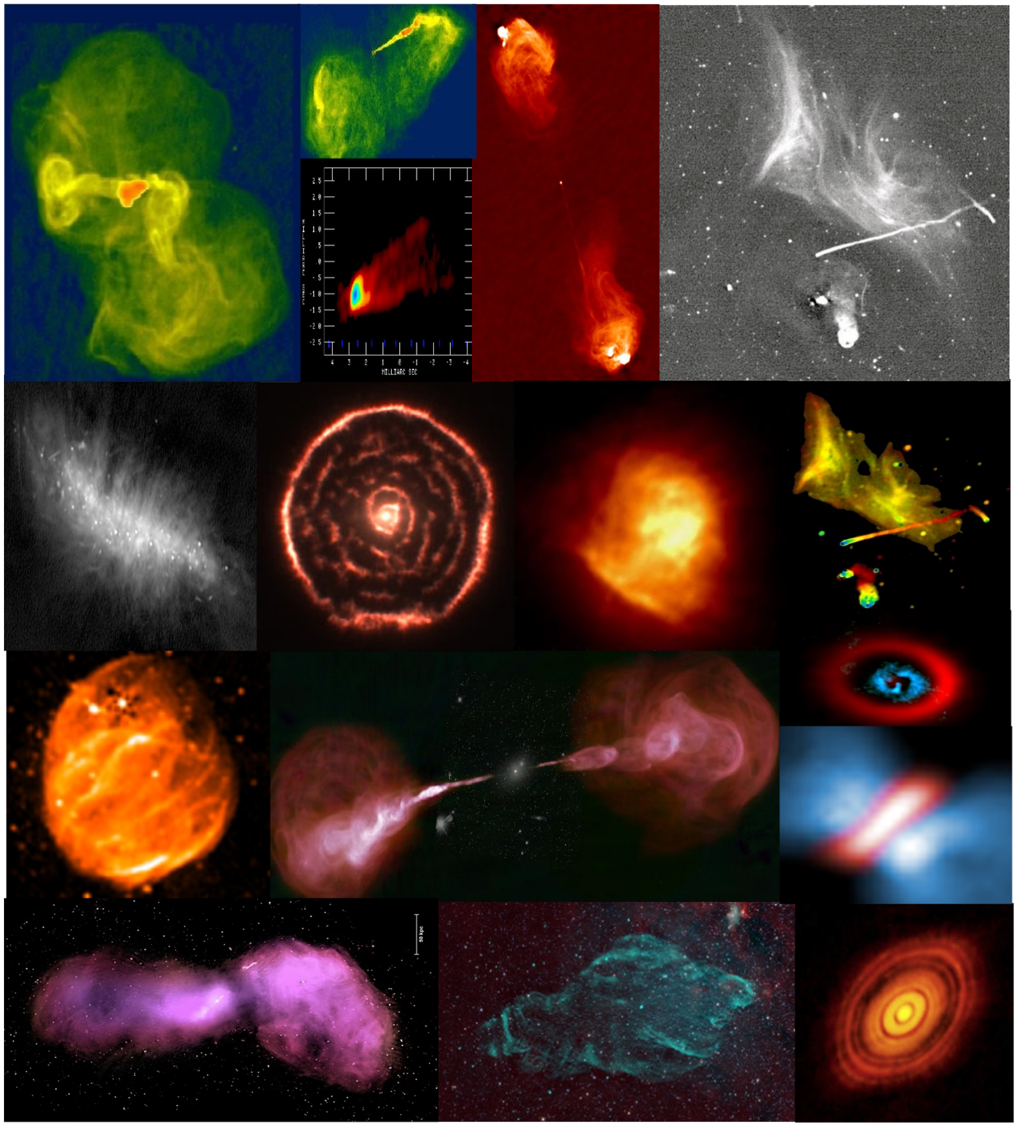
Radio Interferometry - Imaging



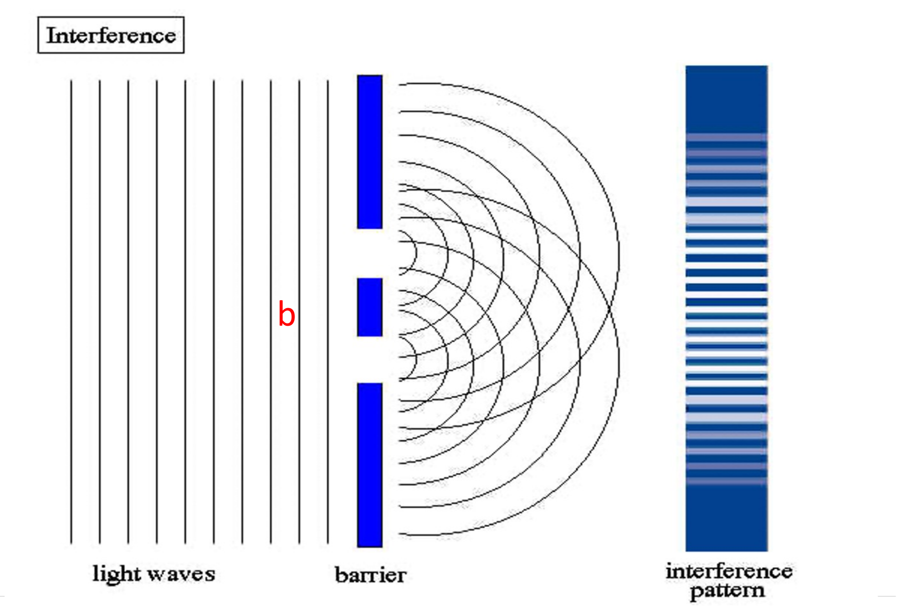
Urvashi Rau

National Radio Astronomy Observatory, Socorro, NM, USA

Image Formation

An interferometer is an indirect imaging device

Young's double slit experiment



2D Fourier transform :

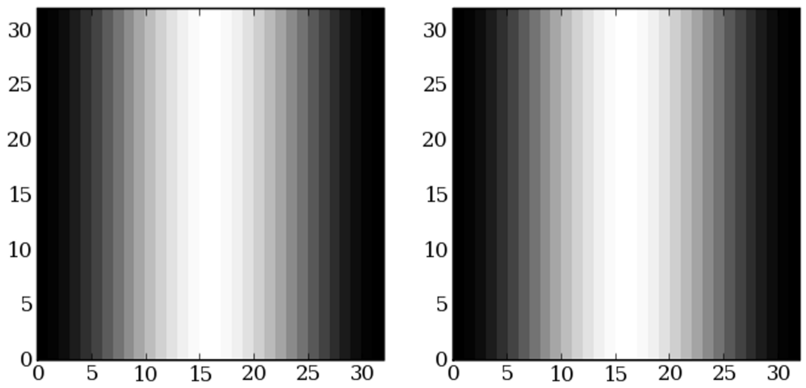
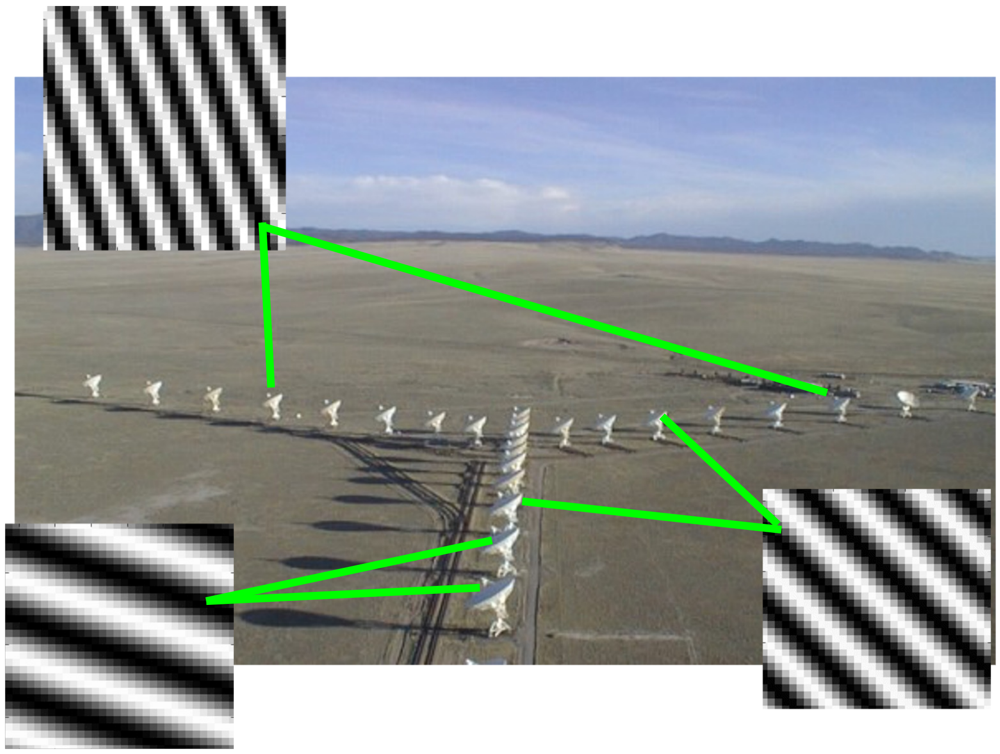


Image = sum of cosine 'fringes'.

Each antenna-pair measures the parameters of one 'fringe'.



Measured Fringe Parameters :

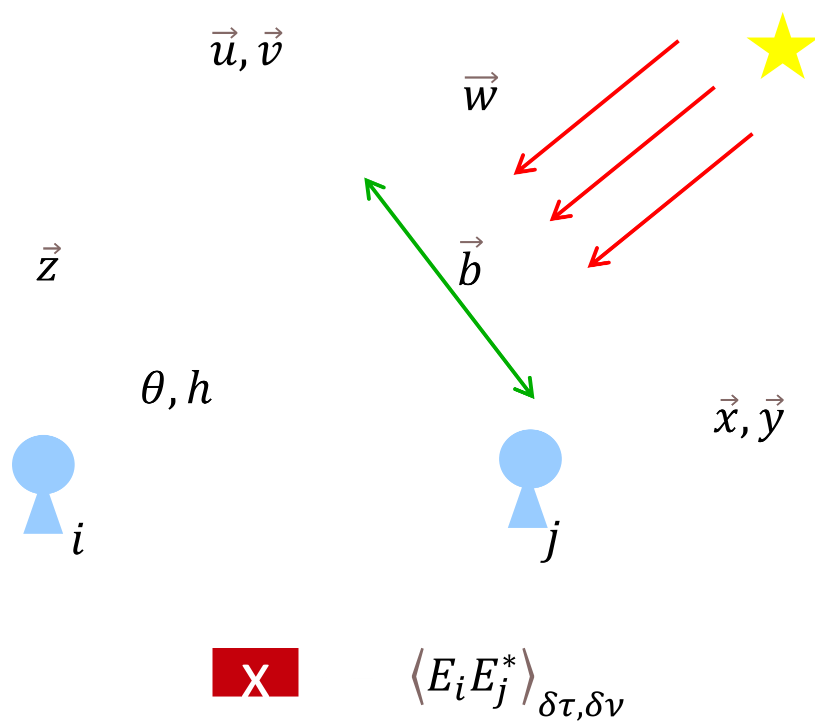
Amplitude, Phase

Orientation, Wavelength

Measurements

Measure the spatial correlation of the E-field incident at each pair of antennas

$$\langle E_i E_j^* \rangle \propto V_{ij}(u,v) = \iint I^{sky}(l,m) e^{2\pi i(ul+vm)} dl dm$$



Measurements

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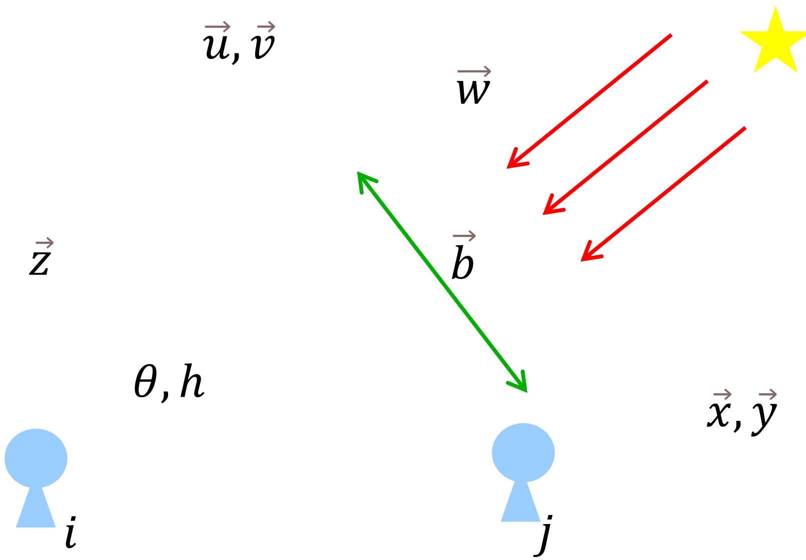
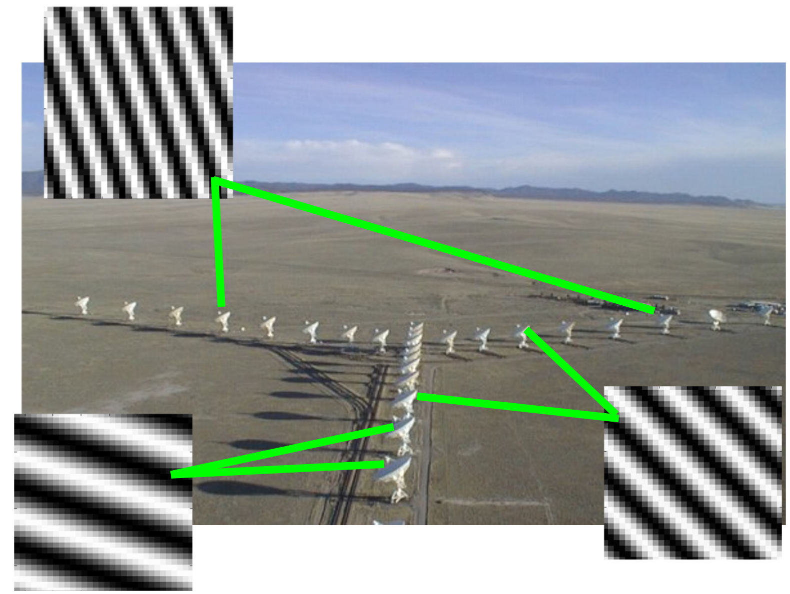


X $\langle E_i E_j^* \rangle_{\delta\tau, \delta\nu}$



Parameters of a Fringe :

Amplitude, Phase :

$\langle E_i E_j^* \rangle$ is a complex number.

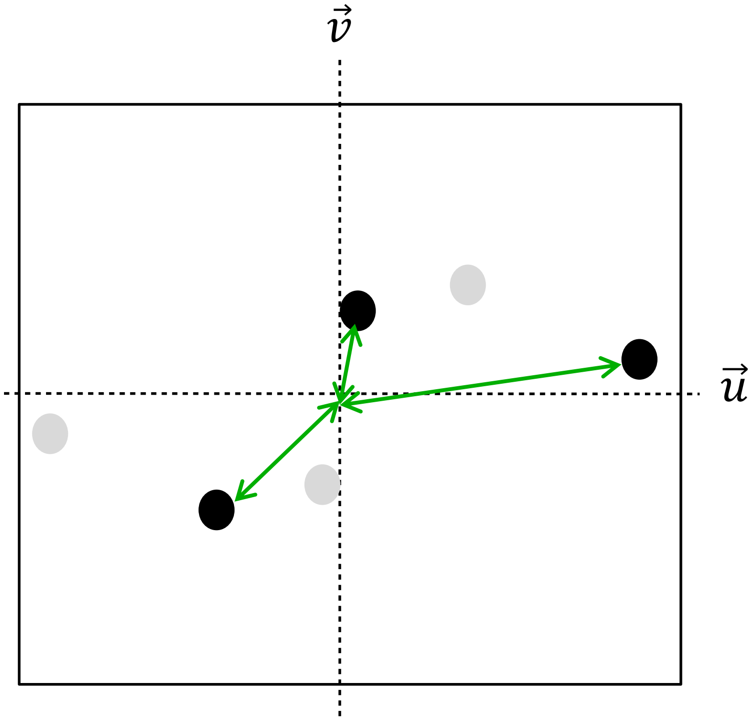
Orientation, Wavelength :

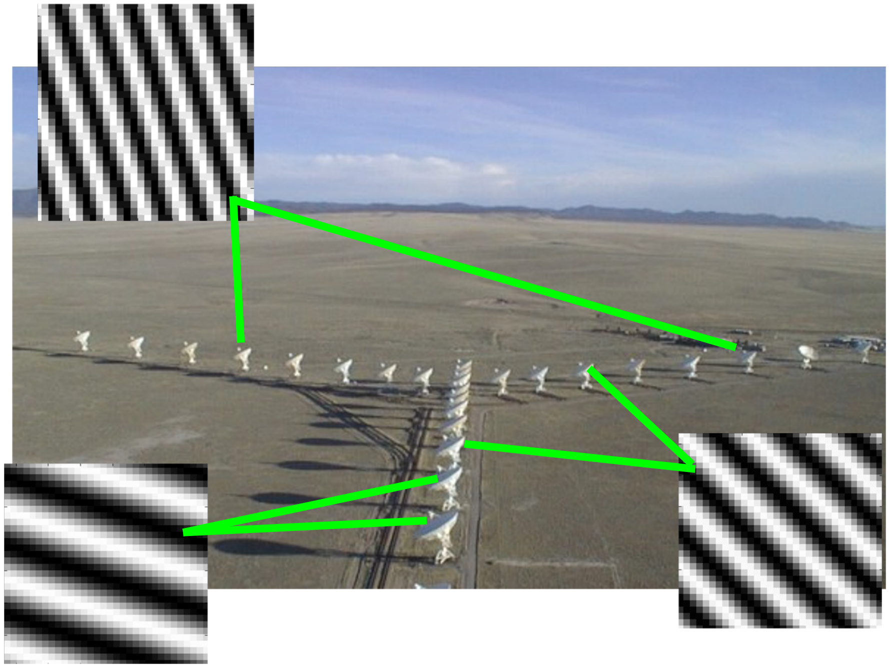
$\vec{u}, \vec{v}, \vec{b}$ (geometry)

Measurements

Measure the spatial correlation of the E-field incident at each pair of antennas

$$\langle E_i E_j^* \rangle \propto V_{ij}(u,v) = \iint I^{sky}(l,m) e^{2\pi i(ul+vm)} dl dm$$





UV plane : Spatial Frequency Domain

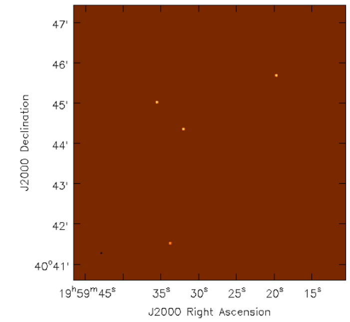
Combine measurements from multiple ‘baselines’ (antenna pairs) on a UV grid.

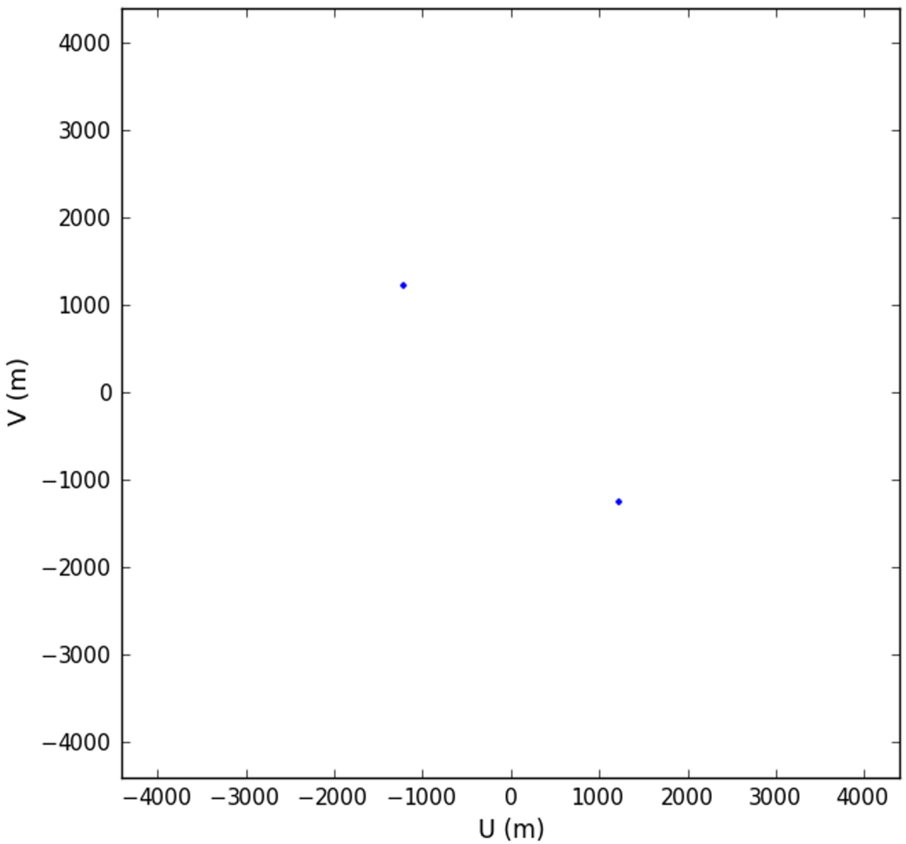
Take the inverse Fourier Transform to construct an image.

Spatial Frequency (uv) coverage + Observed Image

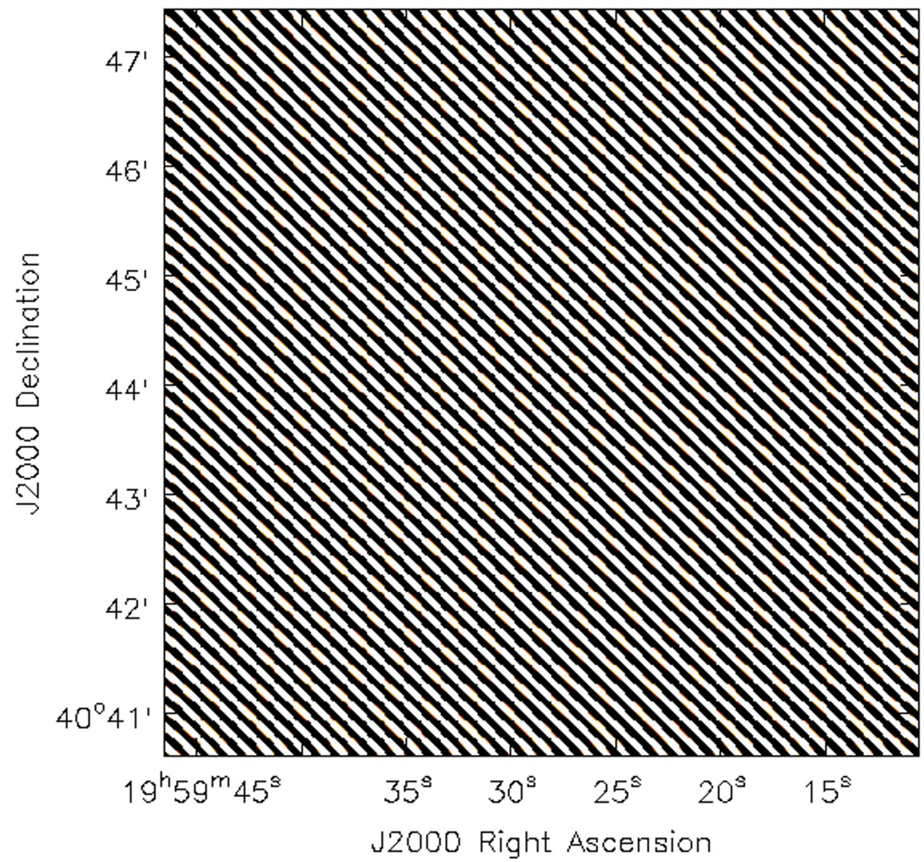
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda}\left[R(h,\theta)\right]\begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 2 antennas





$S(u,v)$

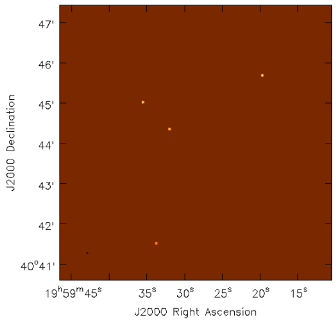


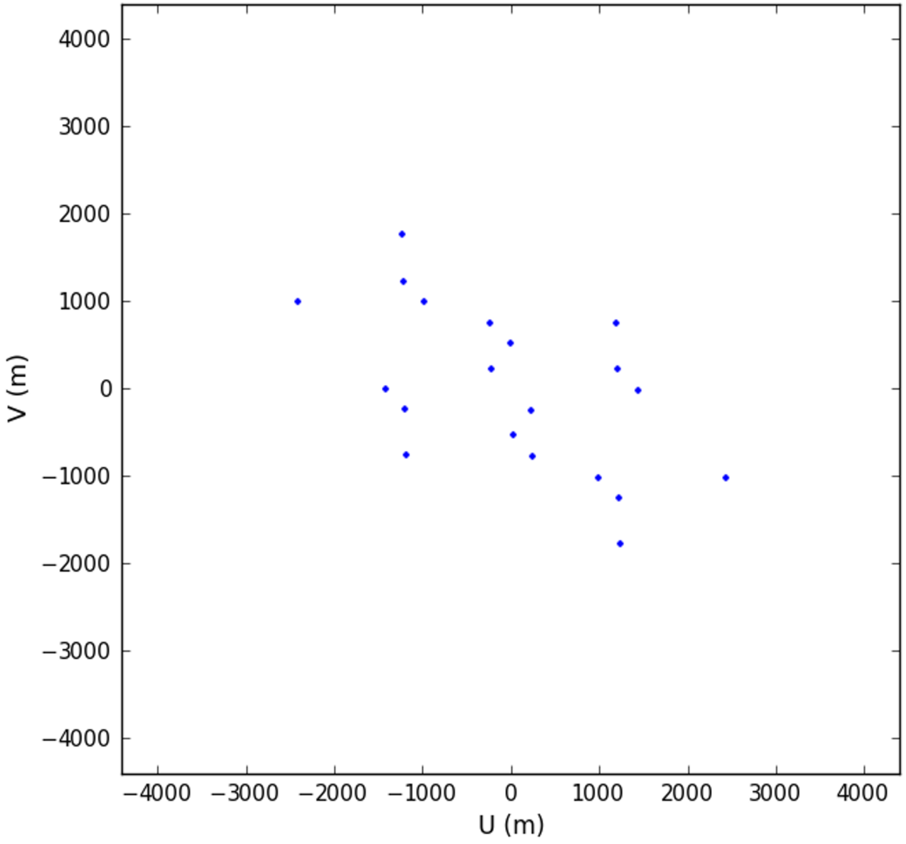
$I^{obs}(l,m)$

Spatial Frequency (uv) coverage + Observed Image

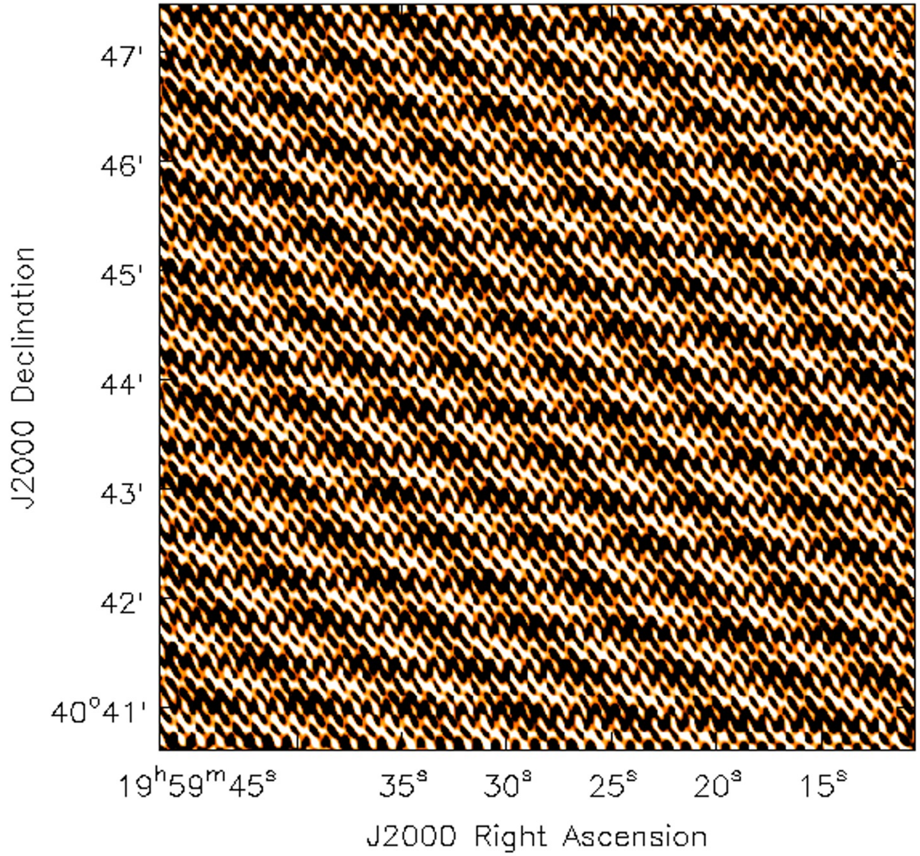
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda}\left[R(h,\theta)\right]\begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 5 antennas





$S(u,v)$

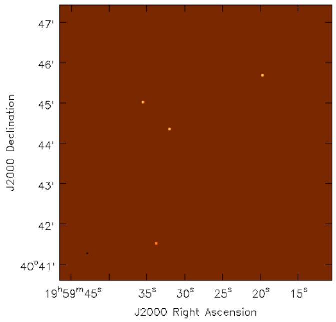


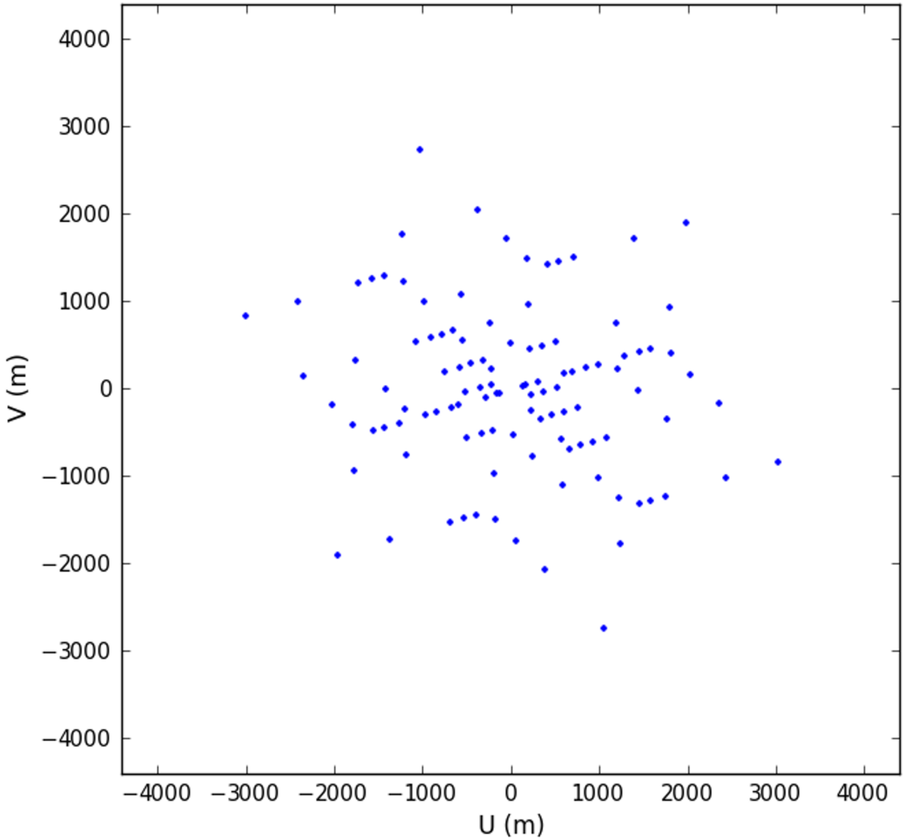
$I^{obs}(l,m)$

Spatial Frequency (uv) coverage + Observed Image

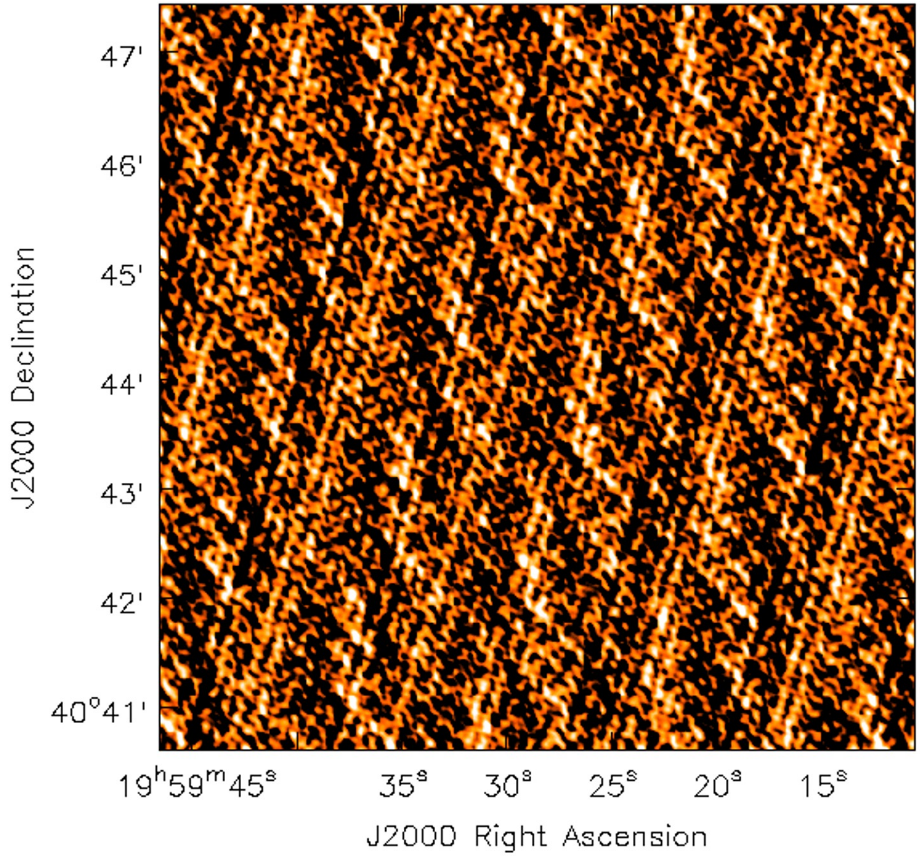
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \left[R(h,\theta) \right] \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 11 antennas





$S(u,v)$

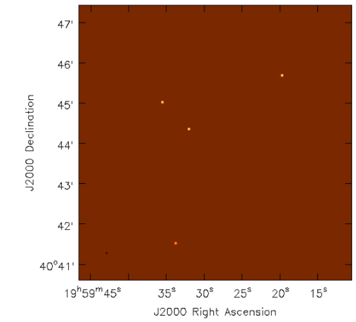


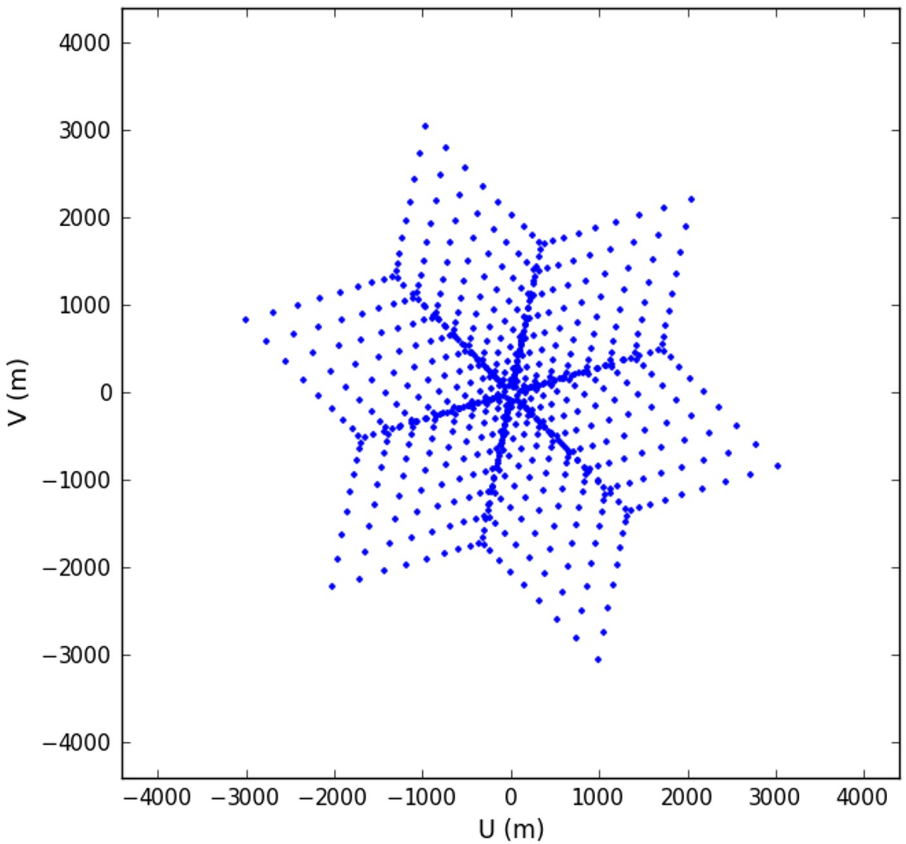
$I^{obs}(l,m)$

Spatial Frequency (uv) coverage + Observed Image

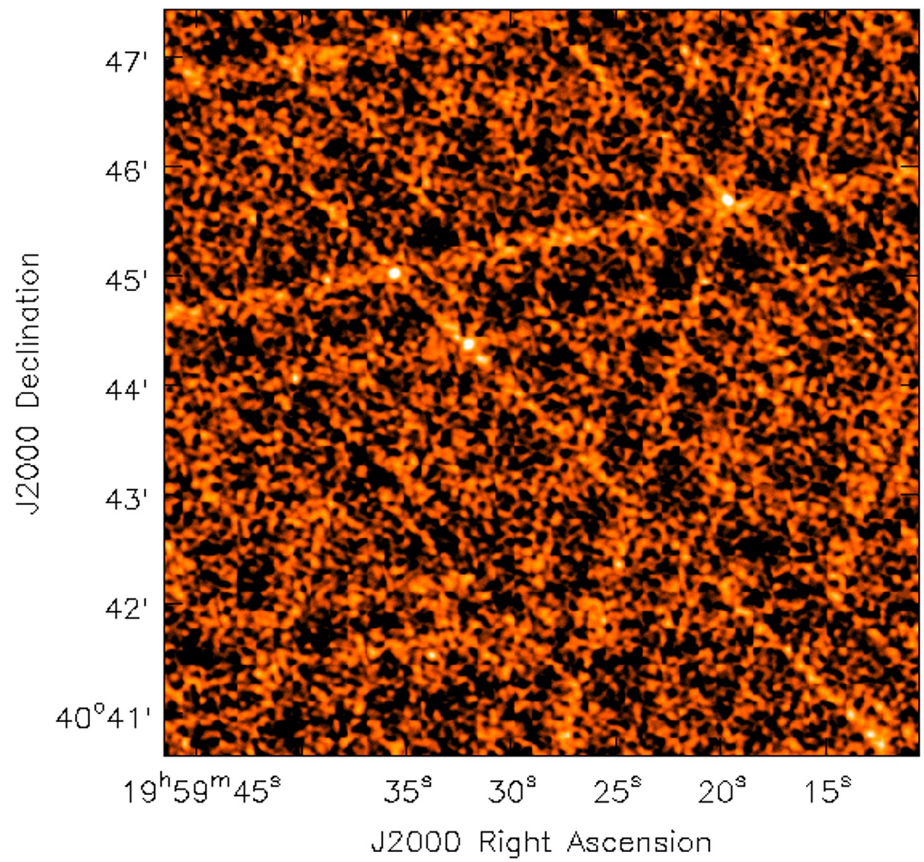
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \left[R(h, \theta) \right] \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 27 antennas





$S(u, v)$

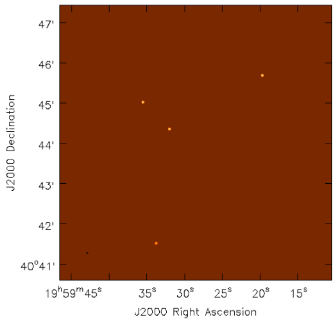


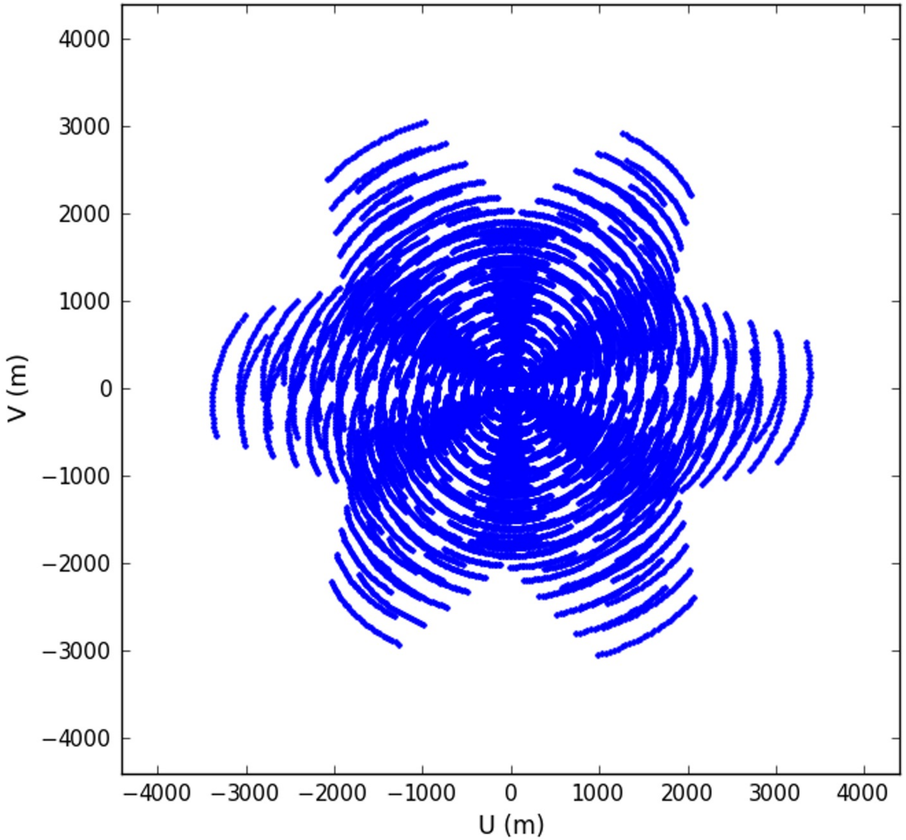
$I^{obs}(l, m)$

Spatial Frequency (uv) coverage + Observed Image

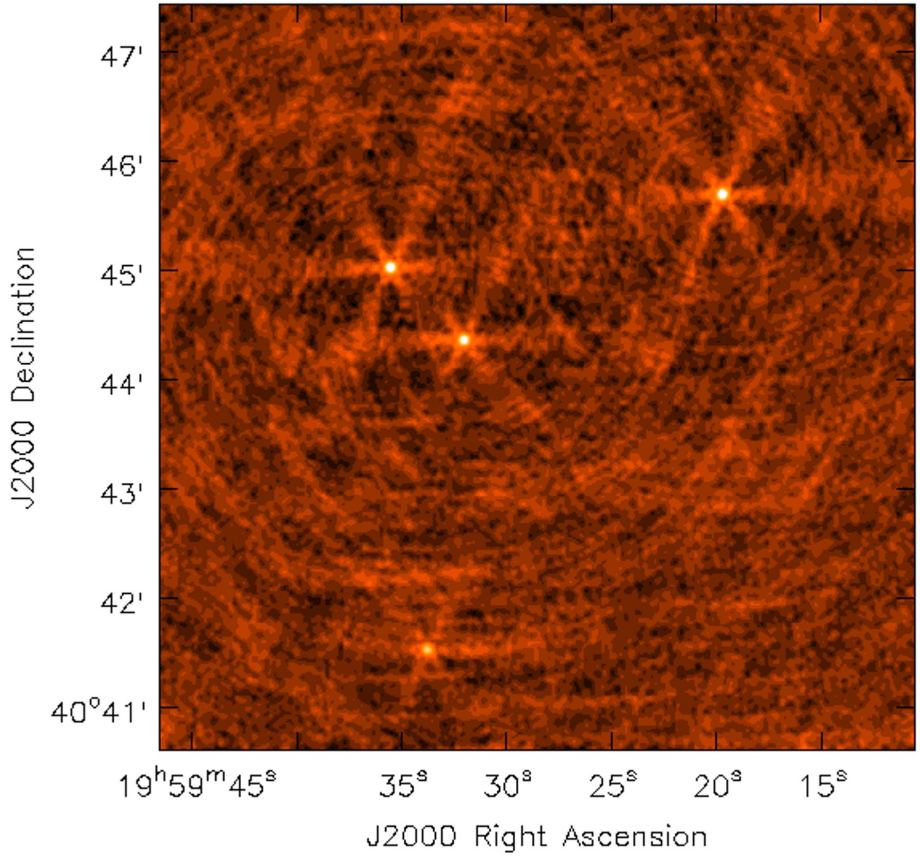
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \left[R(h,\theta) \right] \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky
using 27 antennas
over 2 hours
'Earth Rotation Synthesis'





$S(u,v)$

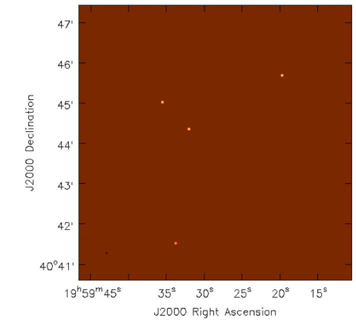


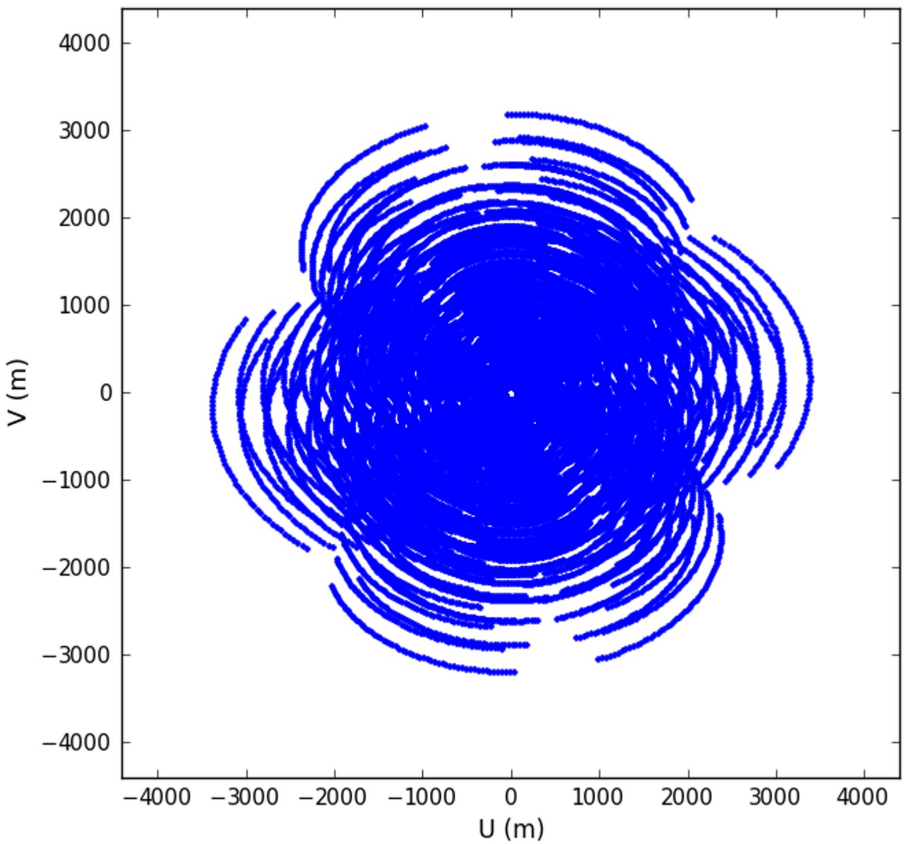
$I^{obs}(l,m)$

Spatial Frequency (uv) coverage + Observed Image

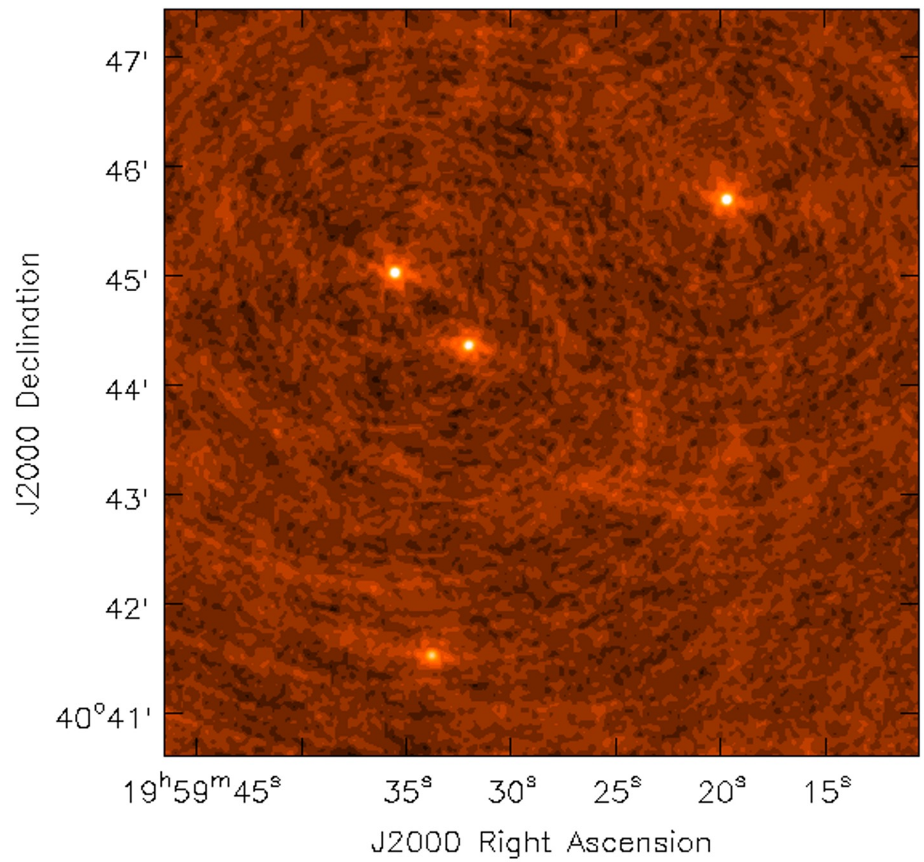
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h,\theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky
using 27 antennas
over 4 hours
'Earth Rotation Synthesis'





$S(u,v)$

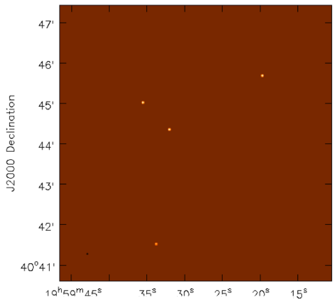


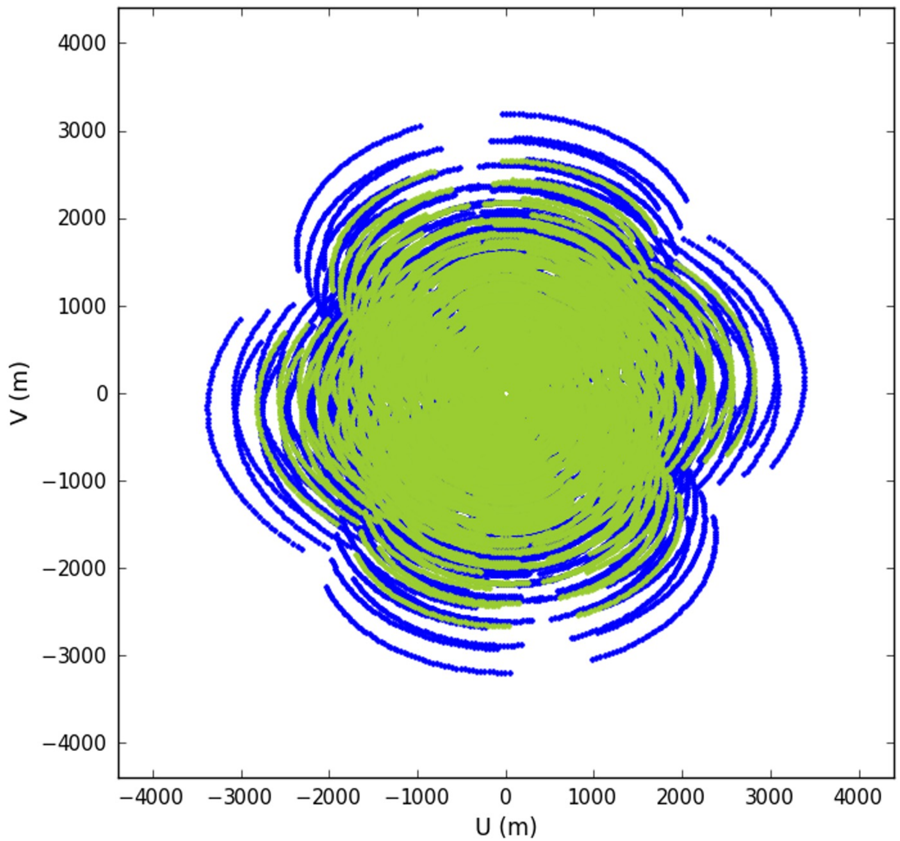
$I^{obs}(l,m)$

Spatial Frequency (uv) coverage + Observed Image

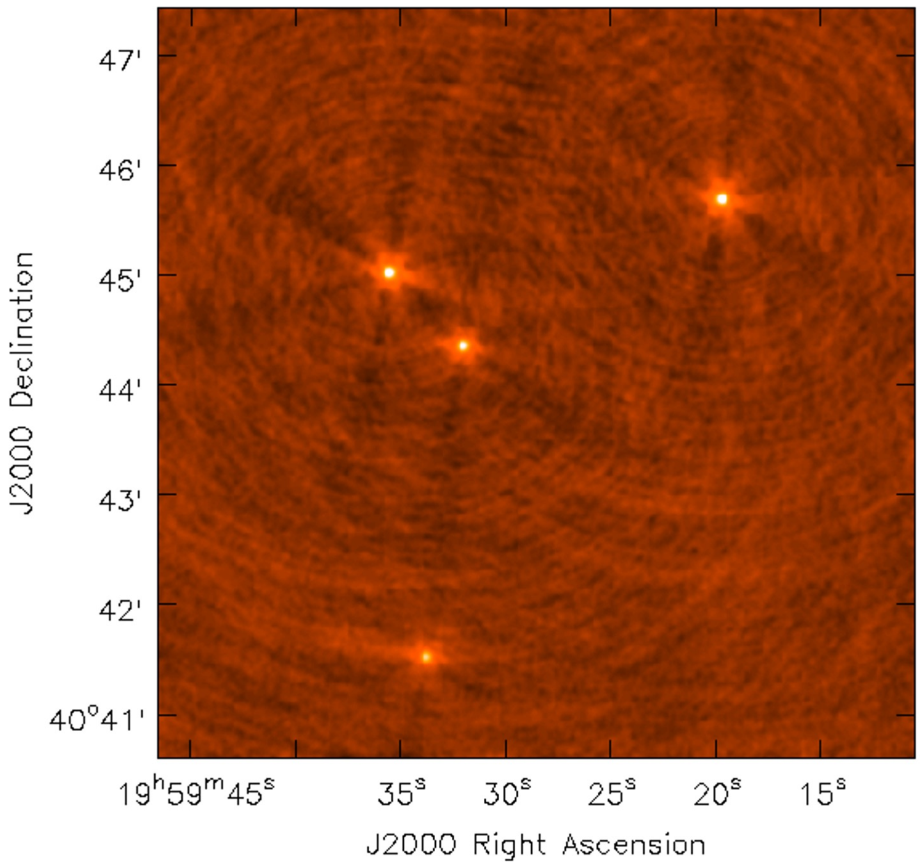
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \left[R(h,\theta) \right] \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky
using 27 antennas
over 4 hours, 2 freqs
'Multi-Frequency Synthesis'





$S(u,v)$

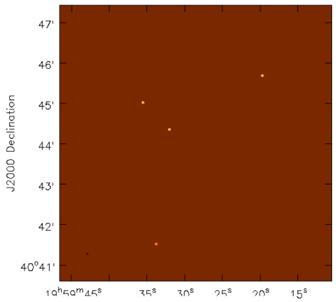


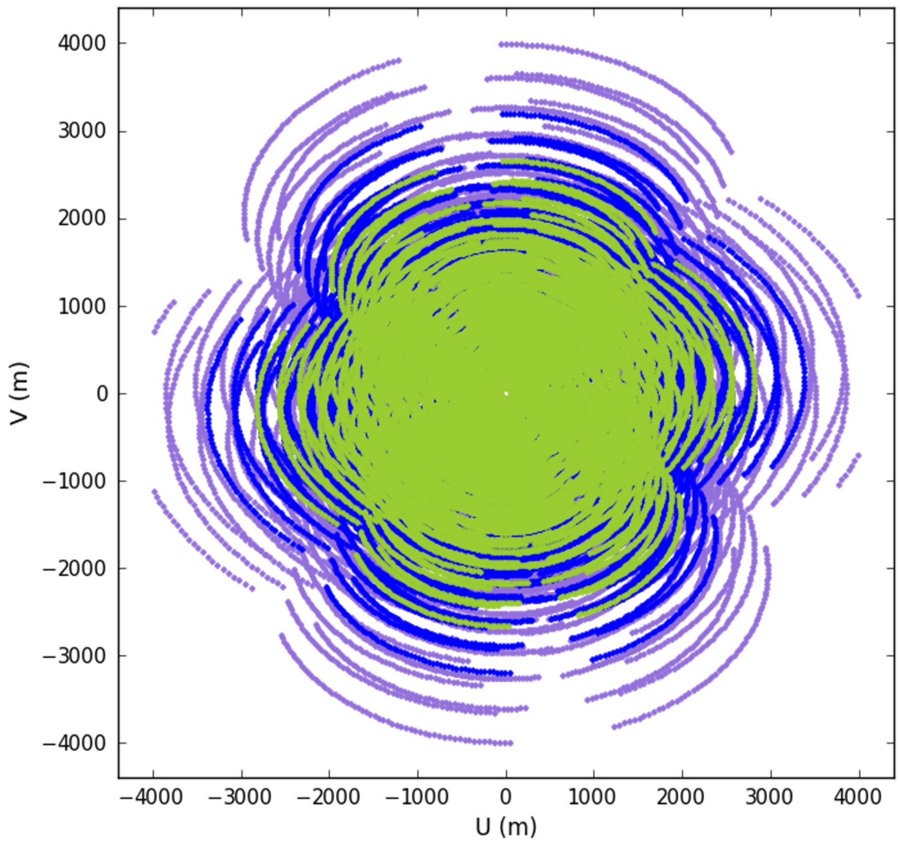
$I^{obs}(l,m)$

Spatial Frequency (uv) coverage + Observed Image

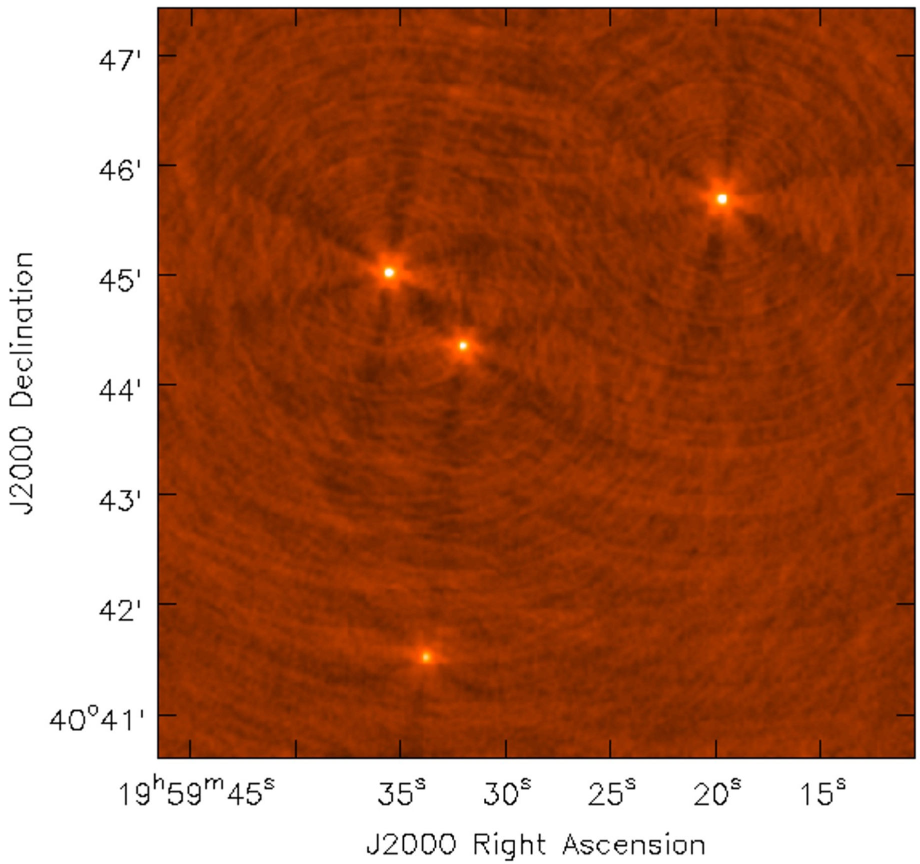
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \left[R(h,\theta) \right] \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky
using 27 antennas
over 4 hours, 3 freqs
'Multi-Frequency Synthesis'





$S(u,v)$



$I^{obs}(l,m)$

Image Reconstruction

Imaging with modern instruments

Basic Imaging :

Narrow-frequency range, Small region of the sky

=> The 2D Fourier Transform relations hold
=> Convolution and deconvolution

Wide-Band Imaging :

=> Sky and instrument change across frequency range

Wide-Field Imaging

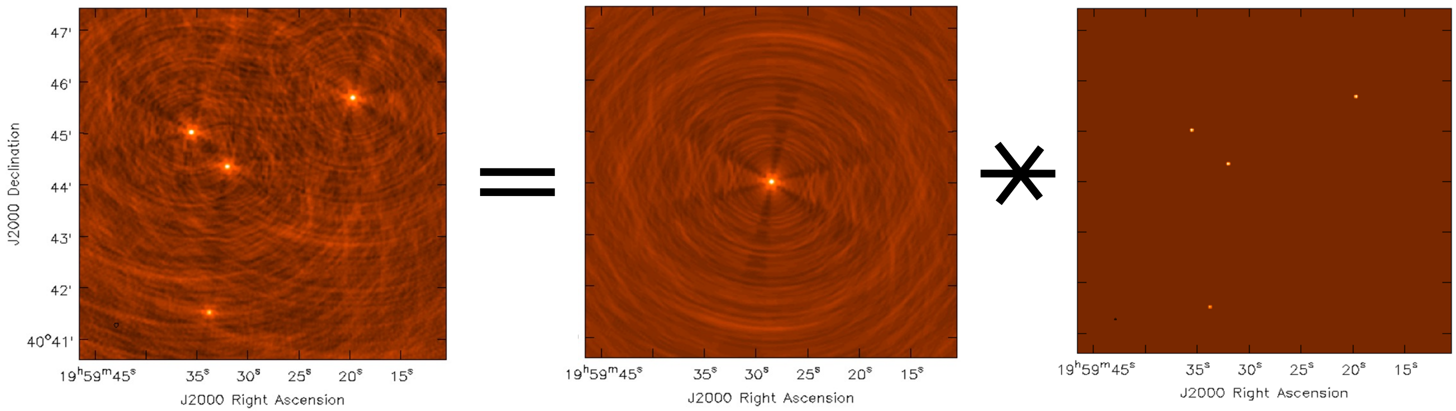
=> The 2D Fourier Transform relation breaks

Mosaic Imaging

=> Image an area larger than what each antenna can see.

Image formed by an interferometer : Convolution Equation

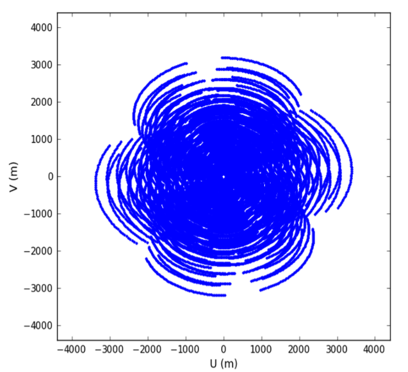
$$I^{obs}(l,m) = I^{PSF}(l,m) * I^{sky}(l,m)$$



You have measured the Convolution of the True Sky with the instrumental PSF.

PSF = Point Spread Function

Inverse Fourier transform of the UV-coverage



$S(u,v)$

Recovering True Sky = DE-convolution

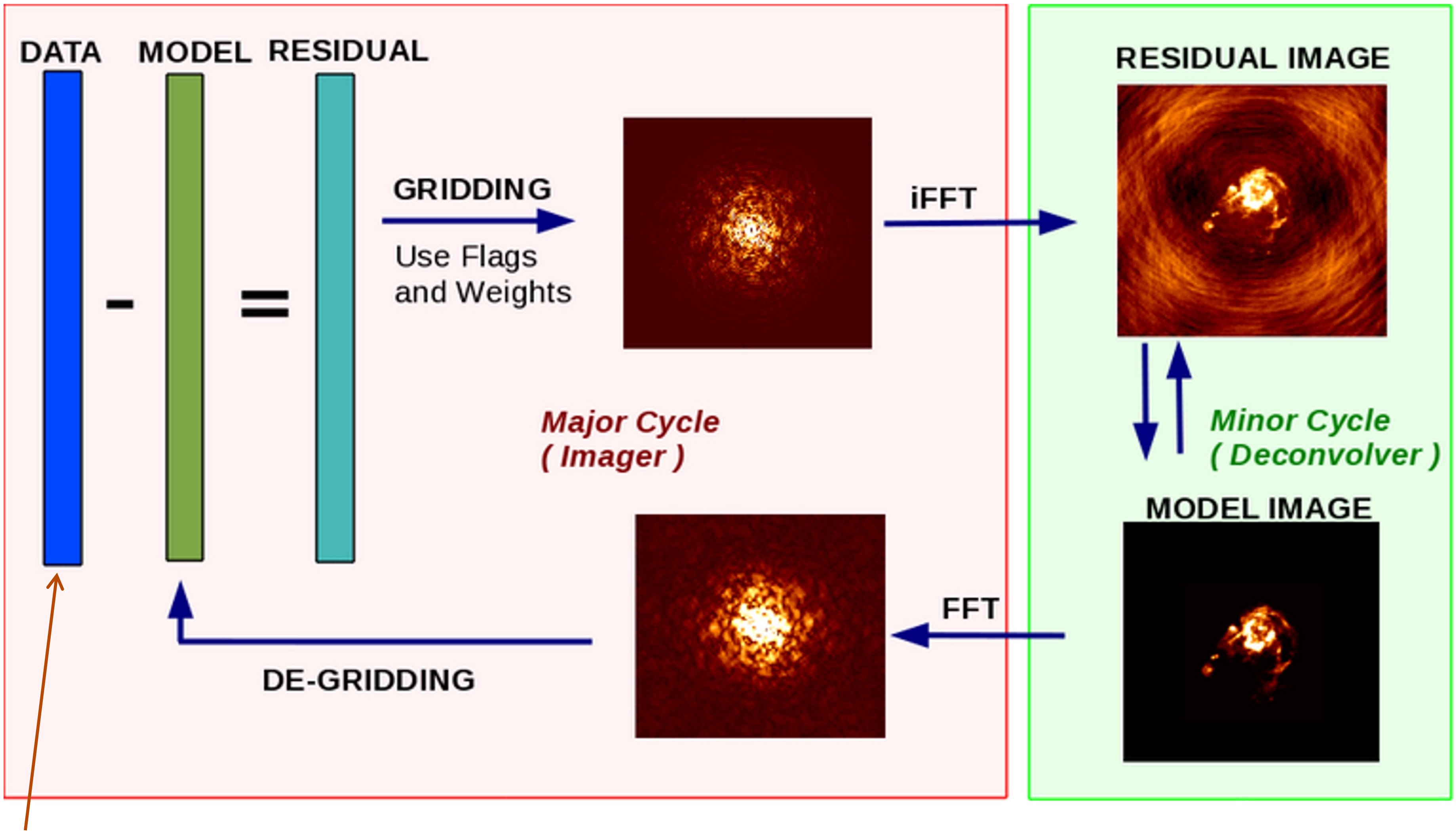
” Model Fitting “

- Parameterize the sky structure
- Iteratively build a “model” of the structure by minimizing Chi-square

Reconstruction = Imaging + Deconvolution

Data = Incomplete set of samples of the true signal

Image Reconstruction = Fit a model (of the sky) to the data



$\langle E_i E_j^* \rangle, \vec{u}, \vec{v}$

Deconvolution – Hogbom CLEAN

Sky Model : List of delta-functions

(1) Construct the observed (dirty) image and PSF

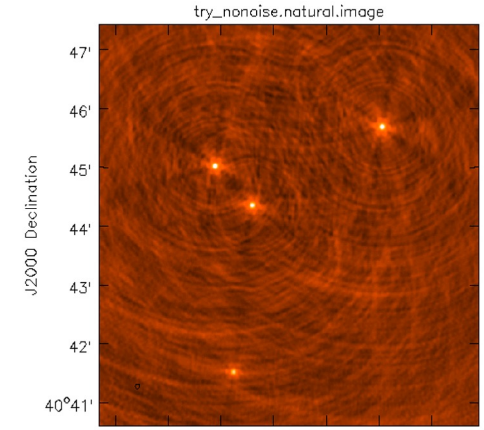
(2) Search for the location of peak amplitude.

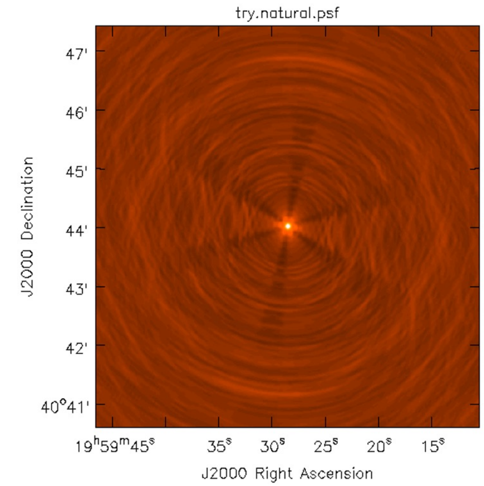
(3) Add a delta-function of this peak/location to the model

(4) Subtract the contribution of this component
from the dirty image - a scaled/shifted copy of the PSF

Repeat steps (2), (3), (4) until a stopping criterion is reached.

(5) Restore : Smooth the model with a 'clean beam' and
add residuals





The CLEAN algorithm can be formally derived as a model-fitting problem

– model parameters : locations and amplitudes of delta functions

– solution process : χ^2 minimization via an iterative steepest-descent algorithm
(method of successive approximation)

Deconvolution – Comparison of Algorithms

CLEAN

Point source model

MEM

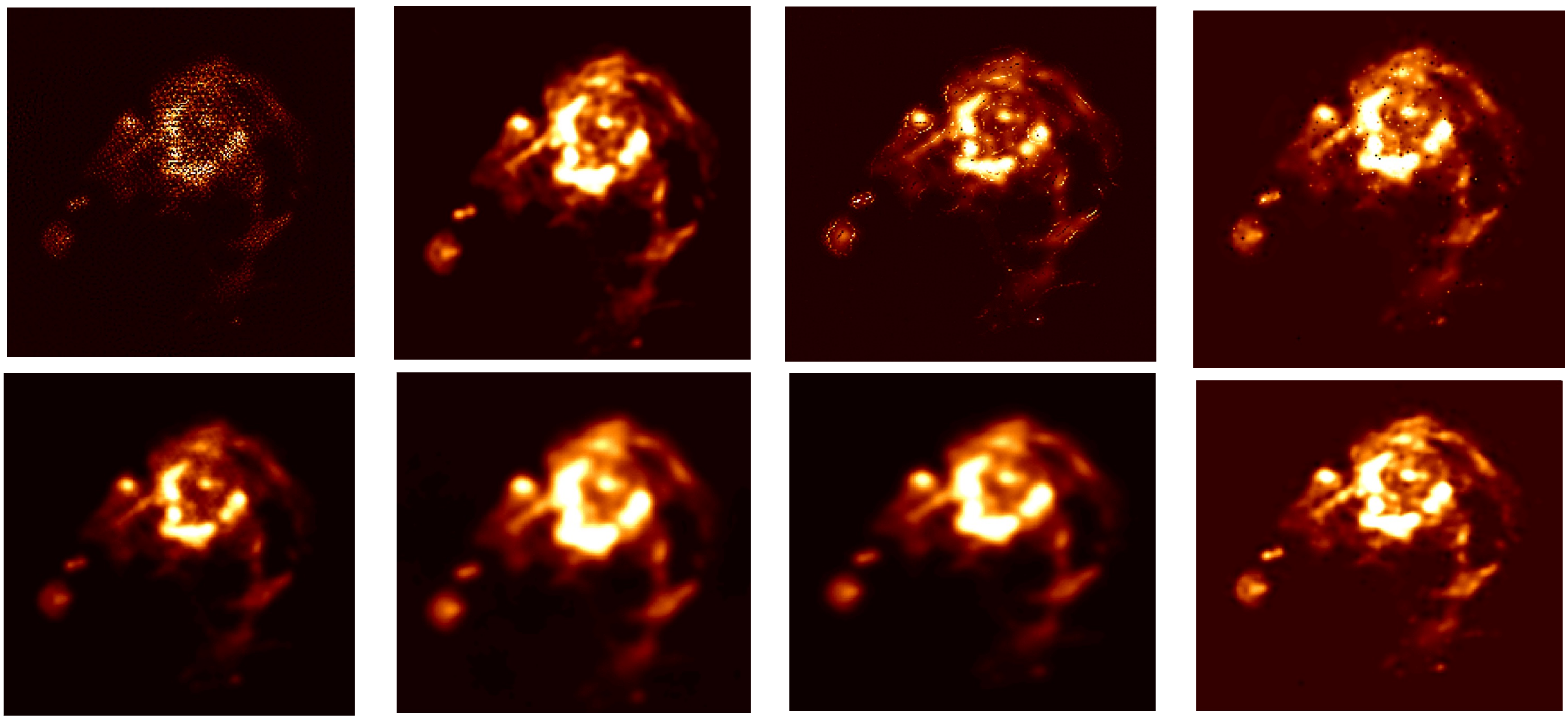
Point source model with a smoothness constraint

MS-CLEAN

Multi-Scale model with a fixed set of scale sizes

ASP

Multi-Scale model with adaptive best-fit scale per component



I^m

I^{out}

Deconvolution – Comparison of Algorithms

CLEAN

Point source model

MEM

Point source model with a smoothness constraint

MS-CLEAN

Multi-Scale model with a fixed set of scale sizes

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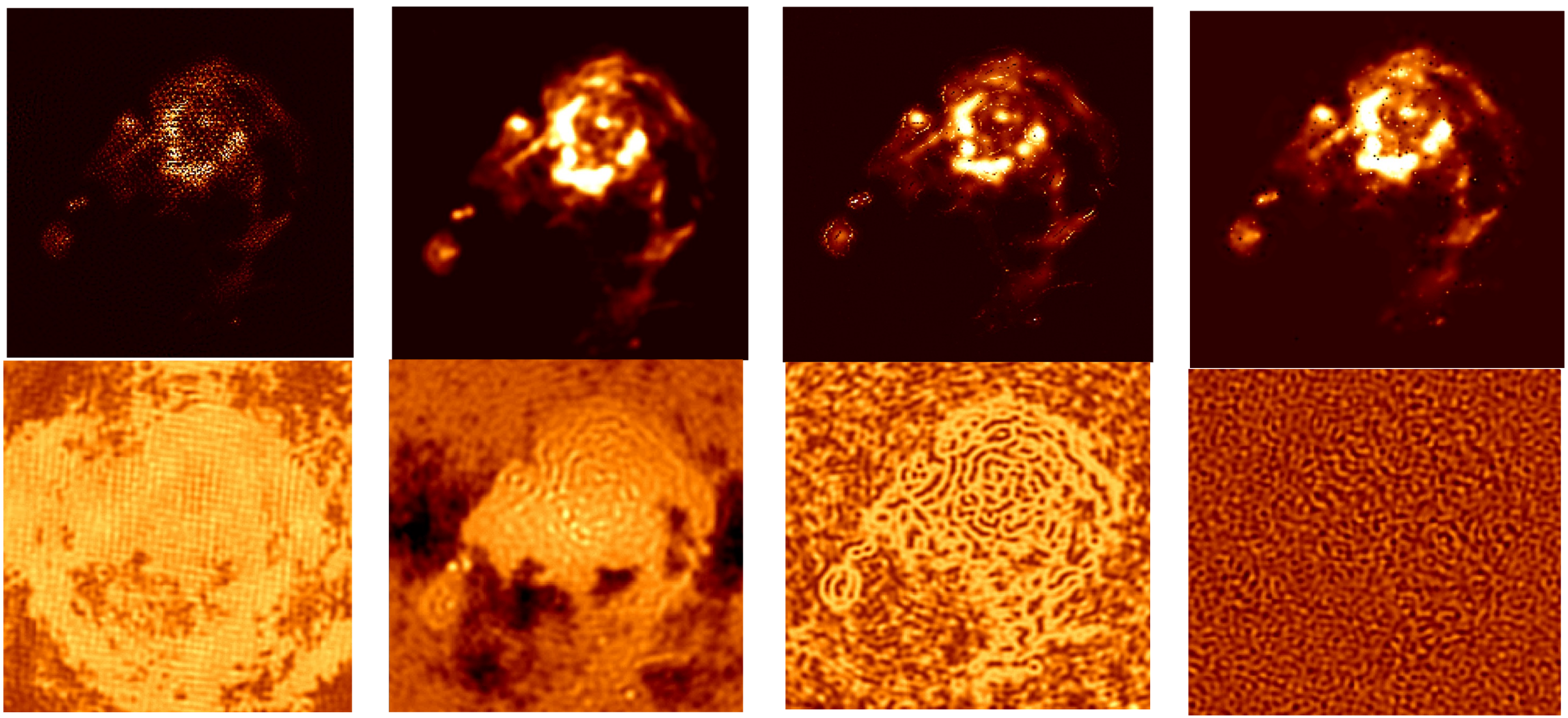
Multi-Scale model with adaptive best-fit scale per component



I^m

I^{res}

Imaging with modern instruments

Basic Imaging :

Narrow-frequency range, Small region of the sky

=> The 2D Fourier Transform relations hold
=> Convolution and deconvolution

Wide-Band Imaging :

=> Sky and instrument change across frequency range

Wide-Field Imaging

=> The 2D Fourier Transform relation breaks

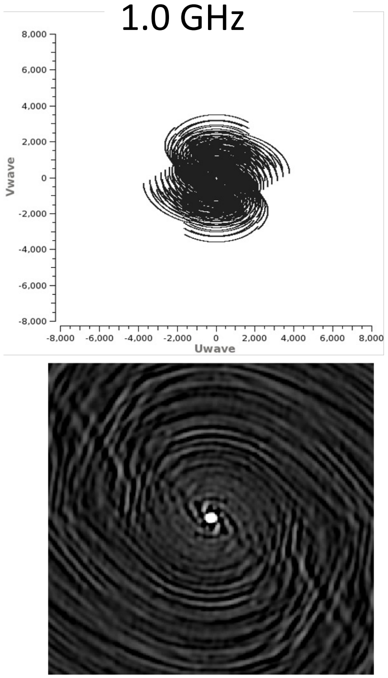
Mosaic Imaging

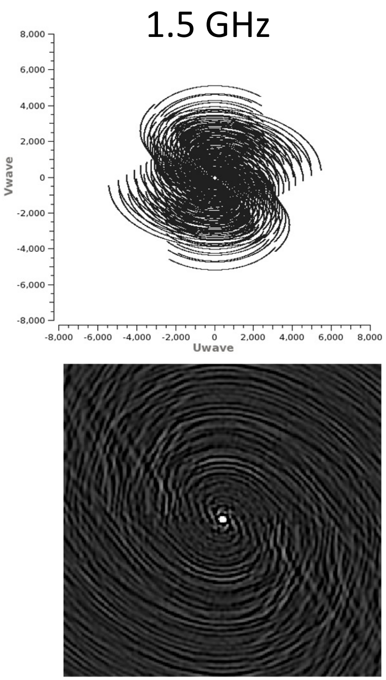
=> Image an area larger than what each antenna can see.

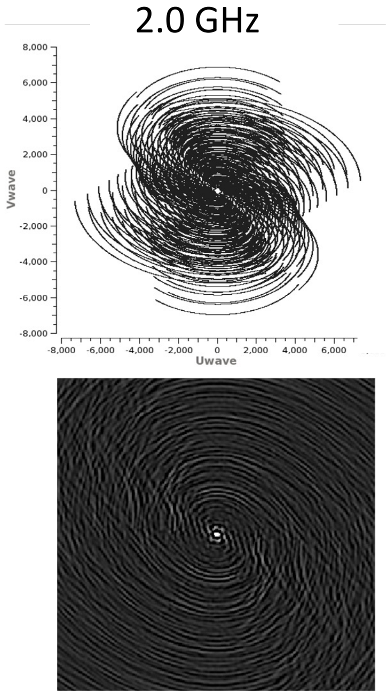
Wide-band Imaging – Sensitivity and Multi-Frequency Synthesis

Frequency Range :	(1 – 2 GHz)	(4 – 8 GHz)	(8 – 12 GHz)
Bandwidth : $\nu_{max} - \nu_{min}$	1 GHz	4 GHz	4 GHz
Bandwidth Ratio : $\nu_{max} : \nu_{min}$	2 : 1	2 : 1	1.5 : 1
Fractional Bandwidth : $(\nu_{max} - \nu_{min}) / \nu_{mid}$	66%	66%	40%

UV-coverage / imaging properties change with frequency

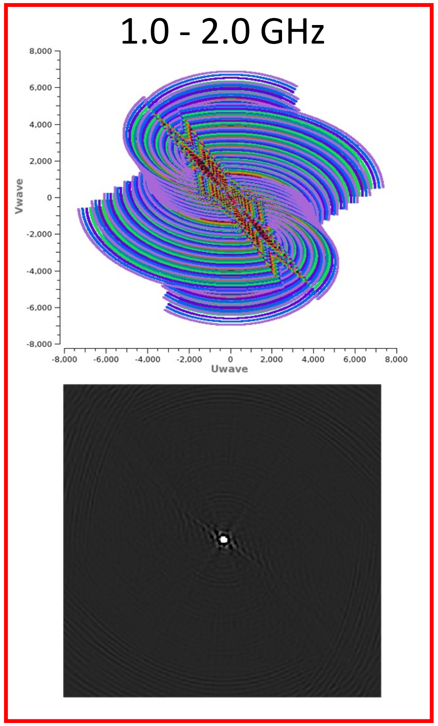






$$S(u,v)_\nu = \frac{\vec{b}}{\lambda} = \frac{\vec{b}\nu}{c}$$

=> combine multi-frequency measurements during imaging



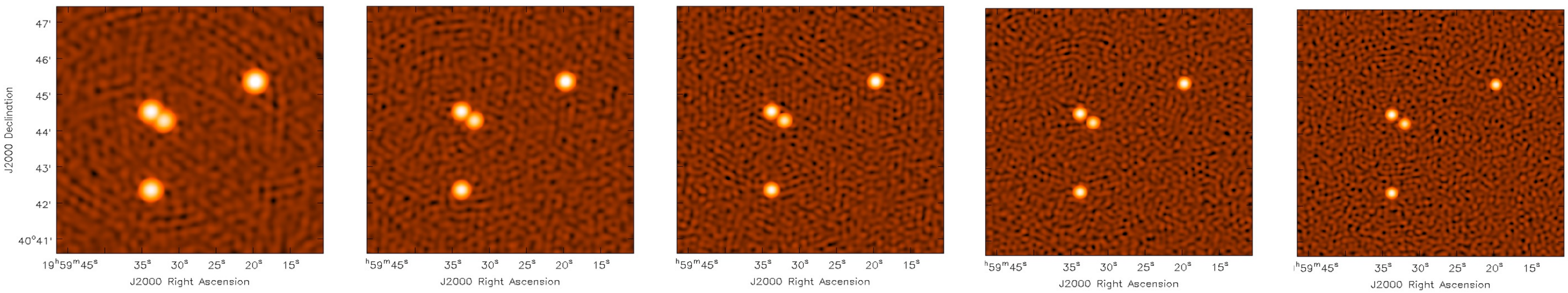
Sky Brightness can also change with frequency → model intensity and spectrum

Spectral Cube (vs) MFS imaging

3 flat-spectrum sources + 1 steep-spectrum source (1-2 GHz VLA observation)

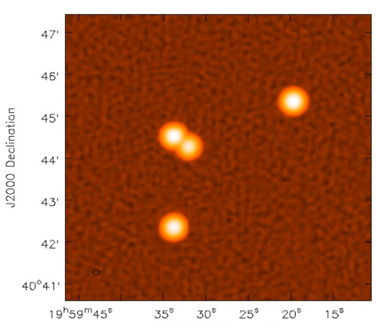
Images made at different frequencies (limited to narrow-band sensitivity)

1 GHz

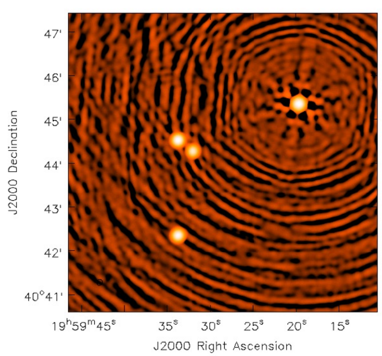


2 GHz

Add all single-frequency images (after smoothing to a low resolution)

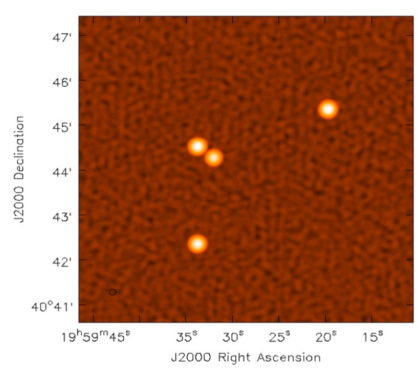


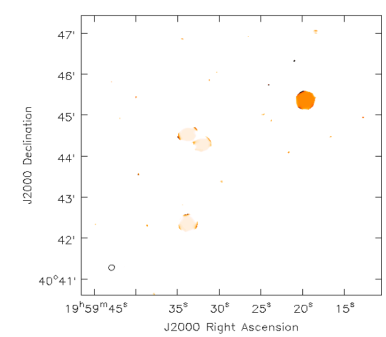
Use wideband UV-coverage, but ignore spectrum (MFS, nterms=1)



Use wideband UV-coverage + Model and fit for spectra too (MT-MFS, nterms > 1)

Output : Intensity and Spectral-Index





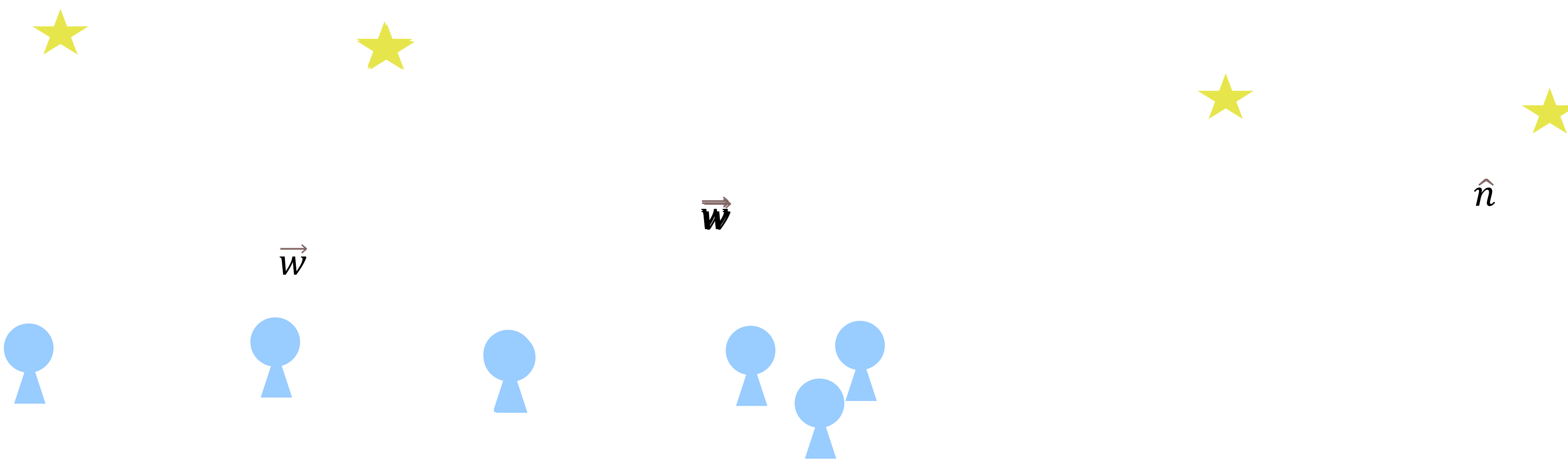
Wide-Field Imaging – W-term

$$V^{obs}(u,v) = S(u,v) \iint I(l,m) e^{2\pi i (ul+vm)} dl dm$$

$$V^{obs}(u,v) = S(u,v) \iiint I(l,m) e^{2\pi i (ul+vm+w(n-1))} dl dm dn$$

The ' w ' of a baseline can be large, away from the image phase center

The ' n ' for a source can be large, away from the image phase center



There are algorithms to account for this : Image Faceting, W-Projection.

Wide-Field Imaging – W-term

$$V^{obs}(u,v) = S(u,v) \iint I\,(l,m) e^{2\pi i (ul+vm)} dldm$$

$$V^{obs}(u,v) = S(u,v) \iiint I\,(l,m) e^{2\pi i (ul+vm+w(n-1))} dldmdn$$

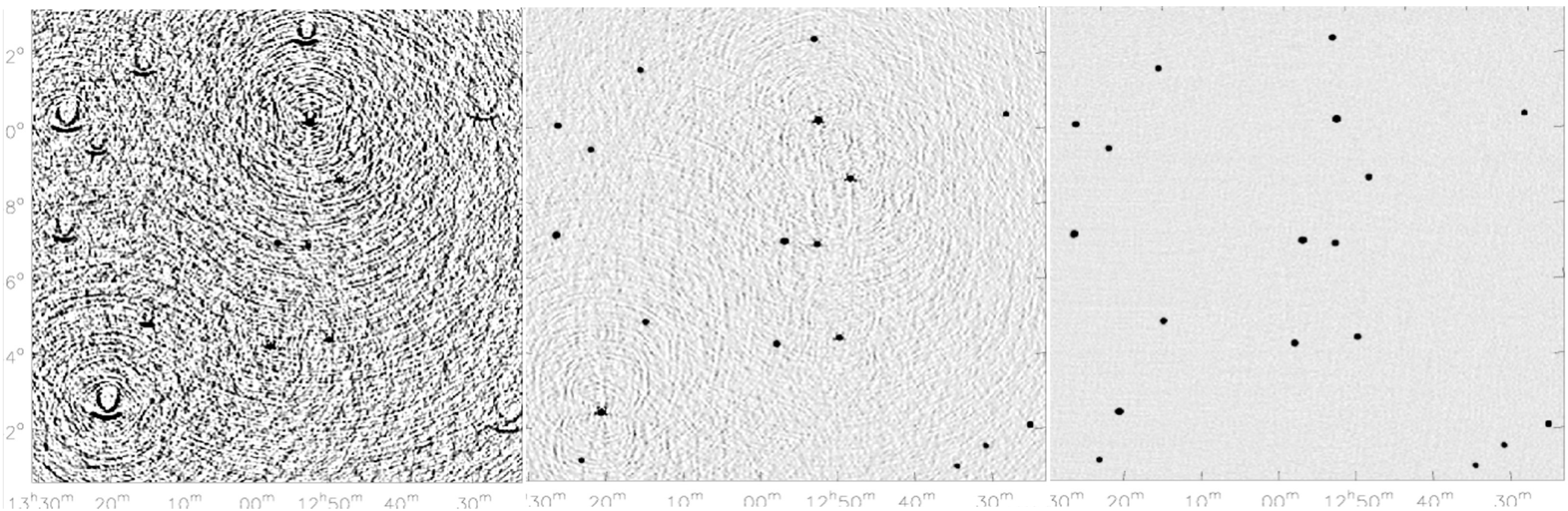
The ' w ' of a baseline can be large, away from the image phase center

The ' n ' for a source can be large, away from the image phase center

2D Imaging

Facet Imaging

W-Projection

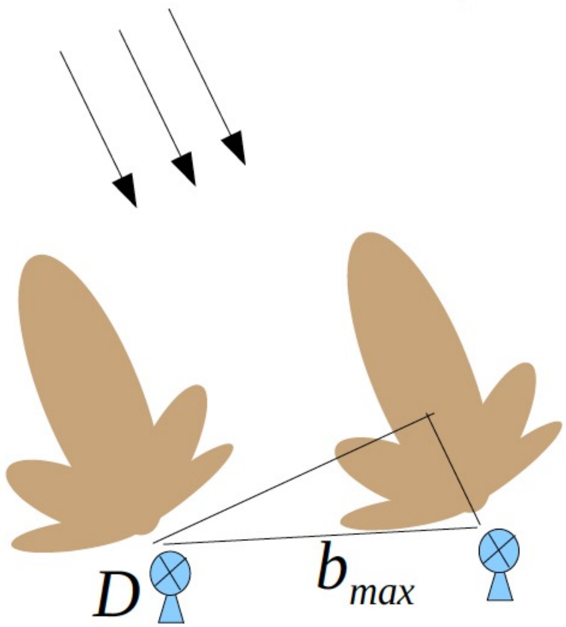


Wide-Field Imaging – Primary Beams

Each antenna has a limited field of view => Primary Beam (gain) pattern

=> Sky is (approx) multiplied by PB, before being sampled by the interferometer

$$I^{obs}(l,m) \approx I^{PSF}(l,m) * [P^{sky}(l,m) \cdot I^{sky}(l,m)]$$

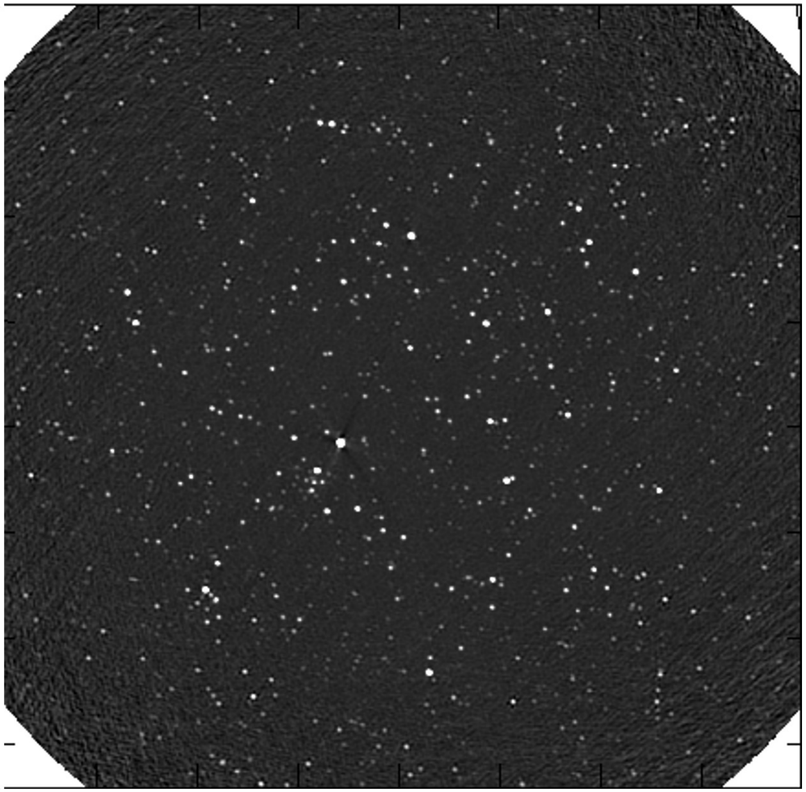


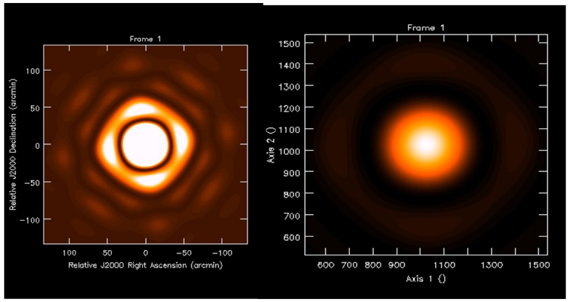
The antenna field of view :
D = antenna diameter

$$\lambda / D$$

Compare with angular resolution of the interferometer :

$$\lambda / b_{max}$$



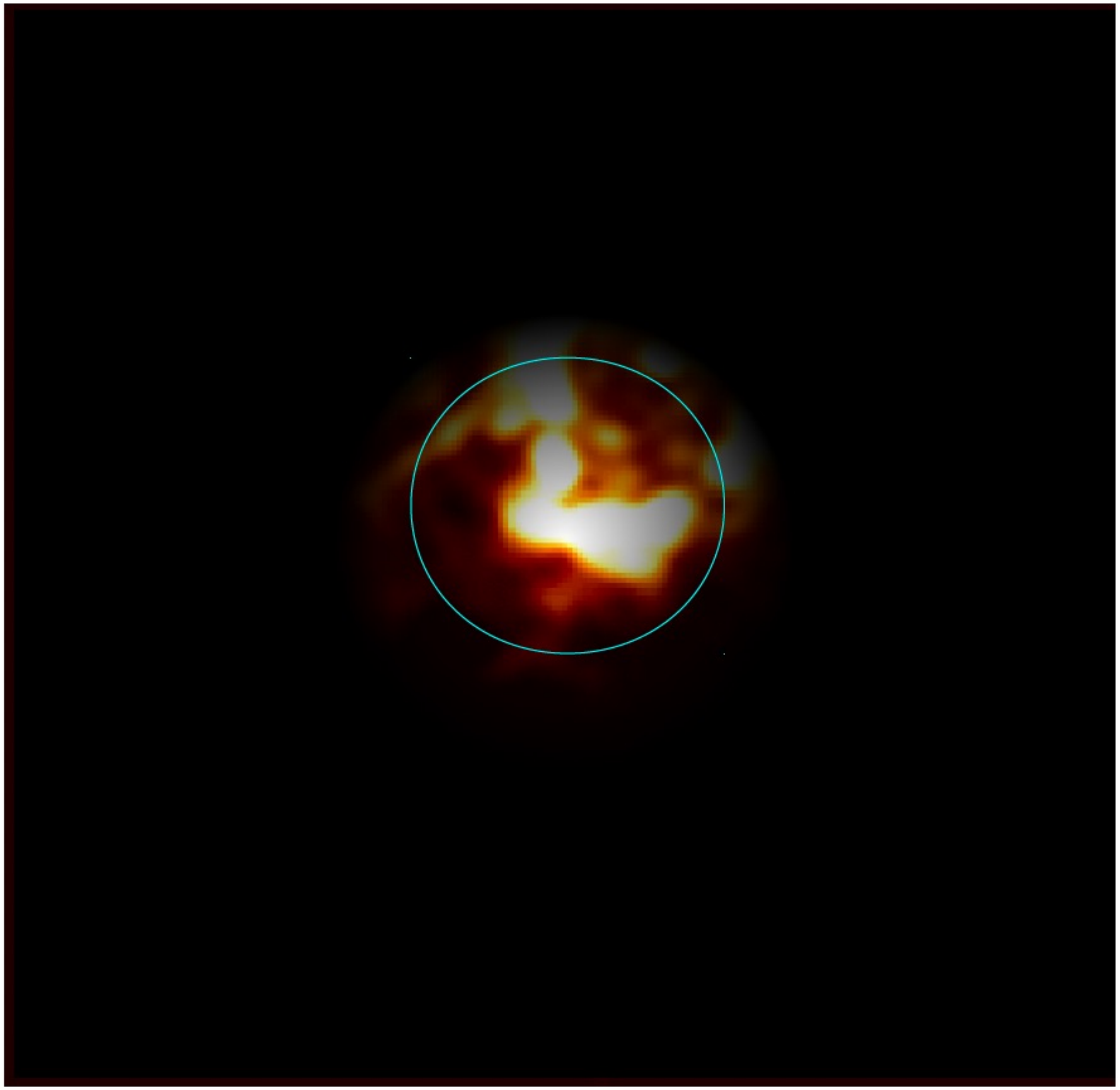


But, in reality, P changes with time, freq, pol and antenna....

=> Ignoring such effects limits dynamic range to 10^4
=> More-accurate method to account for this : A-Projection

Wide-field Imaging -- Mosaics

Combine data from multiple pointings to form one large image.



One Pointing sees only part of the source

Combine pointings either before or after deconvolution.

Stitched mosaic :

-- Deconvolve each pointing separately
-- Divide each image by PB
-- Combine as a weighted avg

Joint mosaic :

-- Combine observed images as a weighted average
(or)
Grid all data onto one UV-grid, and then iFFT

-- Deconvolve as one large image

Wide-field Imaging -- Mosaics

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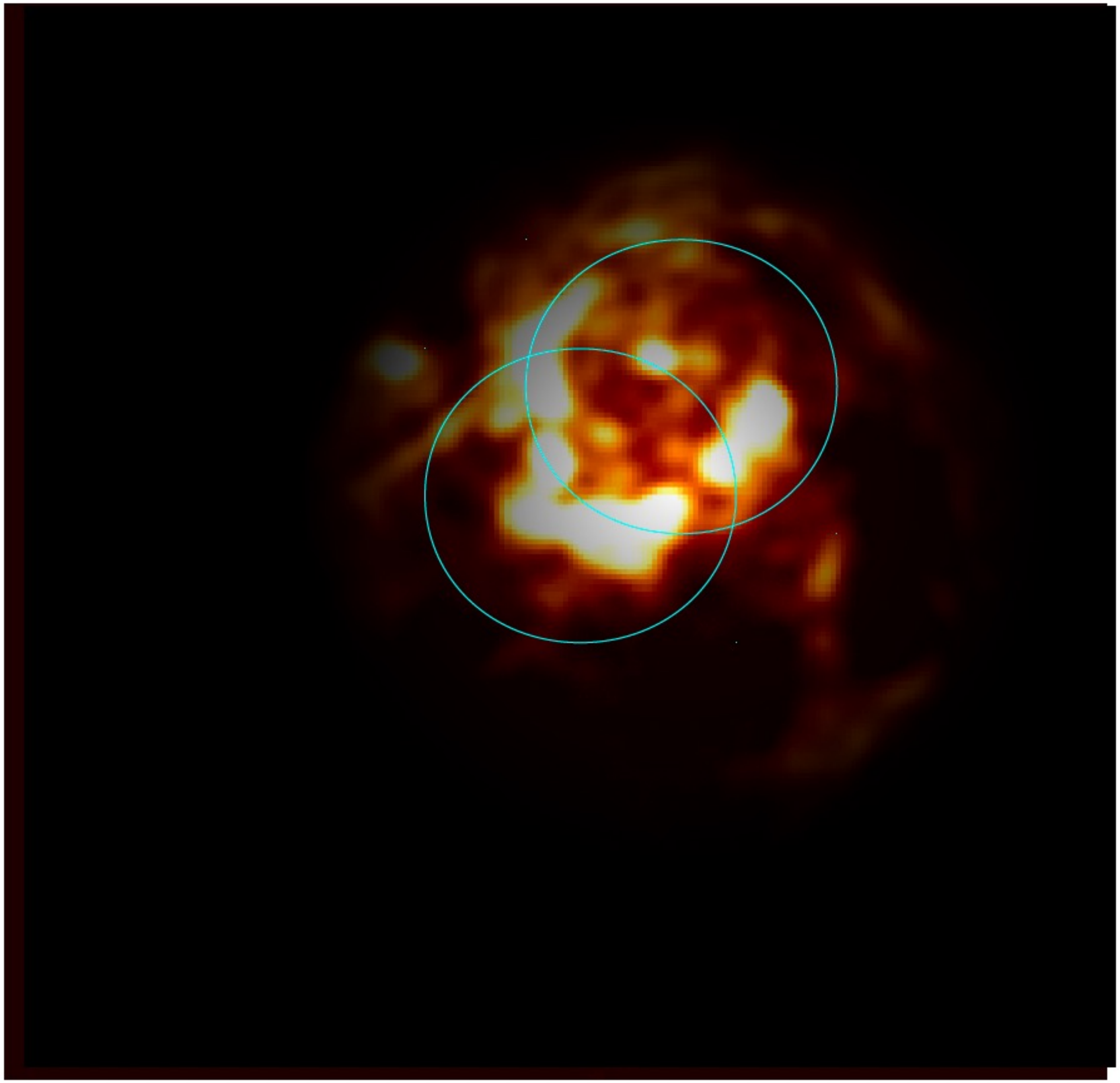

Two Pointings see more.....

Combine pointings either before or after deconvolution.

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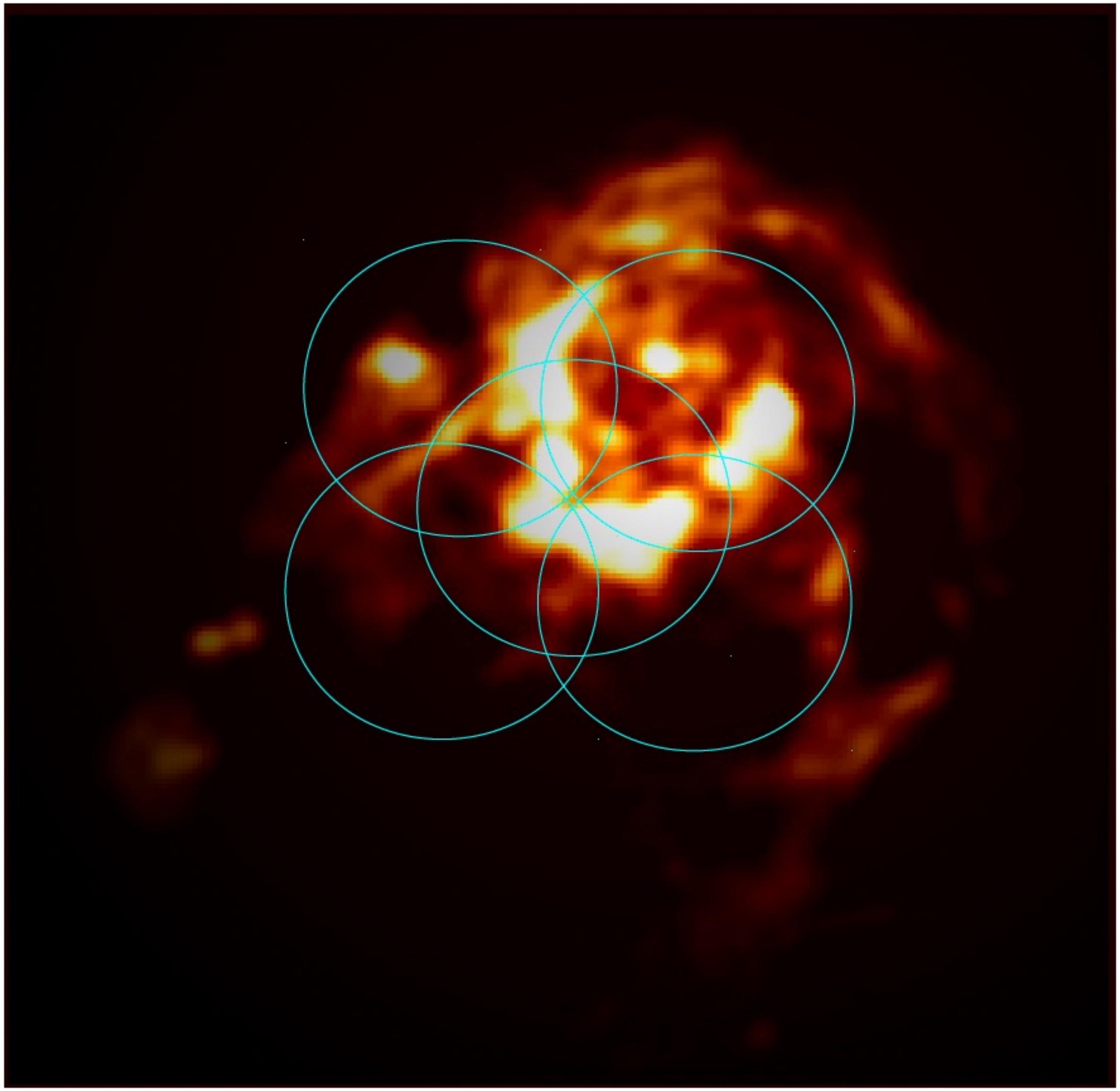

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Joint mosaic :

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(or)
Grid all data onto one UV-grid, and then iFFT

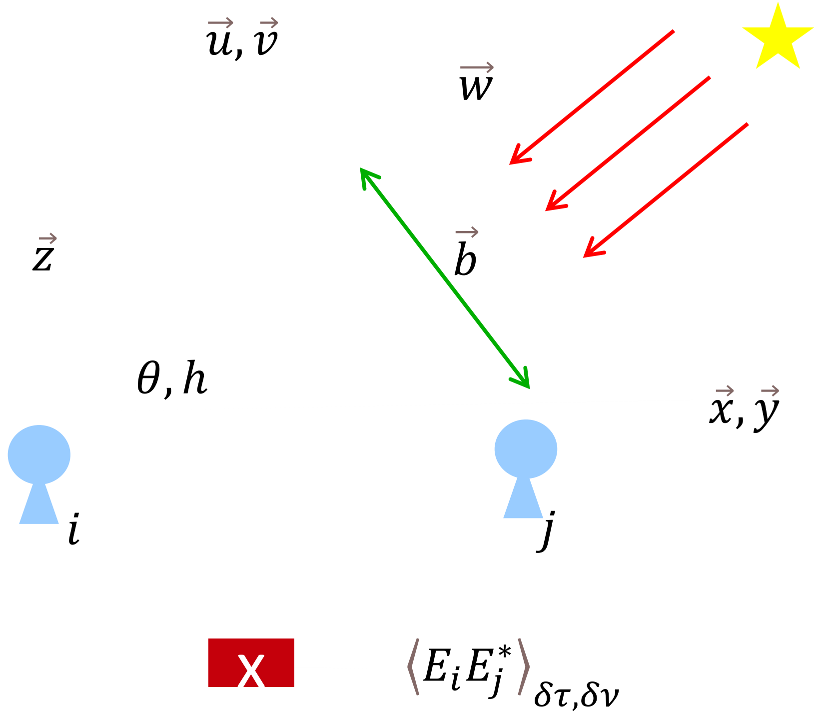
-- Deconvolve as one large image

Use many pointings to cover the source with approximately uniform sensitivity

Data Analysis

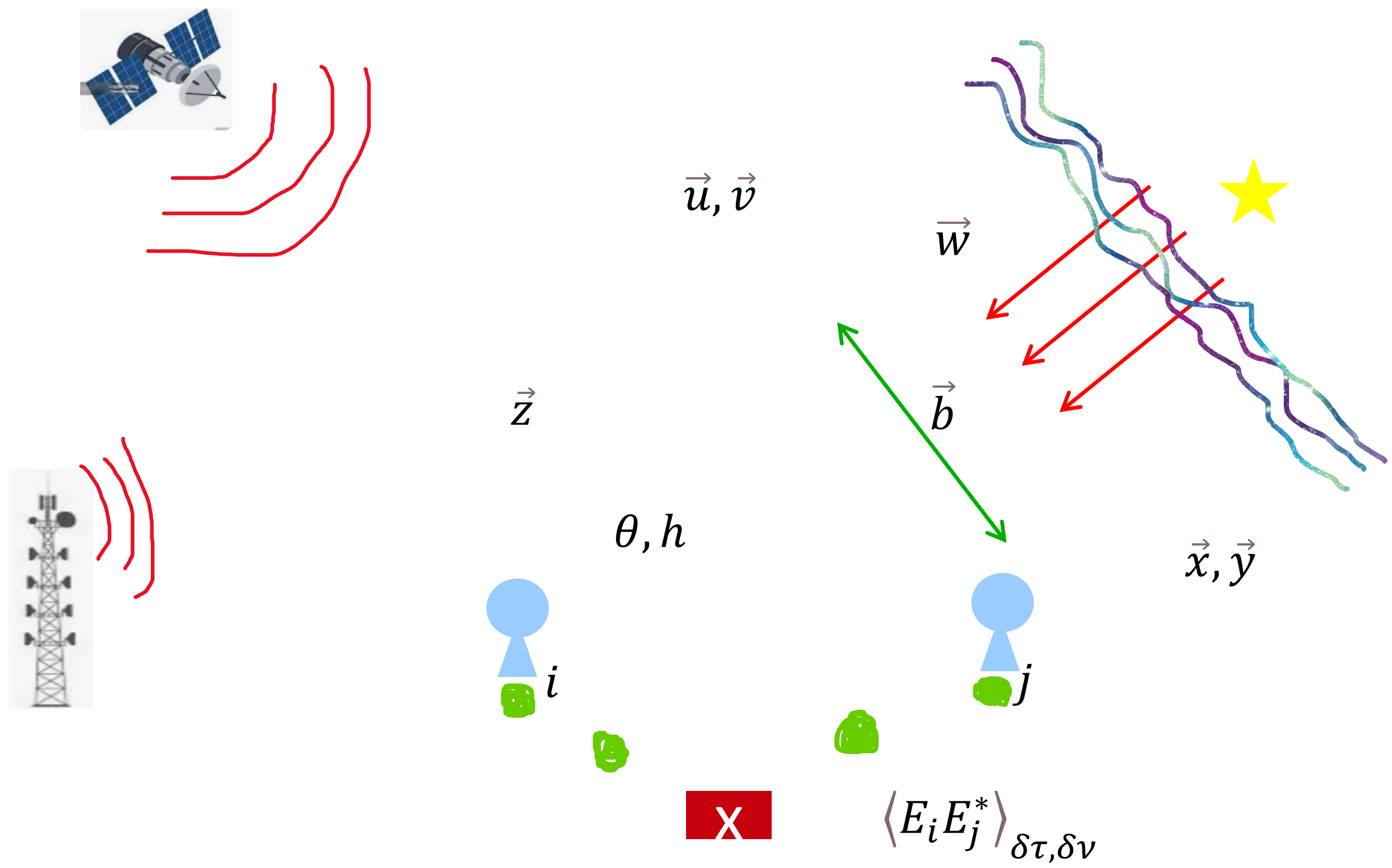
Data Analysis

The measured $\langle E_i E_j^* \rangle$ values are imperfect and incomplete



Data Analysis

The measured $\langle E_i E_j^* \rangle$ values are imperfect and incomplete



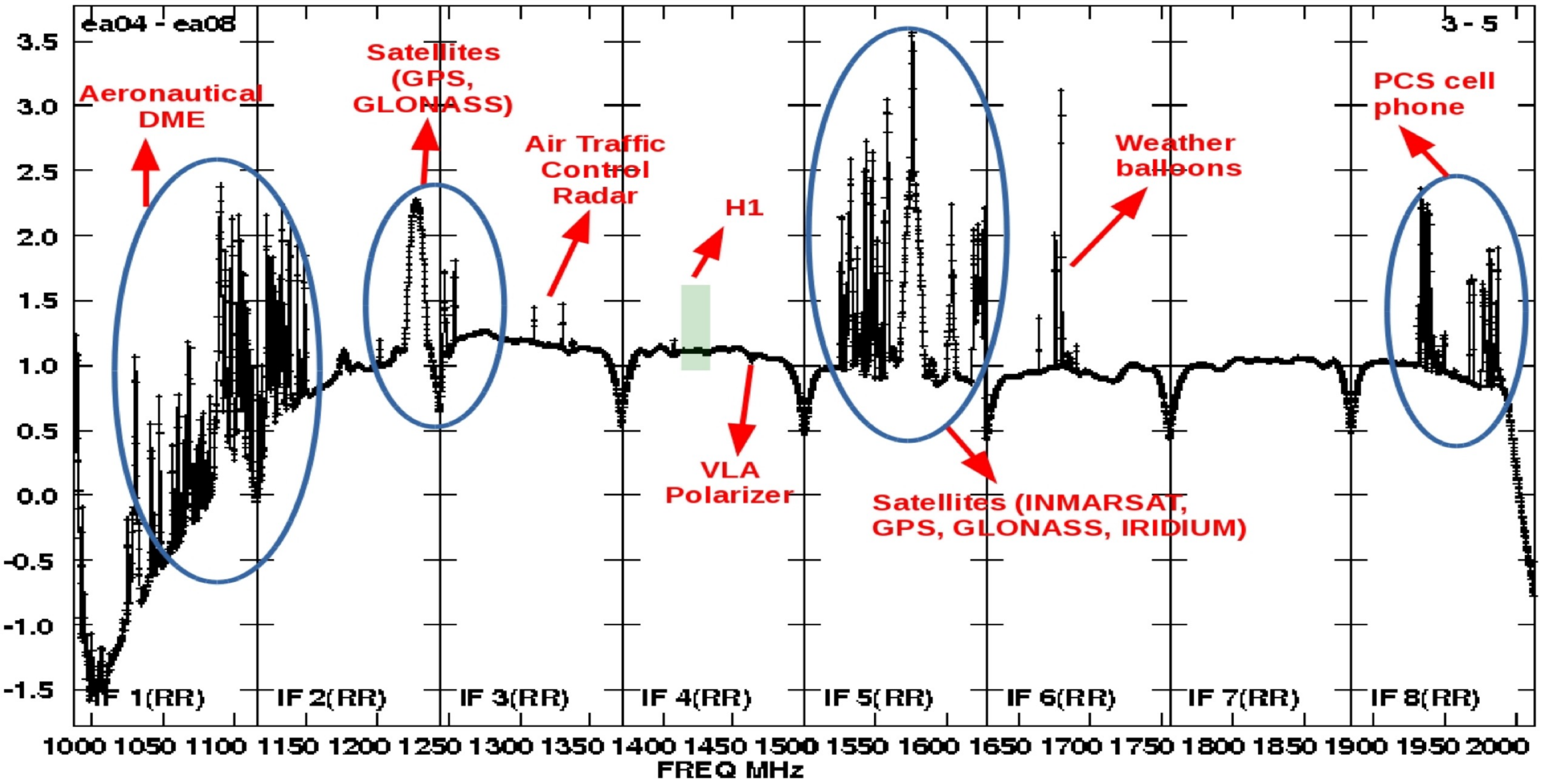
Data Analysis

The measured $\langle E_i E_j^* \rangle$ values are imperfect and incomplete

Flagging

Problem : Radio Frequency Interference (RFI)

Solution : Identify and discard corrupted data samples.



Data Analysis

The measured $\langle E_i E_j^* \rangle$ values are imperfect and incomplete

Flagging

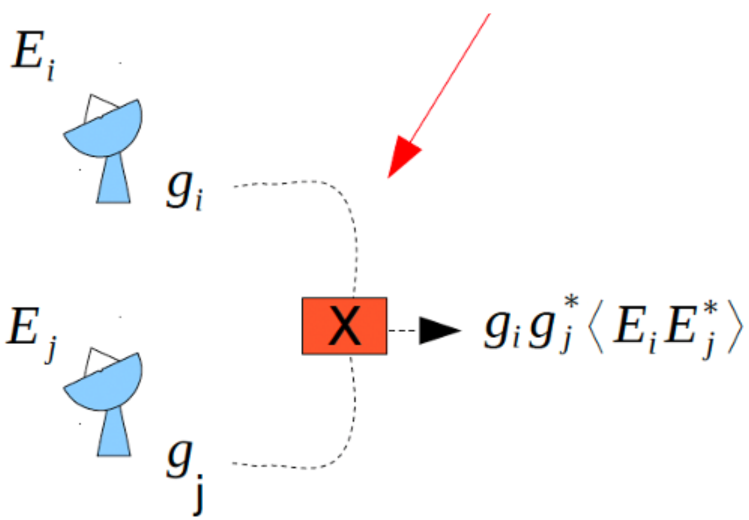
Problem : Radio Frequency Interference (RFI)

Solution : Identify and discard corrupted data samples.

Calibration

Problem : Antenna electronics introduce complex gains g_i

Solution : Numerically solve for antenna gains g_i and apply corrections



- Observe a source where $\langle E_i E_j^* \rangle$ is known
- Use information from all ij to solve for g_i
- Divide out $g_i g_j^*$ from target data

Data Analysis

The measured $\langle E_i E_j^* \rangle$ values are imperfect and incomplete

Flagging

Problem : Radio Frequency Interference (RFI)

Solution : Identify and discard corrupted data samples.

Calibration

Problem : Antenna electronics introduce complex gains g_i

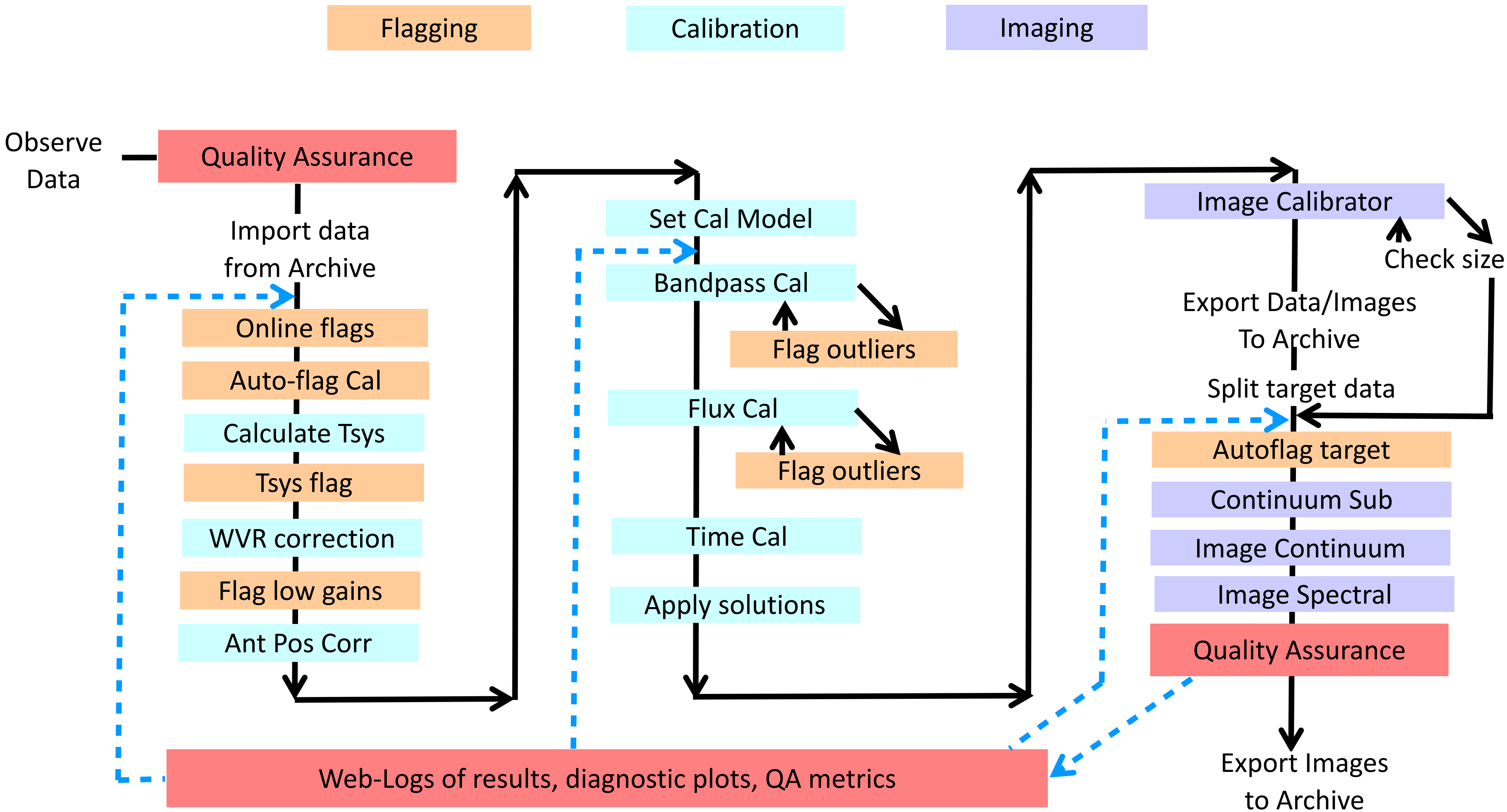
Solution : Numerically solve for antenna gains g_i and apply corrections

Imaging

Problem : Sampling of the 2D Fourier Transform is incomplete
(only N(N-1)/2 measurements per timestep and frequency channel)

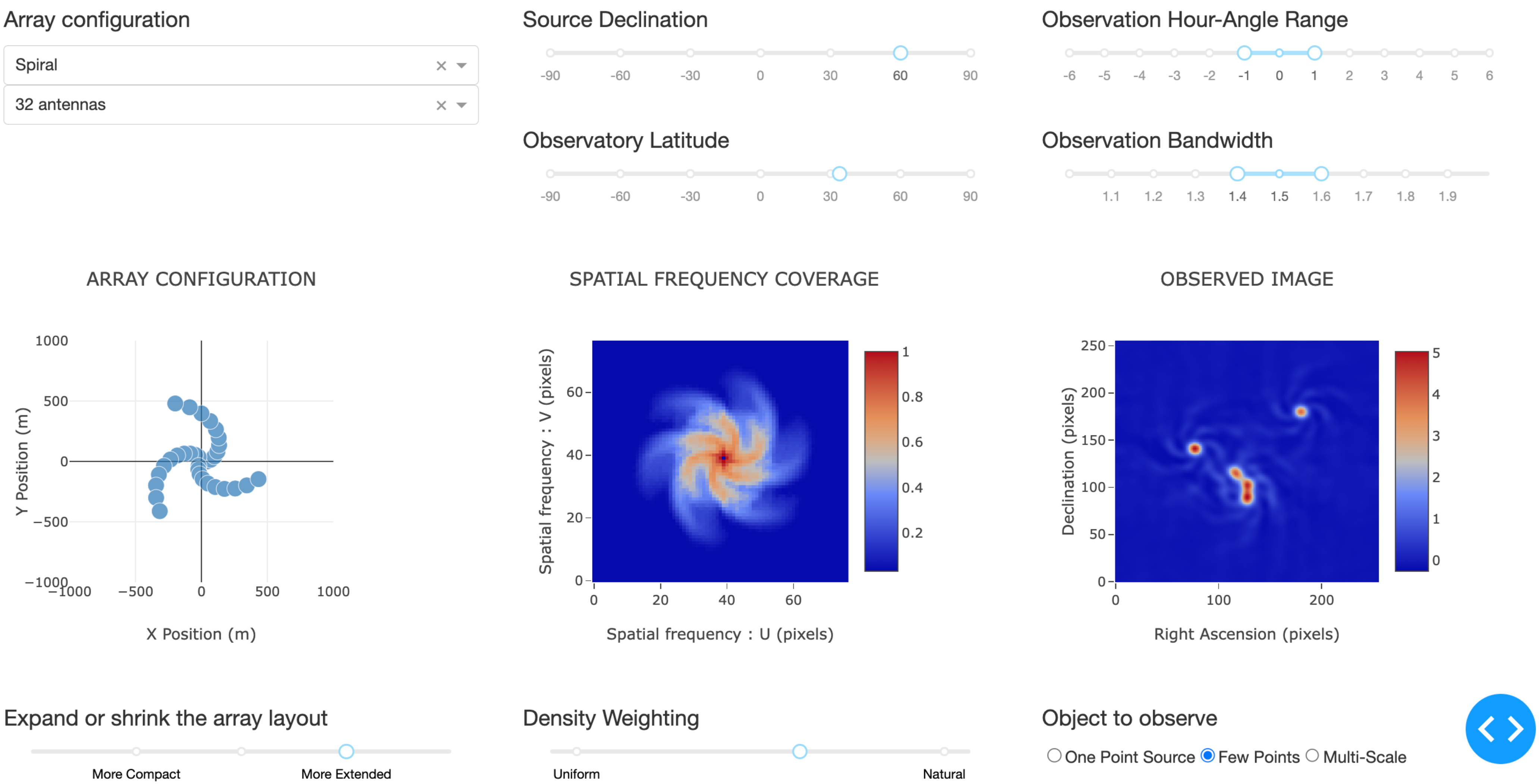
Solution : Fit a model of the sky brightness to the measured data.

Data Analysis - Pipelines



Build your own Interferometer !

Build Your Own Interferometer !



(https://github.com/urvashirau/ImagingSimulator)

Build Your Own Interferometer !

How does the antenna layout and count affect

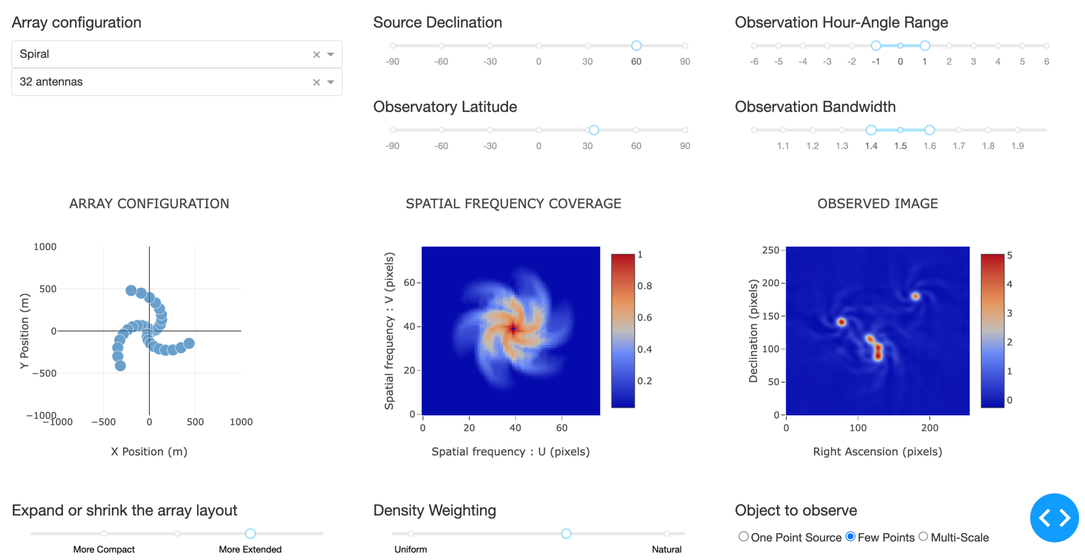
– UV-coverage
– Point Spread Function

Change relative orientation of the array and the object

- Projection effects

Effect of aperture Synthesis

- Change time-ranges
- Change bandwidth



Effect of a compact versus spread-out configuration

– Angular resolution

Different weighting schemes

- What features can be
- emphasized ?

Point sources versus extended emission

- Spatial scales
- Point Spread Function

Questions ?

Some points to remember …

How does an interferometer form an image ?

- Each antenna pair measures one 2D fringe.
  Many antenna pairs => Fourier series

How do you make a raw image from interferometer data ?

- Bin (or resample) the visibility data onto a 2D grid, take a Fourier transform

What does the raw observed image represent ?

- Observed Sky is the convolution of the true sky and the PSF

How do you get a model of the sky ?

- Solve the convolution equation via model-fitting algorithms like Clean

Some points to remember …

What is calibration ?

- Use calibrator data to solve for antenna gains, apply them to target data

How does wide-band data affect the imaging process ?

- Increased sensitivity, but the imaging properties and sky change with frequency

What is an antenna primary beam and what is its effect on an image ?

- Antenna power pattern. It multiplies with the sky,before convolution with the PSF

What is the w-term problem ?

- 2D Fourier transform approximations are invalid far away from the image center