Imaging with Aperture Synthesis Radio Telescopes in the presence of Direction Dependent effects

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Abstract: We describe in this paper results from our work so far on developing algorithms to solve for, and fast application of direction dependent effects while imaging with interferometric radio telescopes.

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1. Introduction

An aperture synthesis telescope measures the coherence function of a spatially incoherent source, which, for a small field of view, is related to the source structure by a Fourier transform relation [1]. A pair of antennas instantaneously measures a spatial frequency proportional to the projected separation between the antennas in the sky plane. The measurement equation for two such ideal antennas i and j, with primary beam pattern given by $P_i(l, m)$ is given by

$$V(u_{ij}, v_{ij}) = \iint P_i(l,m)P_j(l,m)\ I(l,m)e^{2\pi\iota(u_{ij}l+v_{ij}m)}\frac{dldm}{\sqrt{1-l^2-m^2}} \tag{1}$$

where (l, m) are the sky co-ordinates, (u, v) are the conjugate variables in the Fourier space and $I(l, m)$ is the image. V is the measured coherence, also referred to as the Visibility. In general, the entire array is not located on a plane in the uv-space. Consequently the visibility is measured in a 3D space, and is related to the sky as

$$V(u_{ij}, v_{ij}, w_{ij}) = \iint P_i(l,m)P_j(l,m)G(l,m,w_{ij})\ I(l,m)e^{2\pi\iota(u_{ij}l+v_{ij}m)}\frac{dldm}{\sqrt{1-l^2-m^2}} \tag{2}$$

where $G(l, m, w_{ij}) = e^{2\pi\iota w_{ij}\sqrt{1-l^2-m^2}}$.

At high frequencies, where the field of view is narrow, $G \approx 1$. Since the sky is also relatively blank, assuming $P_i = P_j = 1$, a 2D Fourier inversion of Eq.1 is used to estimate the image I convolved with the telescope Point Spread Function (PSF). The PSF is then removed using an iterative deconvolution algorithm. For the next generation radio telescopes now under construction, correcting for the direction dependent terms like the antenna primary beams (P_is) with pointing offsets and the effects of non co-planar array (G) is vital for achieving the ambitious scientific goals. Correcting for the effects of the antenna primary beams involves solving for unknown terms like the antenna pointing offsets and then applying it during imaging and deconvolution. Since deconvolution algorithms as well as solvers for the unknown direction dependent effects are typically iterative and computationally expensive involving several inversion of Eq. 2, developing techniques for the fast application of such terms is crucial.

We are working on developing techniques for the fast application of direction dependent terms during imaging. As a first step towards solving for direction dependent effects, we have used the technique to solve for antenna based pointing offsets. In this paper, we describe an algorithm, inspired by the algorithm to correct for non co-planar effects [2], to solve for antenna based pointing offsets and discuss the results.

2. General approach towards image plane corrections

Ignoring the direction dependent terms, for efficiency reasons, the 2D Fast Fourier Transform (FFT) algorithm is used to transform a model image to the visibility domain and vice versa. The visibility function is not measured on a regular grid by the telescope while the FFT algorithm works on a regular grid. To predict model visibilities using a model image, one therefore requires a re-sampling of the model visibilities using a Gridding Convolution Function (GCF) as

$$V(u_{ij}, v_{ij}) = (GCF(u_{ij}, v_{ij}) \star V^M(\nabla u_{ij}, \nabla v_{ij}))(u_{ij}, v_{ij}) \tag{3}$$

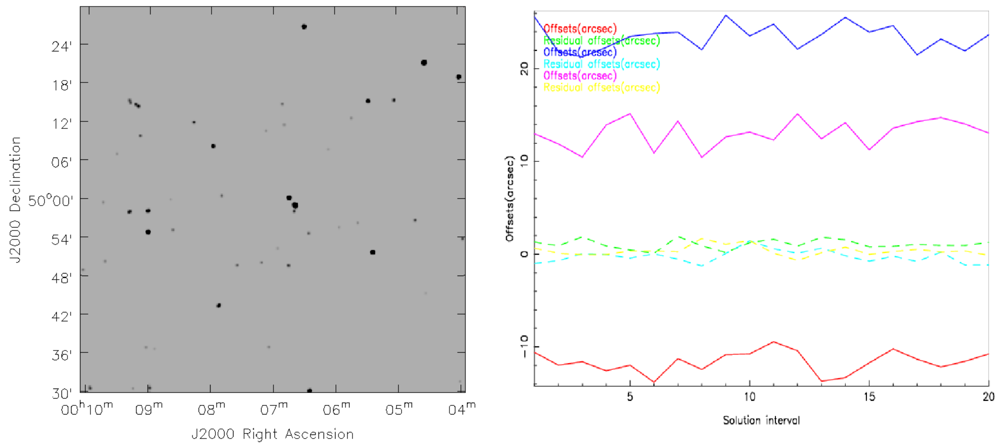

Fig. 1. The image on the left is the model image used in the simulations at 1.4GHz. Plot on the right show the antenna pointing offsets for four antennas (continuous lines) as a function of time. The dashed lines show the residual offsets after solving for the pointing offsets.

where V^M are the model visibilities on a regular grid with a grid size of $(\nabla u_{ij}, \nabla v_{ij})$. Eq. 2 can also be cast as a convolution equation as

$$V(u_{ij}, v_{ij}, w_{ij}) = X(u_{ij}, v_{ij}, w_{ij}) \star V^M(\nabla u_{ij}, \nabla v_{ij})) \tag{4}$$

where X is the Fourier transform of the direction dependent terms. *e. g.*, ignoring the effects of the antenna primary beams, effects of non co-planarity can be incorporated by using X as $W(u_{ij}, v_{ij}, w_{ij}) = \mathcal{FT}(G(l, m, w_{ij}))$ [2]. Similarly the effects of antenna pointing offsets can be incorporated by replacing X by $E_{ij} = E_i \star E_j^*$, where E_i is the complex aperture illumination pattern parameterized by the pointing offsets (l_i, m_i). Note that Eqs. 3 and 4 are similar in form. Since Eq. 3 is evaluated several times for the inversion of the model image during deconvolution as well as calibration process, direction dependent terms can be efficiently incorporated by using the appropriately modified GCF in the re-sampling step.

3. Solving for antenna pointing errors

The measurement equation for a single pointing observation with antenna based pointing offsets is:

$$V_{ij}^{\circ} = E_{ij} \star V_{ij}^M \tag{5}$$

where V_{ij}° and V_{ij}^M is the predicted and the model visibility respectively from antenna i and j. In one dimension, let $P_i(l)$ be the primary beam pattern for antenna i given by $P_i(l) = A_i e^{-(l-l_i)^2 \sigma_i^2}$ where σ_i is the inverse of the width and l_i is the position offset with respect to the true pointing direction. E_{ij} for $\sigma_i = \sigma_j = \sigma$ and $A_i = A_j = 1$ (identical antennas), is given by:

$$E_{ij} = E_{ij}^{\circ} e^{-\frac{(l_i - l_j)^2 \sigma^2}{2}} e^{-\pi \iota u (l_i + l_j)} \tag{6}$$

where u is the conjugate variable in the Fourier space and $E_{ij}^{\circ} = E_i^{\circ} \star E_j^*$, while E_i° is the measured (or ideal) illumination pattern of the antenna (without the pointing offset) . The update direction for an iterative minimization is then given by

$$\frac{\partial \chi^2}{\partial l_i} = -2\Re \sum_j V_{ij}^{R^*} \left[V^M \star \nabla_{l_i} E_{ij} \right]_{ij} \tag{7}$$

where $V_{ij}^R = V_{ij}^{\circ} - E_{ij} \star V_{ij}^M$. V_{ij}^R and $V^M \star \nabla_{l_i} E_{ij}$ can be computed by re-sampling the gridded model visibilities (V_{ij}^M) using E_{ij} and $\nabla_{l_i} E_{ij}$ as the GCF respectively. Similarly the solved pointing offsets can be used during imaging by using them in computing E_{ij}^* and using it as the GCF (with appropriate normalization) for re-sampling the visibilities onto a regular grid.

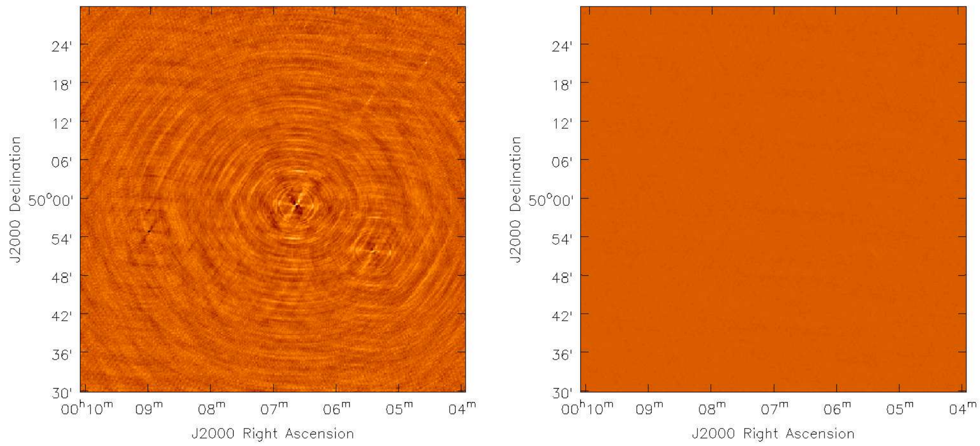

Fig. 2. The image on the left show the residuals before correcting for the antenna pointing offsets, while the image on the right shows the residual after applying the pointing offset corrections. Both images are at the same scale ($\pm 150\mu$Jy). The peak residual is $250\mu Jy$ and $5\mu Jy$ respectively while the RMS is $15\mu Jy/beam$ and $1\mu Jy/beam$ respectively in the two images.

3.1 Simulation and results

The above approach was tested using simulated observations of a typical field at 1.4GHz which is shown in the left panel of Fig. 1. The antenna pointing offsets as a function of time for four typical antennas used in the simulation is shown with continuous lines in the plot on the right. Gaussian random noise of RMS $1\mu Jy/beam$ was added to simulate the telescope thermal noise. The peak residuals without correcting for the pointing offsets is shown in the left panel of Fig. 2. The peak residuals in this image is $\sim 250\mu Jy$ while the RMS noise is $\sim 15\mu Jy/beam$. The pointing offsets were then iteratively solved using the modified GCF to compute the update direction efficiently [3]. The residual image after solving for and applying the solved pointing offsets is shown in the right panel. The RMS noise in this image is $\sim 1\mu Jy/beam$, which meets the EVLA specifications, and the peak residual is $\sim 5\mu Jy$. This represents an improvement of a factor of ~ 15 and ~ 50 for the dynamic range and image fidelity respectively.

4. Conclusions

Solving and correcting for direction dependent effects during imaging is vital to achieve the scientific goals of the next generation radio telescopes, now under construction, like the EVLA (`http://www.aoc.nrao.edu/evla`), and ALMA (`http://www.alma.nrao.edu`). These effects can be efficiently incorporated by including them in the convolution function used for re-sampling. We have used this approach to correct for the effects of non co-planar baselines and for solving and correcting for the antenna pointing offsets [3]. Correcting for the pointing offsets as a function of time is particularly important for mosaicing observations - a major observing mode at very frequencies with ALMA. Further work is in progress to use measured illumination patterns in Eq. 6 to correct for in-beam polarization effects, and other such direction dependent effects, for high dynamic range full beam full polarization imaging.

References

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