Radio Interferometry - Imaging



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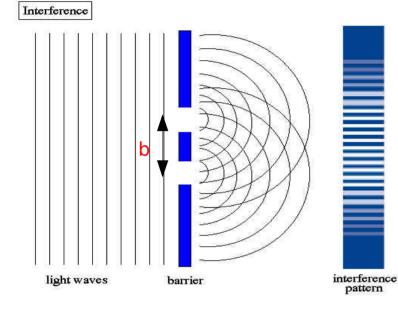
Imaging – Part 1

(1) Concepts

- Image Formation
- Measurement Process
- Data Analysis
- (2) Build your own interferometer
- (3) Imaging in practice
 - Basic Imaging
 - Imaging and Deconvolution
 - How to use reconstruction algorithms
 - Wideband, Wide-field and Mosaic imaging

An interferometer is an indirect imaging device

Young's double slit experiment



2D Fourier transform :

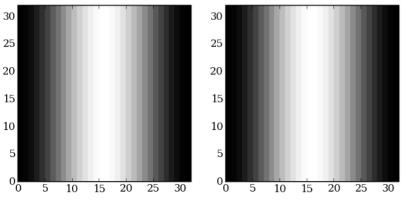
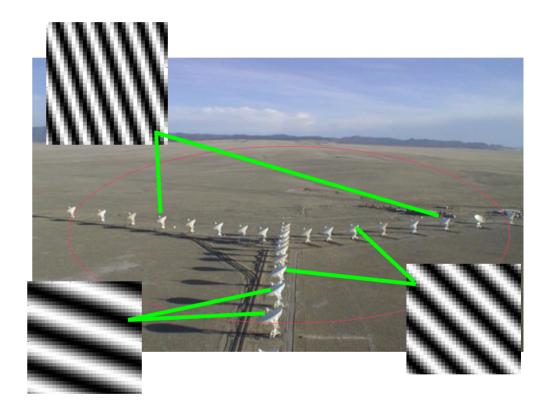


Image = sum of cosine 'fringes'.

Each antenna-pair measures the parameters of one 'fringe'.



Measured Fringe Parameters :

Amplitude, Phase

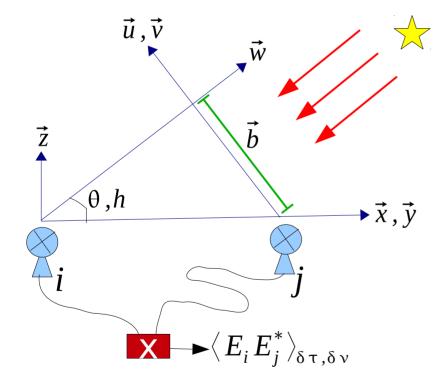
Orientation, Wavelength

Measure the spatial correlation of the E-field incident at each pair of antennas

$$\langle E_i E_j^* \rangle \propto V_{ij}(u,v) = \iint I^{sky}(l,m) e^{2\pi i (ul+vm)} dldm$$

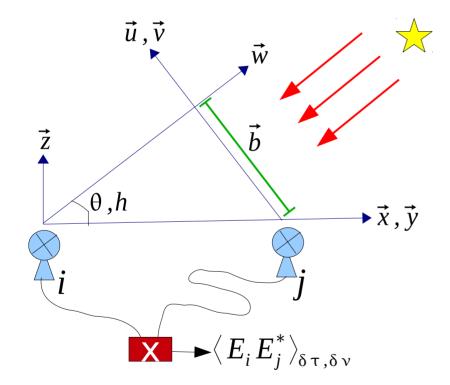
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Parameters of a Fringe :

Amplitude, Phase :

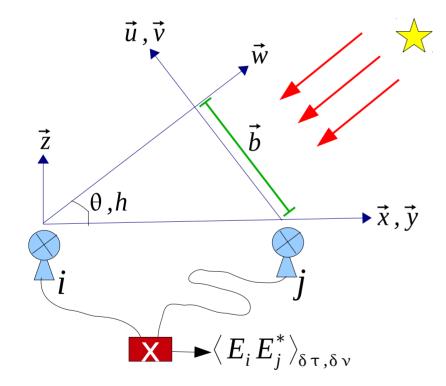
 $\langle E_i E_i^*
angle$ is a complex number.

Orientation, Wavelength :

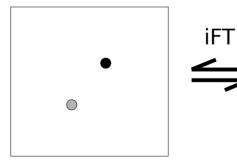
 \vec{u} , \vec{v} , \vec{b} (geometry)

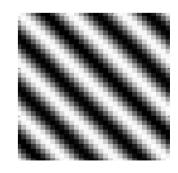
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Parameters of a Fringe : Amplitude, Phase : $\langle E_i E_j^* \rangle$ is a complex number. Orientation, Wavelength : $\vec{u}, \vec{v}, \vec{b}$ (geometry)

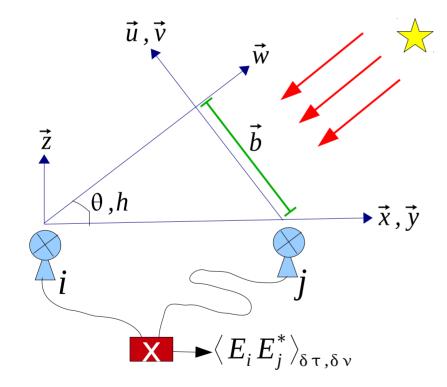




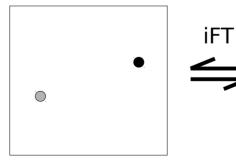
UV domain (spatial frequency) Image domain (directions on sky)

Measure the spatial correlation of the E-field incident at each pair of antennas

$$\langle E_i E_j^* \rangle \propto V_{ij}(u,v) = \iint I^{sky}(l,m) e^{2\pi i (ul+vm)} dldm$$



Parameters of a Fringe : Amplitude, Phase : $\langle E_i E_j^* \rangle$ is a complex number. Orientation, Wavelength : $\vec{u}, \vec{v}, \vec{b}$ (geometry)



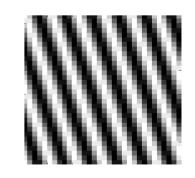


Image domain (directions on sky)

UV domain (spatial frequency)

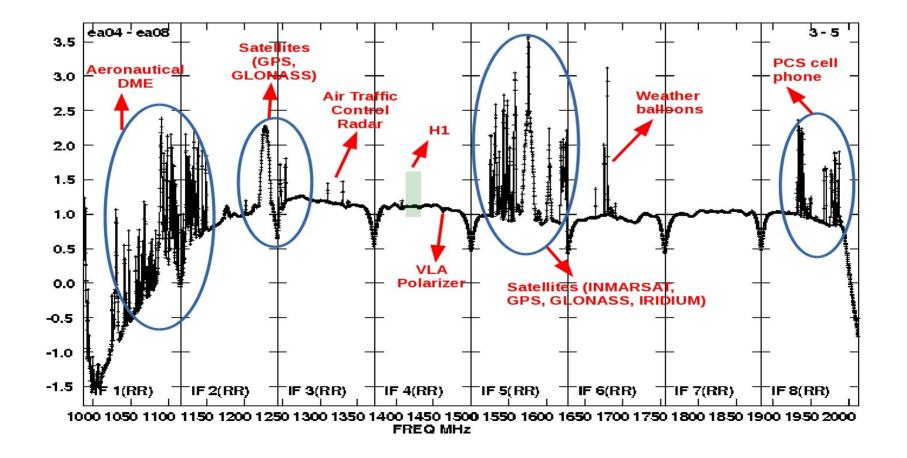
But, the measured $\left< E_i E_j^* \right>$ values are imperfect and incomplete

But, the measured $\langle E_i E_i^* \rangle$ values are imperfect and incomplete

Flagging

Problem : Radio Frequency Interference (RFI)

Solution : Identify and discard corrupted data samples.



But, the measured $\langle E_i E_i^*
angle$ values are imperfect and incomplete

Flagging

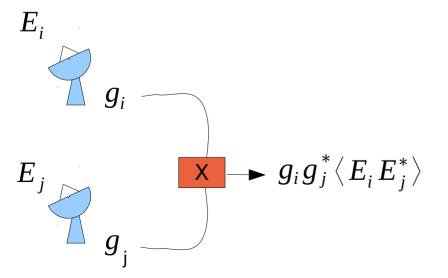
Problem : Radio Frequency Interference (RFI)

Solution : Identify and discard corrupted data samples.

Calibration

Problem : Antenna electronics introduce complex gains g_i

Solution : Numerically solve for antenna gains g_i and apply corrections



- Observe a source where $\langle E_i E_j^*
 angle$ is known
- Use information from all ij to solve for g_i
- Divide out $g_i g_j^*$ from target data

But, the measured $\langle E_i E_i^*
angle$ values are imperfect and incomplete

Flagging

Problem : Radio Frequency Interference (RFI)

Solution : Identify and discard corrupted data samples.

Calibration

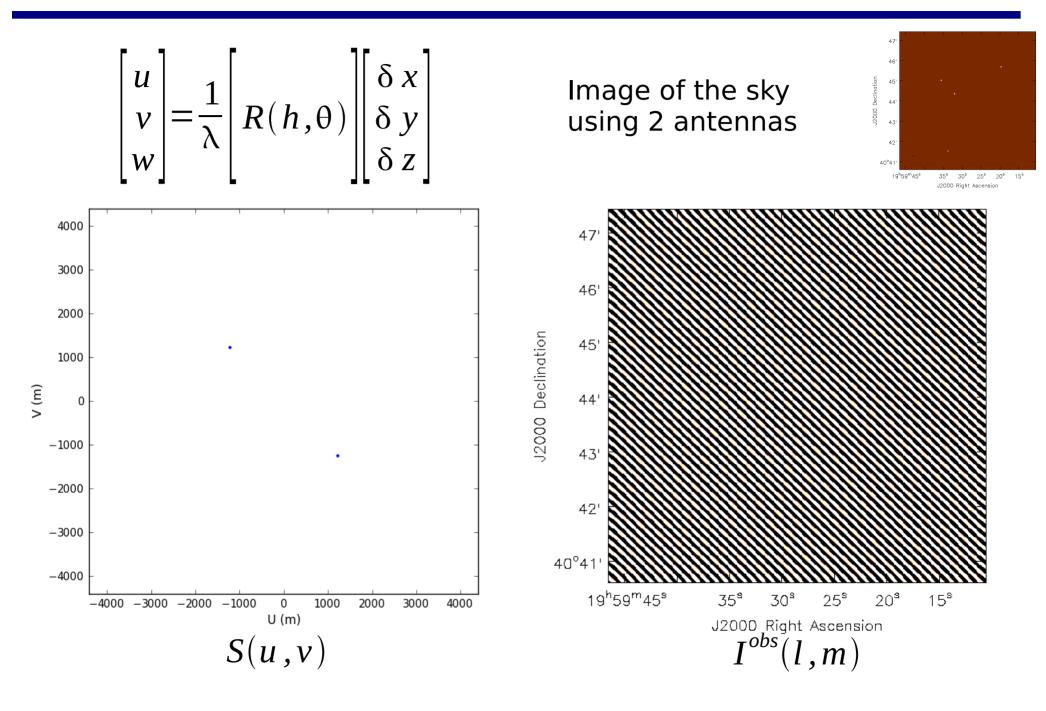
Problem : Antenna electronics introduce complex gains g_i

Solution : Numerically solve for antenna gains G_i and apply corrections

Imaging

Problem : Sampling of the 2D Fourier Transform is incomplete (only N(N-1)/2 measurements per timestep and frequency channel)

Solution : Fit a model of the sky brightness to the measured data.

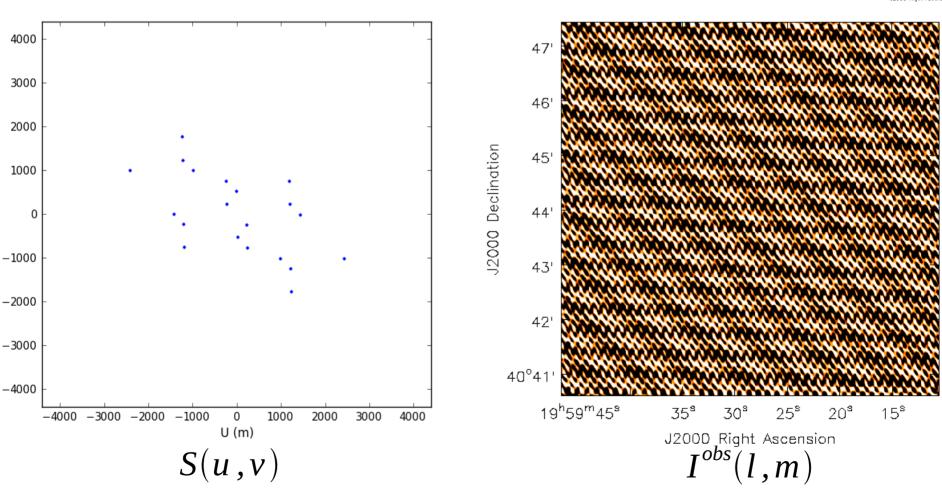


$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h, \theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

V (m)

Image of the sky using 5 antennas

47 46 45 45 45 43 42 40°41' 19^h59^m45^t 35^h 30^h 25^h 20^h 15^h J2000 Right Ascension



$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h, \theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

0

U (m)

4000

3000

2000

1000

-1000

-2000

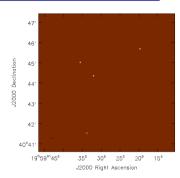
-3000

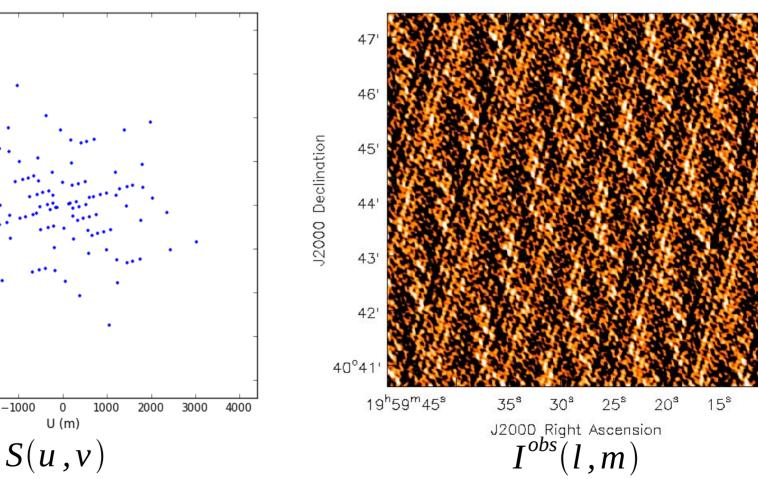
-4000

-4000 -3000 -2000 -1000

(m) V

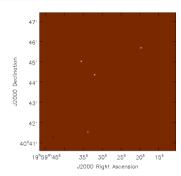
Image of the sky using 11 antennas

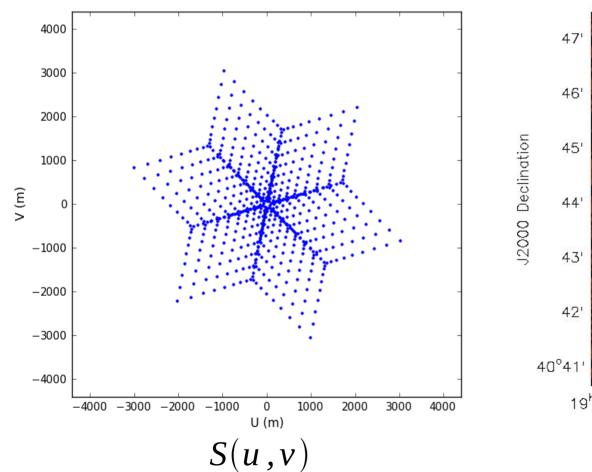


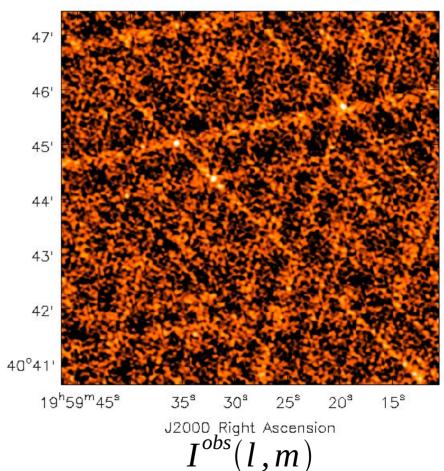


$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h, \theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 27 antennas

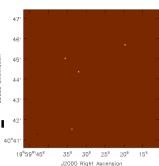


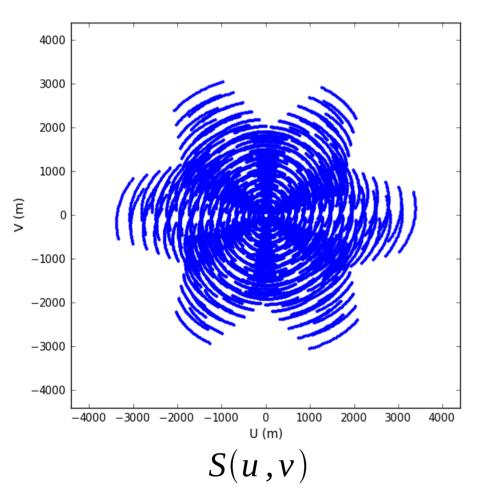


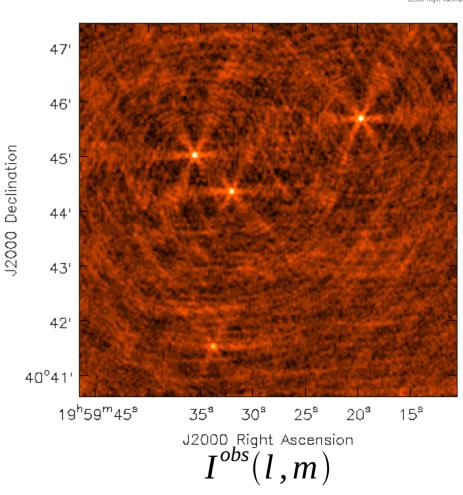


$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h,\theta) \\ \delta y \\ \delta z \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 27 antennas over 2 hours 'Earth Rotation Synthesis'

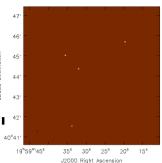


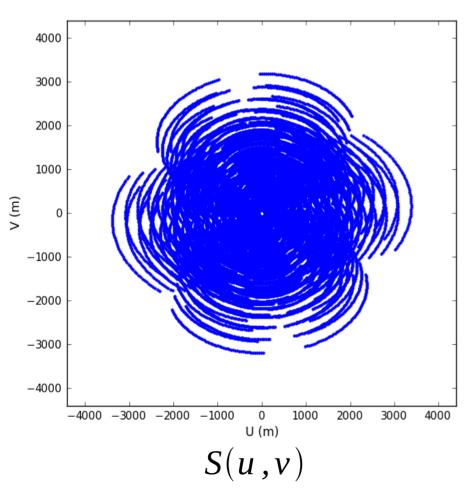


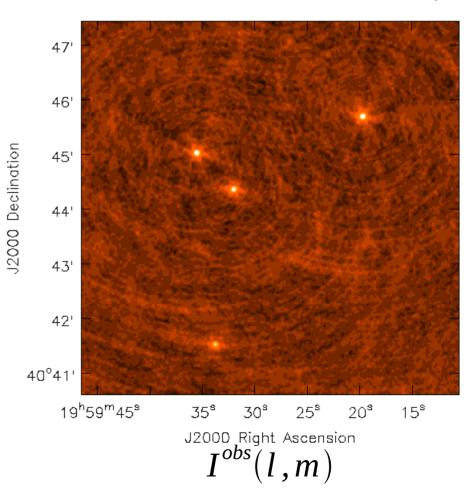


$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h,\theta) \\ \delta y \\ \delta z \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 27 antennas over 4 hours 'Earth Rotation Synthesis'



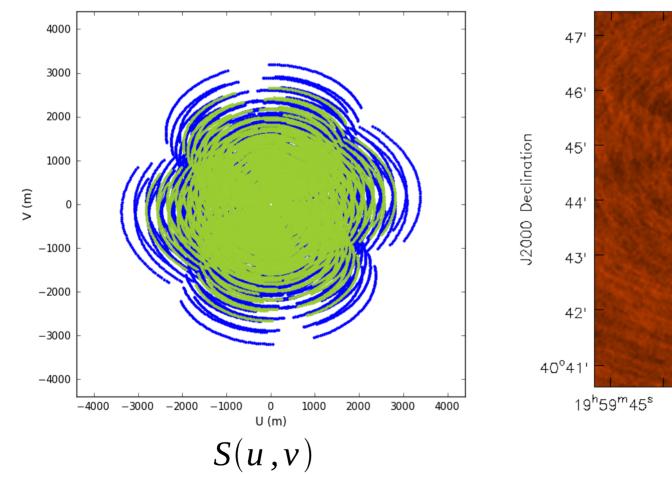


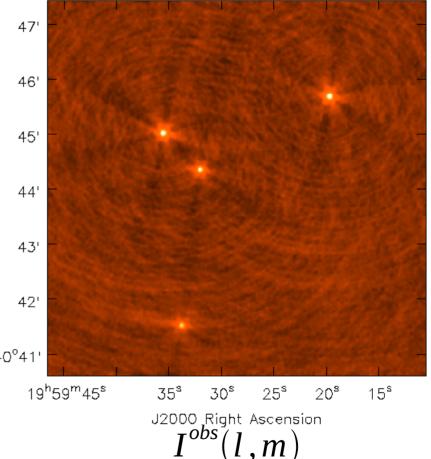


$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h,\theta) \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 27 antennas over 4 hours, 2 freqs 'Multi-Frequency Synthesis

47) 46) 45) 44) 43) 42) 51 10^hc0^m4C^k 75^k 75^k 26^k 26^k 15^k

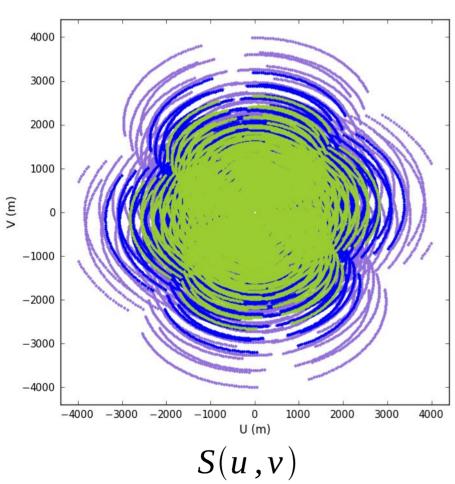




$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h,\theta) \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 27 antennas over 4 hours, 3 freqs 'Multi-Frequency Synthesis

47' 46' 45' 44' 43' 43' 10^hc0^mAc⁵ 76⁵ 20⁵ 20⁵ 16⁵



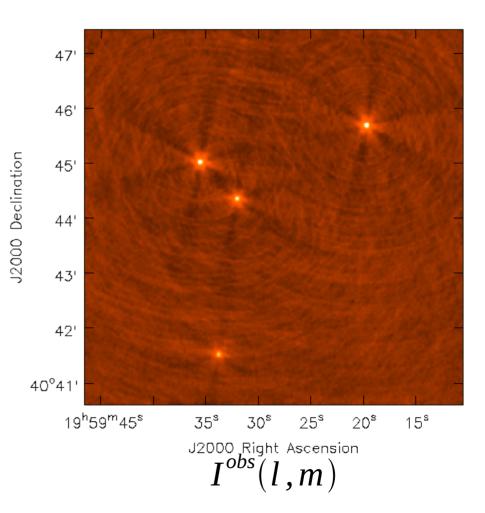
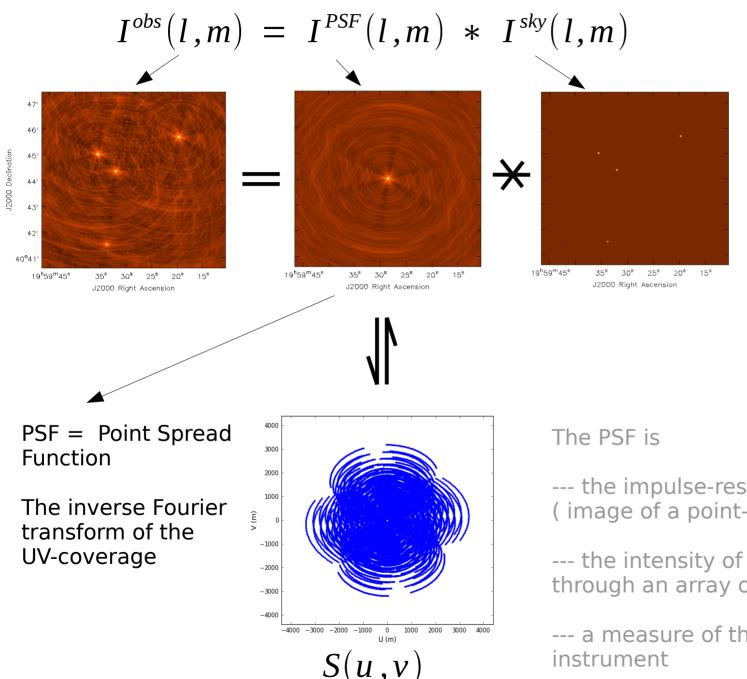


Image formed by an interferometer : Convolution Equation



You have measured the Convolution of the True Sky with the instrumental PSF.

Recovering True Sky = DE-convolution

--- the impulse-response of the instrument (image of a point-source)

--- the intensity of the diffraction pattern through an array of 'slits' (dishes)

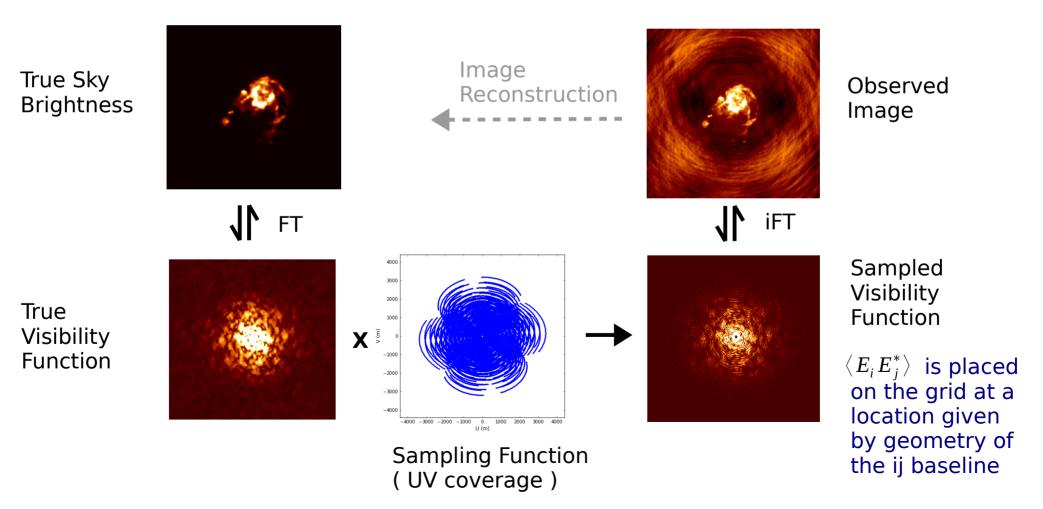
--- a measure of the imaging-properties of the instrument

Image Formation

 $\langle E_i E_j^* \rangle$: The 2D Fourier Transform (FT) of the sky brightness, measured at a spatial frequency given by the geometry of the baseline (antenna pair) ij.

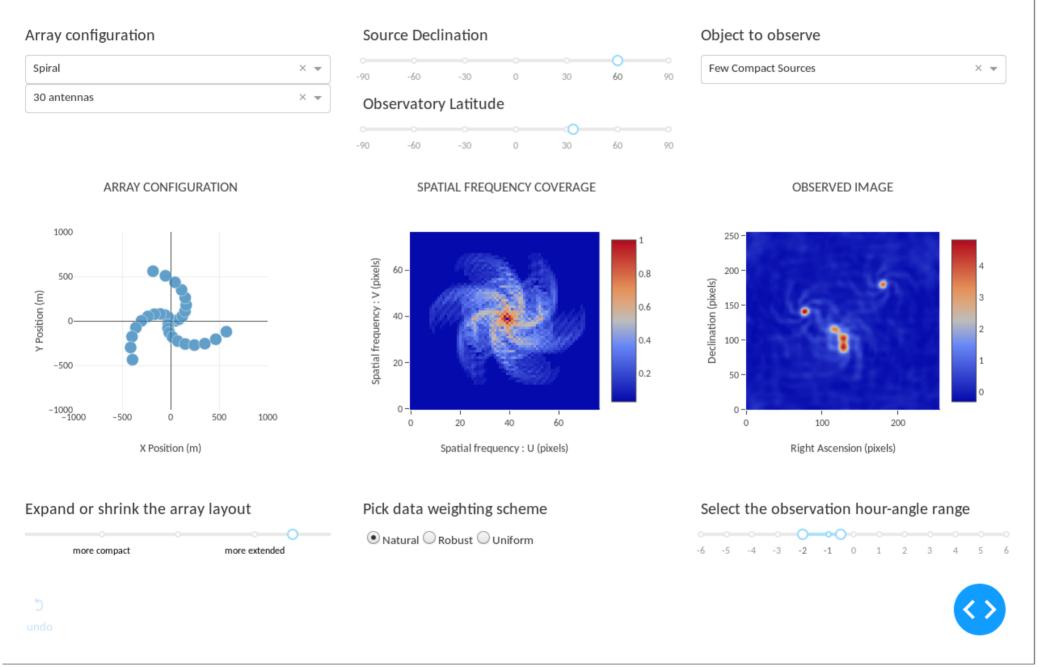
N antennas = N(N-1)/2 samples (for each timestep and channel)

Imaging : Collect all samples onto a 2D spatial frequency grid, and do an iFT.



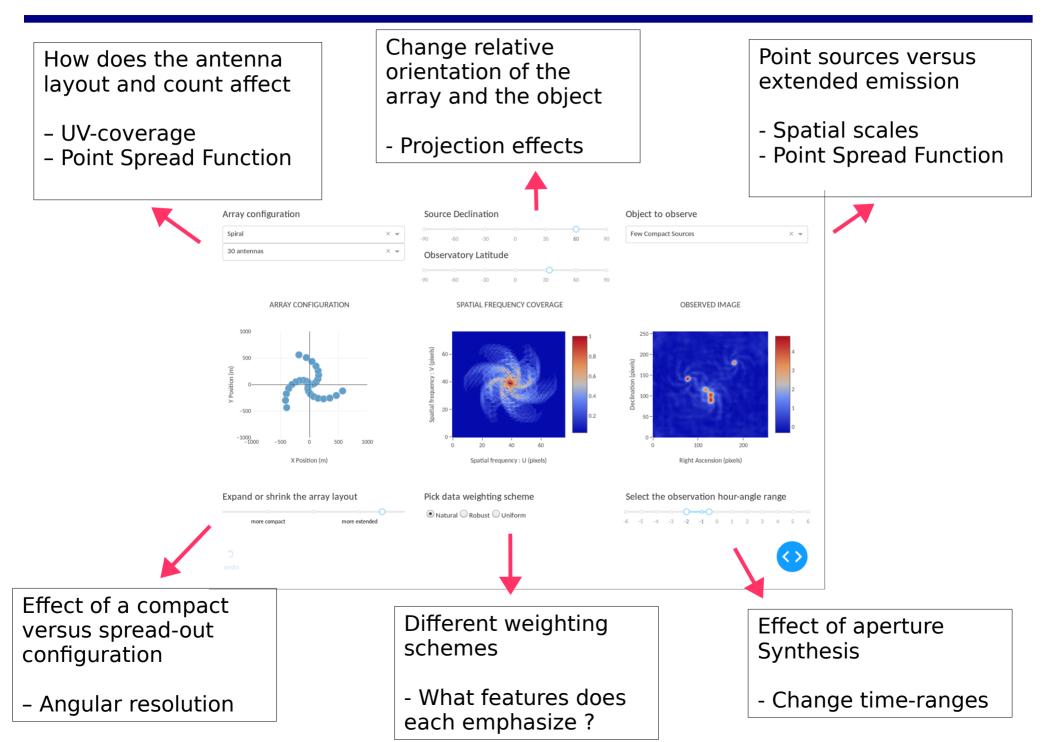
Build Your Own Interferometer !

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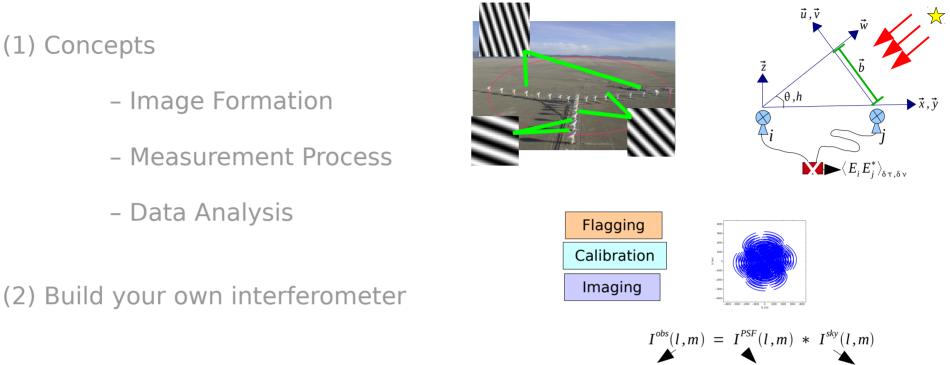


(https://github.com/urvashirau/ImagingSimulator)

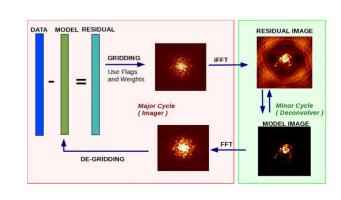
Build Your Own Interferometer !



Imaging – Part 2



- (3) Imaging in practice
 - Basic Imaging
 - Imaging and Deconvolution
 - How to use reconstruction algorithms
 - Wideband, Wide-field and Mosaic imaging



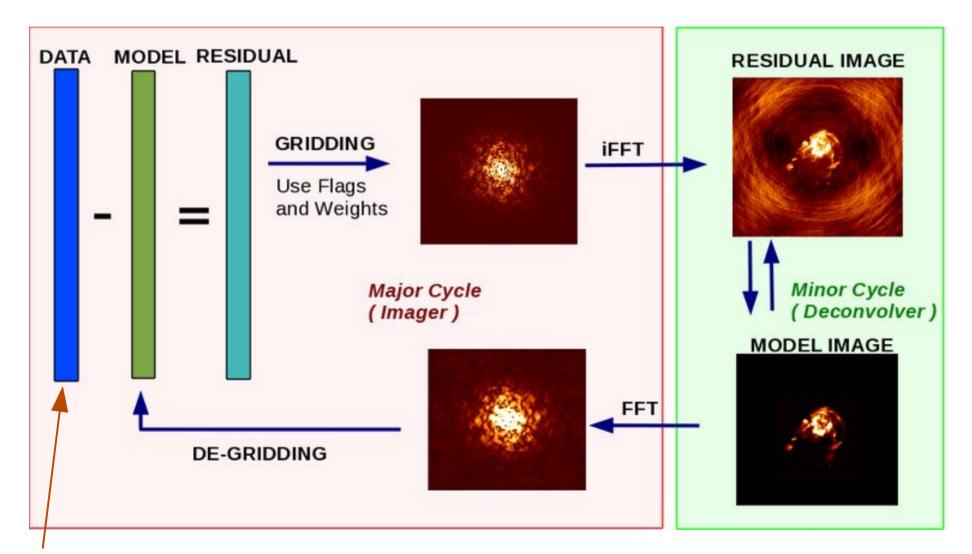
*

35⁸ 30⁸ 25⁸ 20⁸

35" 30" 25" 20' J2000 Right Assension

Data = Incomplete set of samples of the true signal

Image Reconstruction = Fit a model (of the sky) to the data



 $\langle E_i E_j^*
angle$, $ec{u}$, $ec{v}$

Basic Imaging :

- Step 1 : Define image size and cell size
- Step 2 : Gridding, data-weighting and FFT
- Step 3 : Iterative deconvolution

Imaging in practice : Choosing image size, cell-size

- Choosing image 'cell' size : Nyquist-sample the main lobe of the PSF

PSF beam width :
$$\frac{\lambda}{b_{max}} = \frac{1}{u_{max}}$$
 radians (x $\frac{180}{\pi}$ to convert to degrees)

This is the diffraction-limited angular-resolution of the telescope Ex : Max baseline : 10 km. Freq = 1 GHz. Angular resolution : 6 arcsec

Imaging in practice : Choosing image size, cell-size

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- Choosing image field-of-view (npixels) : As much as desired/practical.

$$\frac{1}{fov_{rad}} = \delta u$$

Field of View (fov) controls the uv-grid-cell size $(\delta u, \delta v)$

- Antenna primary-beam limits the field-of-view ('slits' of finite width)

Imaging in practice : Choosing image size, cell-size

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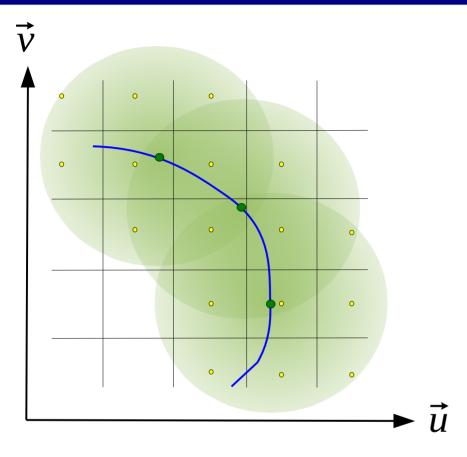
Field of View (fov) controls the uv-grid-cell size $(\delta u, \delta v)$

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- Gridding + FFT :

- An interferometer measures irregularly spaced points on the UV-plane.
- Need to place the visibilities onto a regular grid of UV-pixels, and then take an FFT

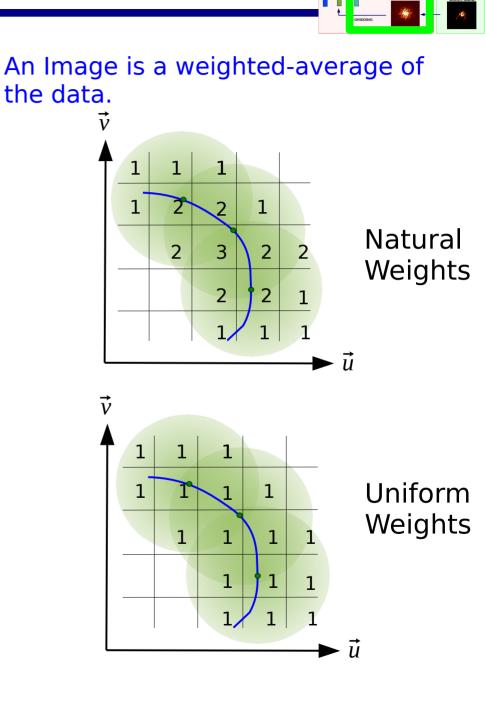
Imaging in practice : Gridding and Weighting



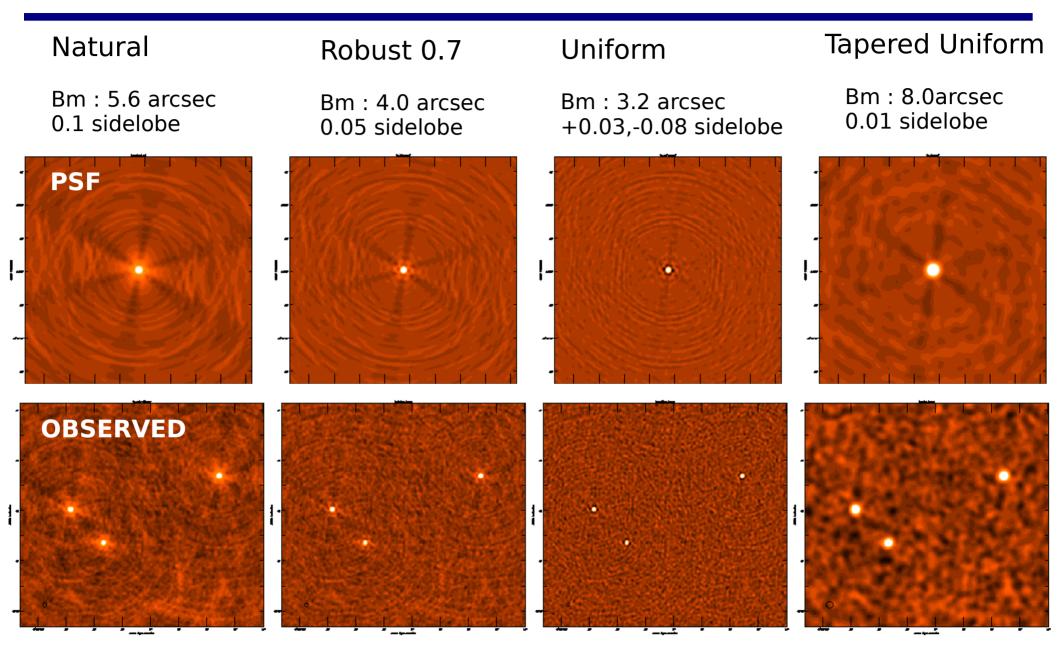
-- Visibility data are placed onto a regular grid before taking an i-FFT

- Convolutional Resampling

=> Use a gridding convolution function
 => Use weights per visibility
 (weighted average of all data points per cell)



Imaging in practice : PSFs and Observed (dirty) Images



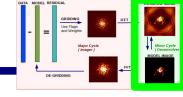
Note the noise-structure. Noise is correlated between pixels by the PSF. Image Units (Jy/beam) ------ All pairs of images satisfy the convolution relation => Need to deconvolve them

An Image is a weighted average of the data.

4000 3000 2000 Weighting-scheme => modify the imaging properties of the instrument => emphasize features/scales of interest => control imaging sensitivity

1000 (E) 0 -1000 -2000 -3000	Uniform/Robust All spatial	Natural/Robust All data points get	UV-Taper Low spatial fregs
-4000	frequencies get equal weight	equal weight	get higher weight than others
Resolution	higher	medium	lower
PSF Sidelobes (VLA)	lower	higher	depends
Point Source Sensitivity	lower	maximum	lower
Extended Source Sensitivity	lower	medium	higher

Imaging in practice : Deconvolution



Observed image = Instrumental Point-Spread-Function convolved with the true sky

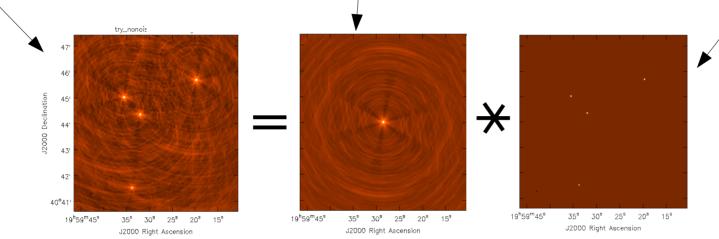


Image reconstruction is typically a "deconvolution" process.

Algorithms : Parameterized models + Iterative model fitting (chi-square minimization)

CLEAN : Model the sky as a collection of delta-function 'flux components'

MEM : Model the sky with delta-functions, and add a smoothness constraint

Multi-Scale-CLEAN : Model the sky as a collection of 'blobs' of different sizes

ASP Clean : Model the sky with a collection of best-fit Gaussians

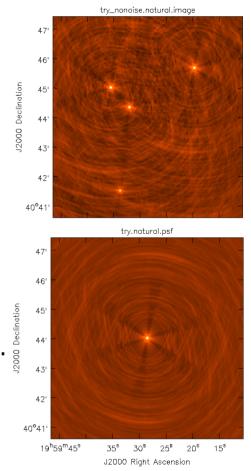
Deconvolution – Hogbom CLEAN

Sky Model : List of delta-functions

- (1) Construct the observed (dirty) image and PSF
- (2) Search for the location of peak amplitude.
- (3) Add a delta-function of this peak/location to the model
- (4) Subtract the contribution of this component from the dirty image - a scaled/shifted copy of the PSF

Repeat steps (2), (3), (4) until a stopping criterion is reached.

(5) Restore : Smooth the model with a 'clean beam' and add residuals



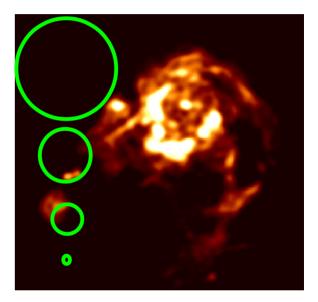
The CLEAN algorithm can be formally derived as a model-fitting problem

- model parameters : locations and amplitudes of delta functions
- solution process : χ^2 minimization via an iterative steepest-descent algorithm (method of successive approximation)

Deconvolution – MultiScale (MS)-CLEAN

Multi-Scale Sky Model : Linear combination of 'blobs' of different scale sizes

- Efficient representation of both compact and extended structure (sparse basis)
- A scale-sensitive algorithm
- (1) Choose a set of scale sizes
- (2) Calculate dirty/residual images smoothed to several scales (basis functions)
 - Normalize by the relative sum-of-weights (instrument's sensitivity to each scale)



(3) Find the peak across all scales, update a single multi-scale model as well as all residual images (using information about coupling between scales)

Iterate, similar to Classic CLEAN, and restore at the end.

The MS-CLEAN algorithm can also be formally derived as a model-fitting problem using χ^2 minimization and a basis set consisting of several 'blob' sizes.

Deconvolution – Comparison of Algorithms

CLEAN

MEM

Point source model

Point source model with a smoothness constraint

MS-CLEAN

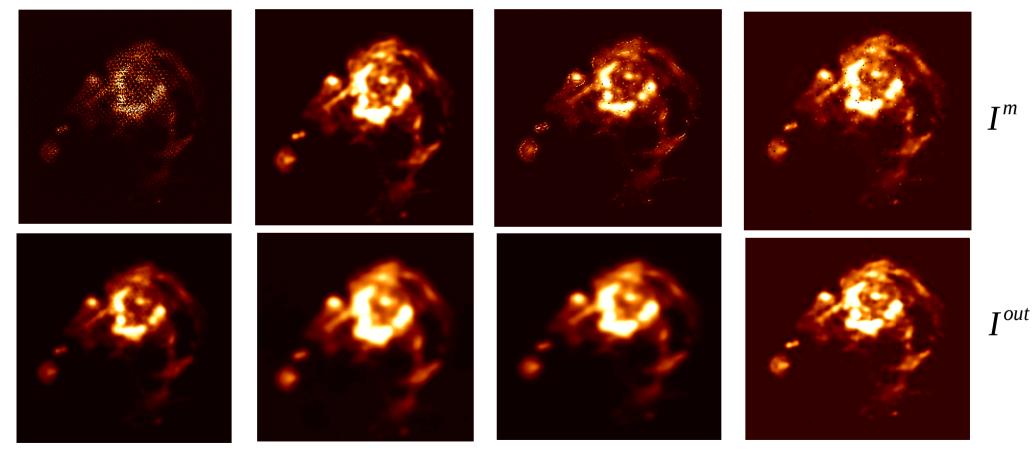
Multi-Scale model

with a fixed set of

scale sizes

ASP

Multi-Scale model with adaptive bestfit scale per component



 I^m

Deconvolution – Comparison of Algorithms

CLEAN

MEM

Point source model

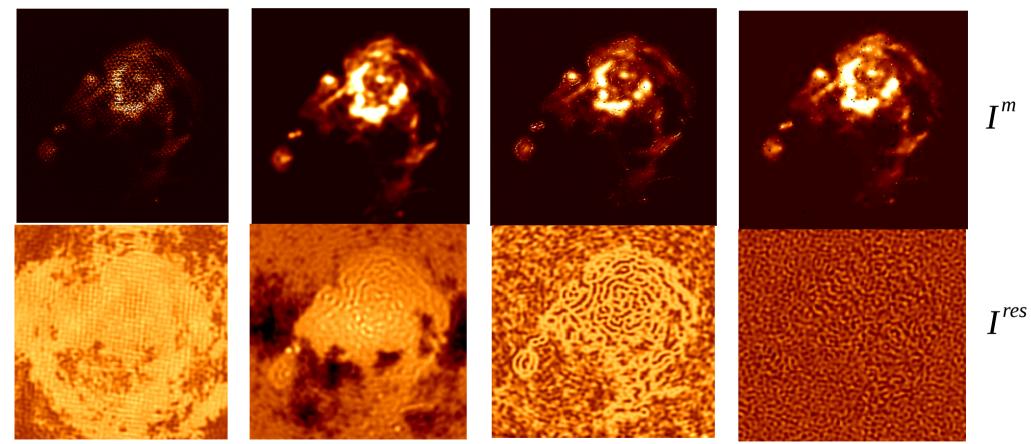
Point source model with a smoothness constraint

MS-CLEAN

Multi-Scale model with a fixed set of scale sizes

Multi-Scale model with adaptive bestfit scale per component

ASP



 I^m

How can you control the quality of image reconstruction ?

(1) Iterations and stopping criterion

'niter' : maximum number of iterations / components 'threshold' : don't search for flux below this level

- minor cycles can be inaccurate, so periodically trigger major cycles

(2) Using masks

Need masks only if the deconvolution is "hard".

- => Bad PSFs with high sidelobes
- => Leftover bad data causing stripes or ripples
- => Extended emission with sharp edges
- => Extended emission that is seen only by very few baselines

Draw interactively (start small, and grow them) or supply final mask.

(3) Self-Calibration

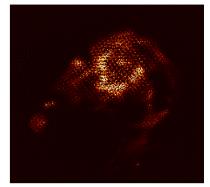
Use your current best estimate of the sky (i.e. the model image) to get new antenna gain solutions. Apply, Image again and repeat.

Image Quality

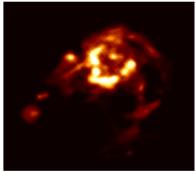
Noise in the image : Measured from restored or residual images

- With perfect reconstruction, The ideal noise level is : $RMS \propto \frac{0.12 \frac{T_{sys}}{\eta_a}}{\sqrt{N_{ant}(N_{ant}-1) \cdot \delta \tau \cdot \delta \nu \cdot N_{pol}}}$
- In reality, measure the RMS of residual pixel amplitudes

Model image



Restored image



Residual image

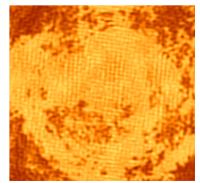


Image Quality

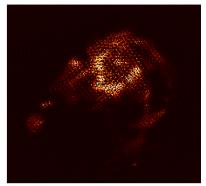
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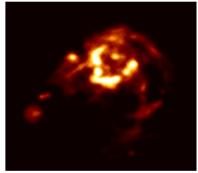
Dynamic Range : Measured from the restored image

- Standard : Ratio of peak brightness to RMS noise in a region devoid of emission.
- More truthful : Ratio of peak brightness to peak error (residual)

Model image



Restored image



Residual image

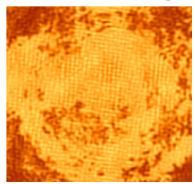


Image Quality

Noise in the image : Measured from restored or residual images

- With perfect reconstruction, The ideal noise level is : $RMS \propto \frac{0.12 \frac{T_{sys}}{\eta_a}}{\sqrt{N_{ant}(N_{ant}-1) \cdot \delta \tau \cdot \delta \nu \cdot N_{pol}}}$
- In reality, measure the RMS of residual pixel amplitudes

Dynamic Range : Measured from the restored image

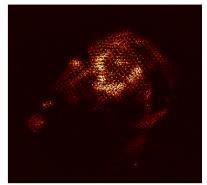
- Standard : Ratio of peak brightness to RMS noise in a region devoid of emission.
- More truthful : Ratio of peak brightness to peak error (residual)

Image Fidelity : Correctness of the reconstruction

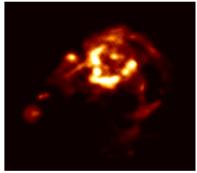
- remember the infinite possibilities that fit the data perfectly ?
- useful only if a comparison image exists.

Inverse of relative error : $\frac{I^m * I^{beam}}{I^m * I^{beam}} - I^{restored}$

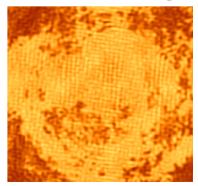
Model image



Restored image



Residual image



Basic Imaging:

Narrow-frequency range, Small region of the sky

=> The 2D Fourier Transform relations hold
=> Convolution and deconvolution

Wide-Band Imaging :

=> Sky and instrument change across frequency range

Wide-Field Imaging

=> The 2D Fourier Transform relation breaks

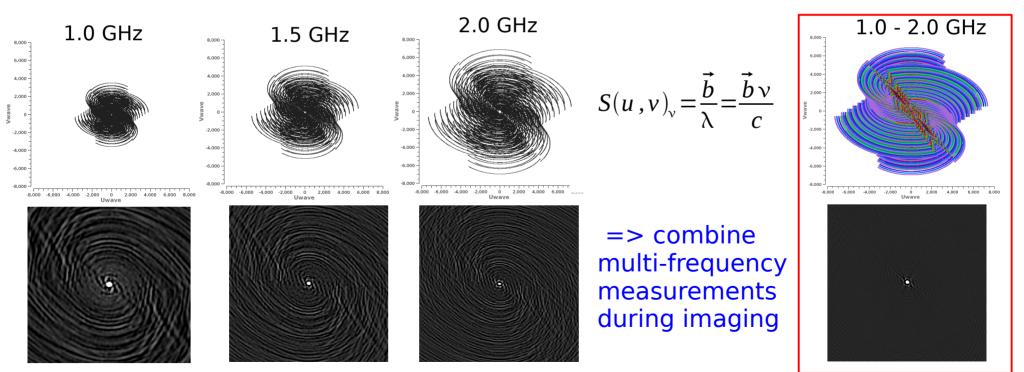
Mosaic Imaging

=> Image an area larger than what each antenna can see.

Wide-band Imaging – Sensitivity and Multi-Frequency Synthesis

Frequency Range :	(1 – 2 GHz)	(4 – 8 GHz)	(8 – 12 GHz)
Bandwidth : $v_{max} - v_{min}$	1 GHz	4 GHz	4 GHz
Bandwidth Ratio : v_{max} : v_{min}	2:1	2:1	1.5 : 1
Fractional Bandwidth : $(v_{max} - v_{min})/v_m$	_{id} 66%	66%	40%

UV-coverage / imaging properties change with frequency

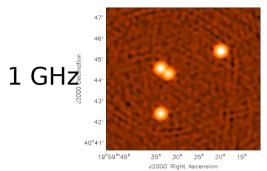


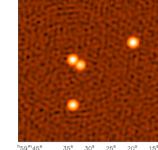
Sky Brightness can also change with frequency \rightarrow model intensity and spectrum

Spectral Cube (vs) MFS imaging

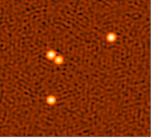
3 flat-spectrum sources + 1 steep-spectrum source (1-2 GHz VLA observation)

Images made at different frequencies (limited to narrow-band sensitivity)





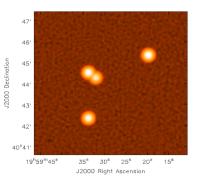
35° 30° 25° 20° 15° J2000 Right Ascension



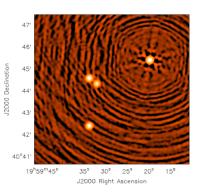
^h59^m45^s 35^s 30^s 25^s 20^s 15^s J2000 Right Ascension •

159^m45^s 35^s 30^s 25^s 20^s 15^s J2000 Right Ascension ^h59^m45^s 35^s 30^s 25^s 20^s 15^s J2000 Right Ascension 2 GHz

Add all singlefrequency images (after smoothing to a low resolution)

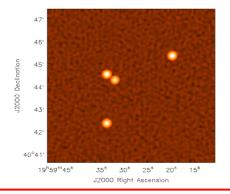


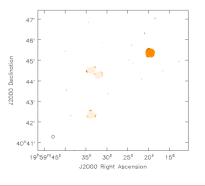
Use wideband UVcoverage, but ignore spectrum (MFS, nterms=1)



Use wideband UV-coverage + Model and fit for spectra too (MT-MFS, nterms > 1)

Output : Intensity and Spectral-Index



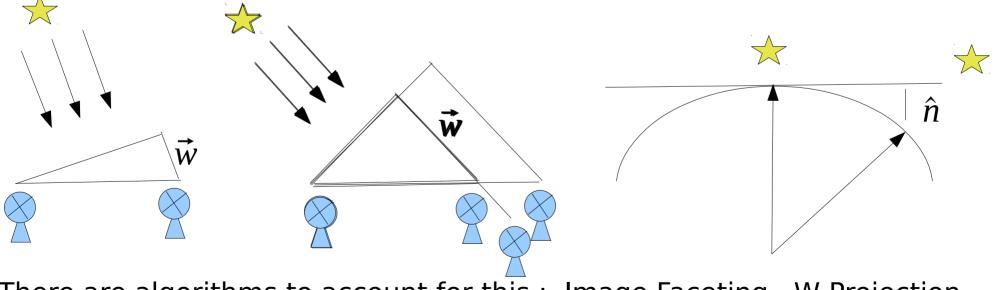


Wide-Field Imaging – W-term

$$V^{obs}(u,v) = S(u,v) \iint I(l,m) e^{2\pi i (ul+vm)} dl dm$$

$$V^{obs}(u,v) = S(u,v) \iiint I(l,m) e^{2\pi i (ul+vm+w(n-1))} dl dm dn$$

The 'w' of a baseline can be large, away from the image phase center The 'n' for a source can be large, away from the image phase center



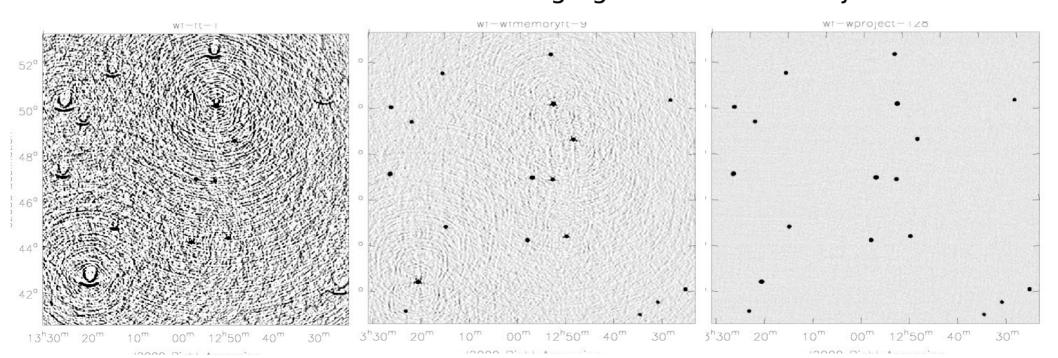
There are algorithms to account for this : Image Faceting, W-Projection.

Wide-Field Imaging – W-term

$$V^{obs}(u,v) = S(u,v) \iint I(l,m) e^{2\pi i (ul+vm)} dl dm$$

$$V^{obs}(u,v) = S(u,v) \iiint I(l,m) e^{2\pi i(ul+vm+w(n-1))} dl dm dn$$

The 'w ' of a baseline can be large, away from the image phase center
The 'n ' for a source can be large, away from the image phase center
2D Imaging Facet Imaging W-Projection

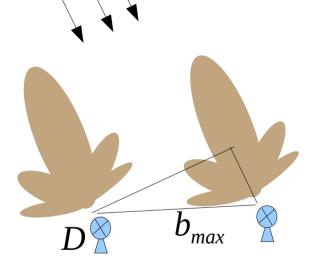


Wide-Field Imaging – Primary Beams

Each antenna has a limited field of view => Primary Beam (gain) pattern

=> Sky is (approx) multiplied by PB, before being sampled by the interferometer

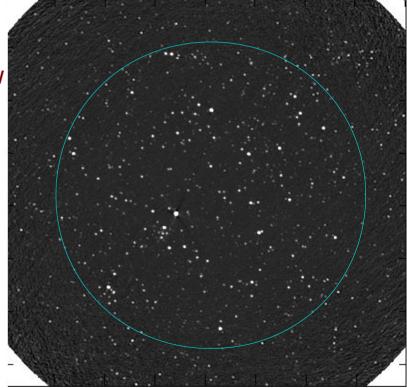
 $I^{obs}(l,m) \approx I^{PSF}(l,m) * \left[P^{sky}(l,m) \cdot I^{sky}(l,m)\right]$

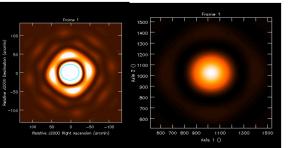


The antenna field of view D = antenna diameter λ/D

Compare with angular resolution of the interferometer :

 λ/b_{max}

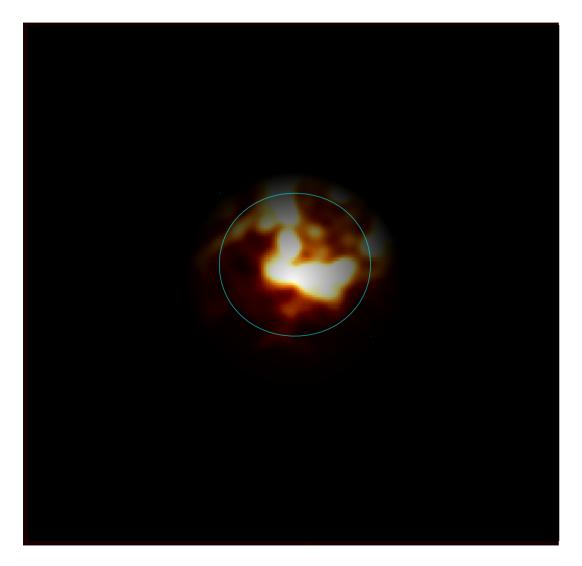




But, in reality, P changes with time, freq, pol and antenna....

=> Ignoring such effects limits dynamic range to 10^4
=> More-accurate method to account for this : A-Projection

Combine data from multiple pointings to form one large image.



One Pointing sees only part of the source

Combine pointings either before or after deconvolution.

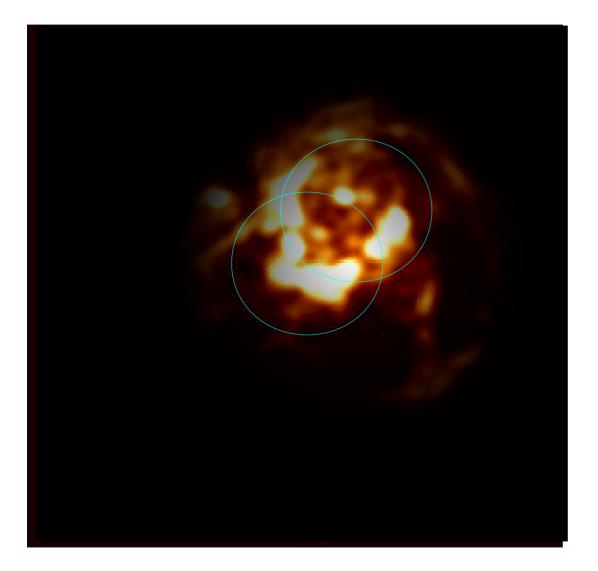
Stitched mosaic :

- -- Deconvolve each pointing separately
- -- Divide each image by PB
- -- Combine as a weighted avg

Joint mosaic :

- Combine observed images as a weighted average (or)
 Grid all data onto one UV-grid, and then iFFT
- -- Deconvolve as one large image

Combine data from multiple pointings to form one large image.



Combine pointings either before or after deconvolution.

Stitched mosaic :

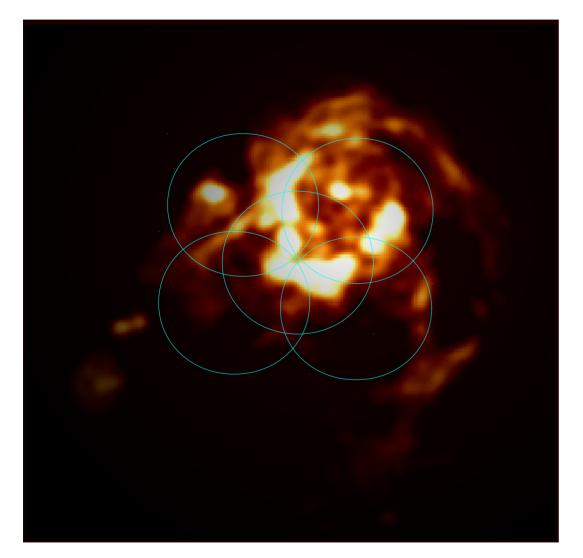
- -- Deconvolve each pointing separately
- -- Divide each image by PB
- -- Combine as a weighted avg

Joint mosaic :

- Combine observed images as a weighted average (or)
 Grid all data onto one UV-grid, and then iFFT
- -- Deconvolve as one large image

Two Pointings see more.....

Combine data from multiple pointings to form one large image.



Use many pointings to cover the source with approximately uniform sensitivity

Combine pointings either before or after deconvolution.

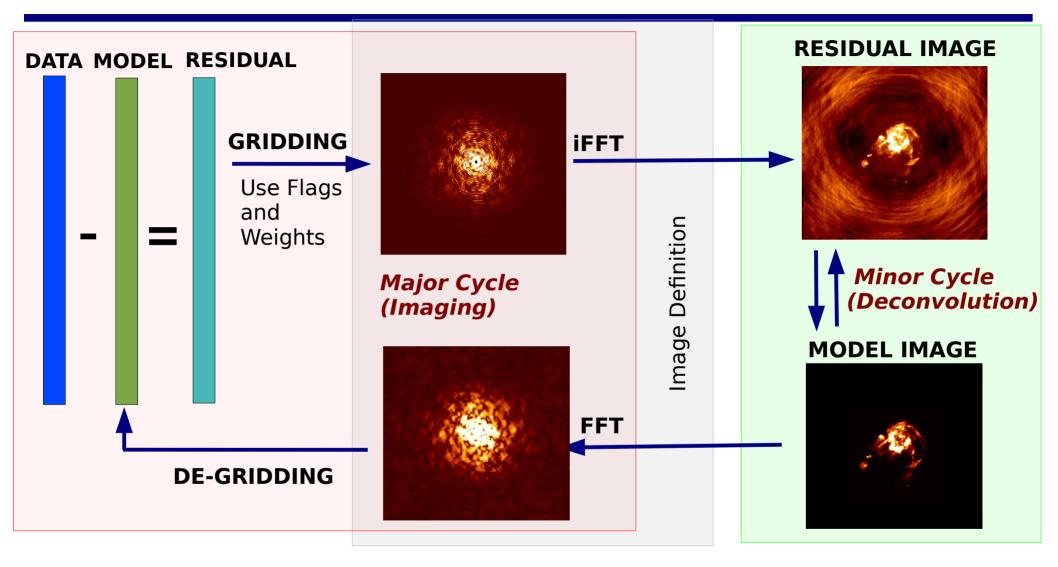
Stitched mosaic :

- -- Deconvolve each pointing separately
- -- Divide each image by PB
- -- Combine as a weighted avg

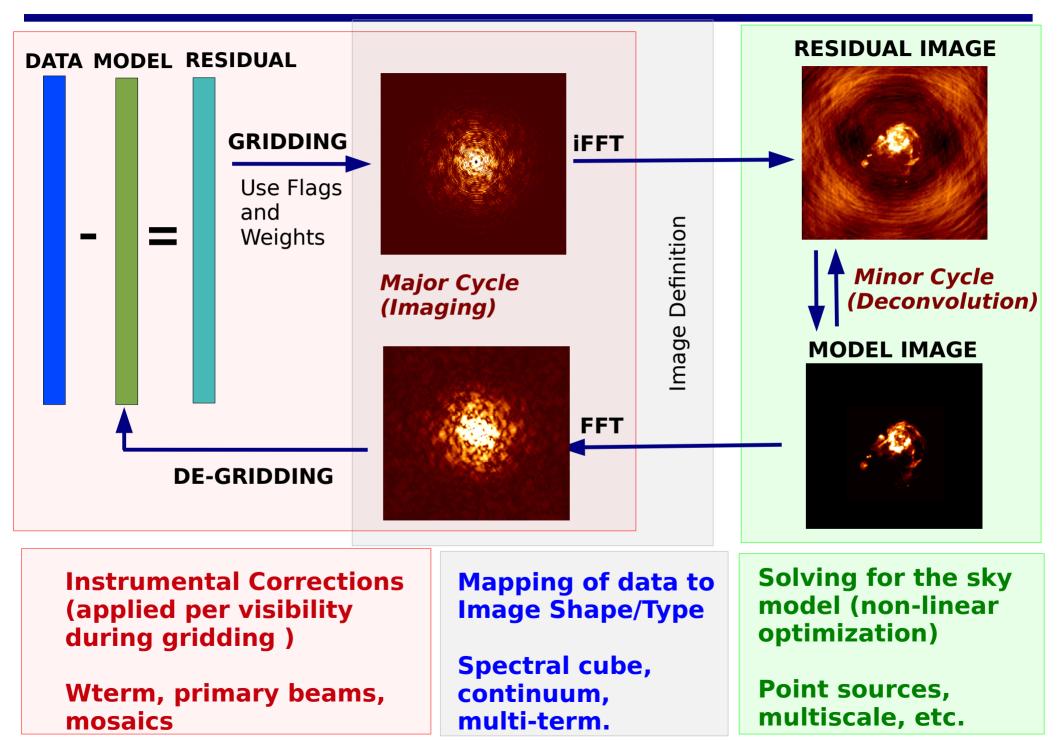
Joint mosaic :

- Combine observed images as a weighted average (or)
 Grid all data onto one UV-grid, and then iFFT
- -- Deconvolve as one large image

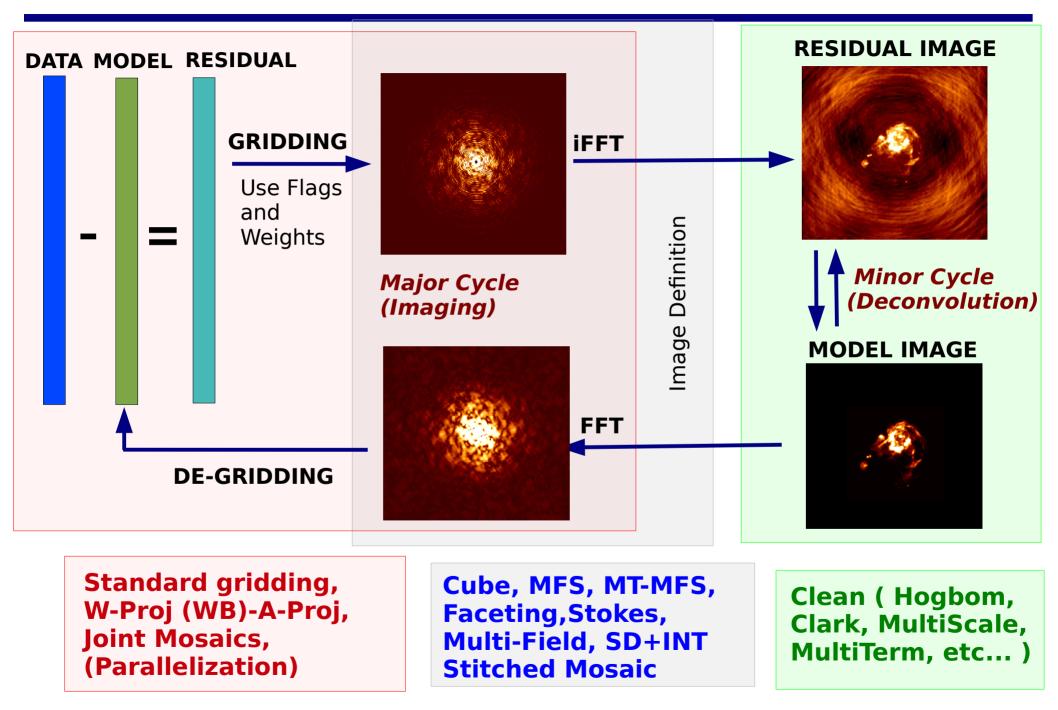
Imaging – Algorithms and Options



Imaging – Algorithms and Options



Imaging – Algorithms and Options



https://casadocs.readthedocs.io/en/v6.2.0/notebooks/synthesis_imaging.html

Some points to remember ...

How does an interferometer form an image ?

Each antenna pair measures one 2D fringe.
 Many antenna pairs => Fourier series

How do you make a raw image from interferometer data ?

- Assign weights to visibilities, grid them, take a Fourier transform

How do you choose the cell-size and image size for imaging ?

- Cell size = (Resolution / 3). Image size = field-of-view / cell size

What does the raw observed image represent ?

- Observed Sky is the convolution of the true sky and the PSF

How do you get a model of the sky ?

- Solve the convolution equation via algorithms like Clean, MS-Clean, MT-Clean...

What is calibration ?

- Use calibrator data to solve for antenna gains, apply them to target data

How does wide-band data affect the imaging process ?

- Increased sensitivity, but the imaging properties and sky change with frequency

How do you image wide-band data ?

- Make a Cube of images, or Multi-Frequency-Synthesis with a spectral fit.

What is an antenna primary beam and what is its effect on an image ?

- Antenna power pattern. It multiplies with the sky, before convolution with the PSF

What is the w-term problem ?

- 2D Fourier transform approximations are invalid far away from the image center