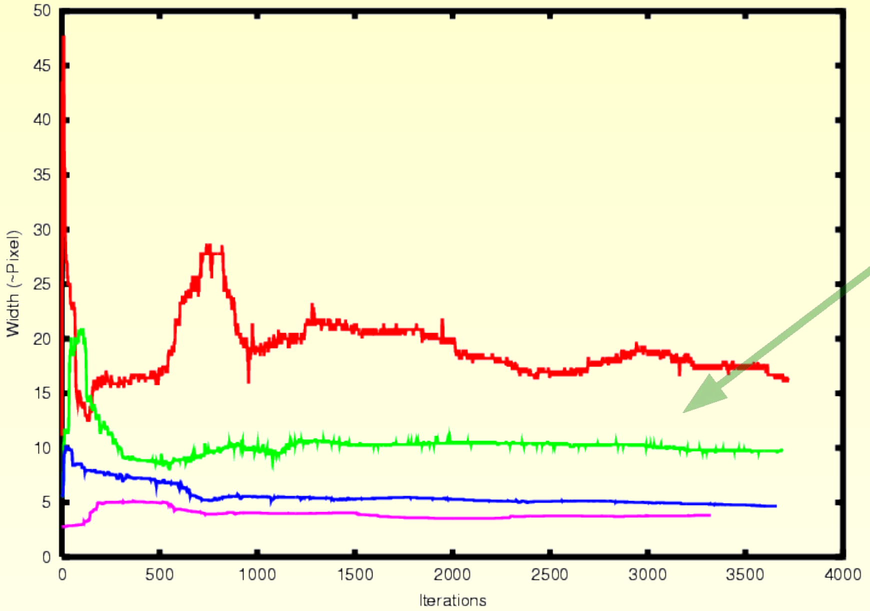
- Uses continuous range of scales and positions



- Uses least DOFs: An order of magnitude less compared to Clean/MSClean (50000/8000 vs. 600)
- Not very sensitive to boxing

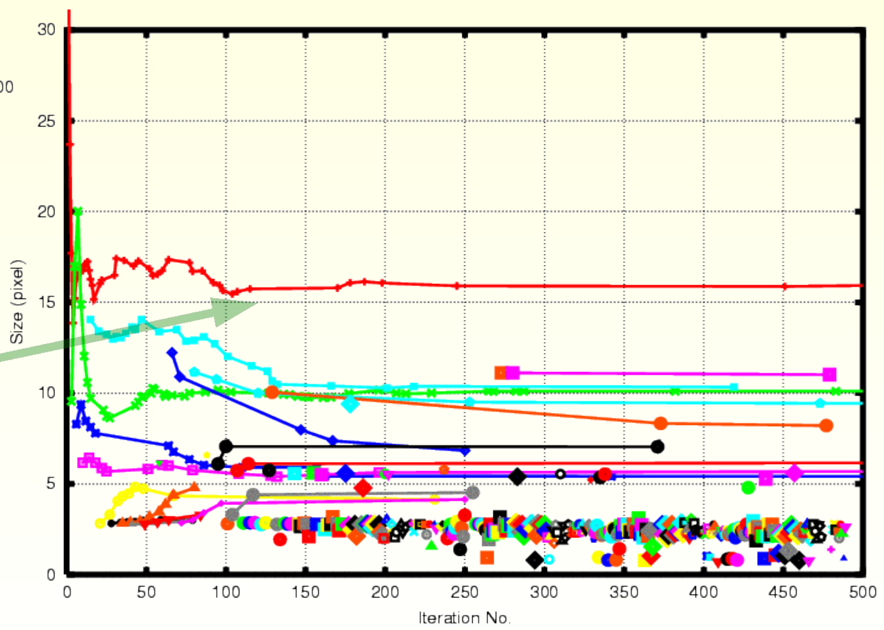
Optimization: Ageing of Aspen

- Not all Aspen remain significant/active



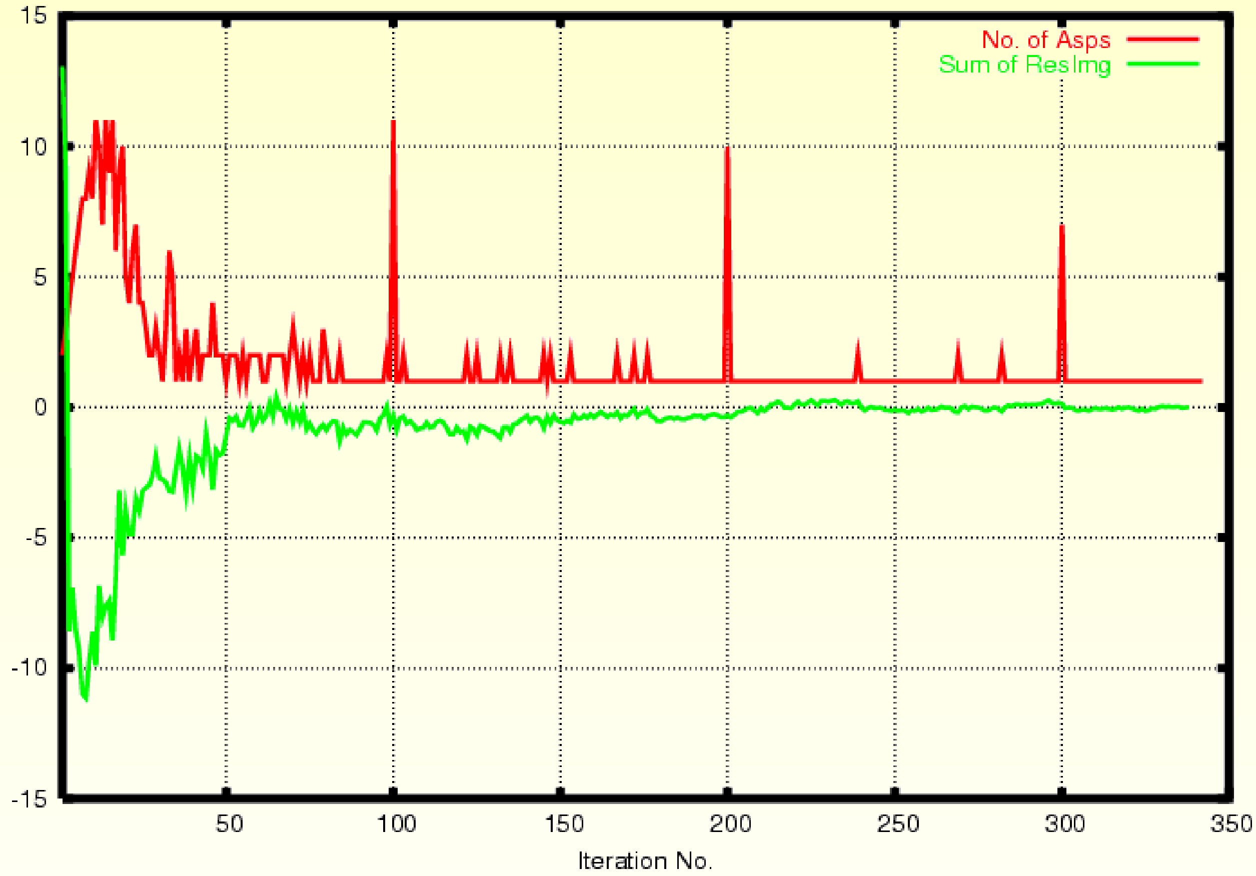
The shape/amplitude does not continously evolve

Drop Aspen which are not evolving



...Optimization: Dimensionality reduction

- Adaptively determine the set of 'active' Aspen



- Merger of Aspen

...Optimization...

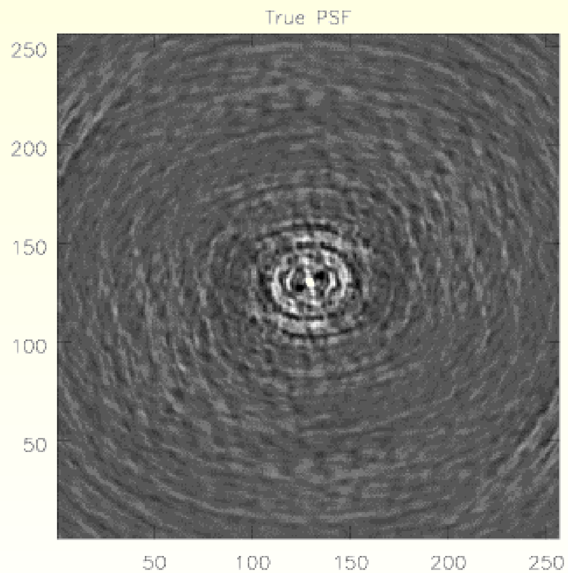
- Use approx. PSF to determine the set of active Asp

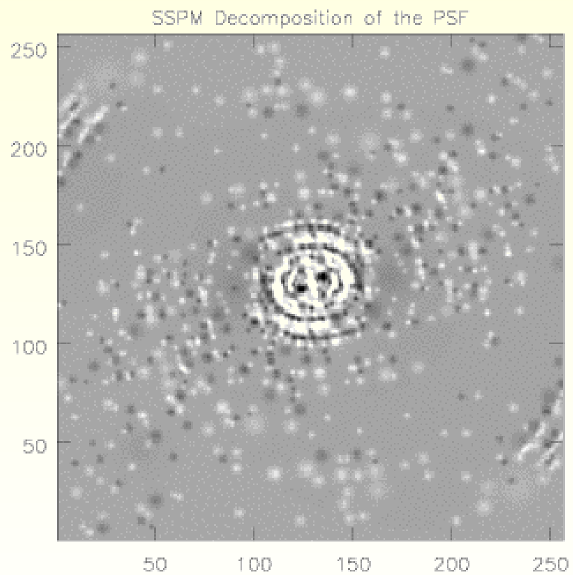
  Approximate PSF $= \sum_b P(p_b)$

- Product and convolution of Aspen is another Asp.

  Approximate H_{ij} can be analytically computed

$$H_{ij} \approx 2 \sum [\sum_b P(p_b)] * [f(p_j) P(p_j)][f(p_k) P(p_k)]$$

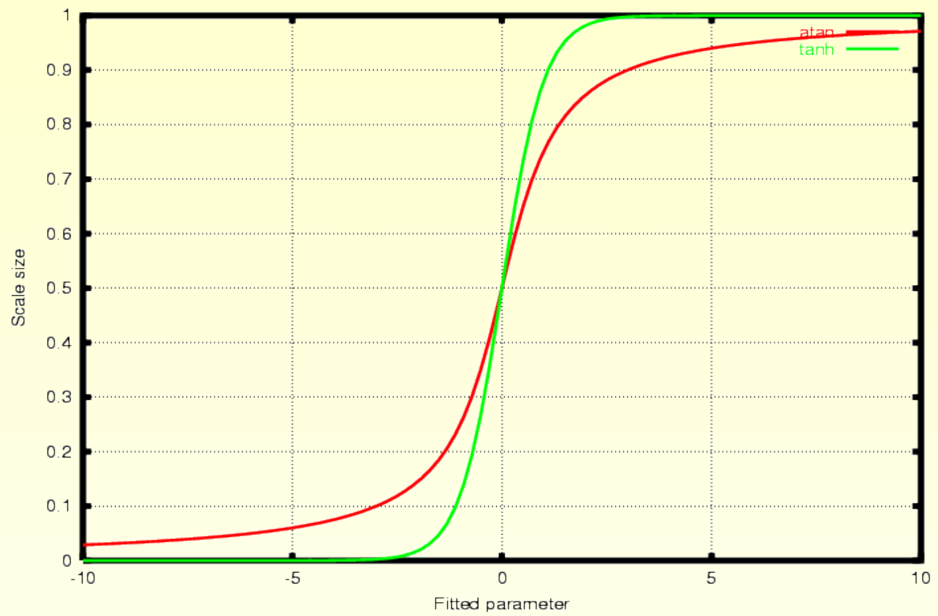




Asp decomp-osition of the PSF

Work in progress:

- Limits on inner and outer scales



- Non-symmetric pixel model with tighter support
- Full Hessian to determine the set of active Aspen
- Include other constraints (e.g. flux at each pixel>0)
- Integrate the code with AIPS++