Solving for Polarization Leakage in Radio Interferometers Using Unpolarized Source

S. Bhatnagar,[1] R. V. Urvashi [2] and R. Nityananda [3]

National Center for Radio Astrophysics (NCRA), Pune, India 411007
Email: sbhatnag@aoc.nrao.edu

Abstract. This paper presents an algorithm for solving antenna based polarization leakage in a radio interferometer using co-polar observations of unpolarized sources. If ignored, polarization leakage manifests itself as closure errors in co-polar visibility measurements (only parallel hand visibilities). Many working radio telescopes, offer observational advantages for observations in non-polar mode (e.g. higher frequency resolution, lower integration time, etc.). Consequently many observations are done in non-polar mode and computation of antenna based leakage gains in co-polar visibilities is therefore scientifically useful for debugging and calibrating the instrument. We also present results from test data taken with the Giant Meterwave Radio Telescope (GMRT) and discuss the degeneracy in the solutions and the equivalence of the leakage induced closure phase and the Pancharatnam phase of optics.

1. Introduction

Co-polar output of an interferometer, can be written as

$$\rho_{ij}^{pp} = \langle (g_i^p E_{i,\circ}^p + \alpha_i^q E_{i,\circ}^q + \epsilon_i)(g_j^p E_{j,\circ}^p + \alpha_j^q E_{j,\circ}^q + \epsilon_j)^\star \rangle \tag{1}$$

where p and q are two orthogonal polarization states (R and L or X and Y), g_i^p the antenna complex gain for the p-channel of antenna i, α_i^q the leakage of q-signal into the p-channel, $E_{i,\circ}^p$ the *ideal* response of the p-channel to the incident radiation, and ϵ_i the antenna based additive noise. For an unpolarized point source $\langle E_{i,\circ}^p E_{j,\circ}^{q^\star} \rangle = \langle E_{i,\circ}^q E_{j,\circ}^{p^\star} \rangle = 0$ and $\langle E_{i,\circ}^p E_{j,\circ}^{p^\star} \rangle = \langle E_{i,\circ}^q E_{j,\circ}^{q^\star} \rangle = \rho_{ij,\circ}^{pp} = I/2$ where I is the total intensity. Writing $X_{ij}^{pp} = \rho_{ij}^{pp}/\rho_{ij,\circ}^{pp}$ we get

$$X_{ij}^{pp} = g_i^p g_j^{p^\star} + \alpha_i^q \alpha_j^{q^\star} + \epsilon_{ij} \tag{2}$$

ϵ_{ij} is the independent baseline based noise. Usually this represents the noise in ρ_{ij}^{pp} after the correlation operation plus the antenna based noise. ϵ_{ij} therefore

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[1] Now at the National Radio Astronomy Observatory (NRAO), Socorro, NM - 87801

[2] Also Birla Institute of Technology & Science (BITS), Pilani, India and now at the department of Computer Science and Engineering, UCSD, CA - 92093. Email: uraovenk@cs.ucsd.edu

[2] Email: rajaram@ncra.tifr.res.in

is a measure of the *true* closure errors in the system and is usually small. Assuming α_i^qs to be negligible, the usual Selfcal algorithm estimates g_i^ps such that $\sum_{\substack{i,j\\ i\neq j}} \left|X_{ij}^{pp} - g_i^p g_j^{p^\star}\right|^2$ is minimized. However, leakage due to mechanical and/or electronic imperfections in the feed, cross talk, squint of *cross-polar* primary beam, off-axis primary beam polarization, etc. is hard to eliminate making α_i^qs potentially non-negligible.

In the presence of significant α_i^qs (compared to $\sqrt{\epsilon_{ij}}$), ignoring the second term in Eq. 2 will be equivalent to a system with *apparent* increased closure noise ($\epsilon_{ij} + \alpha_i^q \alpha_i^{q^\star}$ instead of just ϵ_{ij}). *Hence, polarization leakage manifests as increased closure errors.* This has also been pointed out by Rogers (1983) in the context of VLBA observations, and extensive study by Massi & Aaron (1997) for EVN shows that imaging quality is limited by these errors.

2. Algorithm

Solver for g_i^p for unpolarized sources using co-polar visibilities sees the α_i^qs as increased closure noise and hence return non-optimal solutions. Instead, an optimal solver must solve for g_i^p and α_i^q simultaneously. In the presence of significant polarization leakage, the correct estimator for the *true* closure noise is given by $S = \sum_{\substack{i,j\\ i\neq j}} \left|X_{ij}^{pp} - (g_i^p g_j^{p^\star} + \alpha_i^q \alpha_j^{q^\star})\right|^2 w_{ij}^{pp}$ where w_{ij}^{pp} are the weights. Equating the partial derivatives $\partial S/\partial g_i^{p\star}$, $\partial S/\partial \alpha_i^{q\star}$ to zero, we get a set of non-linear equations for g_i^ps and α_i^qs which can be iteratively solved (Bhatnagar & Nityananda, 2001).

3. Solution degeneracy, Simulations and Experiment with the GMRT

Simulations demonstrate that with the use of the above algorithm, the χ^2 remains constant with increasing leakage, and that it solves for α_i^qs only if they are significant (i.e. distinguishable from $\sqrt{\epsilon_{ij}}$) (Bhatnagar & Nityananda, 2001). The relative decrease in χ^2 compared to that given by Selfcal is due to the use of correct estimator for the closure noise and not just because of extra free parameters (the α_i^qs) thrown in the problem. *The solutions for α_i^qs are therefore also physically meaningful.*

However, an obvious degeneracy is that independent rotation of all α's and all the g's, in general by different amounts, does not affect the left hand side of Eq. 2. This degeneracy can be broken by a suitable transformation which imposes some desirable physical conditions. E.g. if the feeds are nominally linear, one could impose the condition that there is a mean linear basis in which the leakages are smallest. Carrying out the maximization of $\sum |g_i|^2$ by the method of Lagrange multipliers, subject to a constant χ^2, we obtain the condition that $\sum \alpha_i^* g_i = 0$ (implying that the leakage coefficients be orthogonal to the gains) and can be incorporated by first choosing an overall phase for the α's so that $\sum \alpha_i^\star g_i$ is real. Then, carry out a rotation in the $g-\alpha$ plane by an angle θ satisfying $\tan\theta = \sum \alpha_i^* g_i / (\sum (g_i g_i^\star - \alpha_i \alpha_i^\star)$. Results of such a transform on simulated data are shown in Fig. 1. The absolute frame of reference in which α_i^qs are

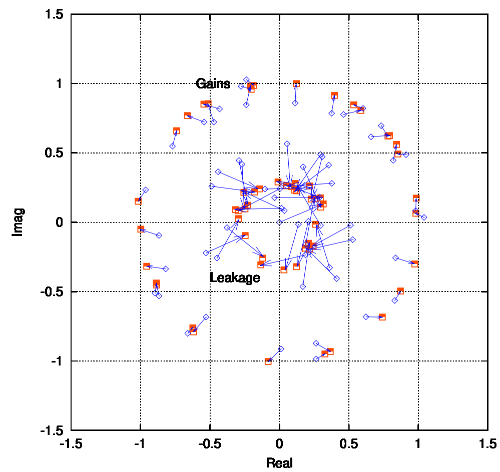

Figure 1. Simulations showing the decoupling of the solutions for g_i^ps and α_i^qs using the transform explained in section 3. The squares are the input test g_i^ps and α_i^qs on the complex plane while the diamonds are the solutions found using the algorithm described in this paper. The arrows show the length and direction of correction due to the transformation.

measured, also remain undetermined since the source is unpolarized. However this degeneracy is same as that in the phase of g_i^ps and is not important for correcting the data.

We used test data at L-band from the GMRT with the feed of only one of the antenna (labeled `C03` in Fig. 2) in circular state and the rest of the antennas with linear feeds. In the mean linear basis of all the antennas, `C03` appears as an antenna with $\alpha_i^q = \alpha_i^p \sim 1$ ($E^R_{C03} = E^X_{C03} e^{-\iota\delta} + E^Y_{C03} e^{\iota\delta}$; ideally $\delta = \pi/4$). The fractional leakage (α_i^q / g_i^p) for all antennas is plotted in the complex plain in Fig. 2[Left panel]. Mean leakage of all the antennas define the reference frame in which the leakage of nominally linear antennas is minimized. All (except one) nominally linearly polarized antenna are at the origin (minimal leakage) and the points corresponding to `C03` are farthest from the center, grouped $\sim 180^\circ$ apart.

4. Poincaré sphere and the Pancharatnam phase

A general elliptically polarized state can be written as a superposition of two states represented by the vector $\begin{pmatrix} \cos\theta/2 \\ \sin\theta/2\ e^{\iota\phi} \end{pmatrix}$ in the basis defined by the left- and right-circular polarization states. Clearly, $\theta = \pi/2$ corresponds to linear polarization and $\theta \neq 0, \pi/2$ to elliptical polarization. The Poincaré sphere representation of the state of polarization maps the general elliptic state to the point (θ, ϕ) on the sphere. It can be shown that the closure phase between three coherent, non-identical, antennas (points I,J and K in Fig. 2[Right panel]) is equal to half the solid angle IJK. This, more famously, goes by the name Pancharatnam/Geometric/Berry's phase in optical literature (Pancharatnam, 1956). *The closure phase due to polarization mis-matches between phased antennas in a radio interferometer therefore naturally measures the Pancharatnam phase.*

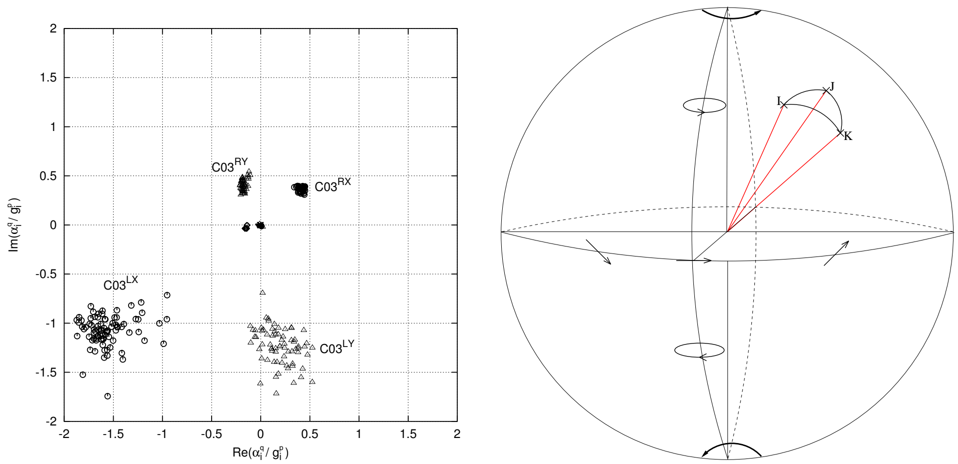

Figure 2. [**Left panel**] Plot of α_i^q/g_i^p for all GMRT antennas in the complex plane. $C03^{LX}$ and $C03^{RX}$ (open circles) are from correlations of R- and L-channel of C03 with X-channel of other linear antennas. Similarly for $C03^{LY}$ and $C03^{RY}$ (triangles). This also shows that one of the linearly polarized antennas is leakier than the others and L-channel of C03 is noisier than its R-channel. [**Right panel**] Poincaré sphere representation of polarization states. Linear states map to the equator, and purely circular states to the poles. Closure phase between three coherent but non-identical antennas represented by the points I,J and K is equal to half the solid angle of IJK.

5. Conclusions

The method described here, to measure the polarization leakage of individual antennas using the *co-polar* visibilities for an unpolarized calibrator, has been shown to be a useful tool for studying the polarization purity of the antennas of radio interferometers. Simultaneous solution of gain and leakage ensures that the method factorizes for leakage only if they are distinguishable from intrinsic *true* closure noise. The degeneracy between solutions of antenna based complex gains and leakage is broken by physically meaningful transform and the solutions can be used to remove leakage induced closure errors. Geometric interpretation of the results on the Poincaré sphere shows that the leakage induced closure phase is same as the Pancharatnam phase and the degeneracy in the solutions can be understood as the rigid rotation of the Poincaré sphere.

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