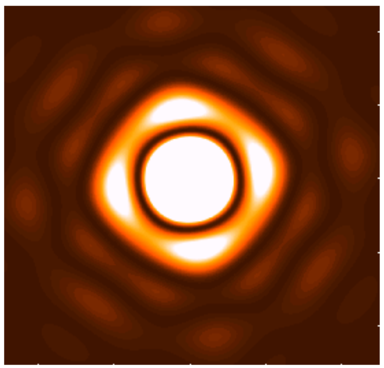
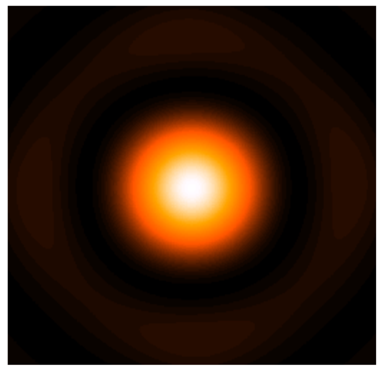
Correcting for wide-band primary-beam effects during imaging and deconvolution

P_t



P_ν



$$I^{obs} = \sum_{\nu,t} I^{psf}_{\nu,t} * \left(P_{\nu,t} \cdot I^{sky}_{\nu,t} \right)$$

Outline :

- Effects of primary-beams that vary with time and frequency

- Algorithms for classical Clean, A-Projection and wideband imaging

- Combining these algorithms to do wide-band primary-beam correction

- Examples

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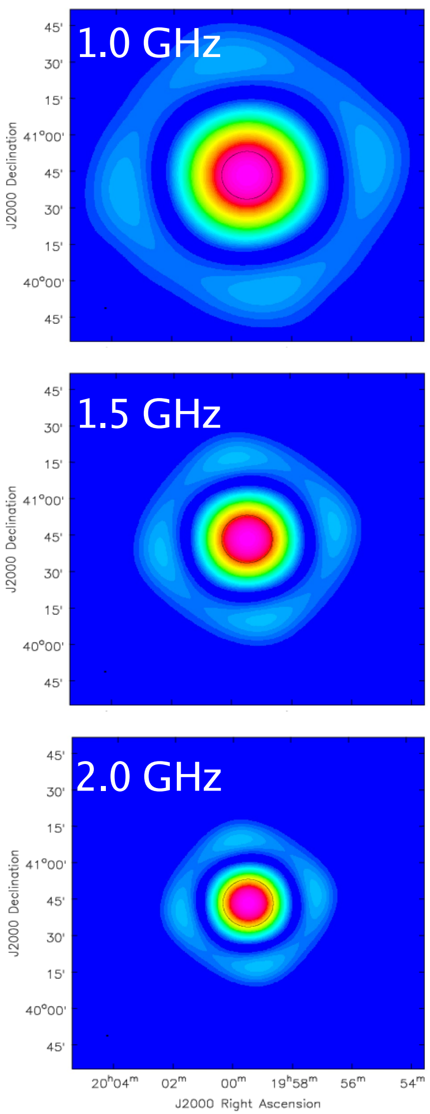
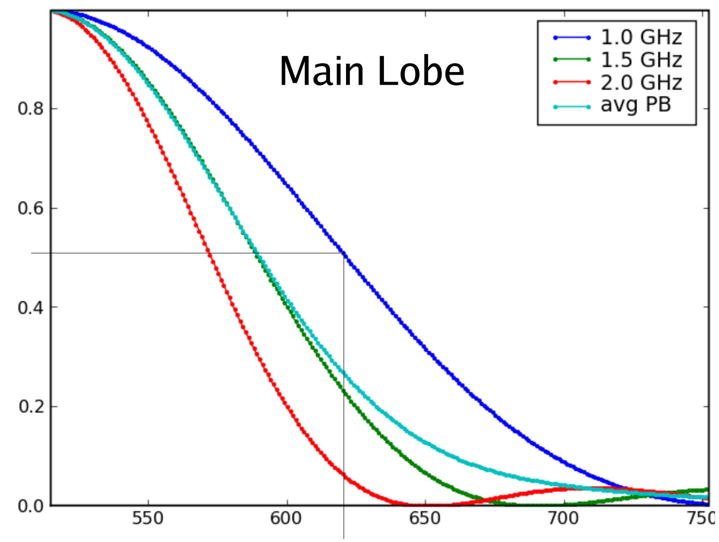
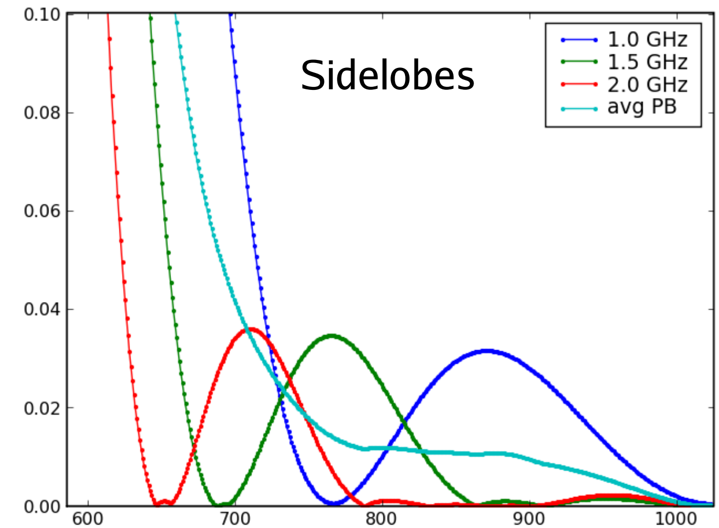
Frequency Dependence of the Primary Beam

EVLA Primary Beams
(simulated)

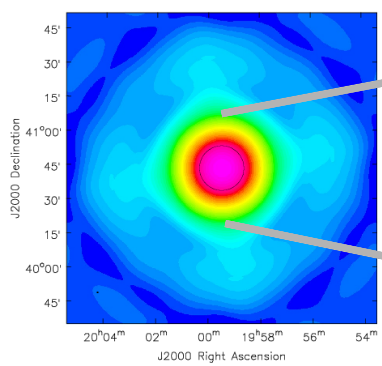


Average beam vs single-freq beams

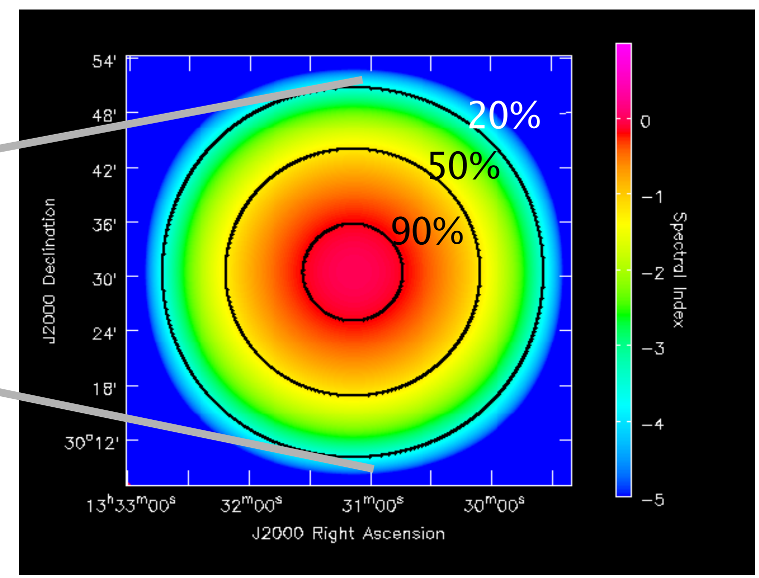




Average Primary Beam



Primary-Beam "Spectral Index" -->



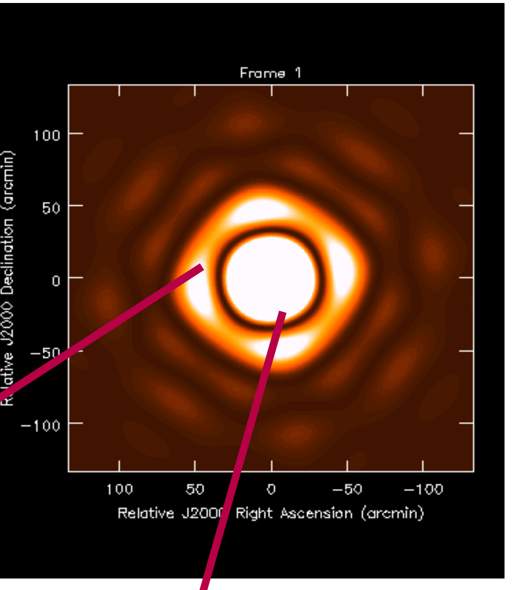
Time-variability and Beam Squint

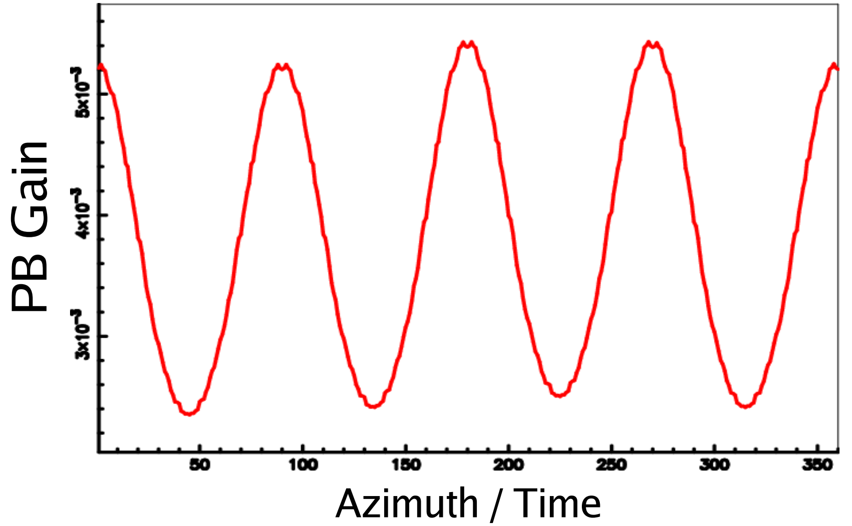
Primary Beams rotate on the sky (effect of Alt-Az mounts)

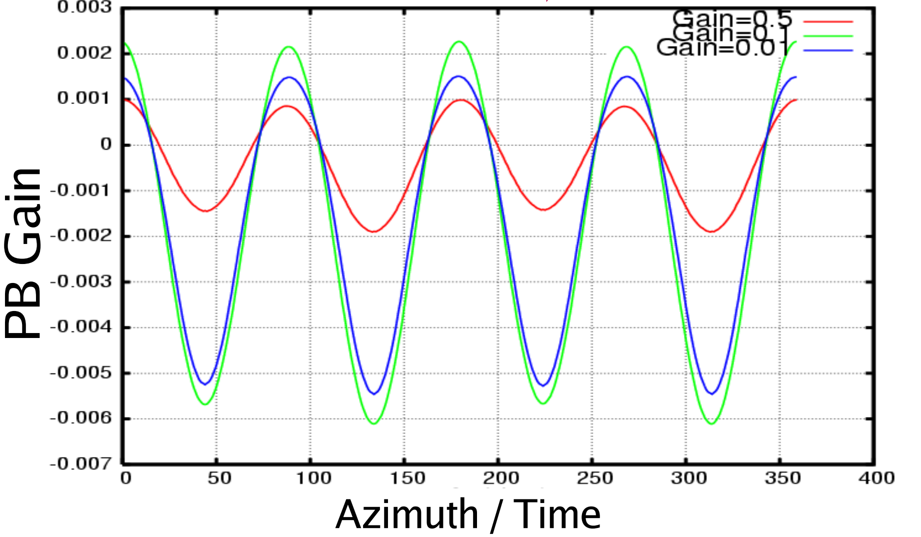
- Azimuthal asymmetry (subreflector supports)
- RR / LL Beam squint (~ 6% of HPBW)
- Pointing errors (10 –20 arcsec)

PB shape and Squint-Angle change (scale) with frequency

=> Direction and magnitude of time-variability of the PB change with frequency : $P_{\nu,t}$







Effects on a continuum image (MFS) and sky spectrum

Continuum Image : $$I^{obs} = \sum_{\nu,t} I^{psf}_{\nu,t} * (P_{\nu,t} \cdot I^{sky}_{\nu,t})$$ Time and Frequency average of snapshot single-channel images.

Continuum sensitivity : $$\frac{\sum_{\nu} w_{\nu} \cdot P_{\nu} \cdot I_{\nu}}{\sum_{\nu} I_{\nu}}$$ Average PB is close to the PB at the middle frequency, but has no obvious 'null'.

Spectrum : $I_{\nu} \cdot P_{\nu}$ A flat-spectrum source at the HPBW will acquire a spectral index of ~ -1.4, and pure MFS imaging (no spectral model) will have a dynamic-range limit of ~ 10^3

Time variability : 100% gain variation in sidelobes, few% gain variation in the main lobe

– Image contains only the average, with a dynamic-range limit of ~ 10^4 to 10^5

Goal : Separate the instrumental time and frequency dependence from the continuum image

Effects on a continuum image - sensitivity

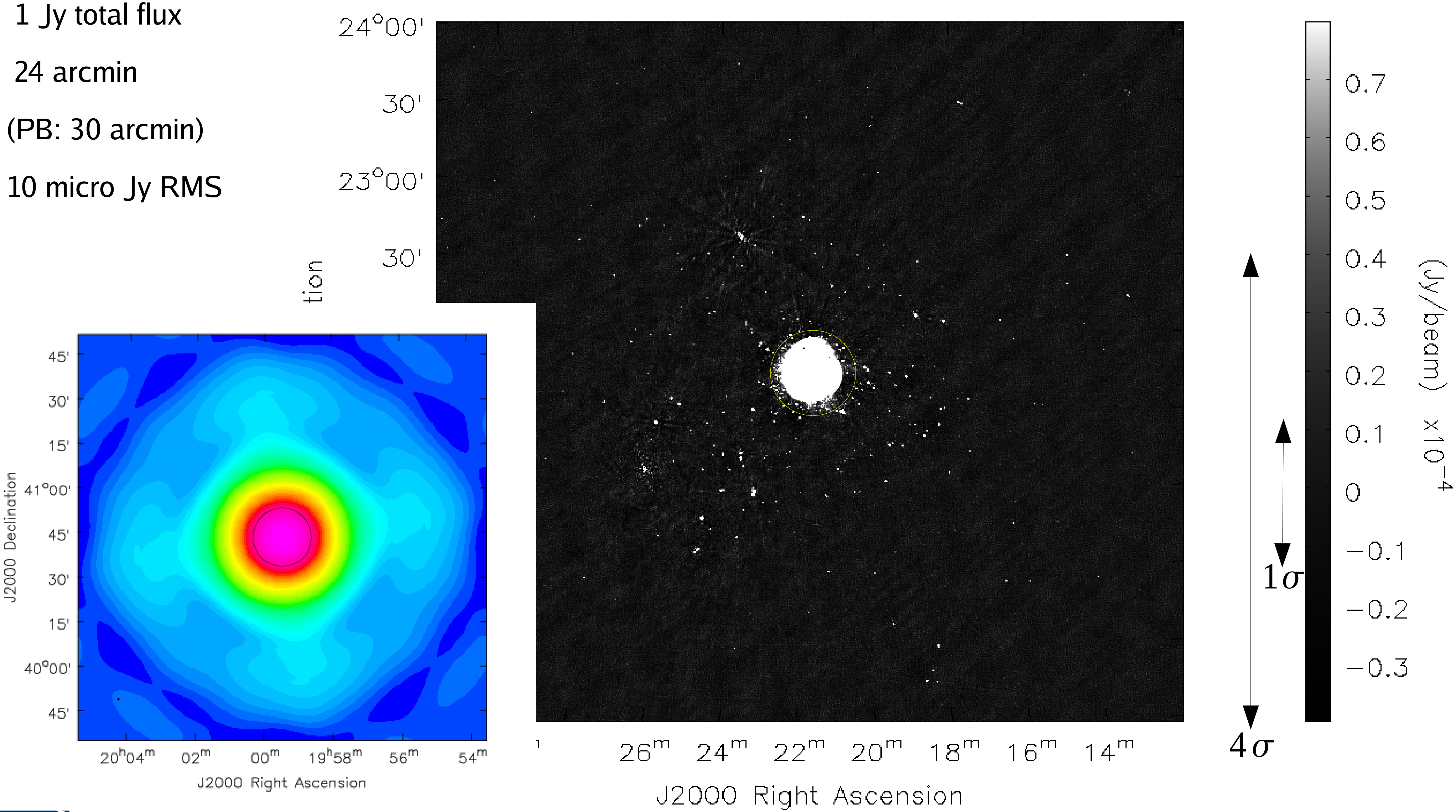
G55.7+3.4 : Galactic supernova remnant : 4 x 4 degree field-of-view from one EVLA pointing

1 Jy total flux

24 arcmin

(PB: 30 arcmin)

10 micro Jy RMS



Effects on a continuum image –PB time/freq variability

Imaging :

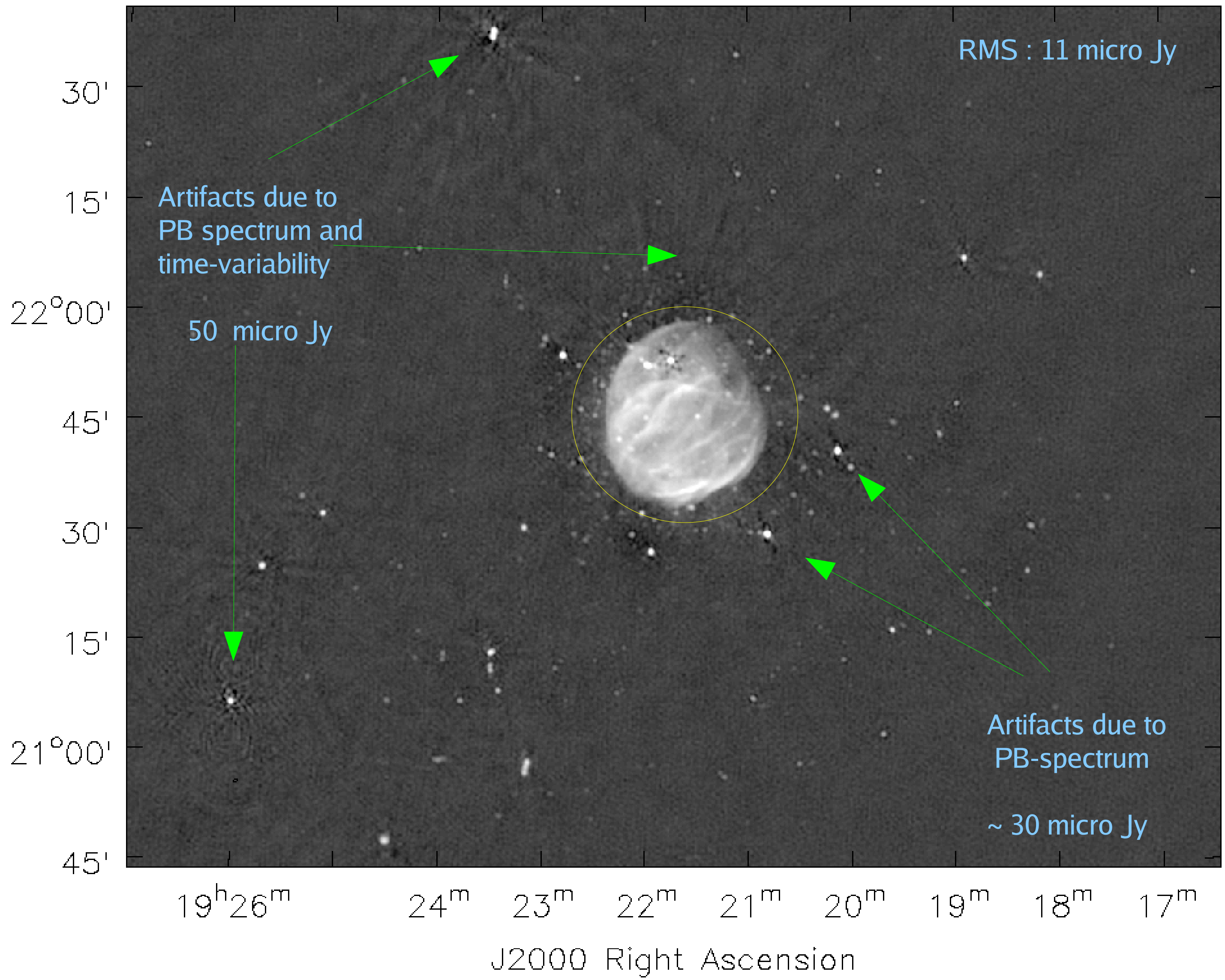
- W-Projection

  – correct for wide-fie w-term effects

- MS-MFS (2 terms)
  – model sky spectrum and (partially) PB spectrum

- MS starting model
  – to prevent ambiguity in the spectral model at very large scales

No PB-Correction yet....



Effects on a continuum image –PB time/freq variability

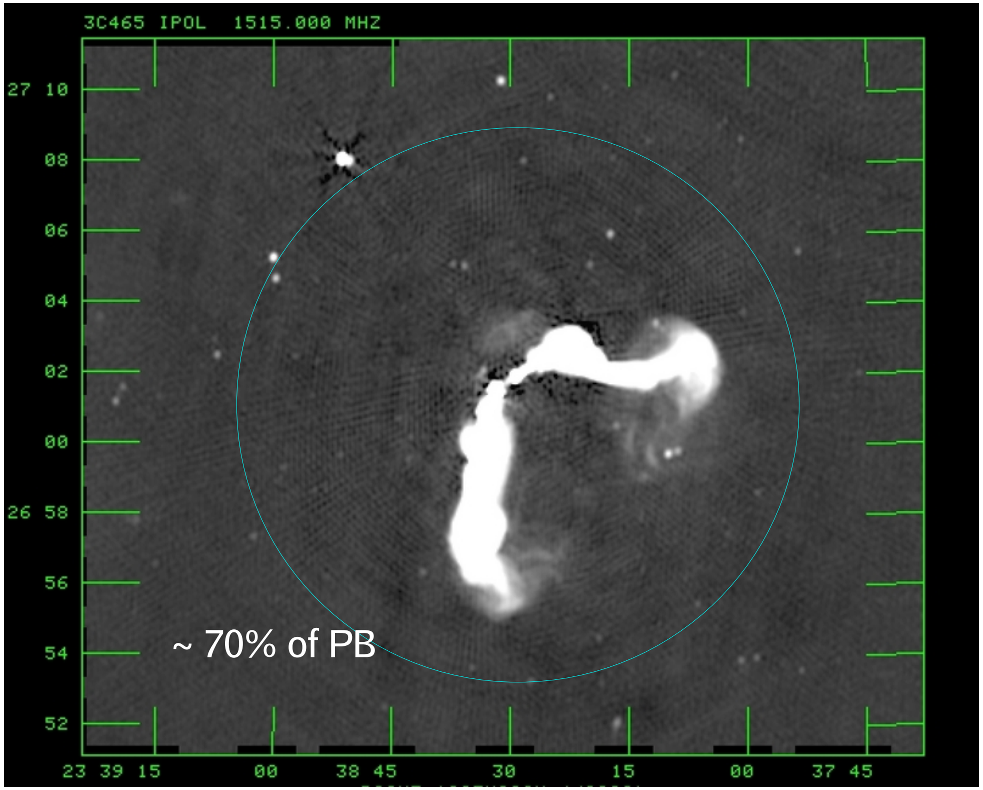
Examples from Frazer Owen :

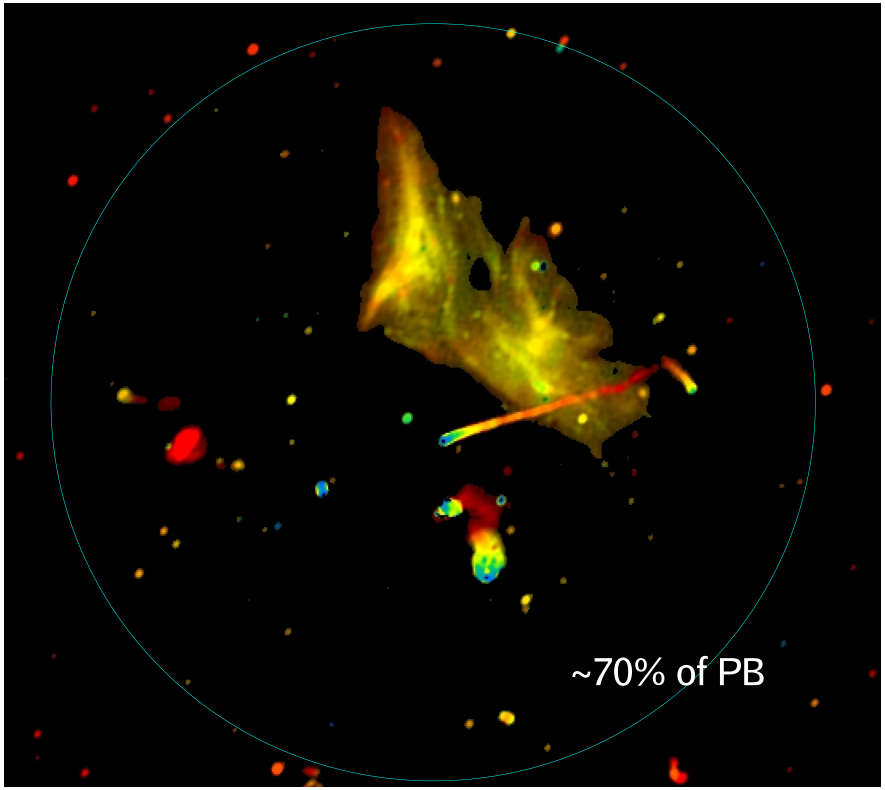
(1) : Time-variability artifacts well within the HPBW

3C465 : Wide-band continuum image

(2) : Artificial steepening of spectral index well-within the HPBW of the primary-beam

Abell 2256 : Intensity-weighted spectral-index (red : steeper)





~ 70% of PB at L-Band : ~ 8 arcmin from the pointing center

Algorithm Recap. : Major cycle (gridding convolution functions)

Image-Reconstruction : Major cycles (gridding/de-gridding) and minor cycles (deconvolution)

– Classical Imaging (FT) : Use a prolate-spheroidal function

– Divide gridded image by F(prolate-spheroidal) before the minor cycle.

$$I^{sky} = \frac{F \sum_{ij} A * V_{ij}}{F(A)}$$

– A-Projection : Use the complex-conjugate of the convolution of the aperture-illumination-functions of two antennas $A_{ij} = E_i * E_j^* = F(P_{ij})$

$$\boxed{P^{-1}} \cdot P \cdot I^{sky} = F \sum_{ij} \boxed{A_{ij}^{-1}} * A_{ij} * V_{ij} = \frac{F \sum_{ij} \boxed{A_{ij}^*} * A_{ij} * V_{ij}}{\boxed{\left| F \sum_{ij} A_{ij}^* * A_{ij} \right|}} = \frac{F \sum_{ij} \boxed{A_{ij}^*} * A_{ij} * V_{ij}}{\boxed{P_{avg}^2}}$$

– Visibility-domain : correct for aperture-domain phase-terms (pointing offset == phase-gradient)
–Image-domain : correct for average amplitude effects (rotational average)

– Flat sky : Divide by P_{avg}^2 ; Minor cycle I^{sky} ; Predict directly. [only for $I^{psf} = \delta$]

– Flat noise : Divide by P_{avg} ; Minor cycle $I^{sky} \cdot P_{avg}$; Divide by P_{avg} before prediction. $P^{-1} \cdot (I^{psf} * (P \cdot I^{sky}))$

Algorithm Recap. : Minor cycle (deconvolution and image-model)

Image-Reconstruction : Major cycles (gridding/de-gridding) and minor cycles (deconvolution)

– Cube Clean : Reconstruct each channel independently (grid each channel separately)

Calculate continuum image and spectra from output. $I_{cont} = \sum_{\nu} I_{\nu}$, $I_{alpha} =$ fit power-law

(single-channel uv-coverage/sensitivity, resolution of lowest frequency, f-o-v of highest frequency)

– Wide-band Clean : Grid all channels together (Multi-Frequency Synthesis)

MT-MFS : Reconstruct continuum intensity and spectrum simultaneously.

$$I_{\nu} = \sum_t C_t \left(\frac{\nu - \nu_{ref}}{\nu_{ref}} \right)^t = \sum_t C_t w_{\nu}^t$$

Calculate C_i from Taylor-weighted residuals by solving normal-equations (χ^2 - minimization)

$$I_{cont} = C_0 \quad , \quad I_{alpha} = C_1 / C_0$$

Interpret Taylor-coefficients

(broad-band uv-coverage/sensitivity, resolution of ~highest frequency, f-o-v of ~middle frequency)

Image Restoration : If the model represents $I_{\nu}^{model} = I_{\nu}^{sky} \cdot P_{avg,\nu}$, divide out $P_{avg,\nu}$ for flat-sky model

Combining FT, A-Proj, Cube, MT-MFS for wide-band PB-correction

(1) FT or A-Proj (flat-noise) with Cube gridding + Cube cleaning + per-channel PB-division

$$I_\nu^{dirty} = \frac{P_\nu^* \cdot P_\nu \cdot I_\nu^{sky}}{|P_\nu|} \qquad I_\nu^{model} = I_\nu^{sky} \cdot P_\nu \qquad I^{sky} = \frac{I_\nu^{model}}{P_\nu}$$

(2) FT gridding + MT-MFS deconvolution + post-deconvolution wideband PB-correction

$$I_\nu^{model} = \sum_t w_\nu^t \cdot I_t^{sky\cdot PB} \qquad \left(I_{t=0}^{sky}, I_{t=1}^{sky}, I_{t=2}^{sky}, \ldots\right) = \frac{\left(I_{t=0}^{sky\cdot PB}, I_{t=1}^{sky\cdot PB}, I_{t=2}^{sky\cdot PB}, \ldots\right)}{\left(P_{t=0}, P_{t=1}, P_{t=2}, \ldots\right)}$$

(minor cycle sees P_ν spectrum)

(3) A-Proj (flat-sky) with Cube gridding + Construct MT-MFS residuals + MT-MFS deconvolution

$$I_\nu^{dirty} = \frac{P_\nu^* \cdot P_\nu \cdot I_\nu^{sky}}{|P_\nu^2|} \qquad I_t^{dirty} = \sum_\nu w_\nu^t I_\nu^{dirty} \qquad I_\nu^{model} = \sum_t w_\nu^t \cdot I_t^{sky}$$

(remove PB frequency-dependence during gridding) (minor cycle sees only sky spectrum)

- (1) and (3) require Cube gridding => expensive (memory and computing).
- (2) sees PB spectrum => requires higher-order terms in the minor cycle + ignores time-variations
- (3) uses flat-sky normalization => requires good PSF

Combining FT, A-Proj, Cube, MT-MFS for wide-band PB-correction

(4) A-Projection for MFS gridding (flat-noise) + MT-MFS deconvolution

$$I_t^{dirty} = \frac{\sum_\nu w_\nu^t \cdot P_\nu^* \cdot P_\nu \cdot I_\nu^{sky}}{\left|\sum_\nu P_\nu\right|}$$

$$I_\nu^{model} = P_{avg} \cdot I_{\nu_{ref}}^{sky} \left(\frac{\nu}{\nu_{ref}}\right)^{\alpha^{sky} + 2\alpha^{PB}}$$

(divide by avg PB, cannot remove frequency-dependence) (minor cycle sees P_ν^2 spectrum)

=> needs more higher-order terms....

(5) Wide-Band A-Projection for MFS gridding (flat-noise) + MT-MFS deconvolution

$$I_t^{dirty} = \frac{\sum_\nu w_\nu^t \cdot P_{\nu(conj)}^* \cdot P_\nu \cdot I_\nu^{sky}}{\left|P_{ref}\right|} \quad where \quad P_{ref}^2 \approx P_{\nu(conj)}^* \cdot P_\nu$$

$$I_\nu^{model} = P_{ref} \cdot I_{\nu_{ref}}^{sky} \left(\frac{\nu}{\nu_{ref}}\right)^{\alpha^{sky}}$$

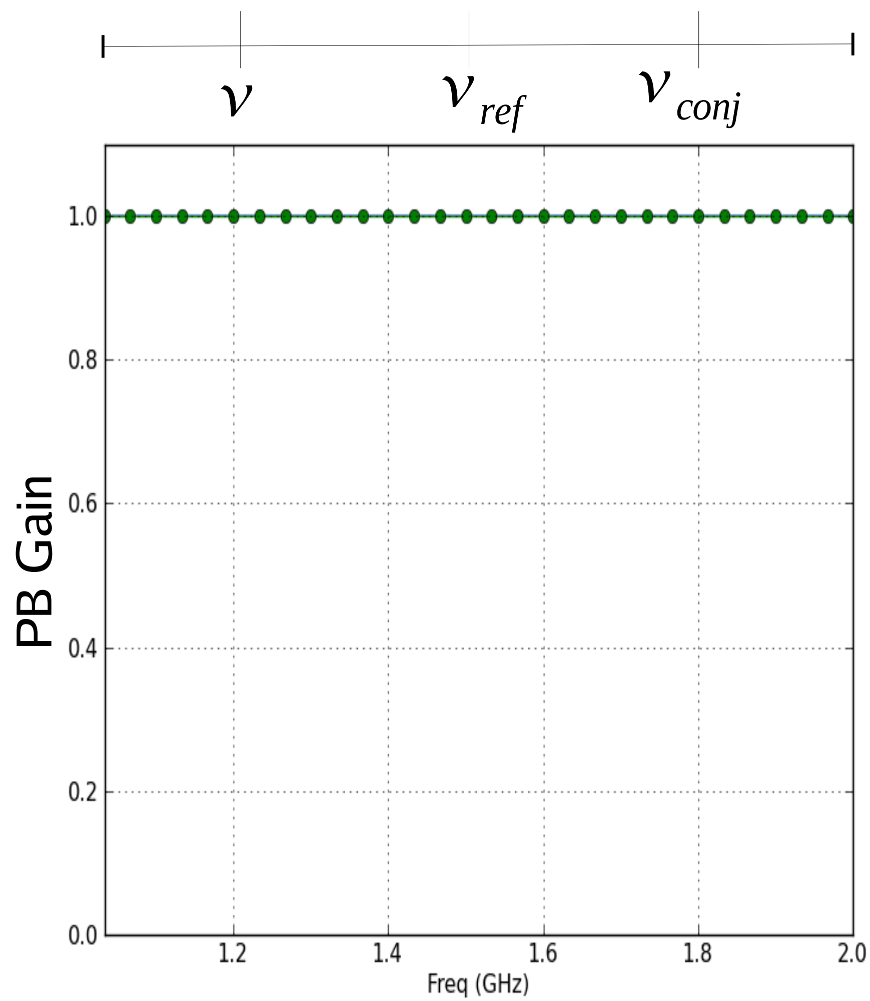
(remove PB frequency-dependence during gridding) (minor cycle sees only sky spectrum)

- Use frequency-conjugate beams during A-Projection gridding to convert all beams to almost the same size/shape, so that there is no frequency-dependence left.

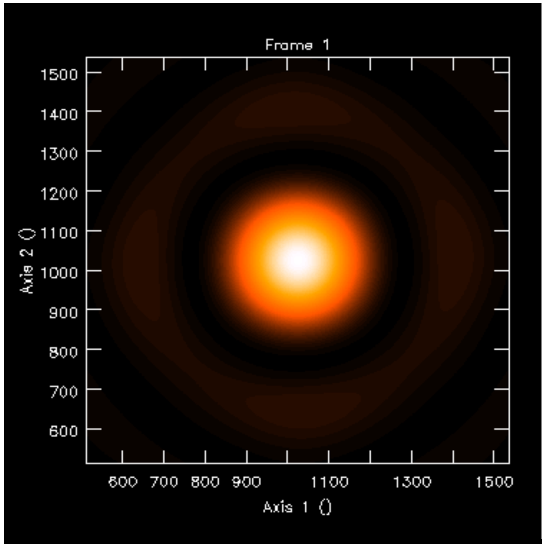
Using frequency-conjugate PBs during gridding

Multiply each $P(\nu)$ with $P(\nu_{conj})$

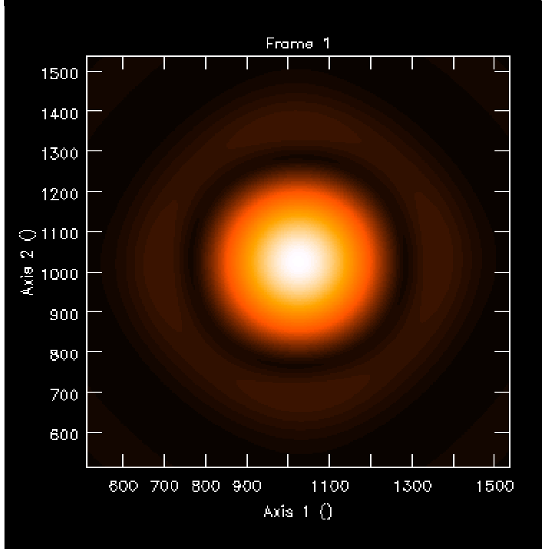
such that the product has a flat spectrum : $P(\nu)\cdot P(\nu_{conj}) \approx P(\nu_{ref})$



$P(\nu)$

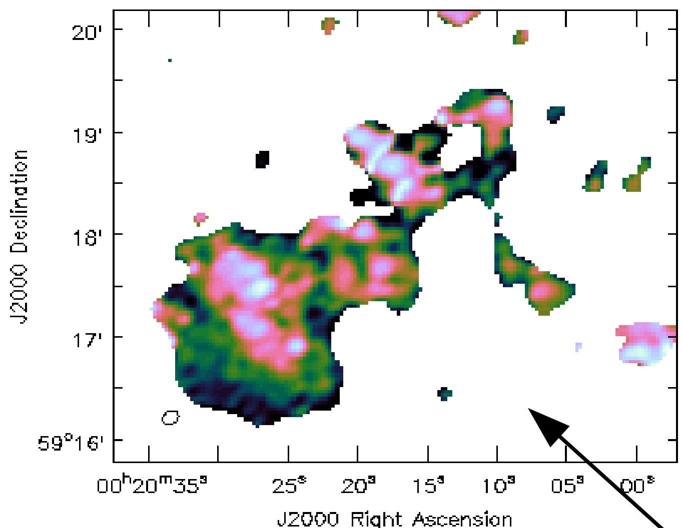


$P(\nu)\cdot P(\nu_{conj})$

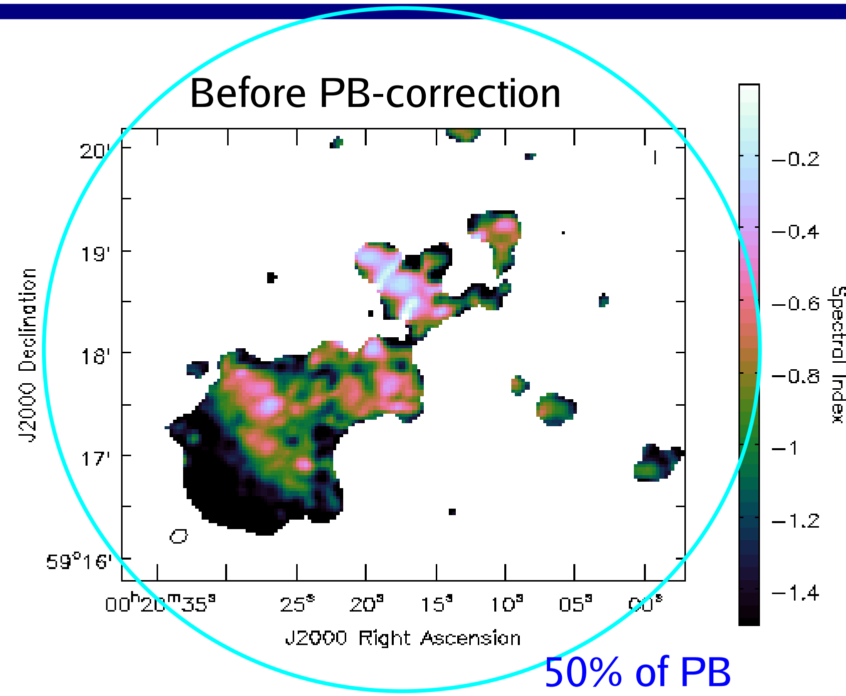


Example with FT : Comparison of MT-MFS vs Cube

After PB-correction

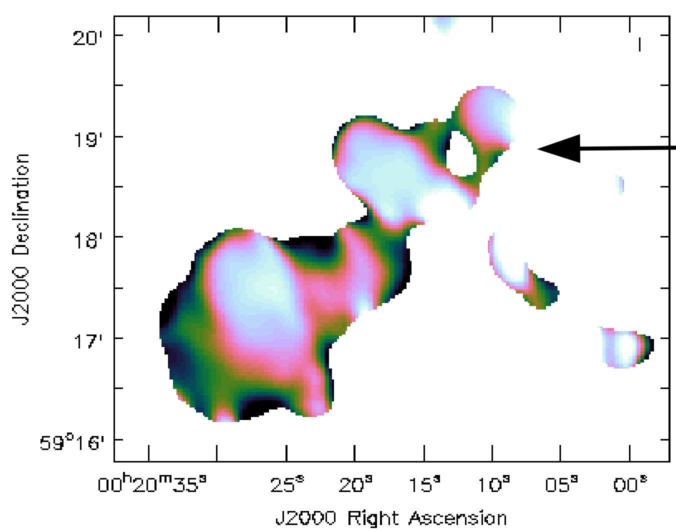


Before PB-correction



IC10 Dwarf Galaxy :

Spectral Index across C-Band.



MT-MFS : Result of wide-band PB-correction after MT-MS-MFS.

Cube : Spectral-index map made by PB-correcting single-SPW images smoothed to the lowest resolution.

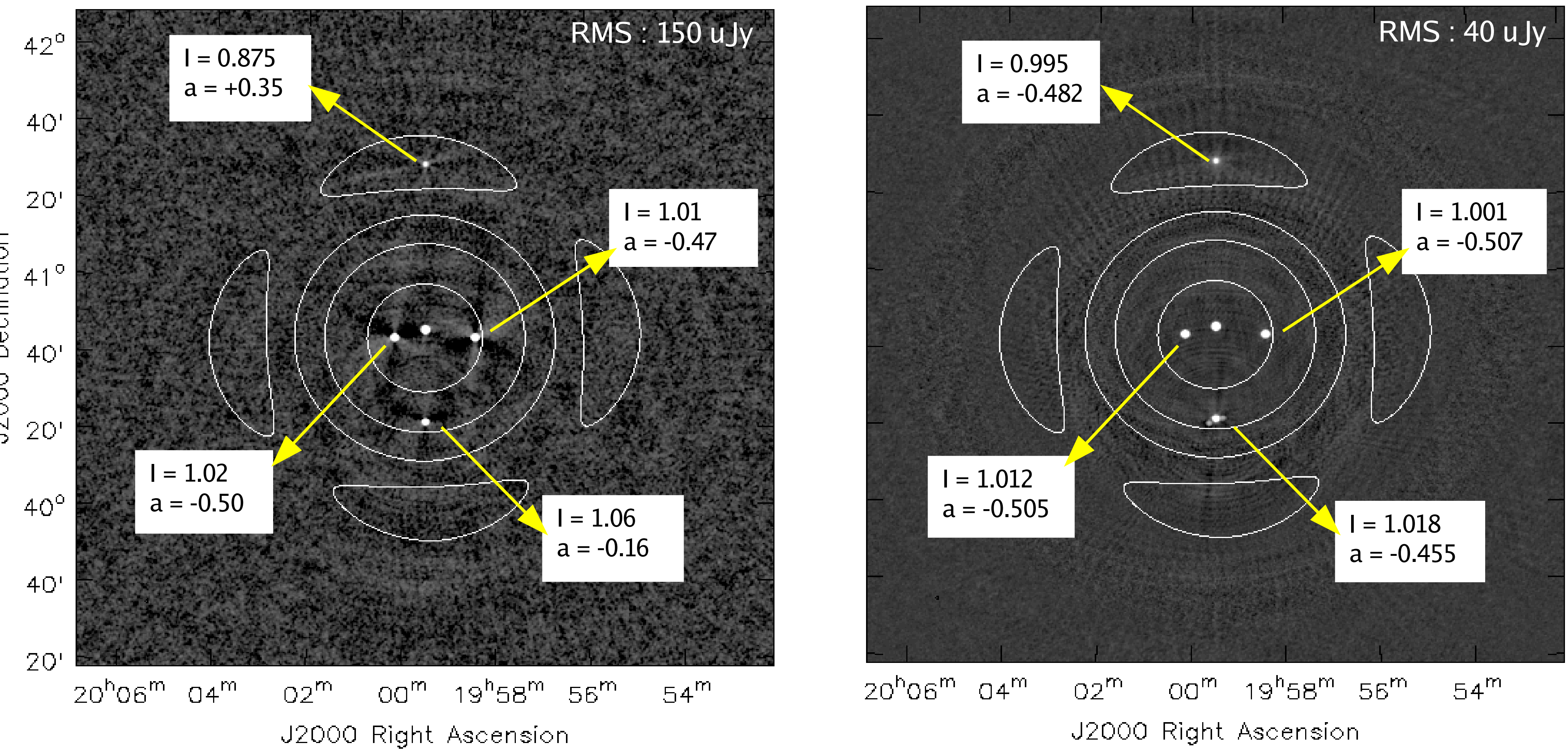
Any post-deconvolution PB-correction assumes that the primary-beam does not vary / rotate during the observation.

=> Dynamic range limit of 10^4 ~ 10^5
=> Valid within ~HPBW (depends on dynamic-range)

Example with MT-MFS : Comparison of FT vs WB-A-Proj

FT +MT-MFS : Model freq-dependence in minor cycle.
Ignore time-variability
Post-deconvolution wideband pbcor

WBAProj + MT-MFS : Minor-cycle sees only sky spectrum
Account for time-variability



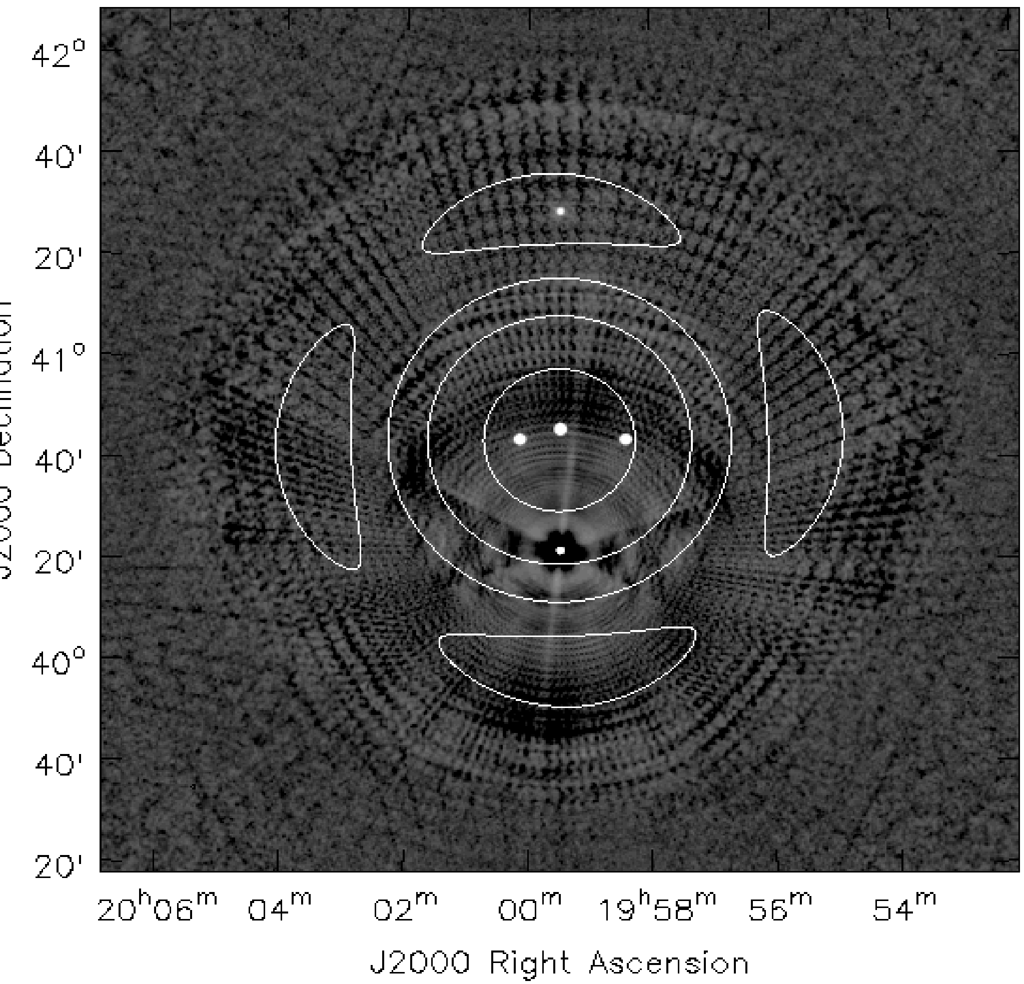
All sources 1 Jy, alpha=-0.5 . Located at PB gains : 0.98, 0.81, 0.60, 0.16, 0.023

Average PB contour levels : 0.5, 0.1, 0.02 . EVLA L-Band D-config simulation (6 hours)

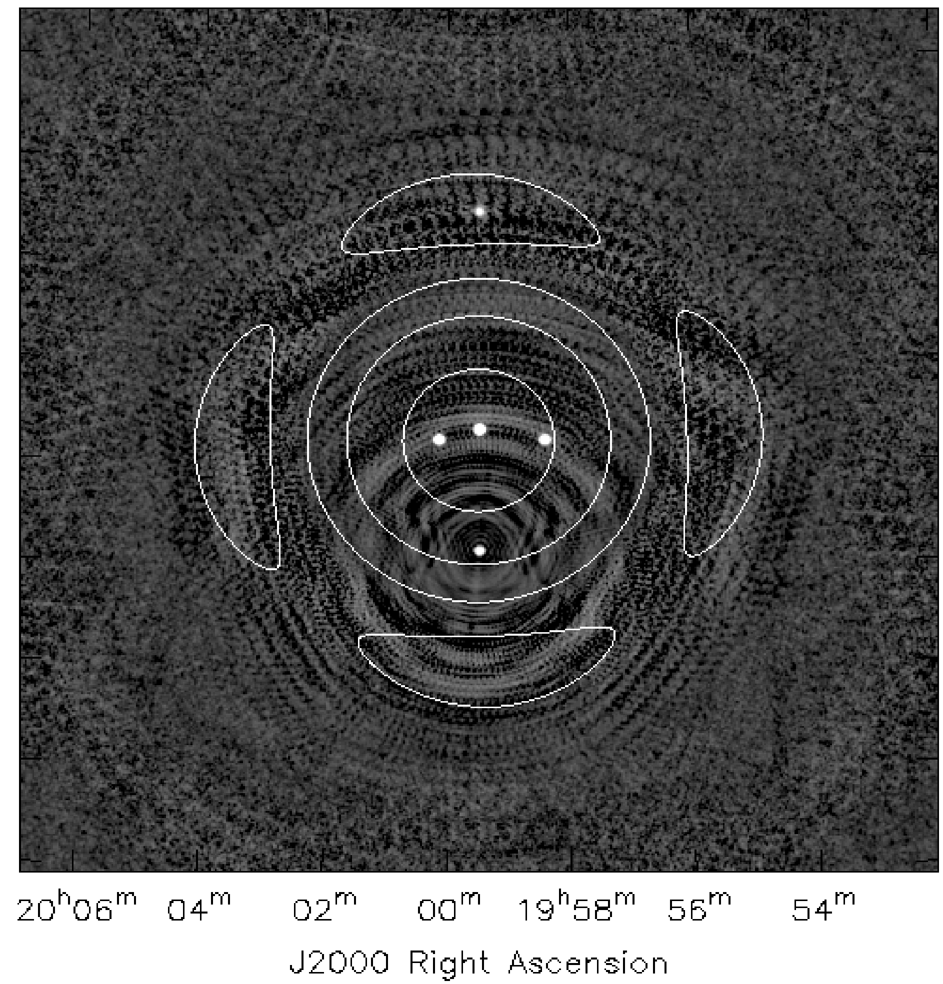
MT-MFS with A-Projection (no conjugate PBs)

Minor Cycle sees spectrum of Sky x PB x PB => Needs more terms in the Taylor polynomial...

N-Terms = 2



N-Terms = 3



Dominant errors are from the source at 15% of the PB : Steep PB gradient ? Reaching null of high-freq PB ? Inaccurate gridding-convolution functions ? Insufficient N-Terms ?

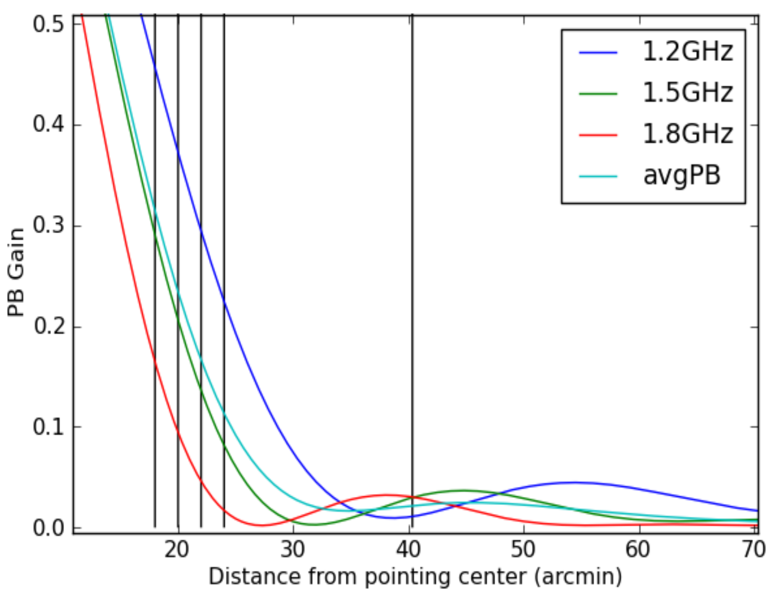
MT-MFS with WB-AProj : Field-of-View Limits ?

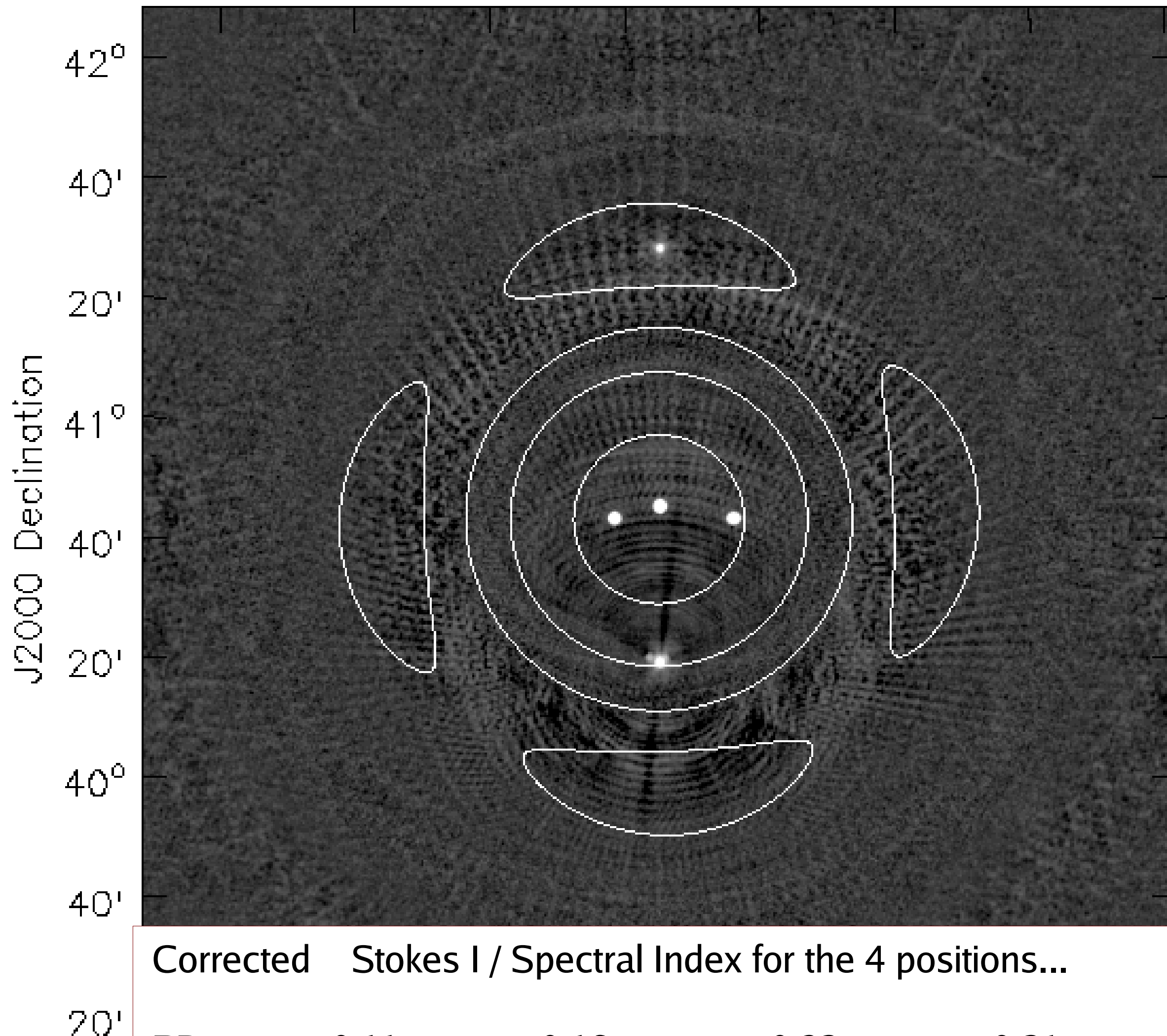
Source was simulated at PB levels :
0.11, 0.16, 0.23, 0.31

MT-MFS with n-terms = 2

Artifacts disappear at ~ 20% PB
(in this simulation)

PB Gain vs Angular Distance from center





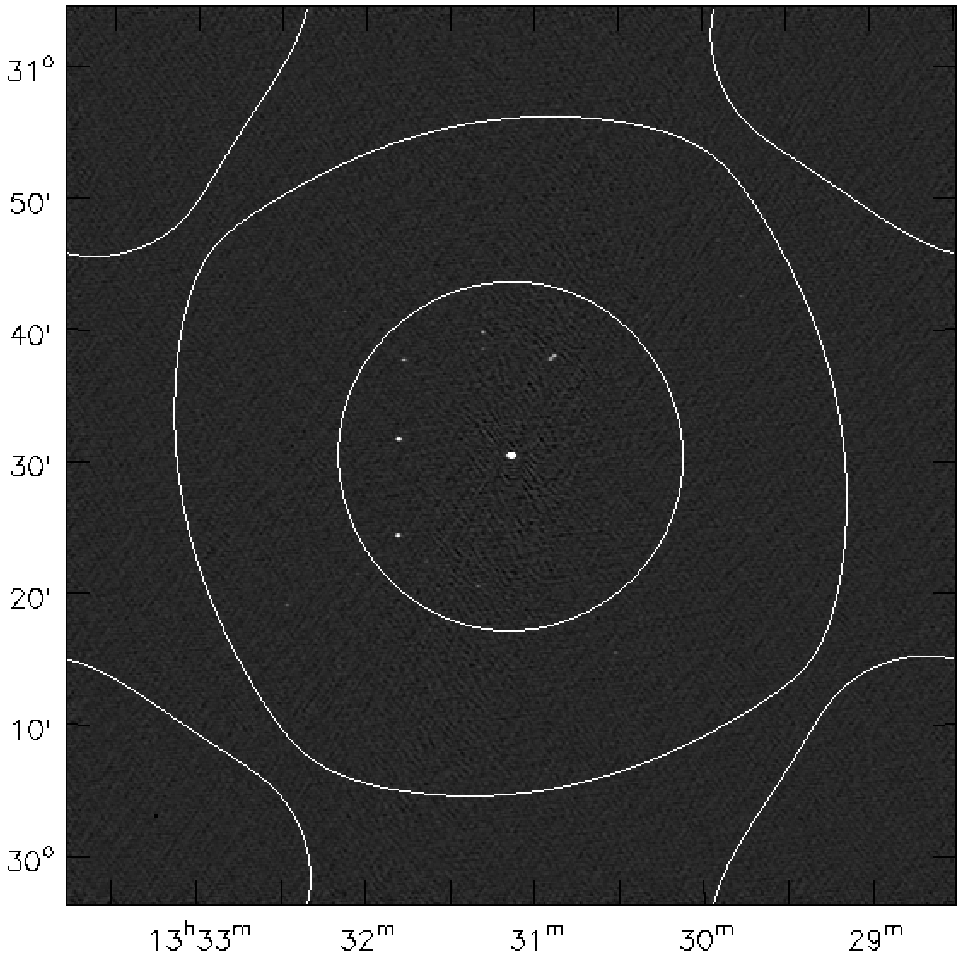
Corrected Stokes I / Spectral Index for the 4 positions...

PB :	0.11	0.16	0.23	0.31
FT :	1.09/-0.12 ,	1.06/-0.32 ,	1.05/-0.38 ,	1.04/-0.43
WBAP :	0.91/-0.6 ,	0.99/-0.45 ,	1.004/-0.506,	1.006/-0.505

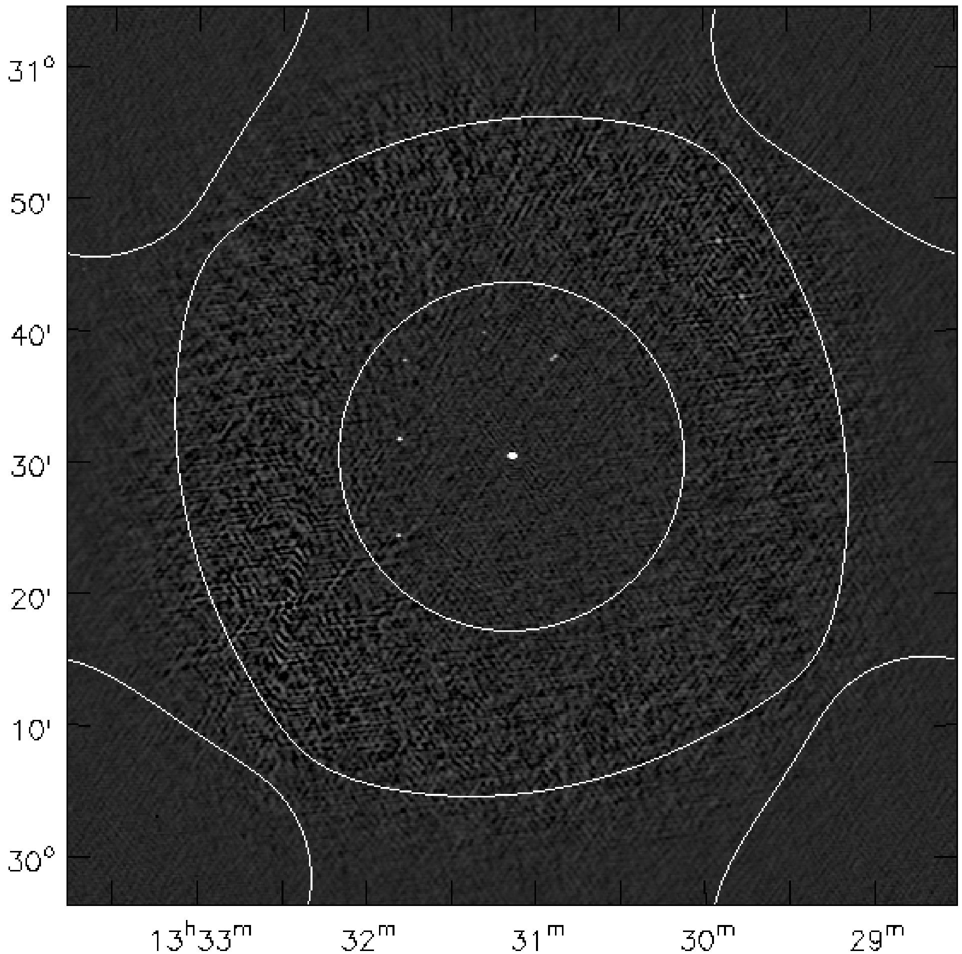
3C286 field EVLA L-Band : MT-MFS with FT vs WBAProj

MT-MFS with 4 terms (required because of 3C286). Theoretical rms : 60 uJy

FT : 0.4 - 0.8 mJy (earlier, 80uJy with blcal)



WBAProj : 0.5 - 1.3 mJy (but the spectrum is right !)



Average PB contour levels : 0.5, 0.1, 0.02 . (Need to make Stokes V image, to check Squint, etc.)

Summary + Future Work

Feasible Options :

- Cube imaging/deconv + post-deconvolution PB-correction per channel
  - Low dynamic-range, low resolution, small FOV → Always an option.
- MT-MFS + FT + post-deconvolution wideband PB-correction
  - Works for frequency-dependent effects but ignores all time-variability.
- MT-MFS with WB-A-Proj : Accounts for time and freq variability before minor cycle.

.... still, work in progress...

Other ideas :

- Variant of 'Peeling' : model and subtract distracting sources via single-channel imaging.
  (decide when to get rid of the source, and not try to reconstruct its flux correctly)
- Variant of MT-MFS : model and solve for residual time and frequency variability in the minor cycle (use time/freq basis functions instead of Taylor-polynomials)

Future Work : Wide-Band Mosaics (MT-MFS + FT or WB-A-Projection)