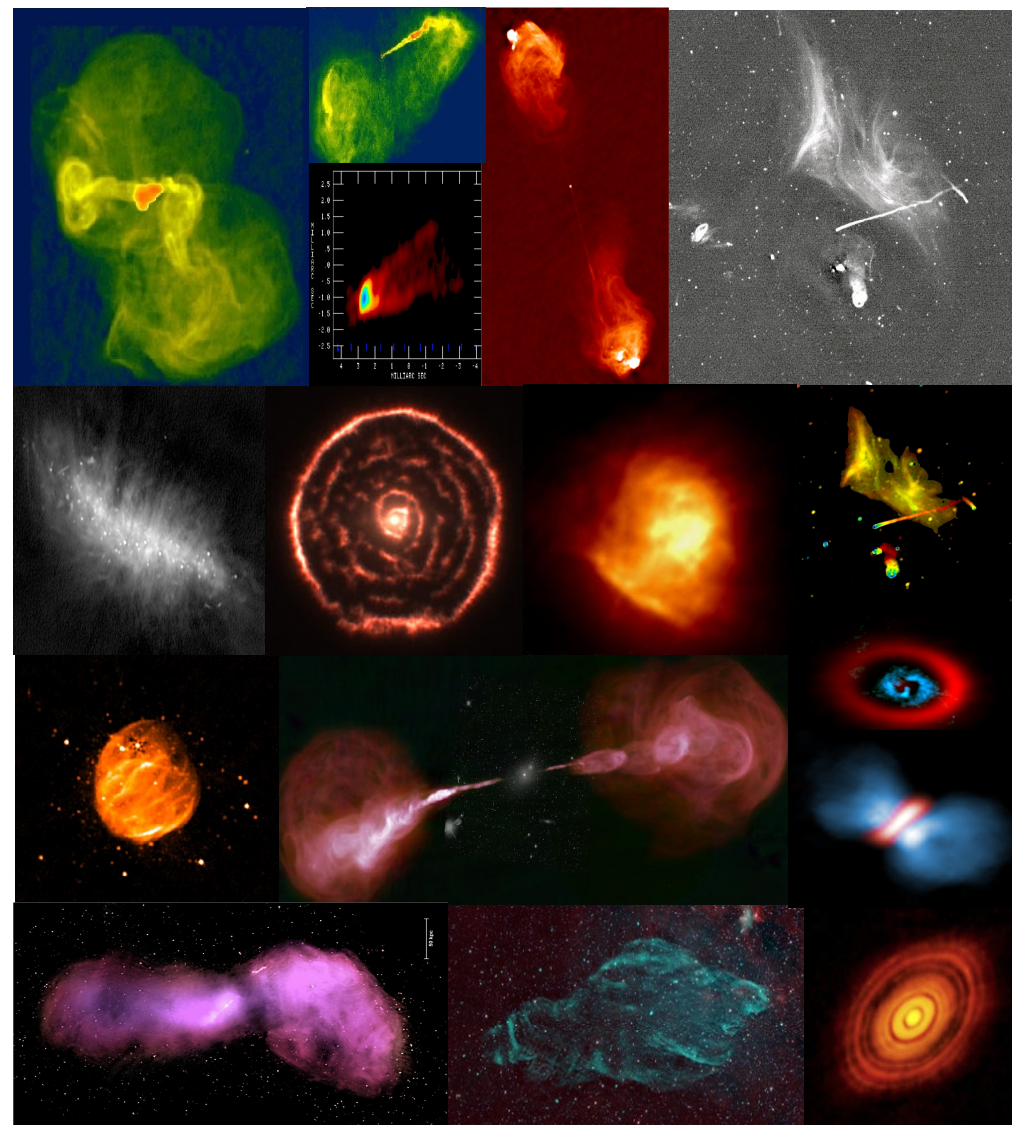


# Radio Interferometry - Imaging



Urvashi Rau

National Radio Astronomy Observatory, Socorro, NM, USA

# Outline

---

## (1) Concepts

- Image Formation
- Measurement Process
- Data Analysis

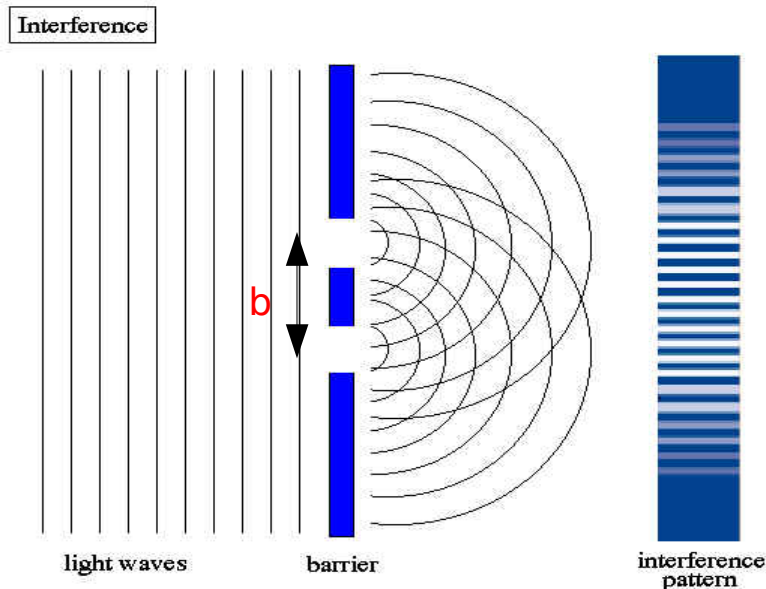
## (2) Build your own interferometer

## (3) Imaging in practice

- Basic Imaging
  - Imaging and Deconvolution
  - How to use reconstruction algorithms
- Wideband, Wide-field and Mosaic imaging

# An interferometer is an indirect imaging device

## Young's double slit experiment



## 2D Fourier transform :

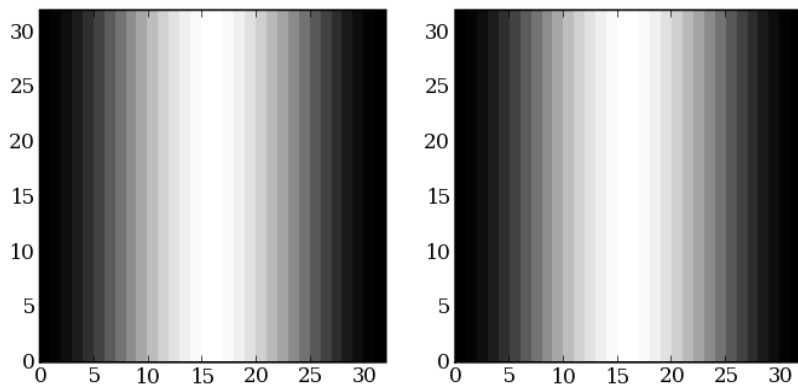
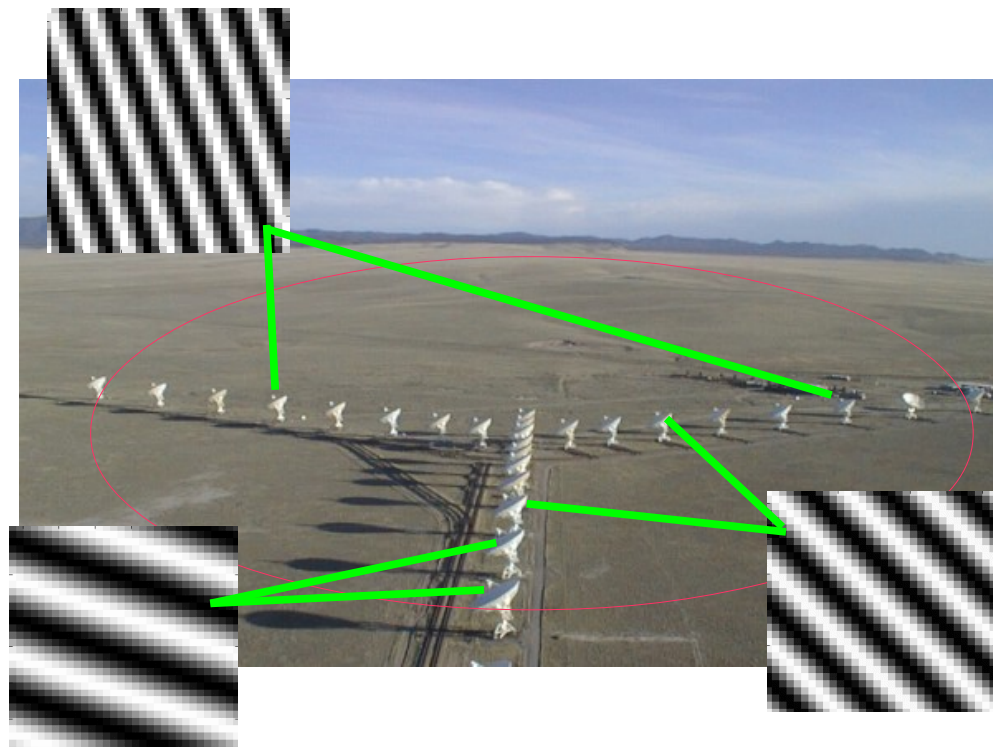


Image = sum of cosine 'fringes'.

Each antenna-pair measures the parameters of one 'fringe'.



Measured Fringe Parameters :

Amplitude, Phase

Orientation, Wavelength

# Measurements

---

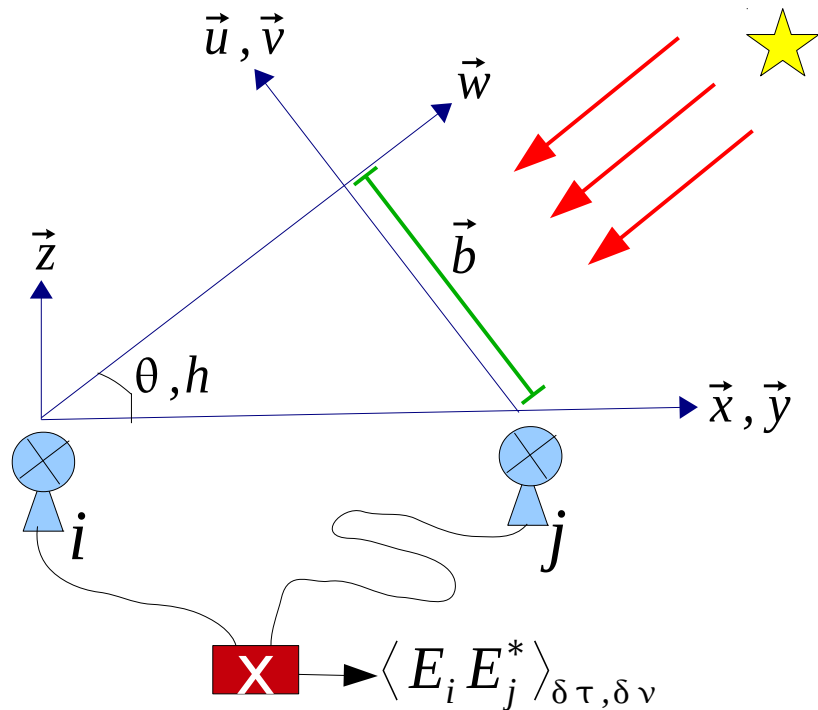
Measure the spatial correlation of the E-field incident at each pair of antennas

$$\langle E_i E_j^* \rangle \propto V_{ij}(u, v) = \iint I^{sky}(l, m) e^{2\pi i(ul+vm)} dl dm$$

# Measurements

Measure the spatial correlation of the E-field incident at each pair of antennas

$$\langle E_i E_j^* \rangle \propto V_{ij}(u, v) = \iint I^{sky}(l, m) e^{2\pi i(ul+vm)} dl dm$$

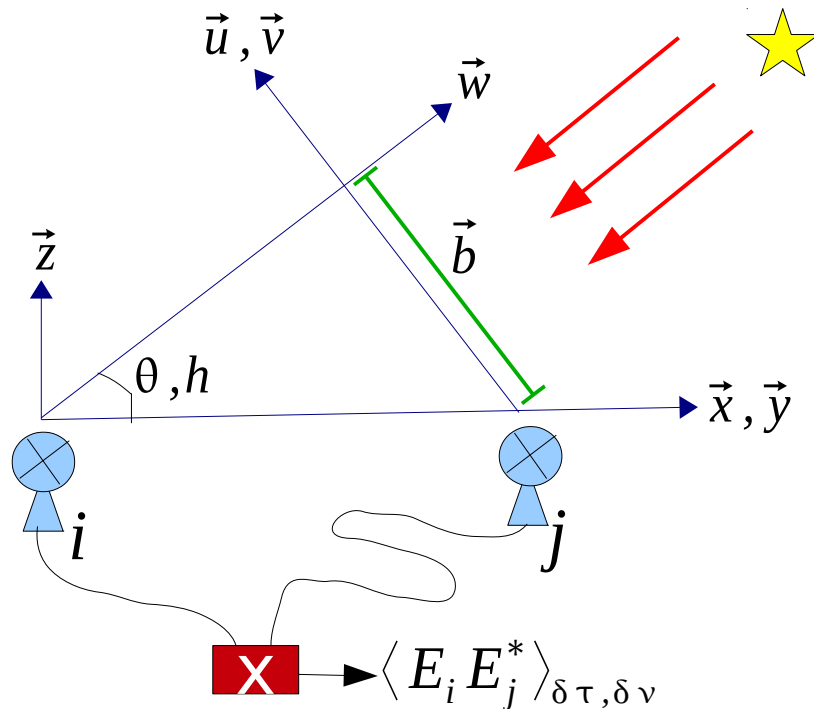




# Measurements

Measure the spatial correlation of the E-field incident at each pair of antennas

$$\langle E_i E_j^* \rangle \propto V_{ij}(u, v) = \iint I^{sky}(l, m) e^{2\pi i(ul+vm)} dl dm$$



Parameters of a Fringe :

Amplitude, Phase :

$\langle E_i E_j^* \rangle$  is a complex number.

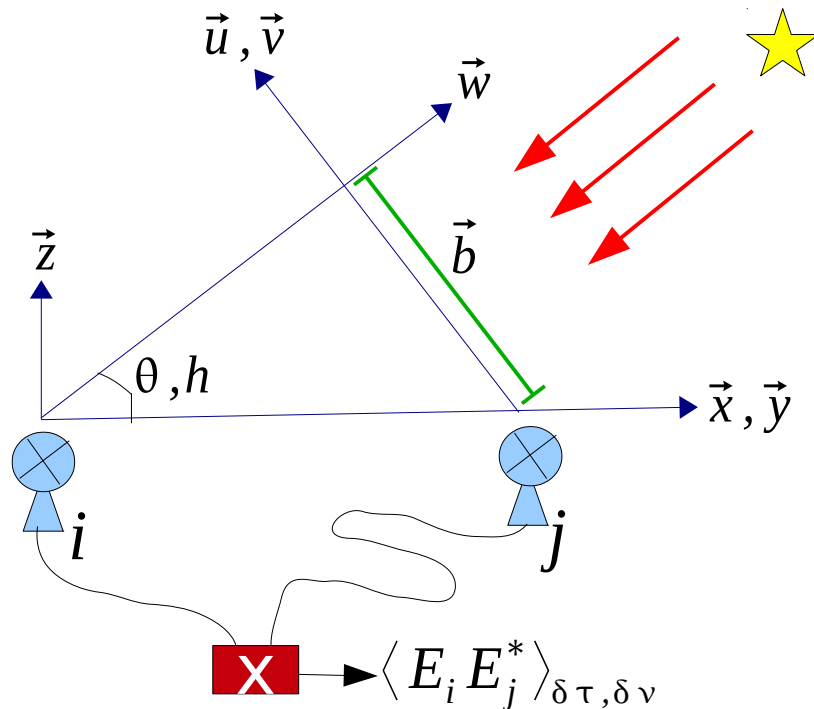
Orientation, Wavelength :

$\vec{u}, \vec{v}, \vec{b}$  (geometry)

# Measurements

Measure the spatial correlation of the E-field incident at each pair of antennas

$$\langle E_i E_j^* \rangle \propto V_{ij}(u, v) = \iint I^{\text{sky}}(l, m) e^{2\pi i(ul+vm)} dl dm$$



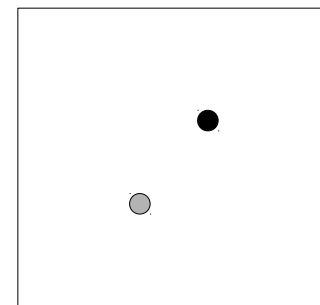
Parameters of a Fringe :

**Amplitude, Phase :**

$\langle E_i E_j^* \rangle$  is a complex number.

**Orientation, Wavelength :**

$\vec{u}, \vec{v}, \vec{b}$  (geometry)



UV domain  
(spatial frequency)

iFT  
 $\rightleftharpoons$

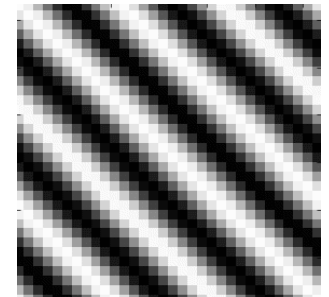
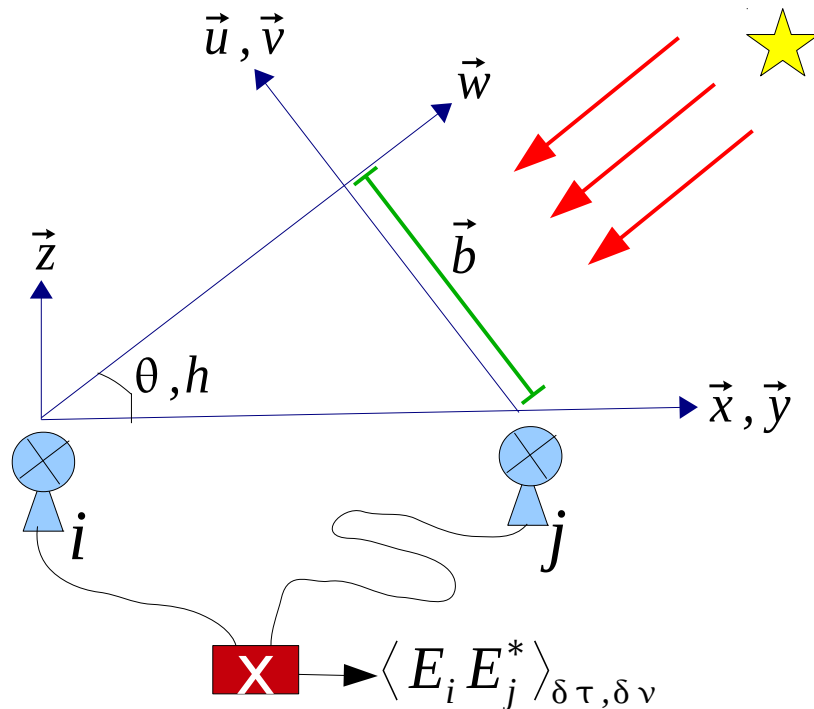


Image domain  
(directions on sky)

# Measurements

Measure the spatial correlation of the E-field incident at each pair of antennas

$$\langle E_i E_j^* \rangle \propto V_{ij}(u, v) = \iint I^{\text{sky}}(l, m) e^{2\pi i(ul+vm)} dl dm$$



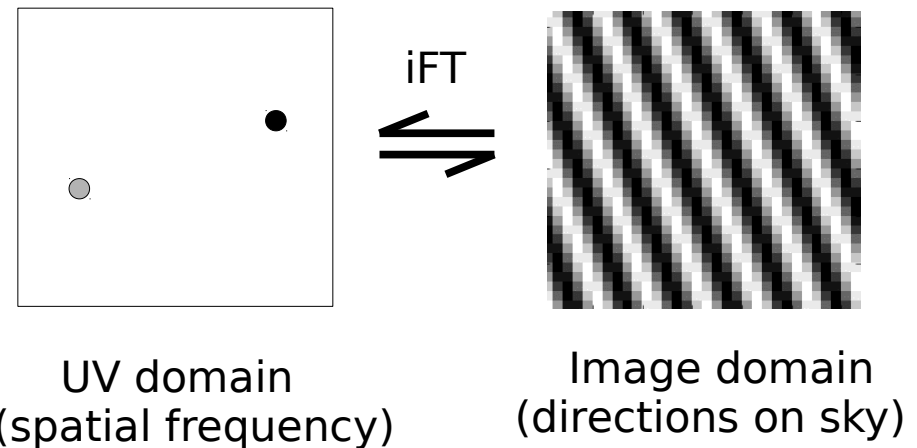
Parameters of a Fringe :

**Amplitude, Phase :**

$\langle E_i E_j^* \rangle$  is a complex number.

**Orientation, Wavelength :**

$\vec{u}, \vec{v}, \vec{b}$  (geometry)





# Data Analysis

---

But, the measured  $\langle E_i E_j^* \rangle$  values are imperfect and incomplete

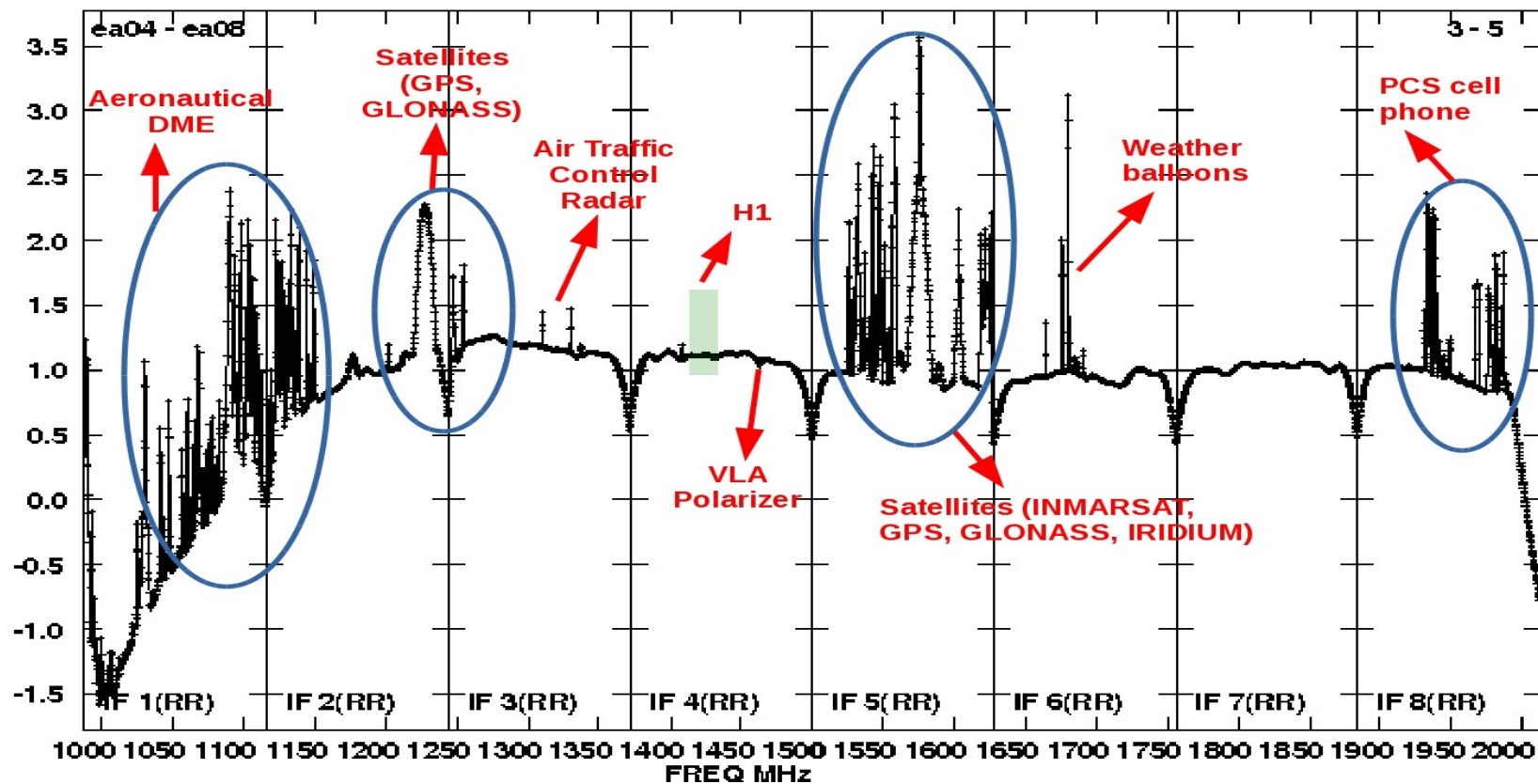
# Data Analysis

But, the measured  $\langle E_i E_j^* \rangle$  values are imperfect and incomplete

## Flagging

Problem : Radio Frequency Interference (RFI)

Solution : Identify and discard corrupted data samples.



# Data Analysis

But, the measured  $\langle E_i E_j^* \rangle$  values are imperfect and incomplete

## Flagging

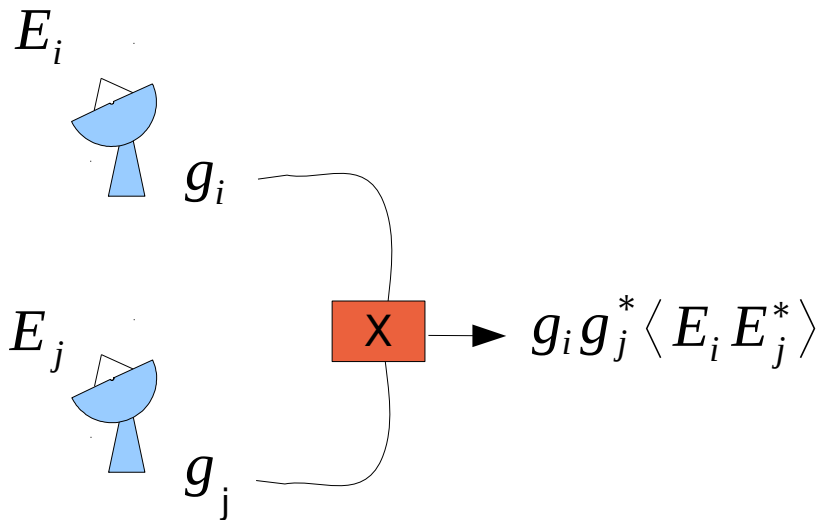
Problem : Radio Frequency Interference (RFI)

Solution : Identify and discard corrupted data samples.

## Calibration

Problem : Antenna electronics introduce complex gains  $g_i$

Solution : Numerically solve for antenna gains  $g_i$  and apply corrections



- Observe a source where  $\langle E_i E_j^* \rangle$  is known
- Use information from all  $ij$  to solve for  $g_i$
- Divide out  $g_i g_j^*$  from target data

# Data Analysis

---

But, the measured  $\langle E_i E_j^* \rangle$  values are imperfect and incomplete

## Flagging

Problem : Radio Frequency Interference (RFI)

Solution : Identify and discard corrupted data samples.

## Calibration

Problem : Antenna electronics introduce complex gains  $g_i$

Solution : Numerically solve for antenna gains  $g_i$  and apply corrections

## Imaging

Problem : Sampling of the 2D Fourier Transform is incomplete  
( only  $N(N-1)/2$  measurements per timestep and frequency channel )

Solution : Fit a model of the sky brightness to the measured data.

# Image Formation

---

$\langle E_i E_j^* \rangle$  : The 2D Fourier Transform (FT) of the sky brightness, measured at a spatial frequency given by the geometry of the baseline (antenna pair)  $ij$ .

N antennas =  $N(N-1)/2$  samples (for each timestep and channel)

Imaging : Collect all samples onto a 2D spatial frequency grid, and do an iFT.

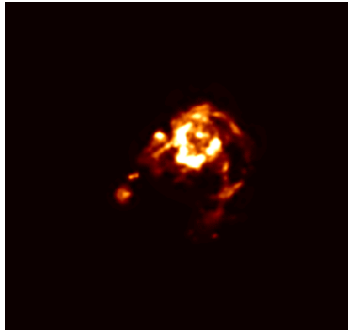
# Image Formation

$\langle E_i E_j^* \rangle$  : The 2D Fourier Transform (FT) of the sky brightness, measured at a spatial frequency given by the geometry of the baseline (antenna pair)  $ij$ .

N antennas =  $N(N-1)/2$  samples (for each timestep and channel)

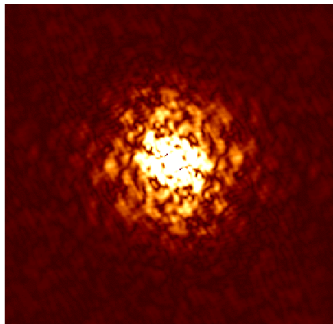
Imaging : Collect all samples onto a 2D spatial frequency grid, and do an iFT.

True Sky  
Brightness



↕↗ FT

True  
Visibility  
Function





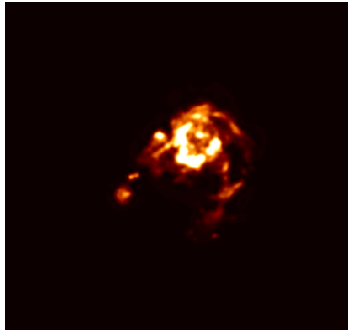
# Image Formation

$\langle E_i E_j^* \rangle$  : The 2D Fourier Transform (FT) of the sky brightness, measured at a spatial frequency given by the geometry of the baseline (antenna pair)  $ij$ .

N antennas =  $N(N-1)/2$  samples (for each timestep and channel)

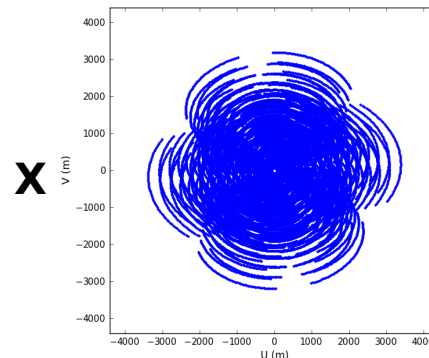
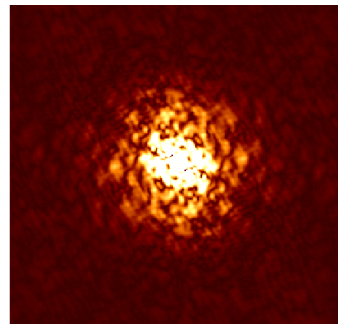
Imaging : Collect all samples onto a 2D spatial frequency grid, and do an iFT.

True Sky  
Brightness



↕ FT

True  
Visibility  
Function



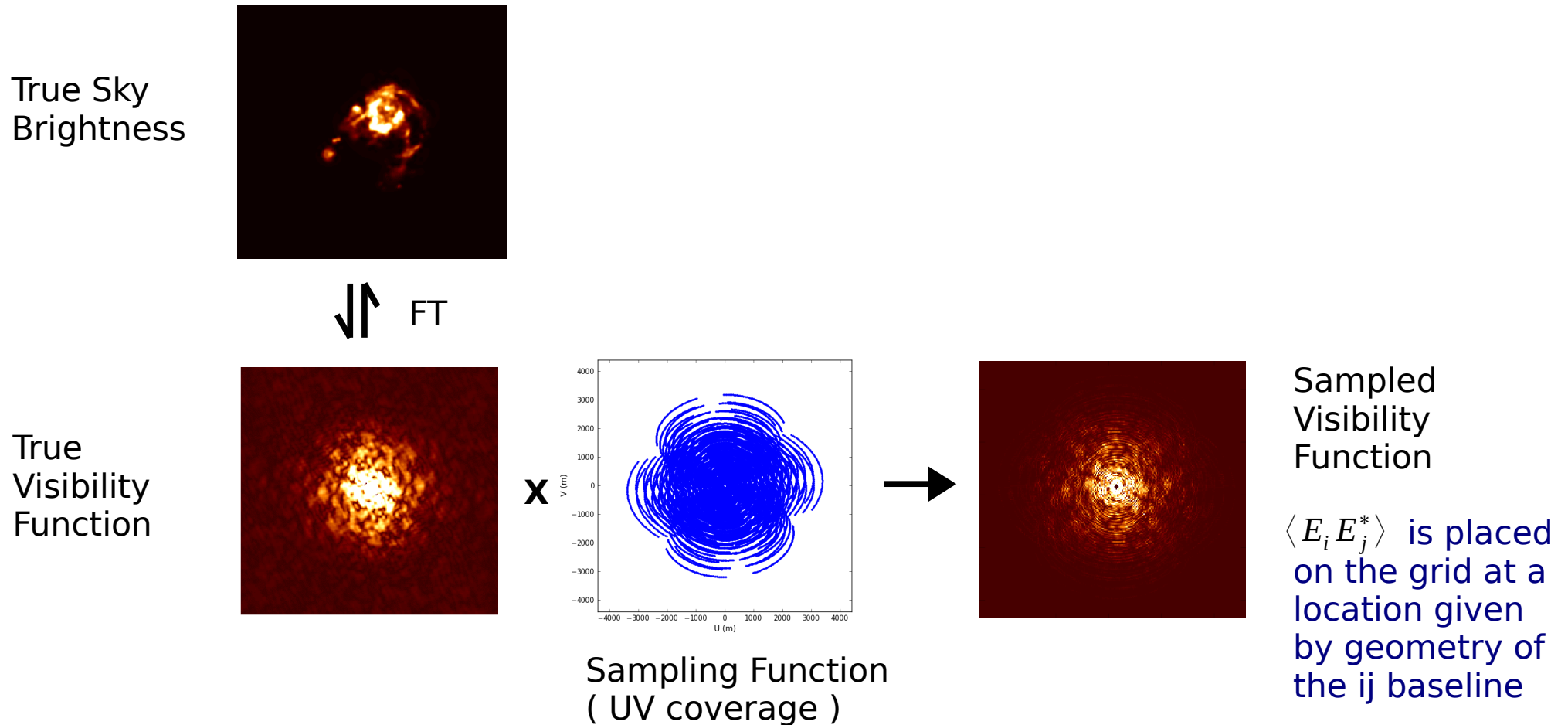
Sampling Function  
( UV coverage )

# Image Formation

$\langle E_i E_j^* \rangle$  : The 2D Fourier Transform (FT) of the sky brightness, measured at a spatial frequency given by the geometry of the baseline (antenna pair)  $ij$ .

$N$  antennas =  $N(N-1)/2$  samples (for each timestep and channel)

Imaging : Collect all samples onto a 2D spatial frequency grid, and do an iFT.

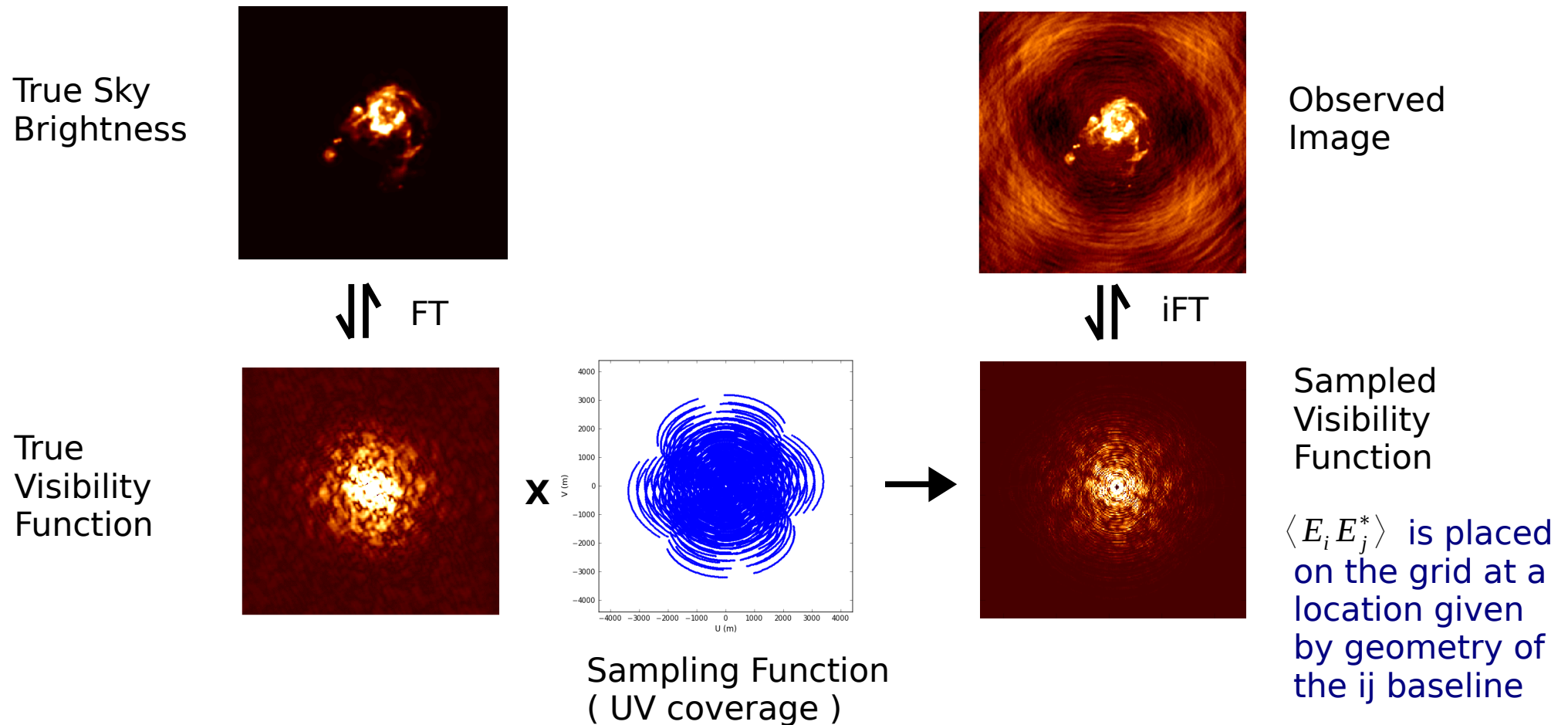


# Image Formation

$\langle E_i E_j^* \rangle$  : The 2D Fourier Transform (FT) of the sky brightness, measured at a spatial frequency given by the geometry of the baseline (antenna pair)  $ij$ .

N antennas =  $N(N-1)/2$  samples (for each timestep and channel)

Imaging : Collect all samples onto a 2D spatial frequency grid, and do an iFT.

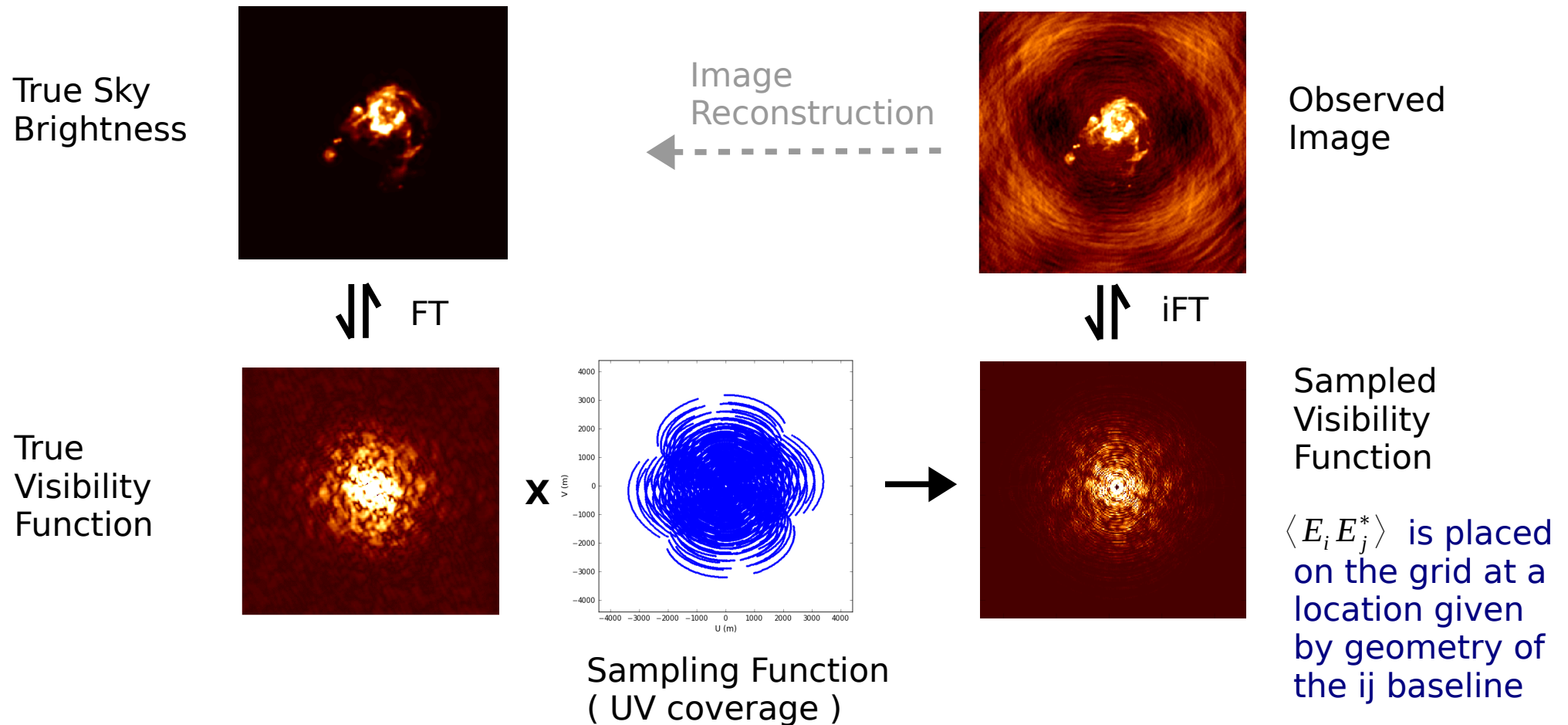


# Image Formation

$\langle E_i E_j^* \rangle$  : The 2D Fourier Transform (FT) of the sky brightness, measured at a spatial frequency given by the geometry of the baseline (antenna pair)  $ij$ .

N antennas =  $N(N-1)/2$  samples (for each timestep and channel)

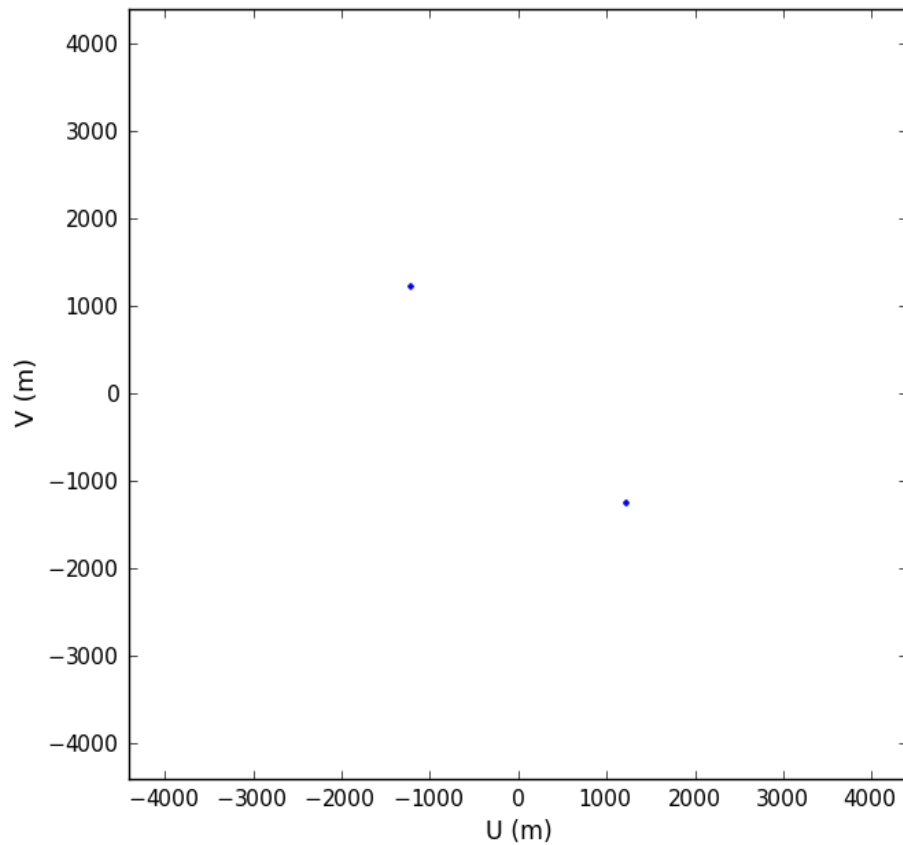
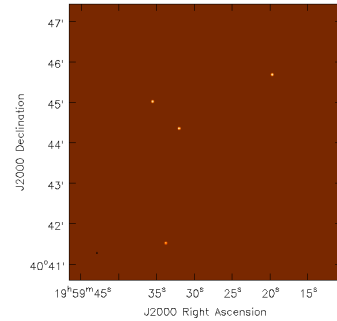
Imaging : Collect all samples onto a 2D spatial frequency grid, and do an iFT.



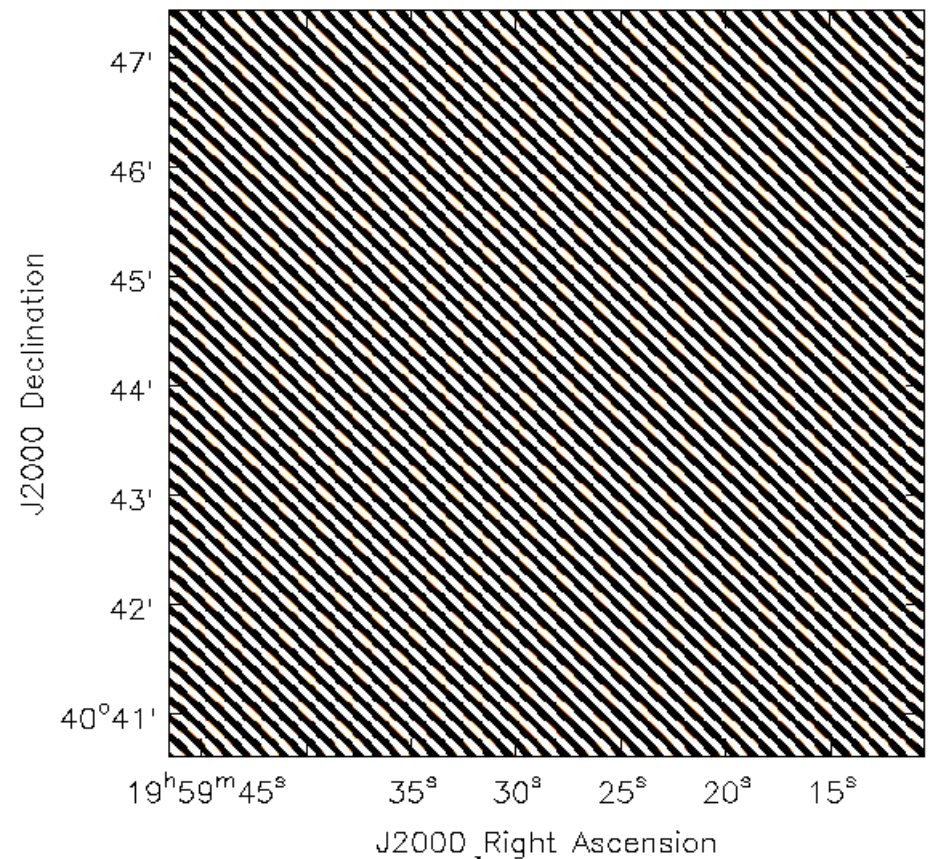
# Spatial Frequency (uv) coverage + Observed Image

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h, \theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky  
using 2 antennas



$S(u, v)$



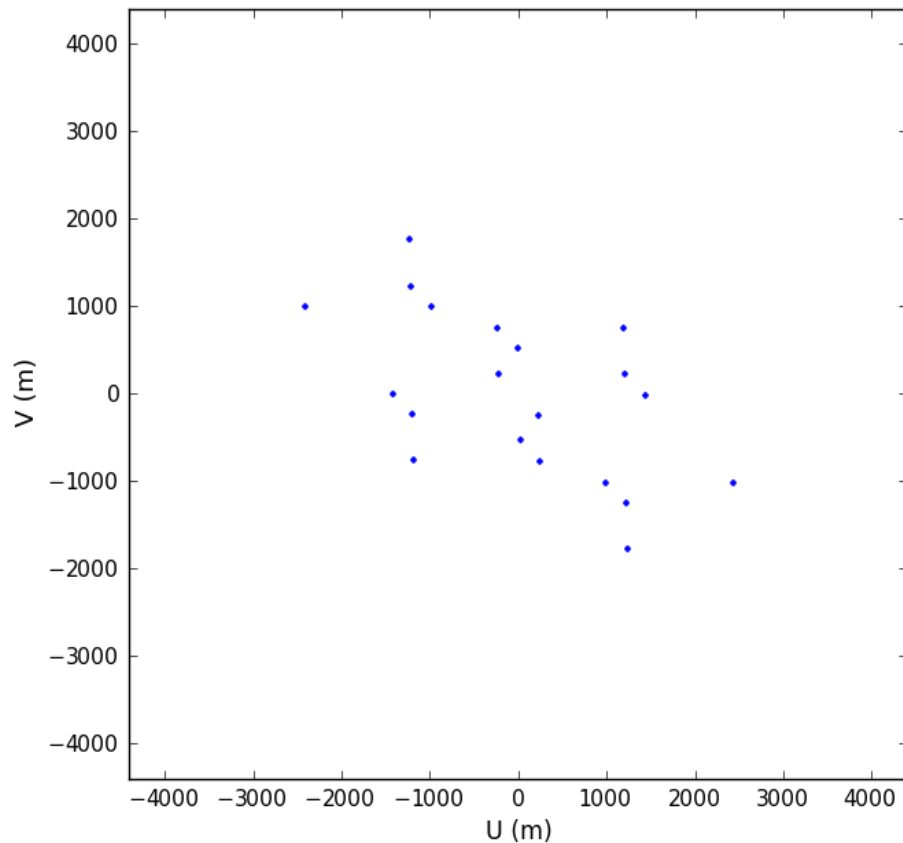
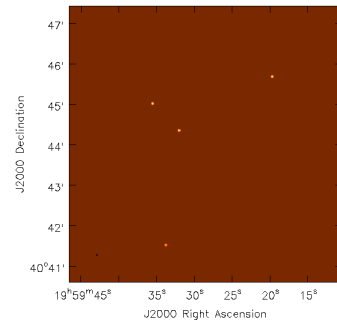
$I^{obs}(l, m)$



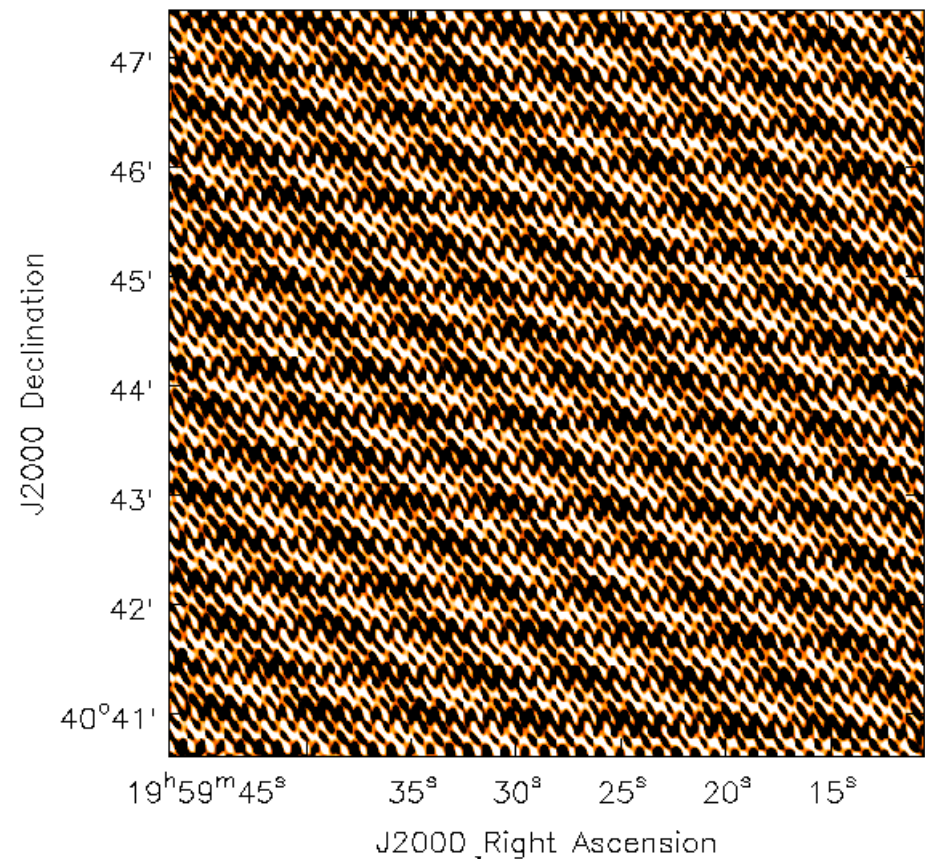
# Spatial Frequency (uv) coverage + Observed Image

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} R(h, \theta) \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky  
using 5 antennas



$S(u, v)$



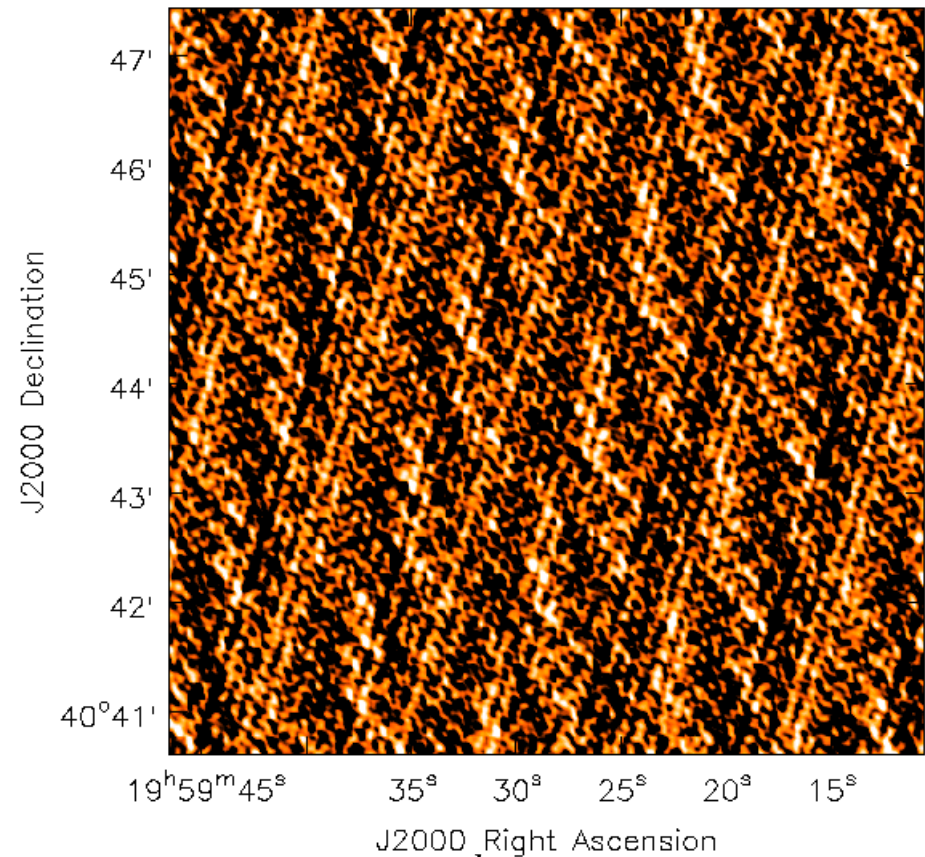
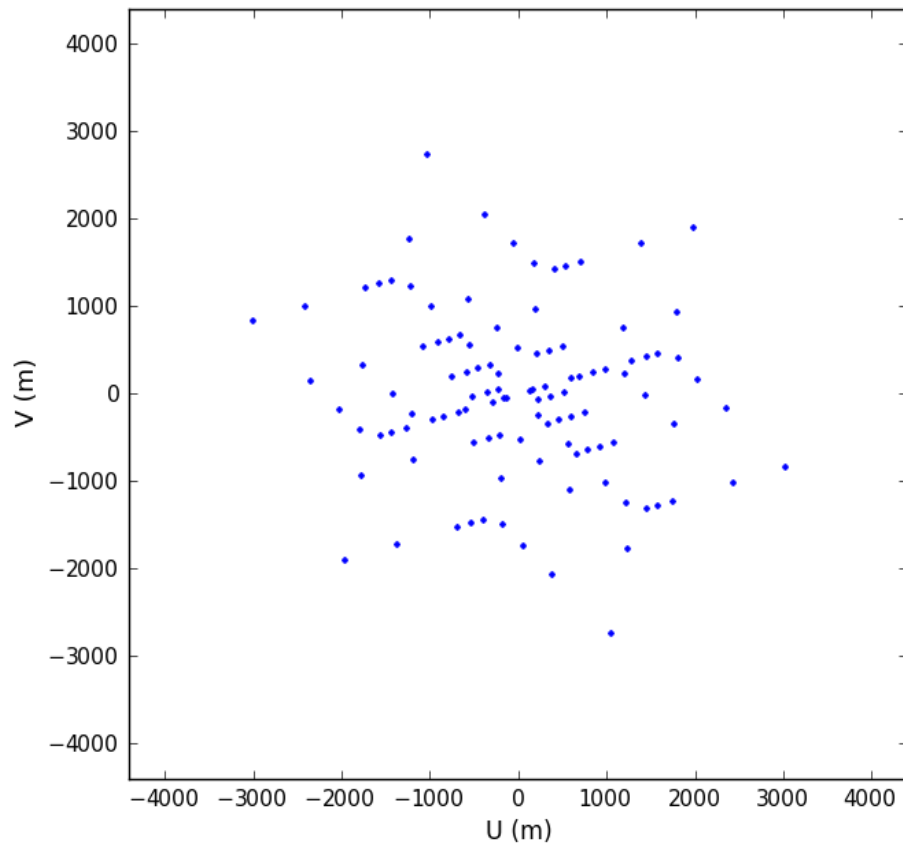
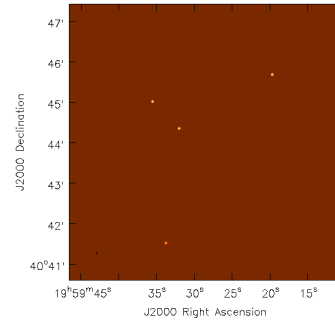
$I^{obs}(l, m)$



# Spatial Frequency (uv) coverage + Observed Image

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} R(h, \theta) \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky  
using 11 antennas



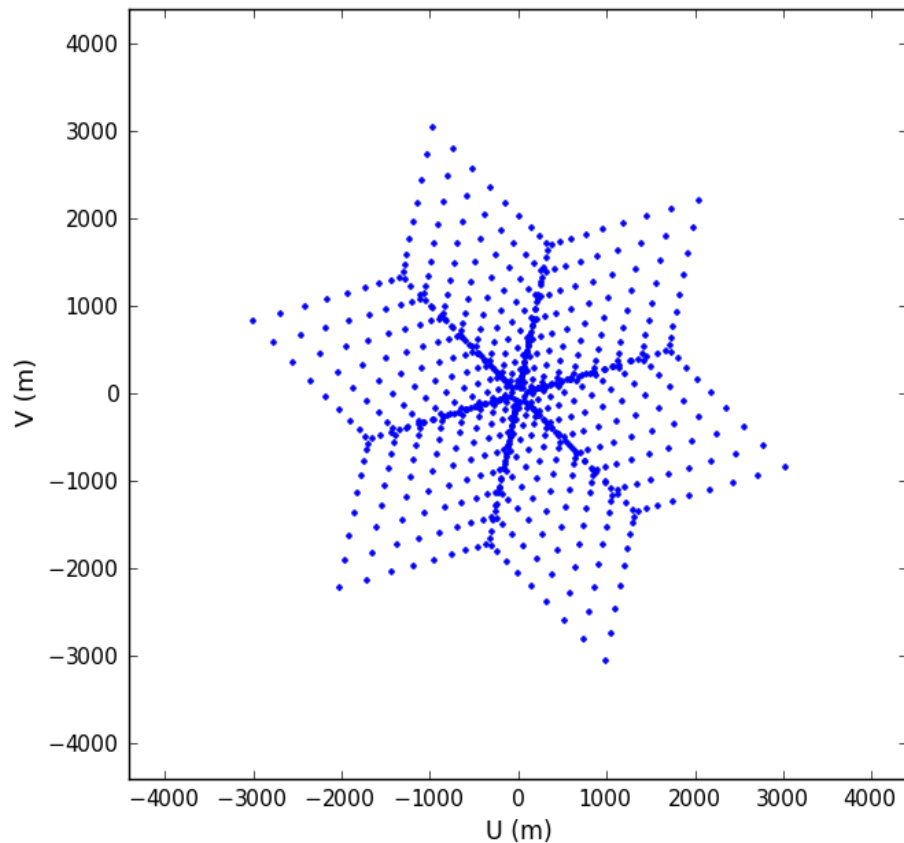
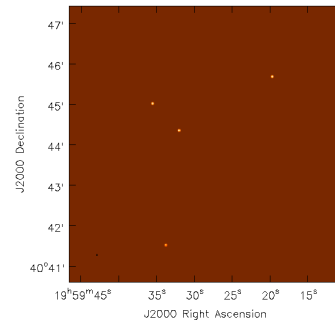
$S(u, v)$

$I^{obs}(l, m)$

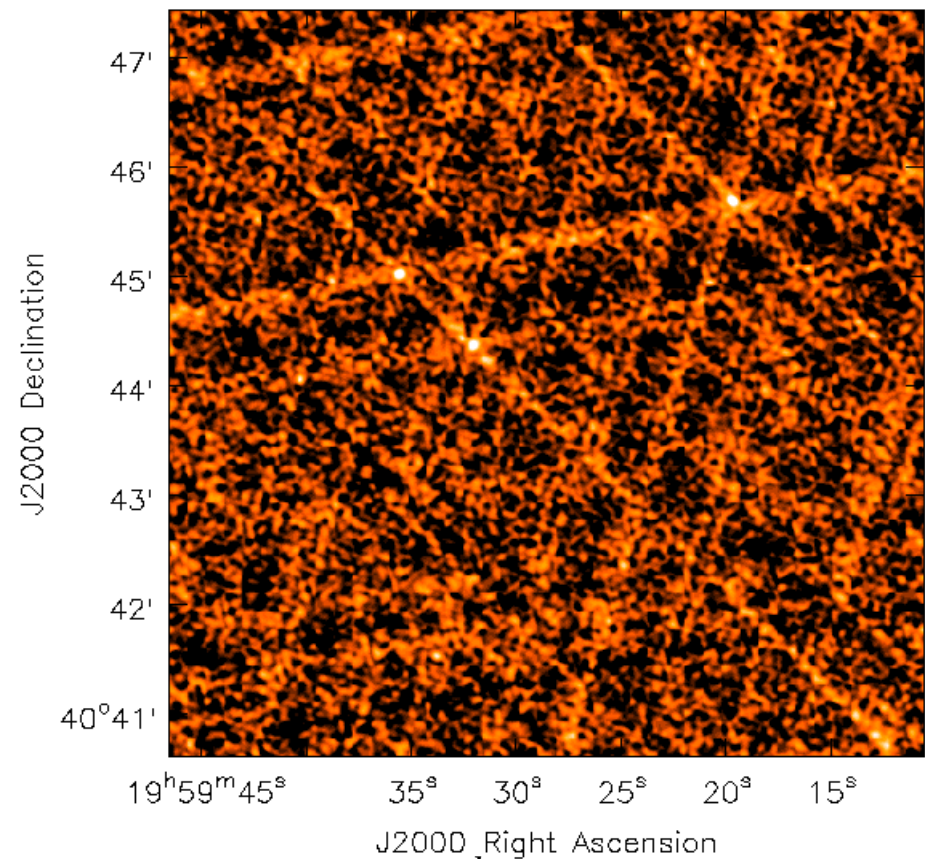
# Spatial Frequency (uv) coverage + Observed Image

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h, \theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky  
using 27 antennas



$S(u, v)$

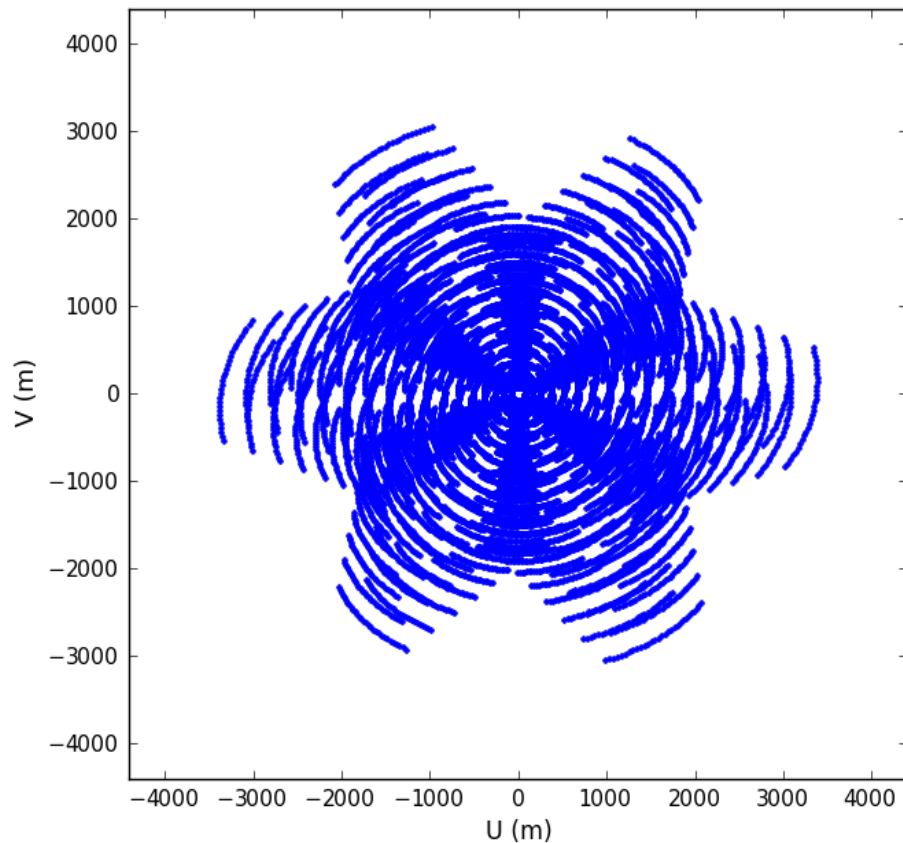
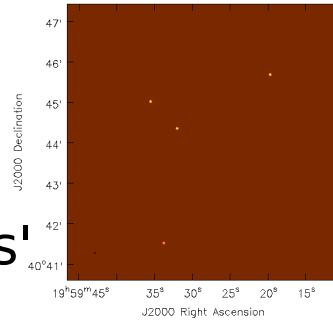


$I^{obs}(l, m)$

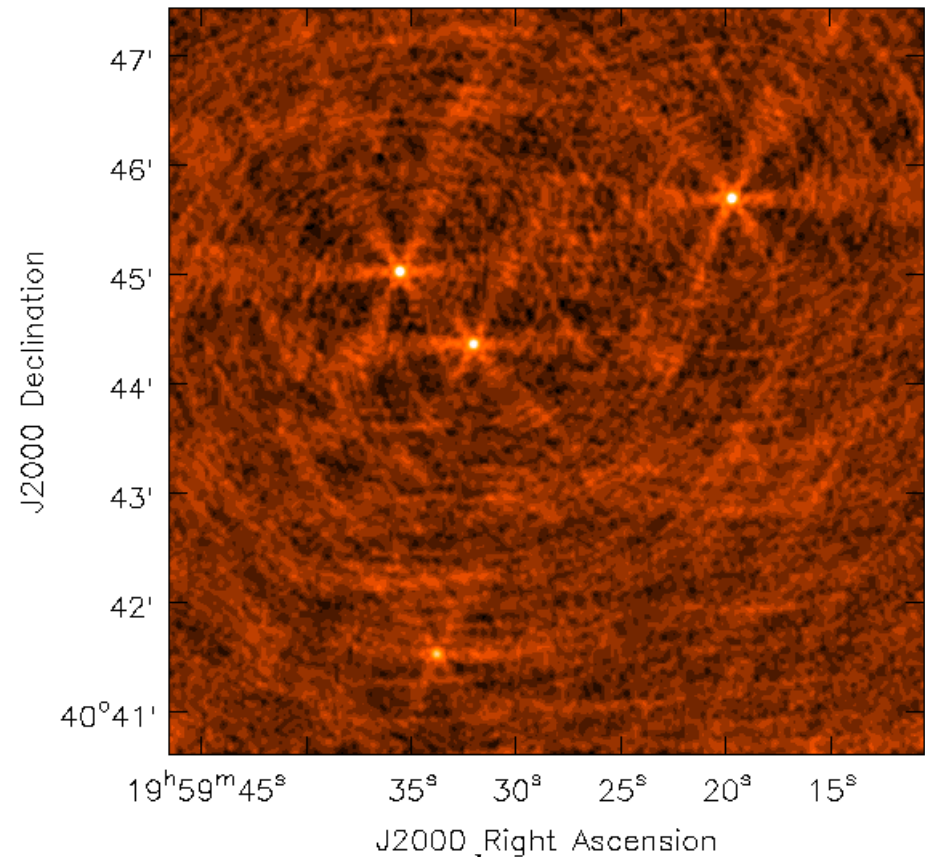
# Spatial Frequency (uv) coverage + Observed Image

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h, \theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky  
using 27 antennas  
over 2 hours  
'Earth Rotation Synthesis'



$$S(u, v)$$

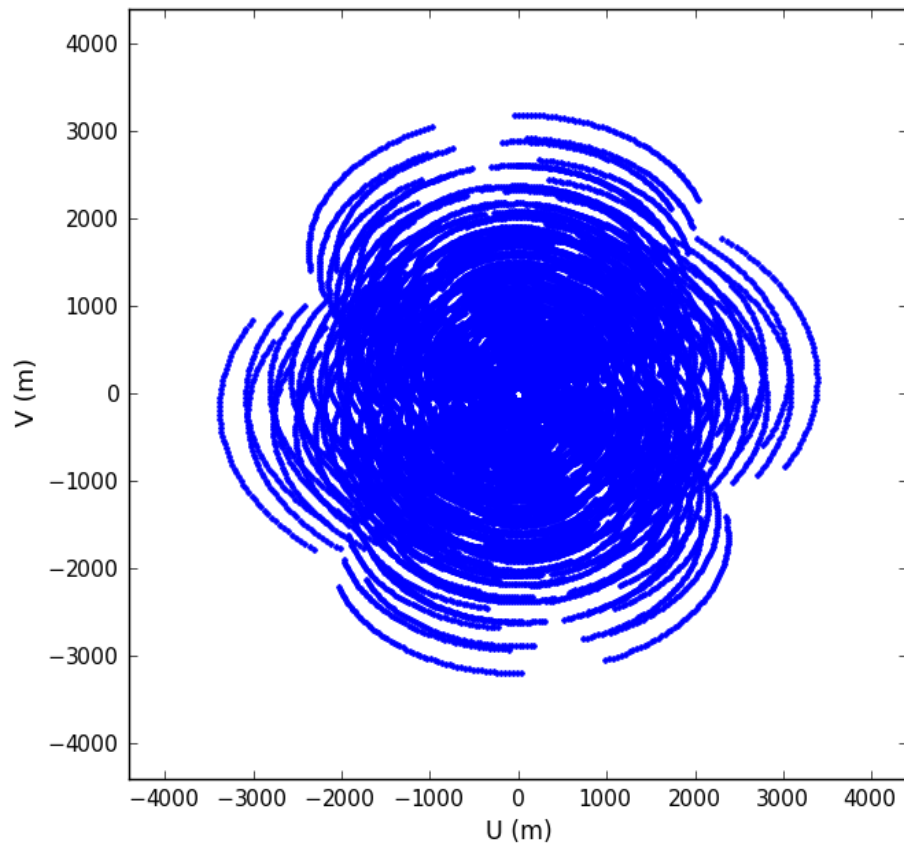
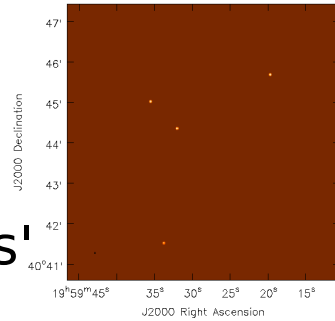


$$I^{obs}(l, m)$$

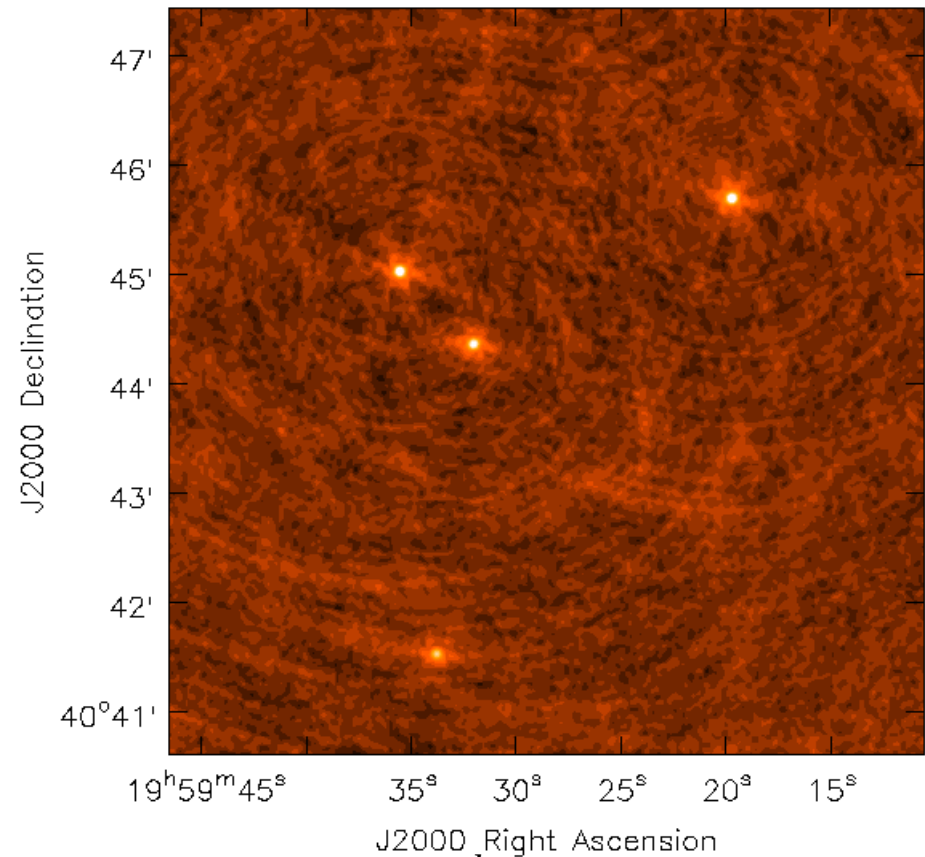
# Spatial Frequency (uv) coverage + Observed Image

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h, \theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky  
using 27 antennas  
over 4 hours  
'Earth Rotation Synthesis'



$S(u, v)$



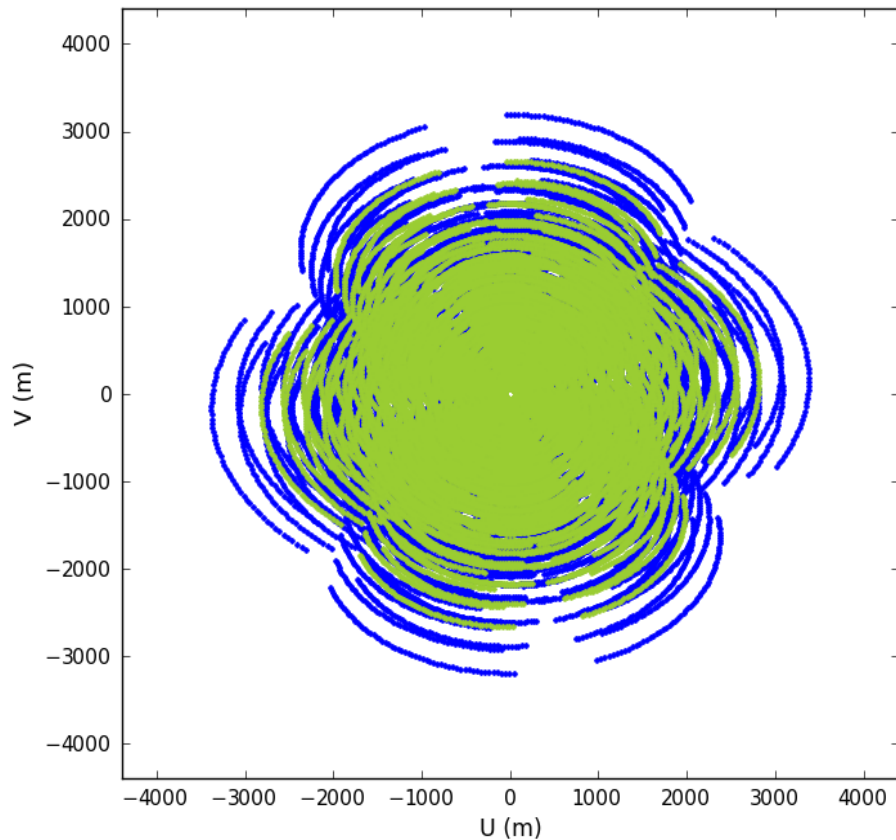
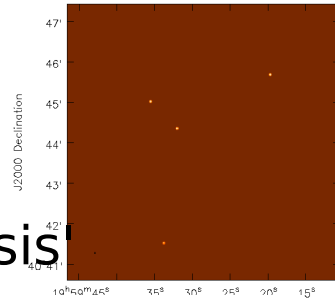
$I^{obs}(l, m)$



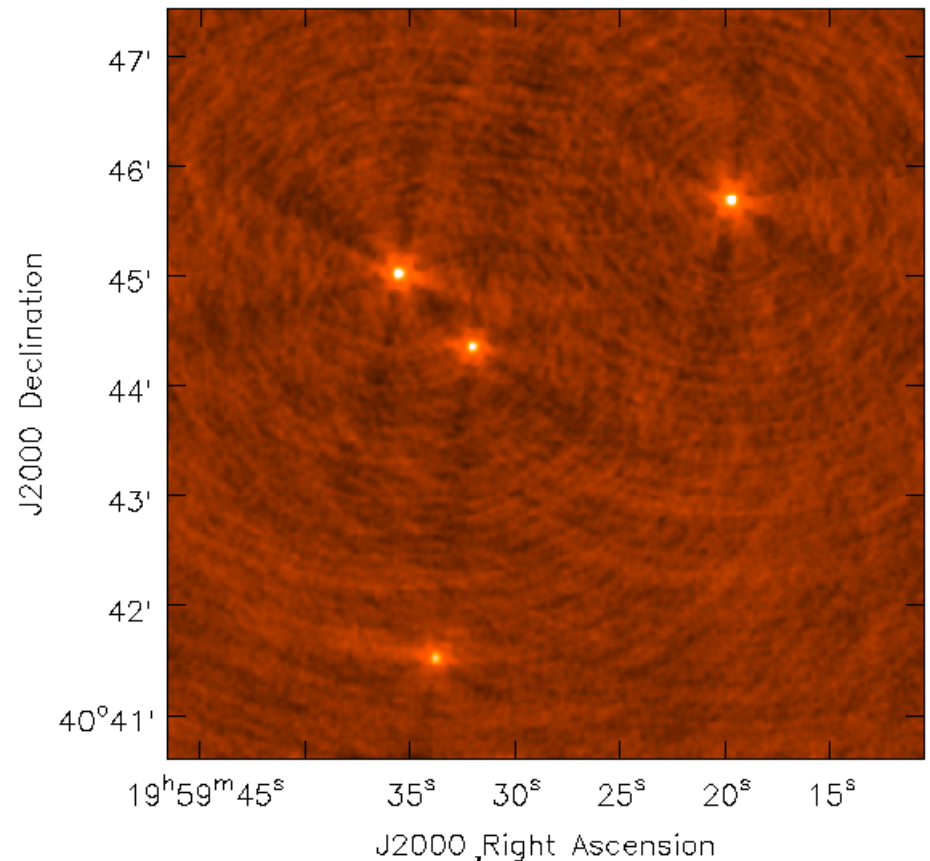
# Spatial Frequency (uv) coverage + Observed Image

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h, \theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky  
using 27 antennas  
over 4 hours, 2 freqs  
'Multi-Frequency Synthesis'



$S(u, v)$

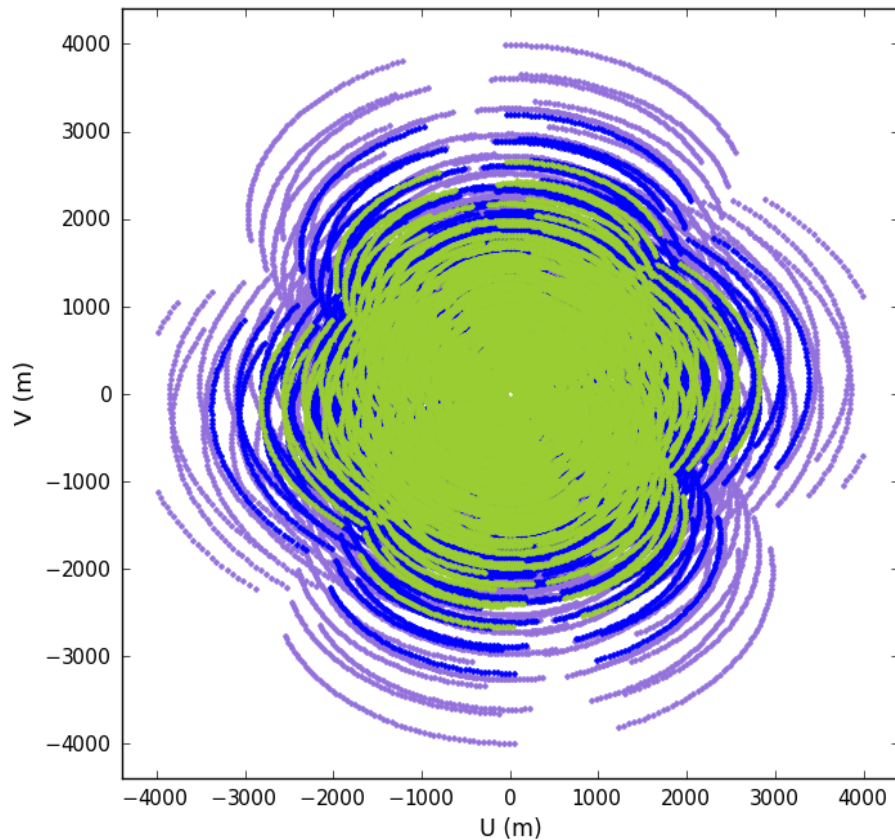
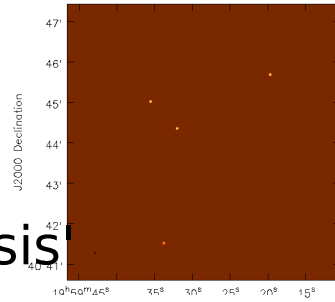


$I^{obs}(l, m)$

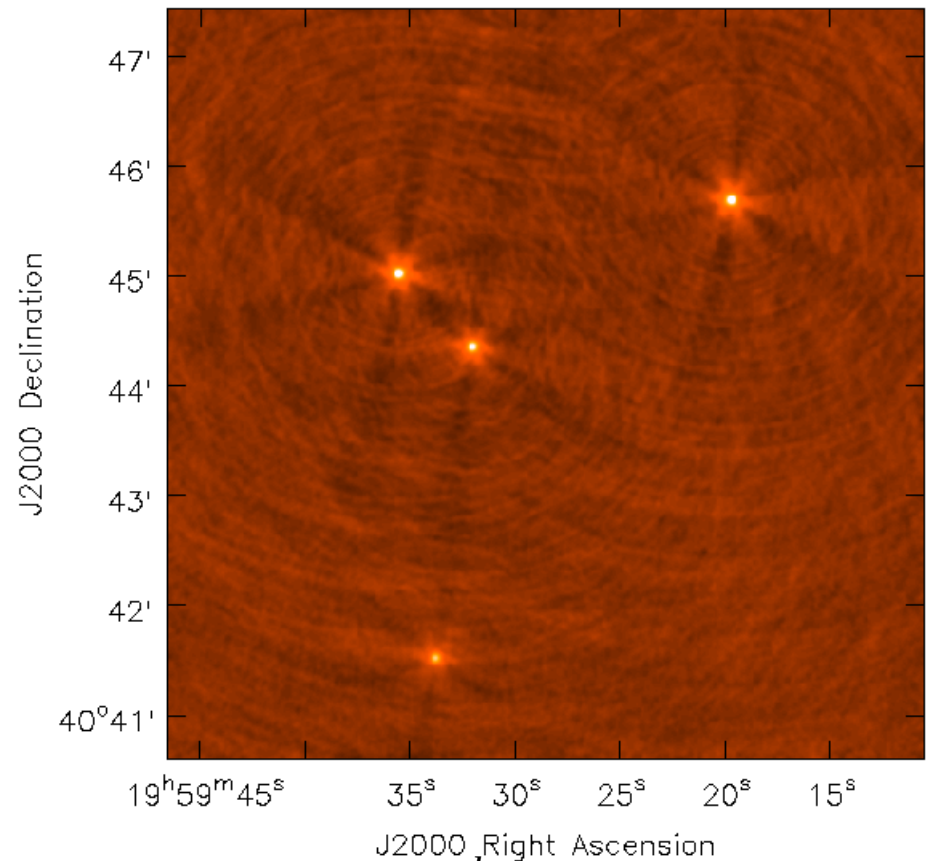
# Spatial Frequency (uv) coverage + Observed Image

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h, \theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky  
using 27 antennas  
over 4 hours, 3 freqs  
'Multi-Frequency Synthesis'



$S(u, v)$

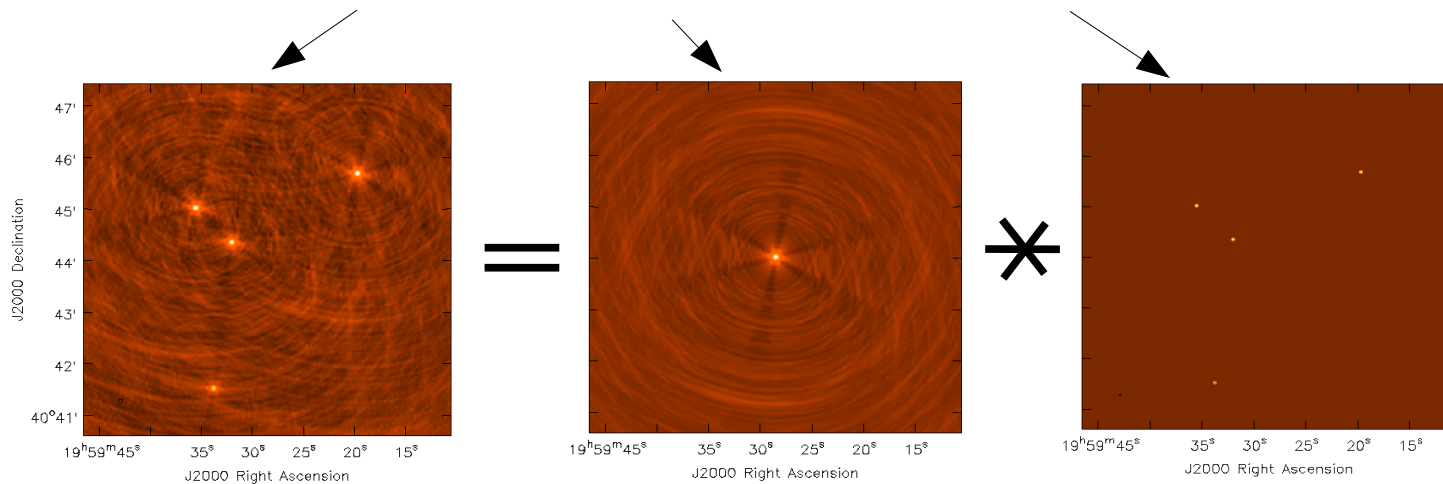


$I^{obs}(l, m)$



# Image formed by an interferometer : Convolution Equation

$$I^{obs}(l, m) = I^{PSF}(l, m) * I^{sky}(l, m)$$

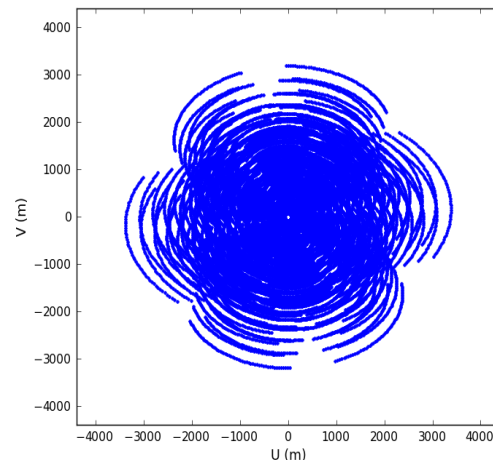


You have measured the  
Convolution of the True  
Sky with the  
instrumental PSF.

Recovering True Sky  
=  
DE-convolution

PSF = Point Spread  
Function

The inverse Fourier  
transform of the  
UV-coverage



$$S(u, v)$$

The PSF is

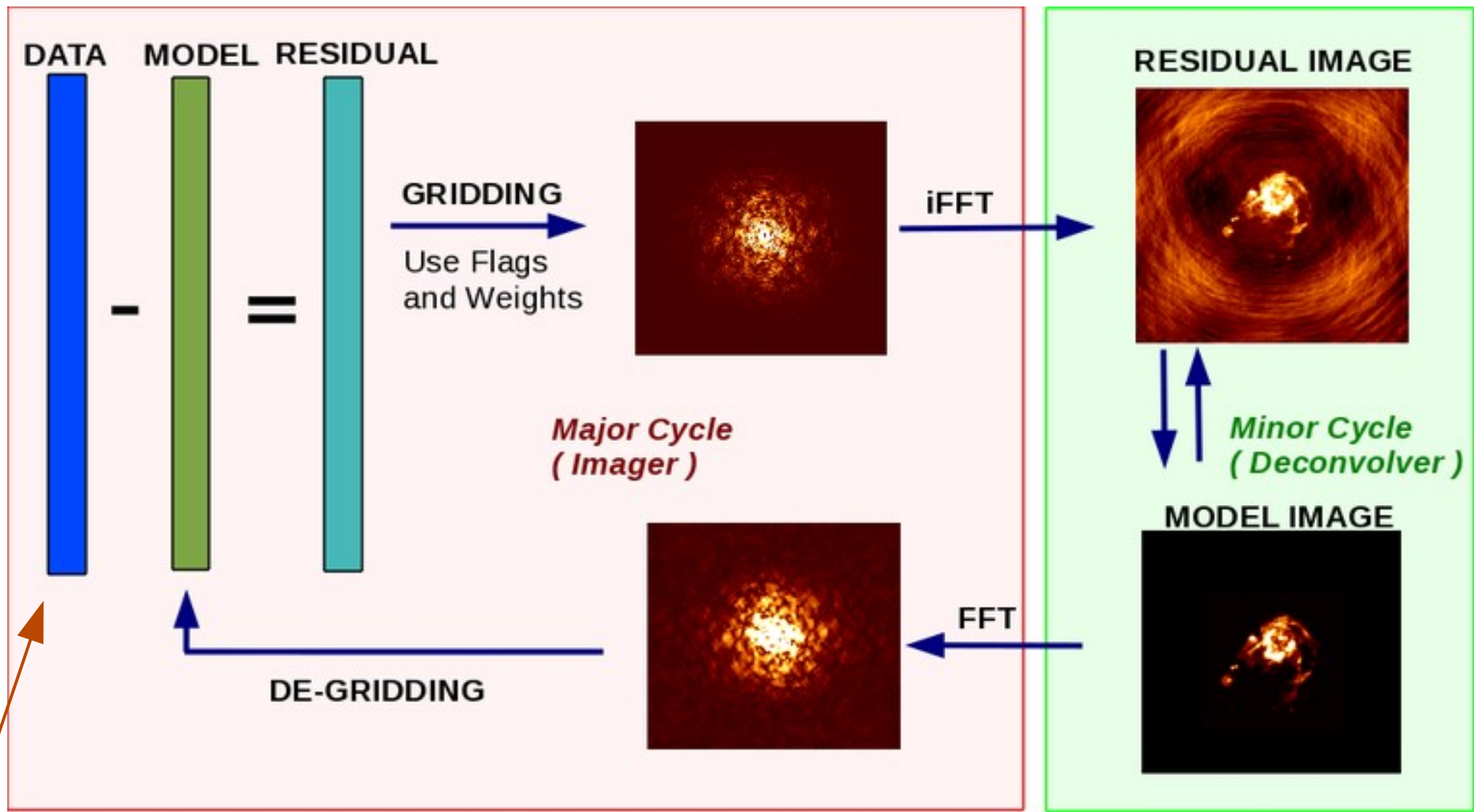
--- the impulse-response of the instrument  
( image of a point-source )

--- the intensity of the diffraction pattern  
through an array of 'slits' ( dishes )

--- a measure of the imaging-properties of the  
instrument

# Imaging + Deconvolution

Image reconstruction is an iterative model-fitting / optimization problem



$$\langle E_i E_j^* \rangle, \vec{u}, \vec{v}$$

# Build Your Own Interferometer !

---

# Build Your Own Interferometer !

## Array configuration

Spiral

×

▼

30 antennas

×

▼

## Source Declination



## Observatory Latitude



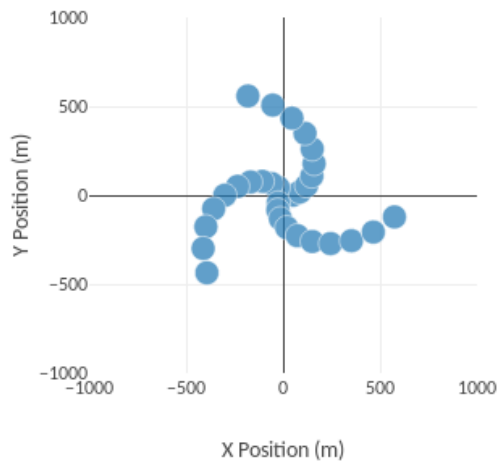
## Object to observe

Few Compact Sources

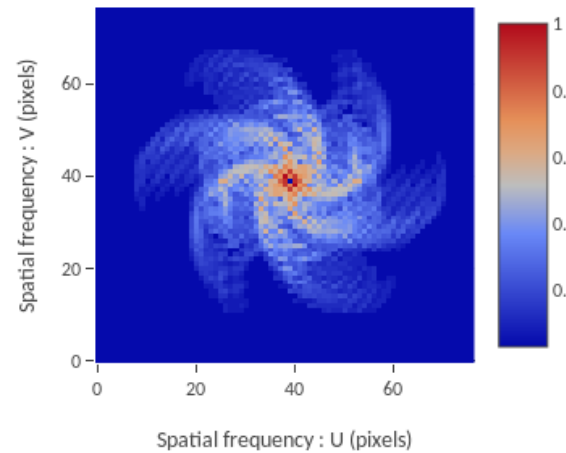
×

▼

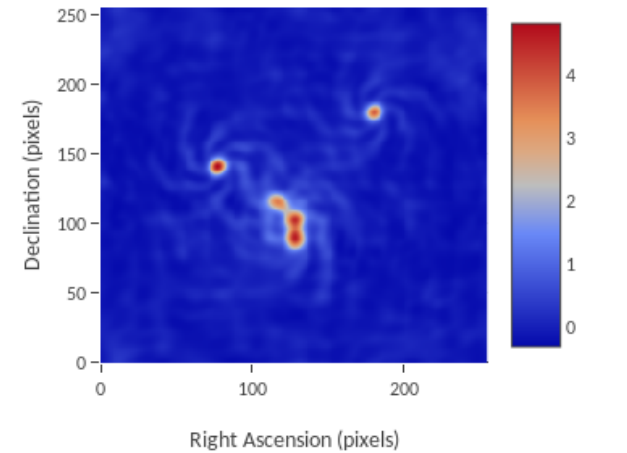
ARRAY CONFIGURATION



SPATIAL FREQUENCY COVERAGE



OBSERVED IMAGE



## Expand or shrink the array layout



## Pick data weighting scheme

☒ Natural

☐ Robust

☐ Uniform

## Select the observation hour-angle range



undo



# Build Your Own Interferometer !

How does the antenna layout and count affect

- UV-coverage
- Point Spread Function

Change relative orientation of the array and the object

- Projection effects

Point sources versus extended emission

- Spatial scales
- Point Spread Function

Array configuration

Spiral  
30 antennas

Source Declination



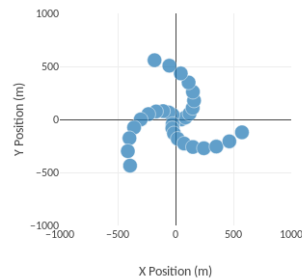
Observatory Latitude



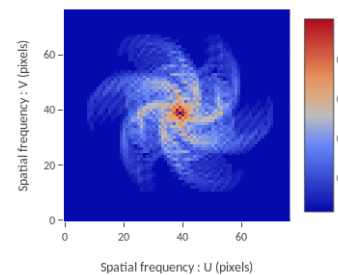
Object to observe

Few Compact Sources

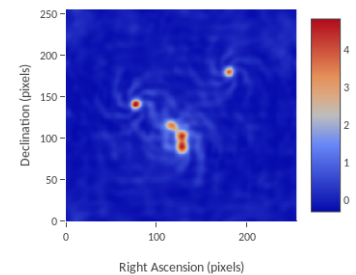
ARRAY CONFIGURATION



SPATIAL FREQUENCY COVERAGE



OBSERVED IMAGE



Expand or shrink the array layout

more compact more extended

Pick data weighting scheme

☒ Natural ☐ Robust ☐ Uniform

Select the observation hour-angle range

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

Effect of a compact versus spread-out configuration

- Angular resolution

Different weighting schemes

- What features does each emphasize ?

Effect of aperture Synthesis

- Change time-ranges

# Imaging in Practice

---

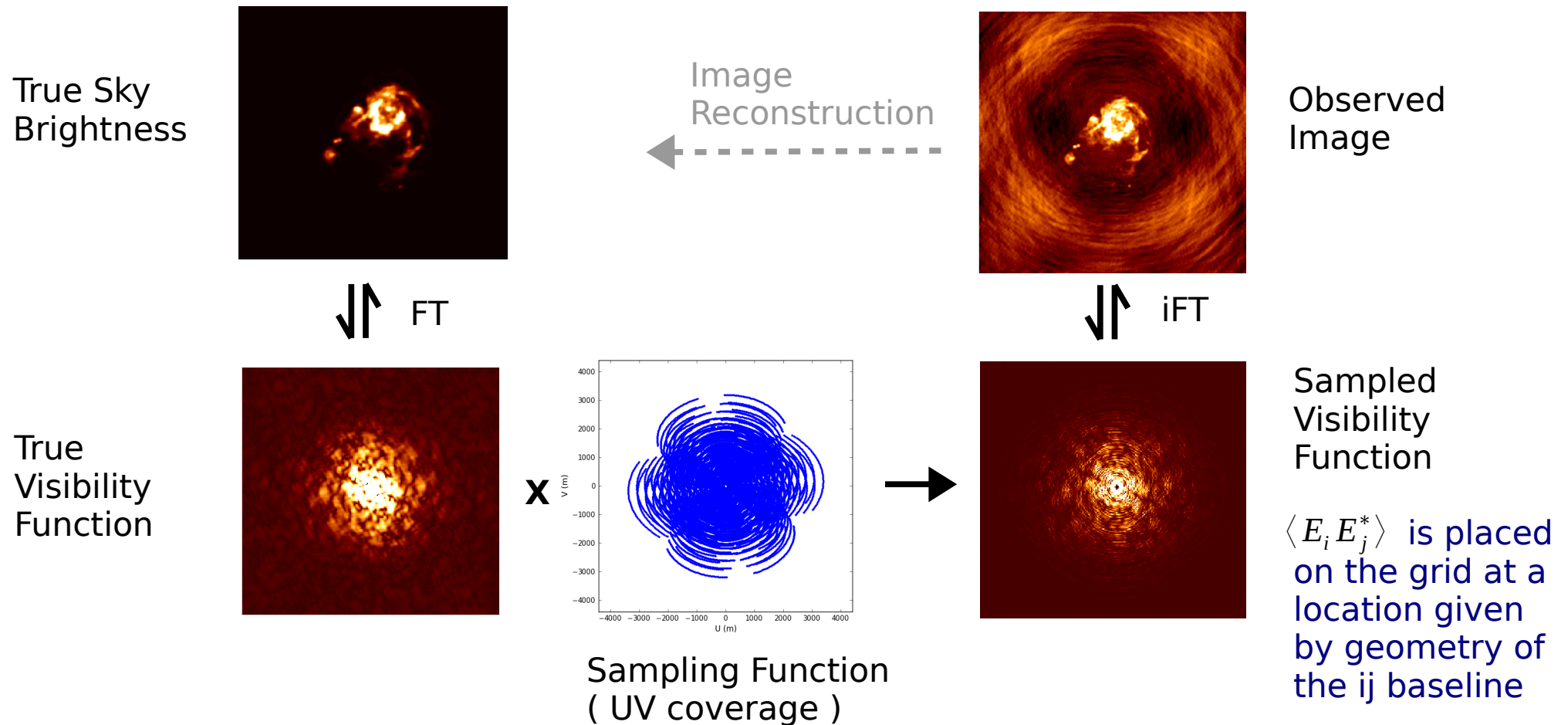
## Part II



$\langle E_i E_j^* \rangle$  : The 2D Fourier Transform (FT) of the sky brightness, measured at a spatial frequency given by the geometry of the baseline (antenna pair)  $ij$ .

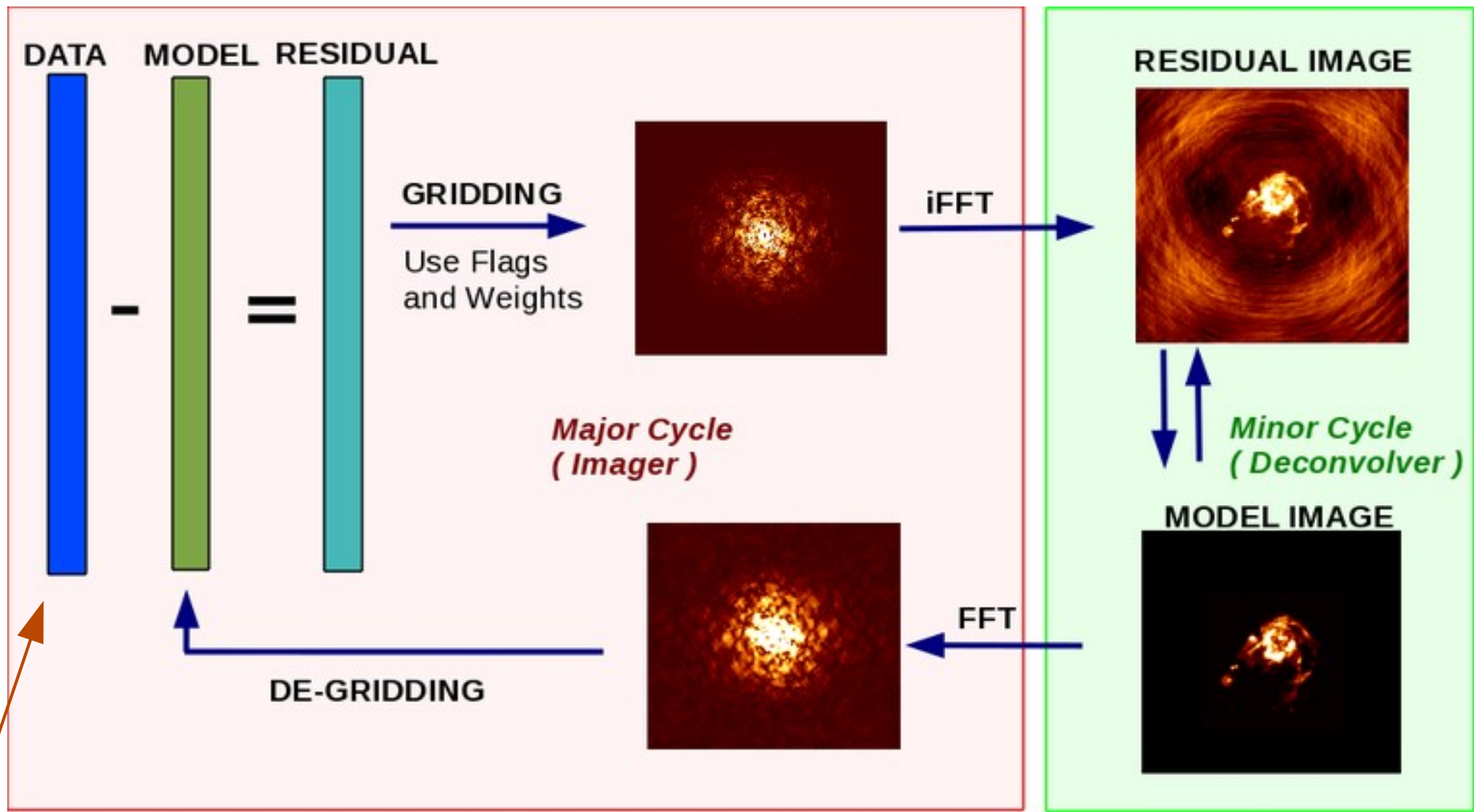
N antennas =  $N(N-1)/2$  samples (for each timestep and channel)

Imaging : Collect all samples onto a 2D spatial frequency grid, and do an iFT.



# Imaging + Deconvolution

Image reconstruction is an iterative model-fitting / optimization problem



$$\langle E_i E_j^* \rangle, \vec{u}, \vec{v}$$

# Imaging in practice

---

## Basic Imaging :

Step 1 : Define image size and cell size

Step 2 : Gridding, data-weighting and FFT

Step 3 : Iterative deconvolution

# Imaging in practice : Choosing image size, cell-size

---

- Choosing image 'cell' size : Nyquist-sample the main lobe of the PSF

$$\text{PSF beam width } \frac{\lambda}{b_{\max}} = \frac{1}{u_{\max}} \text{ radians} \quad \left( \times \frac{180}{\pi} \text{ to convert to degrees} \right)$$

This is the diffraction-limited angular-resolution of the telescope

Ex : Max baseline : 10 km. Freq = 1 GHz. Angular resolution : 6 arcsec

# Imaging in practice : Choosing image size, cell-size

---

- Choosing image 'cell' size : Nyquist-sample the main lobe of the PSF

$$\text{PSF beam width } \frac{\lambda}{b_{\max}} = \frac{1}{u_{\max}} \text{ radians} \quad \left( \times \frac{180}{\pi} \text{ to convert to degrees} \right)$$

This is the diffraction-limited angular-resolution of the telescope

Ex : Max baseline : 10 km. Freq = 1 GHz. Angular resolution : 6 arcsec

- Choosing image field-of-view (npixels) : As much as desired/practical.

$$\frac{1}{fov_{\text{rad}}} = \delta u \quad \text{Field of View (fov) controls the uv-grid-cell size } (\delta u, \delta v)$$

- Antenna primary-beam limits the field-of-view ( 'slits' of finite width )

# Imaging in practice : Choosing image size, cell-size

---

- Choosing image 'cell' size : Nyquist-sample the main lobe of the PSF

$$\text{PSF beam width } \frac{\lambda}{b_{\max}} = \frac{1}{u_{\max}} \text{ radians} \quad \left( \times \frac{180}{\pi} \text{ to convert to degrees} \right)$$

This is the diffraction-limited angular-resolution of the telescope

Ex : Max baseline : 10 km. Freq = 1 GHz. Angular resolution : 6 arcsec

- Choosing image field-of-view (npixels) : As much as desired/practical.

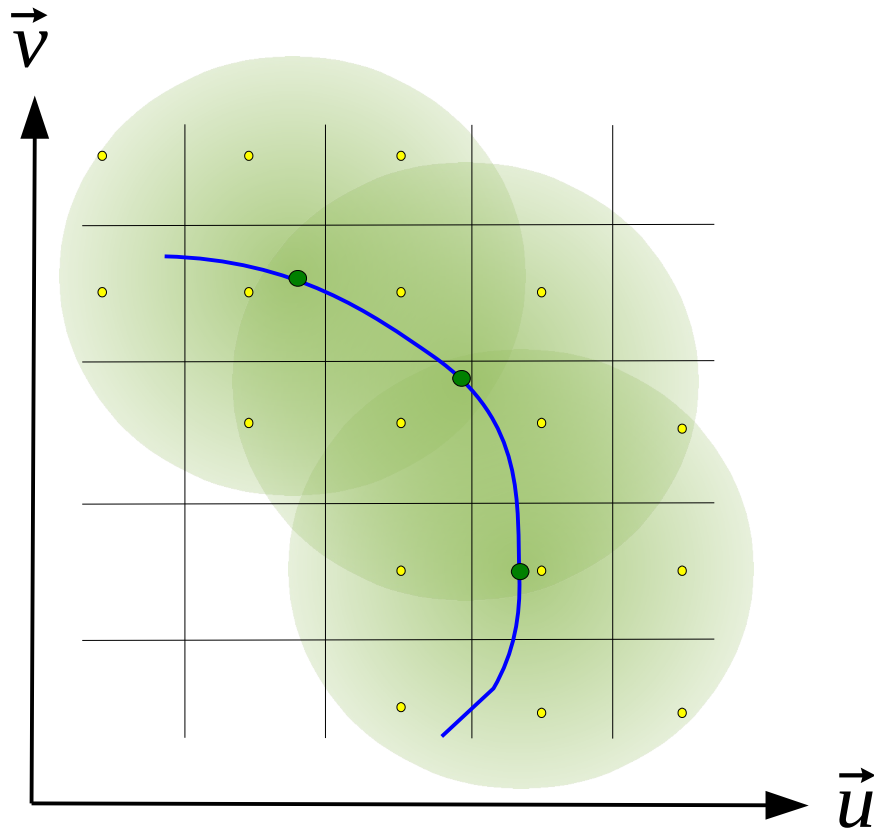
$$\frac{1}{fov_{\text{rad}}} = \delta u \quad \text{Field of View (fov) controls the uv-grid-cell size } (\delta u, \delta v)$$

- Antenna primary-beam limits the field-of-view ( 'slits' of finite width )

- Gridding + FFT :

- An interferometer measures irregularly spaced points on the UV-plane.
- Need to place the visibilities onto a regular grid of UV-pixels, and then take an FFT

# Imaging in practice : Gridding and Weighting



-- Visibility data are placed onto a regular grid before taking an i-FFT

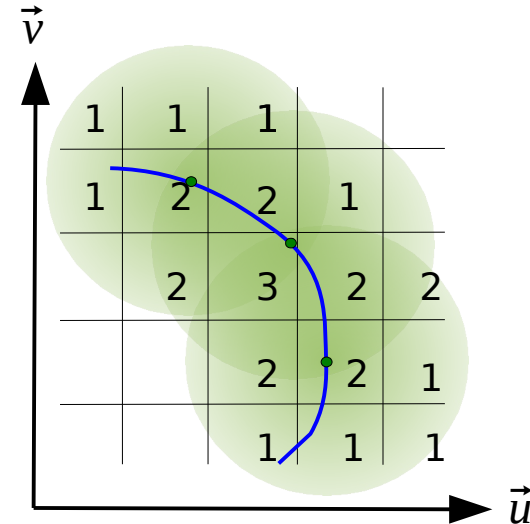
- Convolutional Resampling

=> Use a gridding convolution function

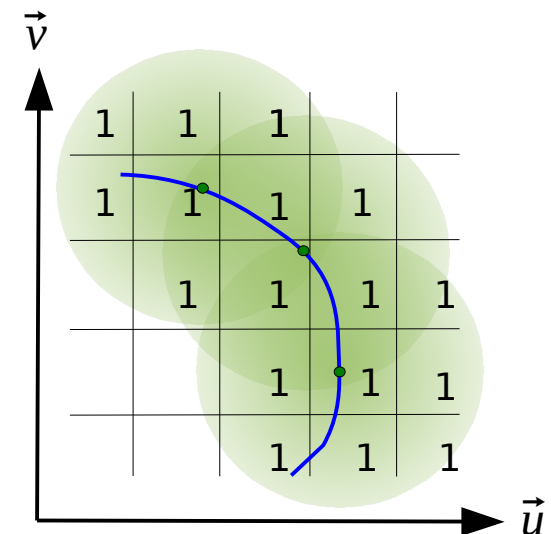
=> Use weights per visibility

(weighted average of all data points per cell)

An Image is a weighted-average of the data.



Natural  
Weights



Uniform  
Weights

# Imaging in practice : PSFs and Observed (dirty) Images

Natural

Bm : 5.6 arcsec  
0.1 sidelobe

Robust 0.7

Bm : 4.0 arcsec  
0.05 sidelobe

Uniform

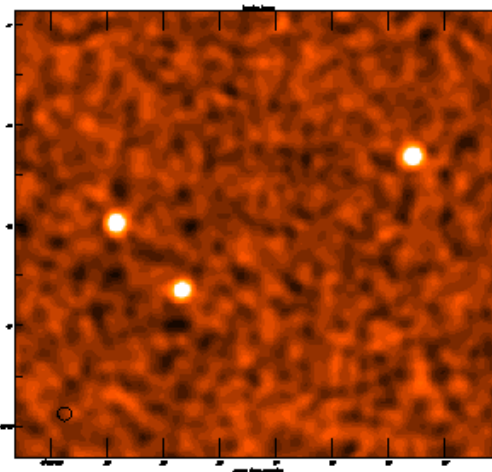
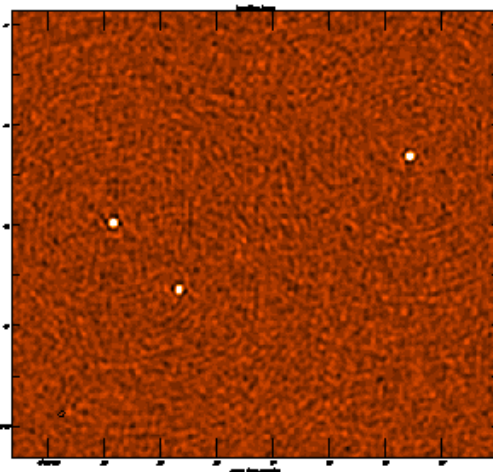
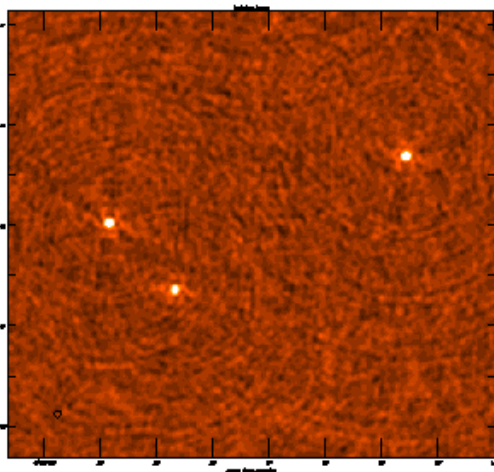
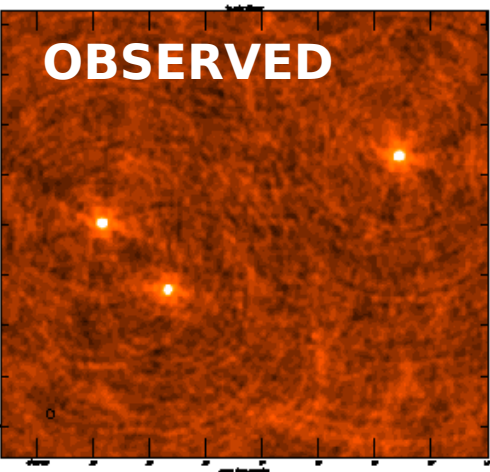
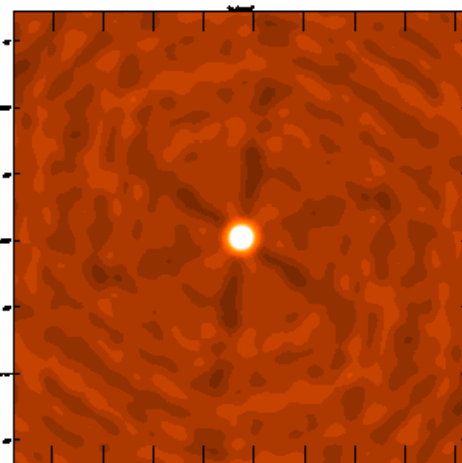
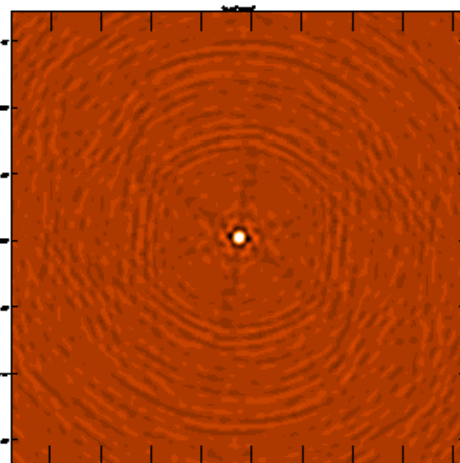
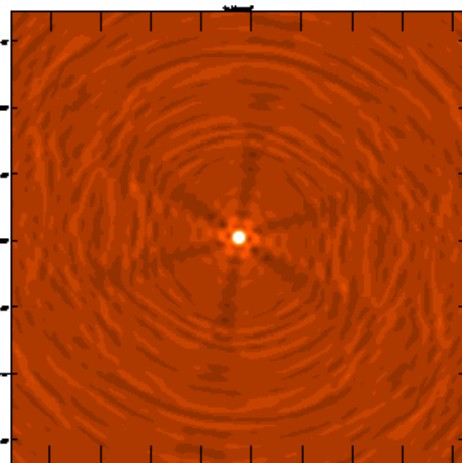
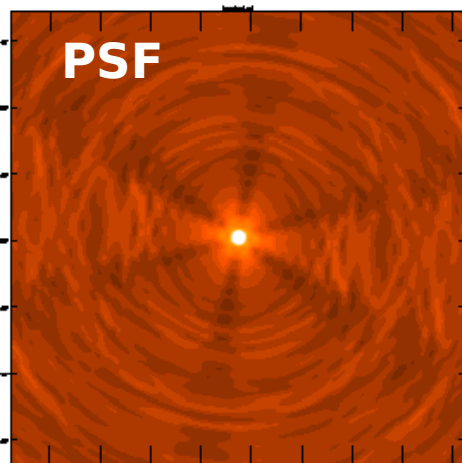
Bm : 3.2 arcsec  
+0.03,-0.08 sidelobe

Tapered Uniform

Bm : 8.0 arcsec  
0.01 sidelobe

PSF

OBSERVED



Note the noise-structure. Noise is correlated between pixels by the PSF. Image Units (Jy/beam)

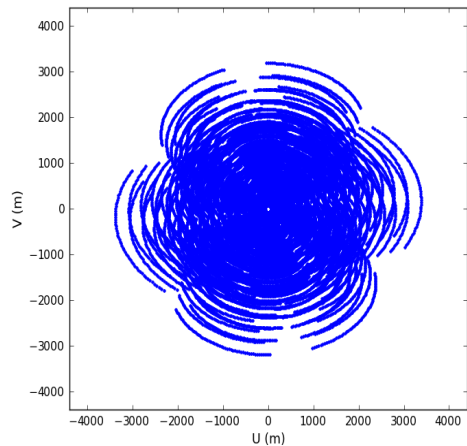
----- All pairs of images satisfy the convolution relation => Need to deconvolve them



# Imaging in practice : Weighting schemes

An Image is a weighted average of the data.

Weighting-scheme => modify the imaging properties of the instrument  
=> emphasize features/scales of interest  
=> control imaging sensitivity



	Uniform/Robust	Natural/Robust	UV-Taper
	All spatial frequencies get equal weight	All data points get equal weight	Low spatial freqs get higher weight than others
Resolution	higher	medium	lower
PSF Sidelobes (VLA)	lower	higher	depends
Point Source Sensitivity	lower	maximum	lower
Extended Source Sensitivity	lower	medium	higher

# Imaging in practice : Deconvolution

Observed image = Instrumental Point-Spread-Function convolved with the true sky

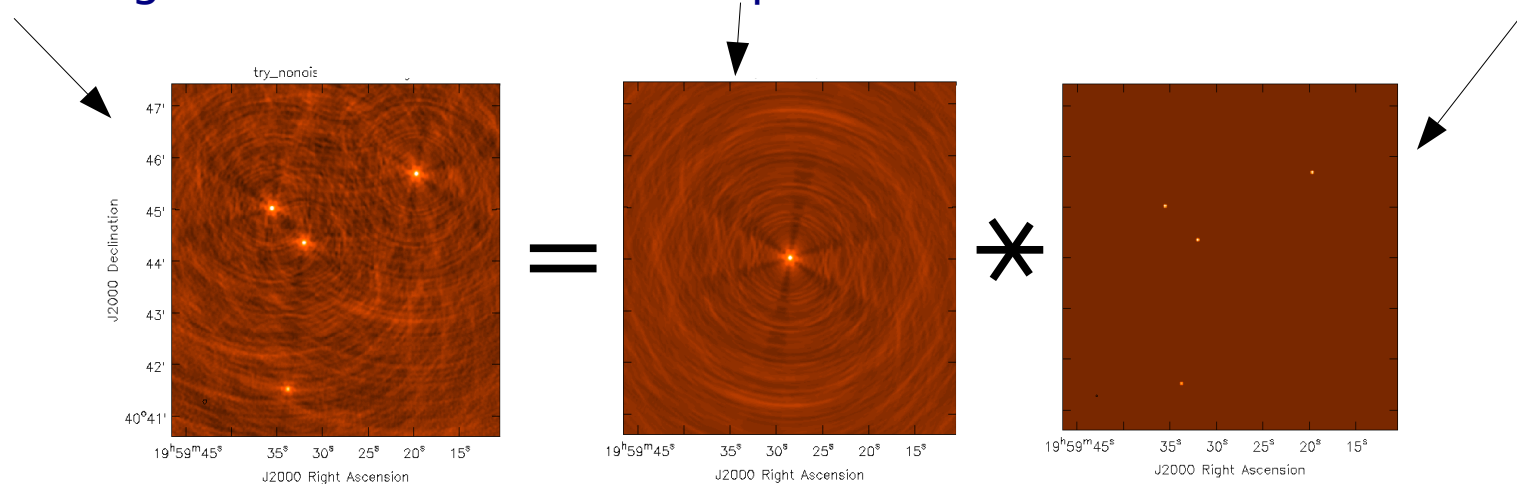


Image reconstruction is typically a “deconvolution” process.

**Algorithms** : Parameterized models + Iterative model fitting  
( chi-square minimization )

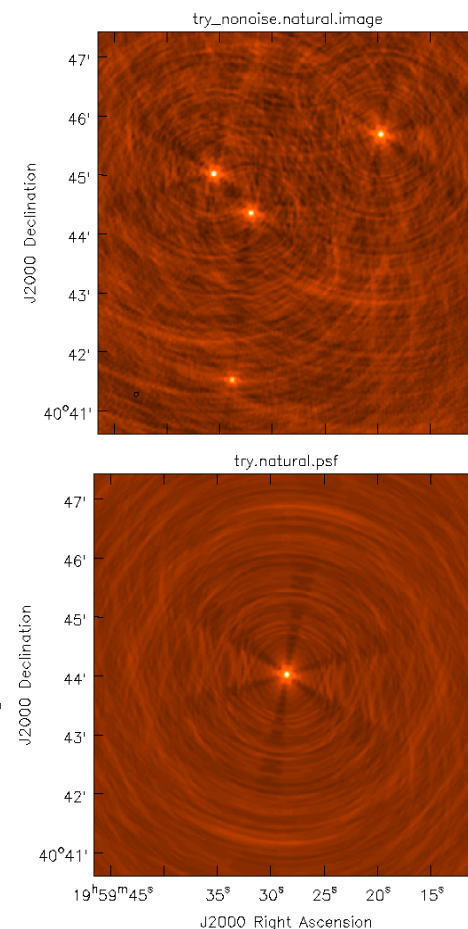
**CLEAN** : Model the sky as a collection of delta-function ‘flux components’

**Multi-Scale-CLEAN** : Model the sky as a collection of ‘blobs’ of different sizes

# Deconvolution – Hogbom CLEAN

## Sky Model : List of delta-functions

- (1) Construct the observed (dirty) image and PSF
  - (2) Search for the location of peak amplitude.
  - (3) Add a delta-function of this peak/location to the model
  - (4) Subtract the contribution of this component from the dirty image - a scaled/shifted copy of the PSF
- Repeat steps (2), (3), (4) until a stopping criterion is reached.
- (5) Restore : Smooth the model with a 'clean beam' and add residuals



The CLEAN algorithm can be formally derived as a model-fitting problem

- model parameters : locations and amplitudes of delta functions
- solution process :  $\chi^2$  minimization via an iterative steepest-descent algorithm ( method of successive approximation )

# Deconvolution – MultiScale (MS)-CLEAN

**Multi-Scale Sky Model** : Linear combination of 'blobs' of different scale sizes

- Efficient representation of both compact and extended structure (sparse basis)

A scale-sensitive algorithm

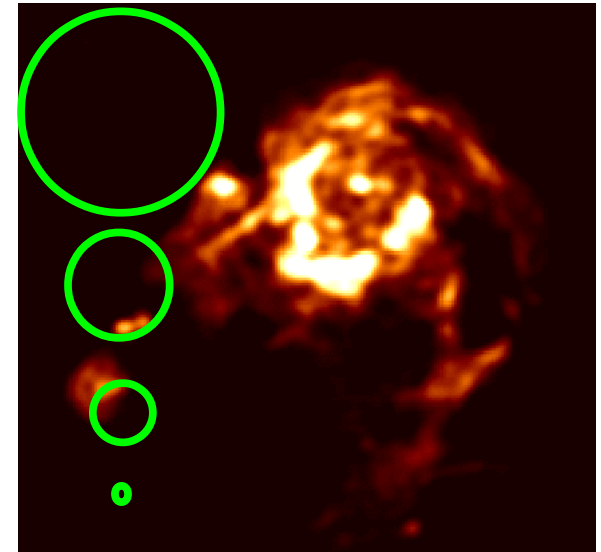
(1) Choose a set of scale sizes

(2) Calculate dirty/residual images  
smoothed to several scales (basis functions)

- Normalize by the relative sum-of-weights  
(instrument's sensitivity to each scale)

(3) Find the peak across all scales, update a single multi-scale model as well as all residual images (using information about coupling between scales)

Iterate, similar to Classic CLEAN, and restore at the end.



The MS-CLEAN algorithm can also be formally derived as a model-fitting problem using  $\chi^2$  minimization and a basis set consisting of several 'blob' sizes.

# Deconvolution – Comparison of Algorithms

CLEAN

MEM

MS-CLEAN

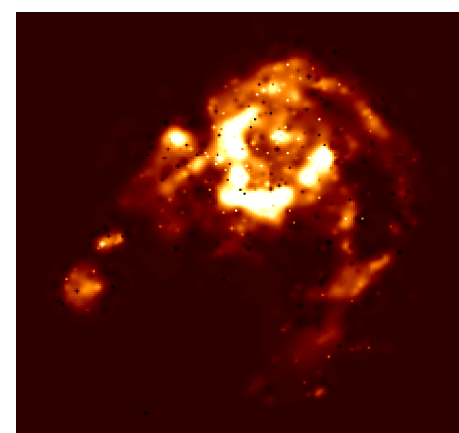
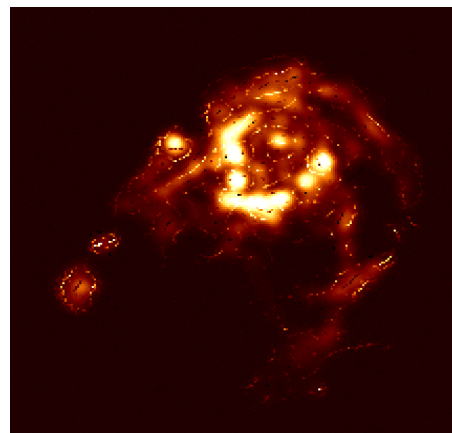
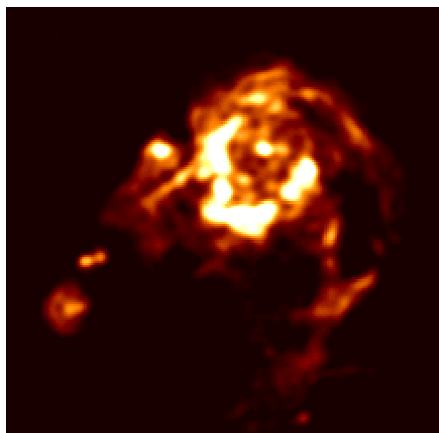
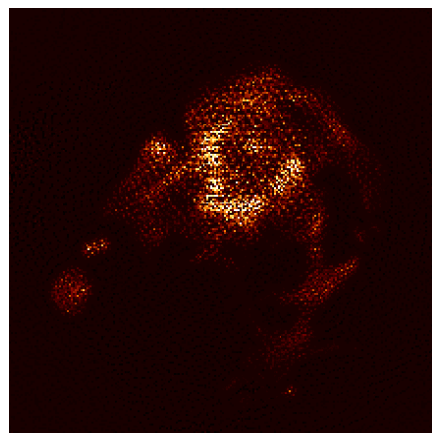
ASP

Point source  
model

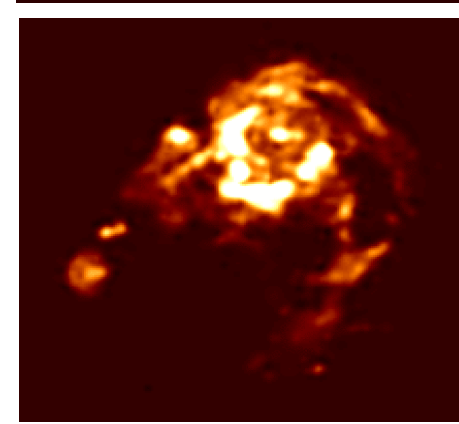
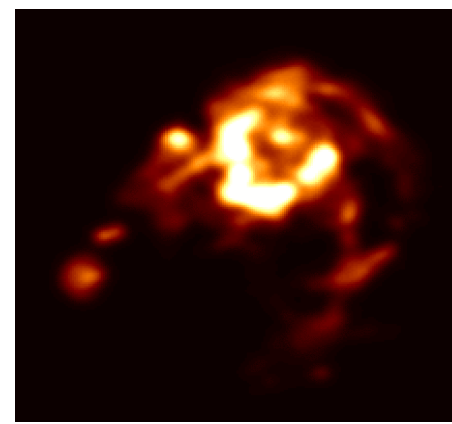
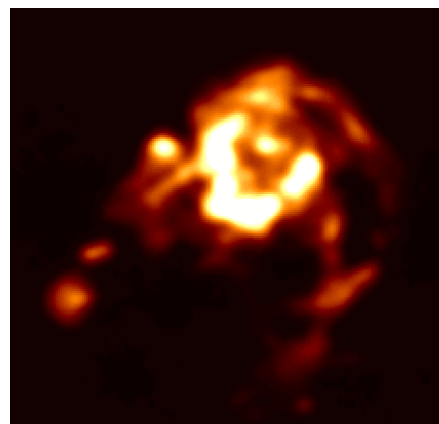
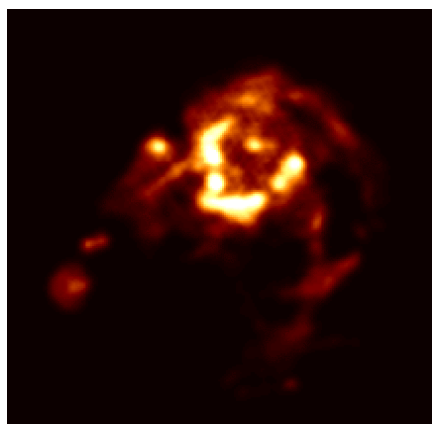
Point source  
model with a  
smoothness  
constraint

Multi-Scale model  
with a fixed set of  
scale sizes

Multi-Scale model  
with adaptive best-  
fit scale per  
component



$I^m$



$I^{out}$

# Deconvolution – Comparison of Algorithms

CLEAN

MEM

MS-CLEAN

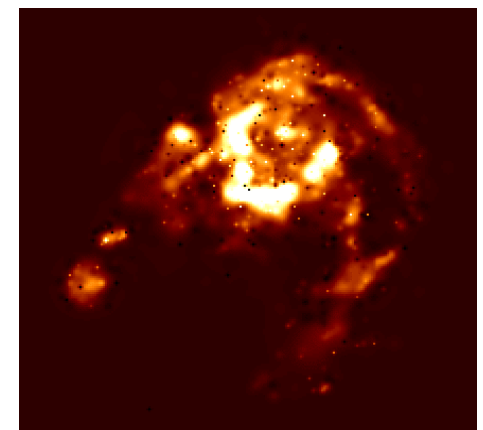
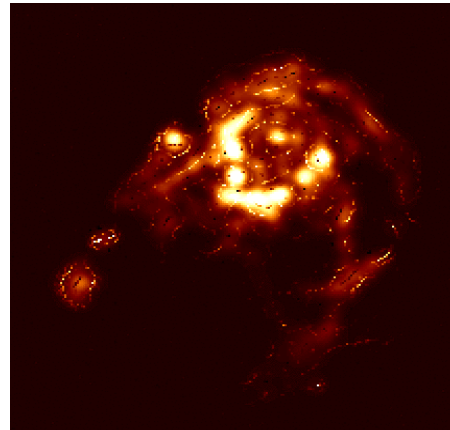
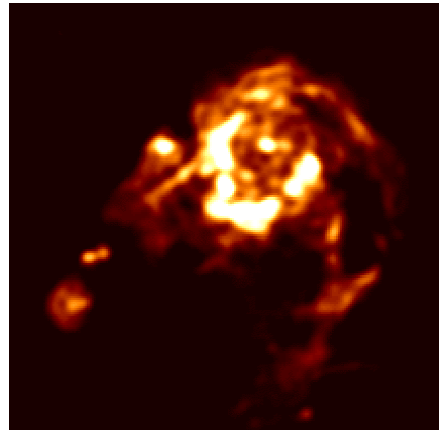
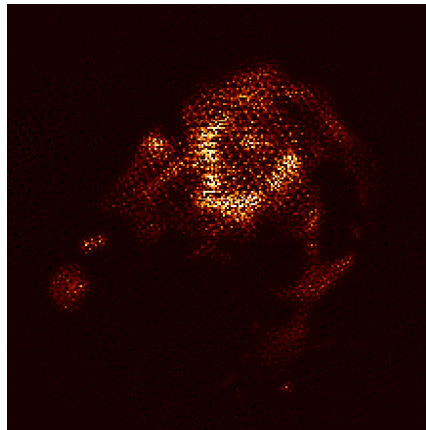
ASP

Point source  
model

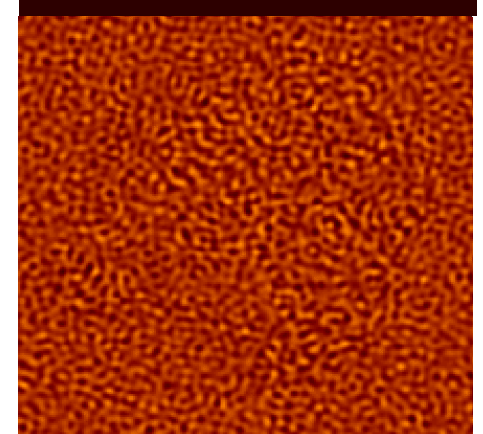
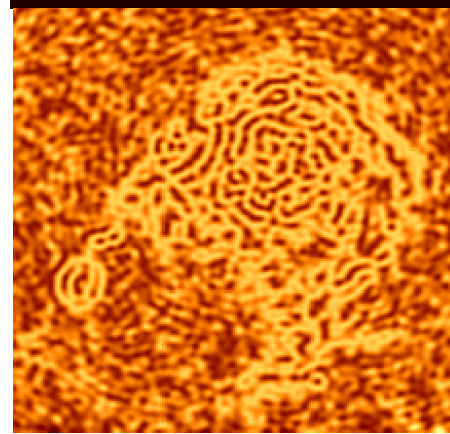
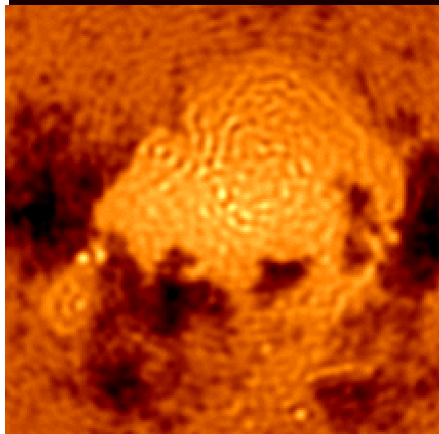
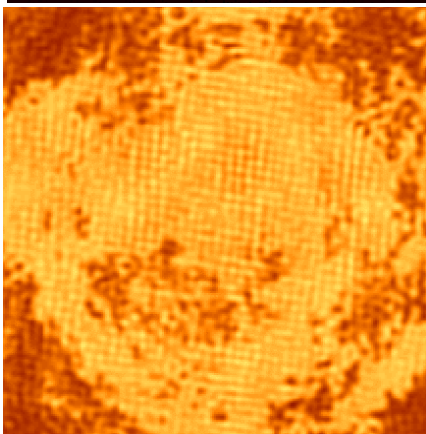
Point source  
model with a  
smoothness  
constraint

Multi-Scale model  
with a fixed set of  
scale sizes

Multi-Scale model  
with adaptive best-  
fit scale per  
component



$I^m$



$I^{res}$

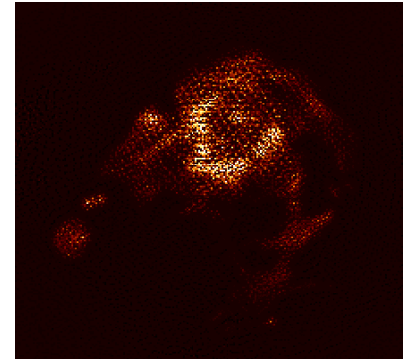


# Image Quality

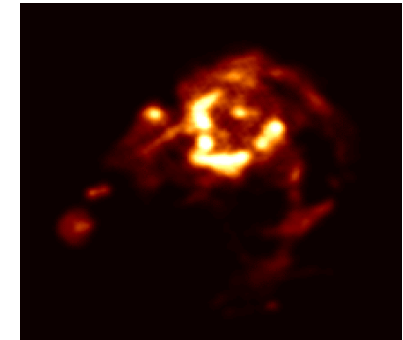
Noise in the image : Measured from restored or residual images

- With perfect reconstruction,  
The ideal noise level is : 
$$RMS \propto \frac{0.12 \frac{T_{sys}}{\eta_a}}{\sqrt{N_{ant}(N_{ant}-1) \cdot \delta \tau \cdot \delta \nu \cdot N_{pol}}}$$
- In reality, measure the RMS of residual pixel amplitudes

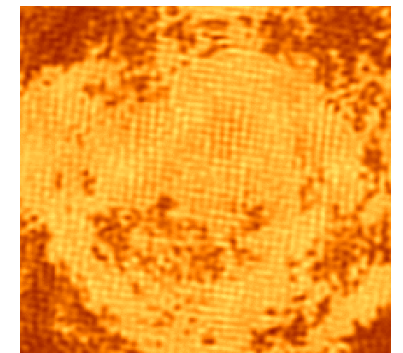
Model image



Restored image



Residual image





# Image Quality

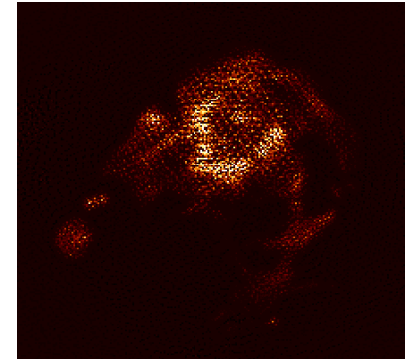
Noise in the image : Measured from restored or residual images

- With perfect reconstruction,  
The ideal noise level is : 
$$RMS \propto \frac{0.12 \frac{T_{sys}}{\eta_a}}{\sqrt{N_{ant}(N_{ant}-1) \cdot \delta \tau \cdot \delta \nu \cdot N_{pol}}}$$
- In reality, measure the RMS of residual pixel amplitudes

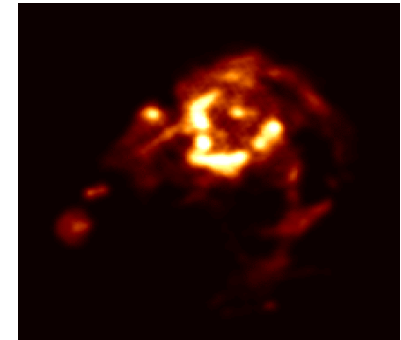
Dynamic Range : Measured from the restored image

- Standard : Ratio of peak brightness to RMS noise in a region devoid of emission.
- More truthful : Ratio of peak brightness to peak error (residual)

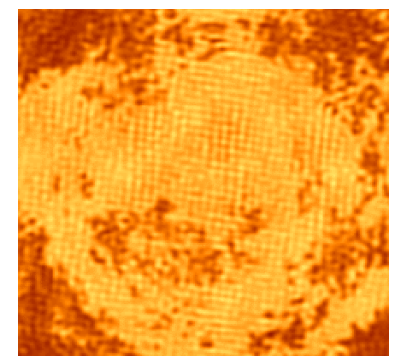
Model image



Restored image



Residual image



# Image Quality

Noise in the image : Measured from restored or residual images

- With perfect reconstruction,  
The ideal noise level is : 
$$RMS \propto \frac{0.12 \frac{T_{sys}}{\eta_a}}{\sqrt{N_{ant}(N_{ant}-1) \cdot \delta\tau \cdot \delta\nu \cdot N_{pol}}}$$
- In reality, measure the RMS of residual pixel amplitudes

Dynamic Range : Measured from the restored image

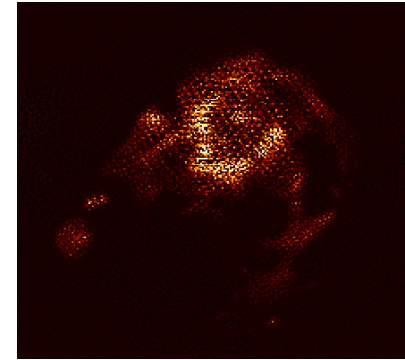
- Standard : Ratio of peak brightness to RMS noise in a region devoid of emission.
- More truthful : Ratio of peak brightness to peak error (residual)

Image Fidelity : Correctness of the reconstruction

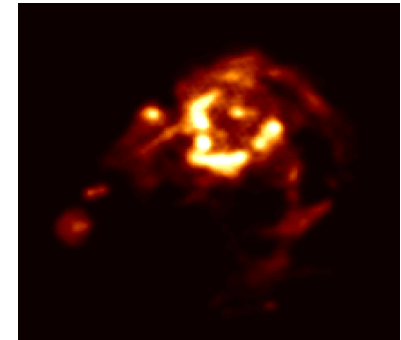
- remember the infinite possibilities that fit the data perfectly ?
- useful only if a comparison image exists.

$$\text{Inverse of relative error : } \frac{I^m * I^{beam}}{I^m * I^{beam} - I^{restored}}$$

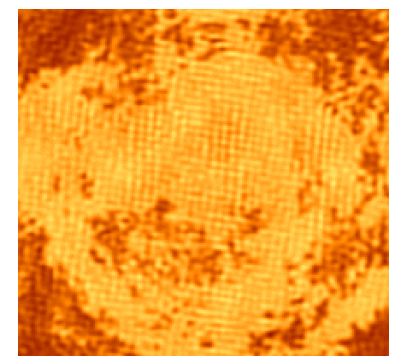
Model image



Restored image



Residual image



# Imaging with modern instruments

---

## Basic Imaging :

Narrow-frequency range, Small region of the sky

=> The 2D Fourier Transform relations hold

=> Convolution and deconvolution

## Wide-Band Imaging :

=> Sky and instrument change across frequency range

## Wide-Field Imaging

=> The 2D Fourier Transform relation breaks

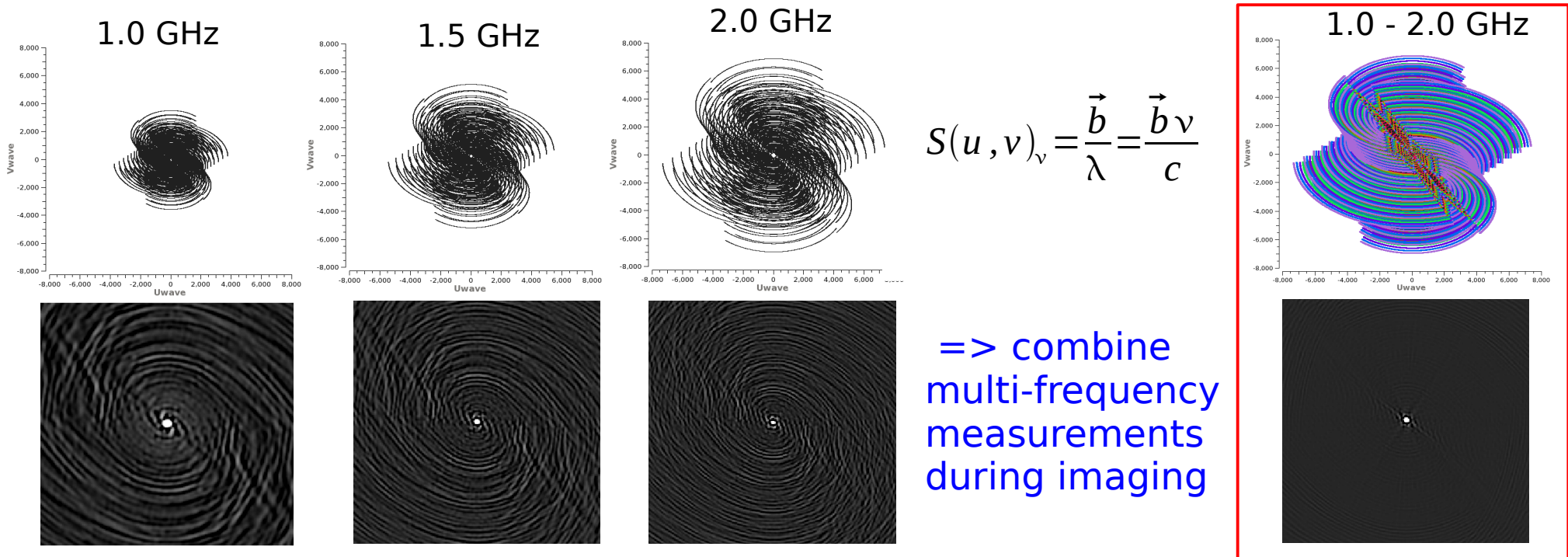
## Mosaic Imaging

=> Image an area larger than what each antenna can see.

# Wide-band Imaging – Sensitivity and Multi-Frequency Synthesis

Frequency Range :	(1 – 2 GHz)	(4 – 8 GHz)	(8 – 12 GHz)
Bandwidth : $\nu_{max} - \nu_{min}$	1 GHz	4 GHz	4 GHz
Bandwidth Ratio : $\nu_{max} : \nu_{min}$	2 : 1	2 : 1	1.5 : 1
Fractional Bandwidth : $(\nu_{max} - \nu_{min}) / \nu_{mid}$	66%	66%	40%

UV-coverage / imaging properties change with frequency

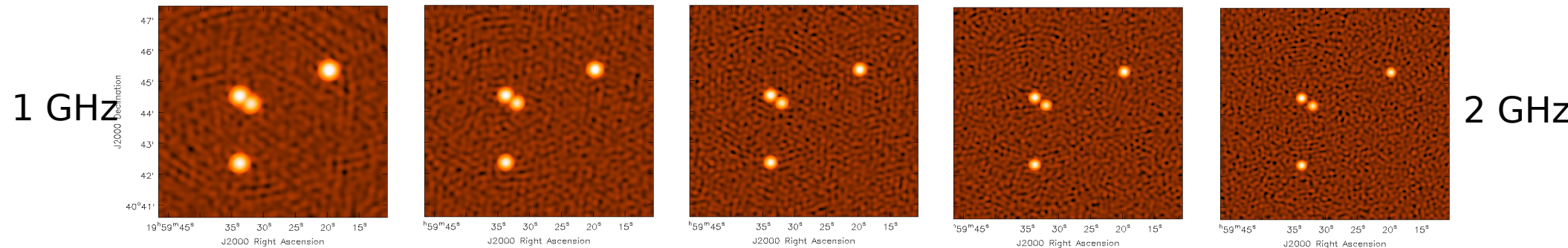


Sky Brightness can also change with frequency → model intensity and spectrum

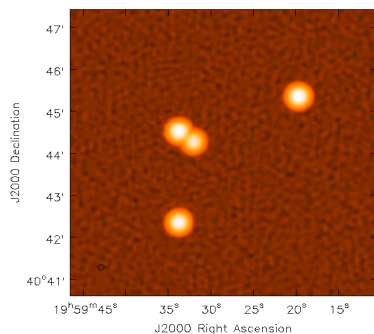
# Spectral Cube (vs) MFS imaging

3 flat-spectrum sources + 1 steep-spectrum source ( 1-2 GHz VLA observation )

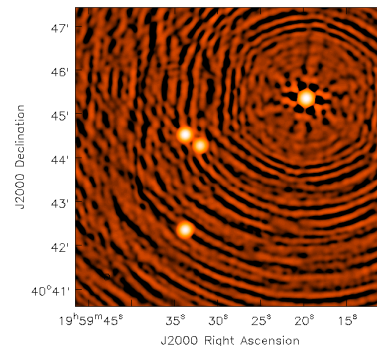
Images made at different frequencies ( limited to narrow-band sensitivity )



Add all single-frequency images (after smoothing to a low resolution)

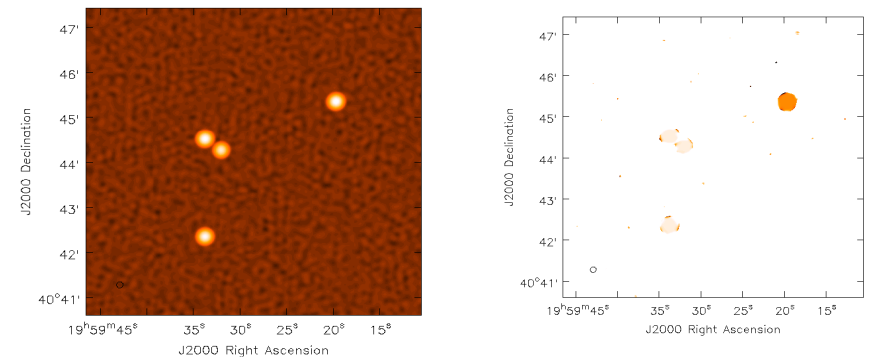


Use wideband UV-coverage, but **ignore spectrum** ( MFS, nterms=1)



Use wideband UV-coverage + Model and fit for spectra too ( MT-MFS, nterms > 1 )

Output : Intensity and Spectral-Index



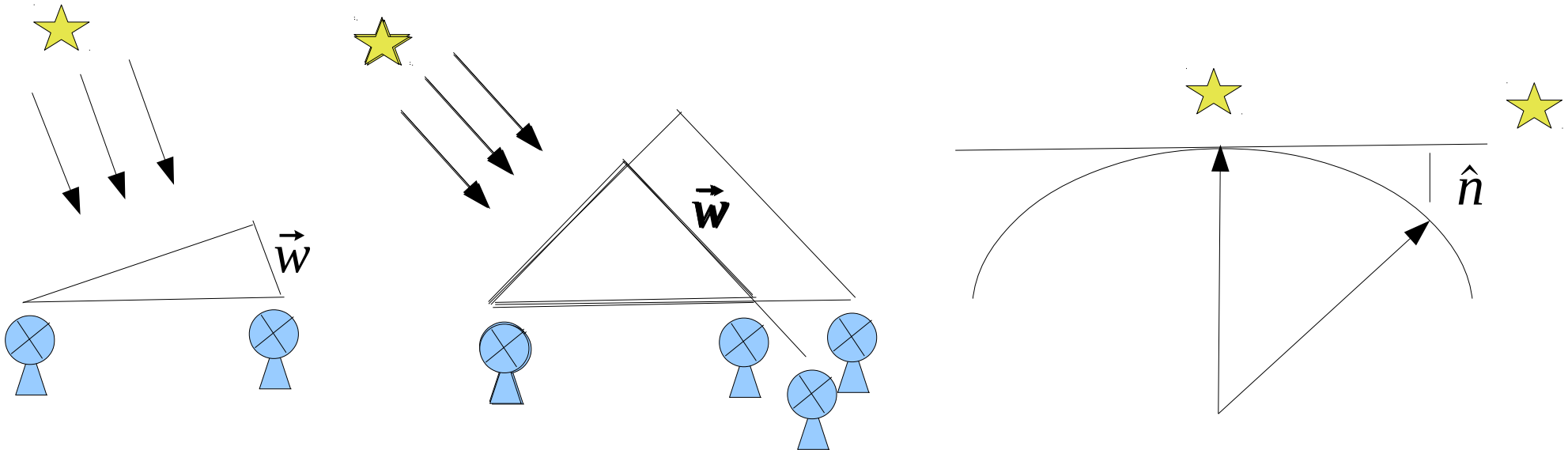
## Wide-Field Imaging – W-term

$$V^{obs}(u, v) = S(u, v) \iint I(l, m) e^{2\pi i (ul + vm)} dl dm$$

$$V^{obs}(u, v) = S(u, v) \iiint I(l, m) e^{2\pi i (ul + vm + \boxed{w(n-1)})} dl dm dn$$

The ' w ' of a baseline can be large, away from the image phase center

The ' n ' for a source can be large, away from the image phase center



There are algorithms to account for this : Image Faceting, W-Projection.



# Wide-Field Imaging – W-term

$$V^{obs}(u, v) = S(u, v) \iint I(l, m) e^{2\pi i (ul + vm)} dl dm$$

$$V^{obs}(u, v) = S(u, v) \iiint I(l, m) e^{2\pi i (ul + vm + w(n-1))} dl dm dn$$

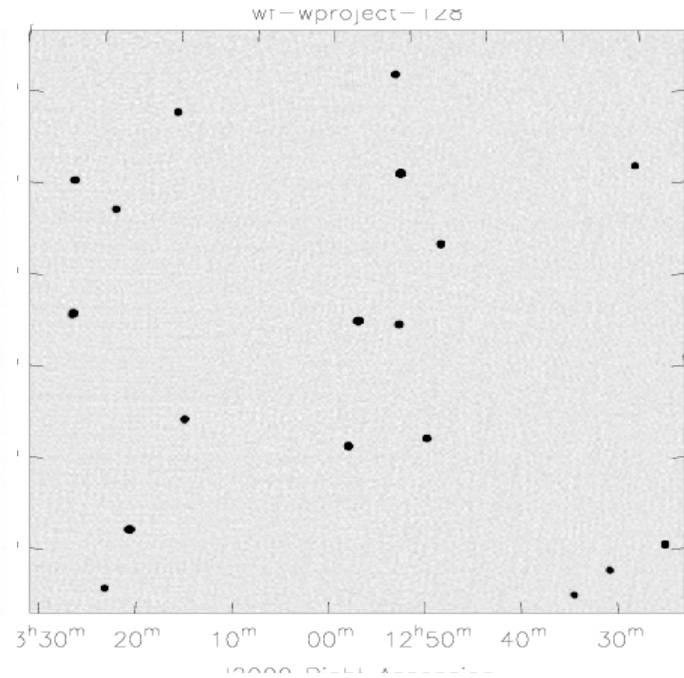
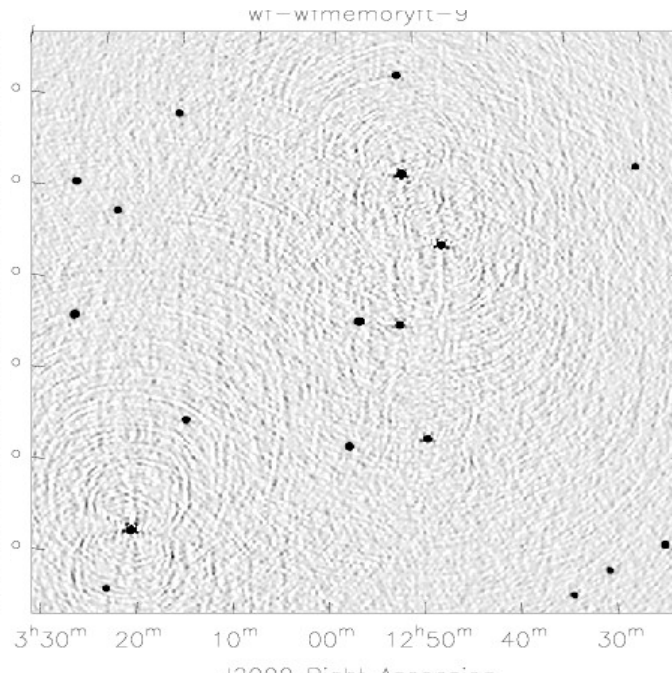
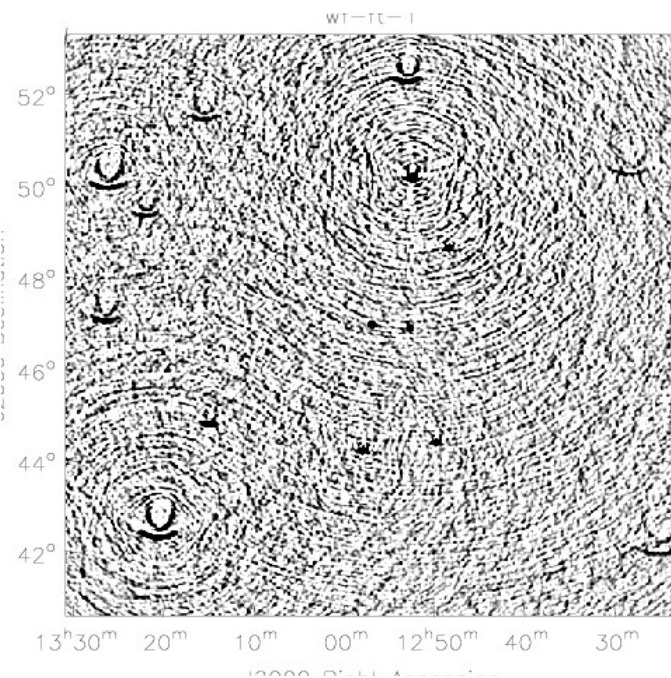
The ' w ' of a baseline can be large, away from the image phase center

The ' n ' for a source can be large, away from the image phase center

2D Imaging

Facet Imaging

W-Projection



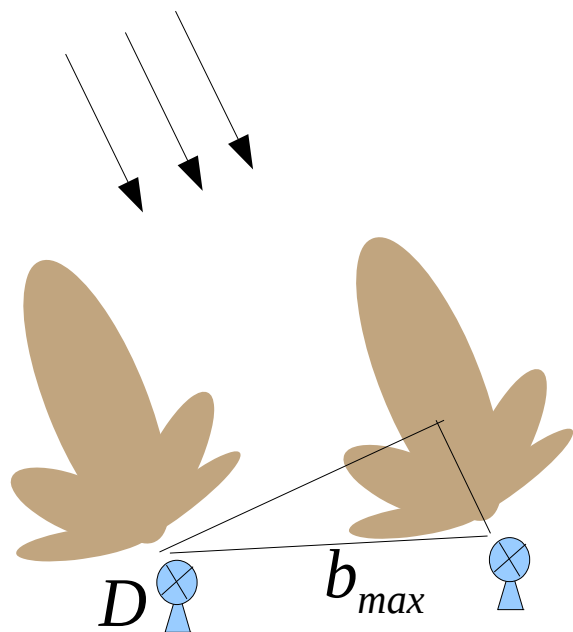


# Wide-Field Imaging – Primary Beams

Each antenna has a limited field of view => Primary Beam (gain) pattern

=> Sky is (approx) multiplied by PB, before being sampled by the interferometer

$$I^{obs}(l, m) \approx I^{PSF}(l, m) * [P^{sky}(l, m) \cdot I^{sky}(l, m)]$$

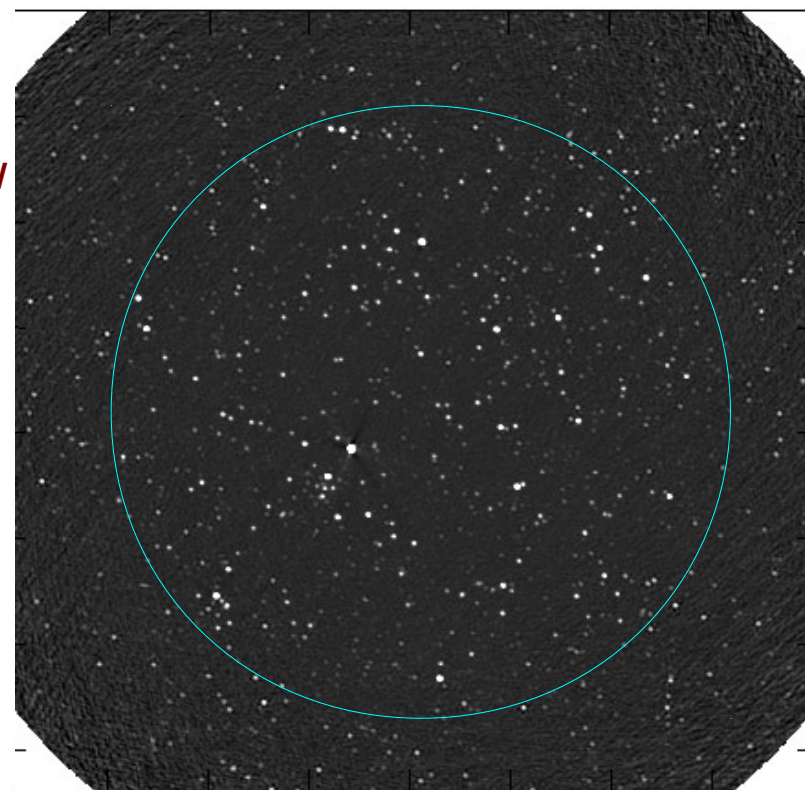


The antenna field of view  
D = antenna diameter

$$\lambda/D$$

Compare with angular  
resolution of the  
interferometer :

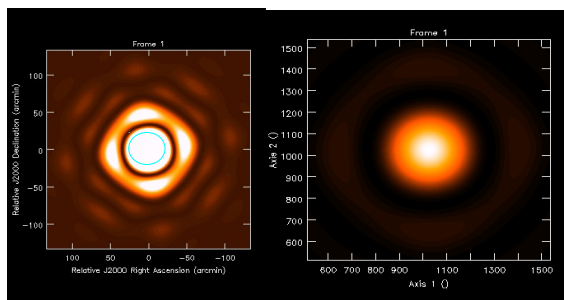
$$\lambda/b_{max}$$



But, in reality, P changes with time, freq, pol and antenna....

=> Ignoring such effects limits dynamic range to  $10^4$

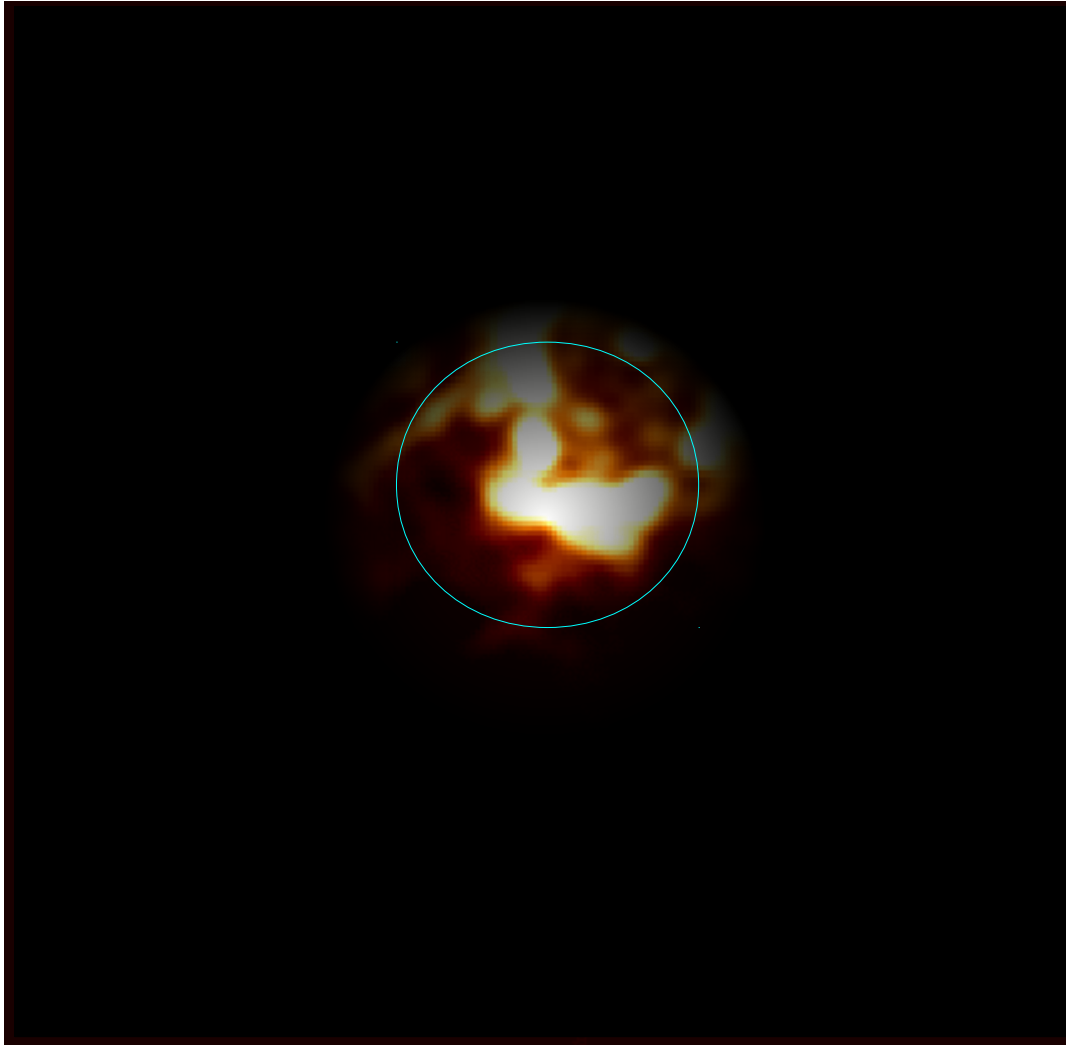
=> More-accurate method to account for this : A-Projection



# Wide-field Imaging -- Mosaics

---

Combine data from multiple pointings to form one large image.



One Pointing sees only part of the source

Combine pointings either before or after deconvolution.

## **Stitched mosaic :**

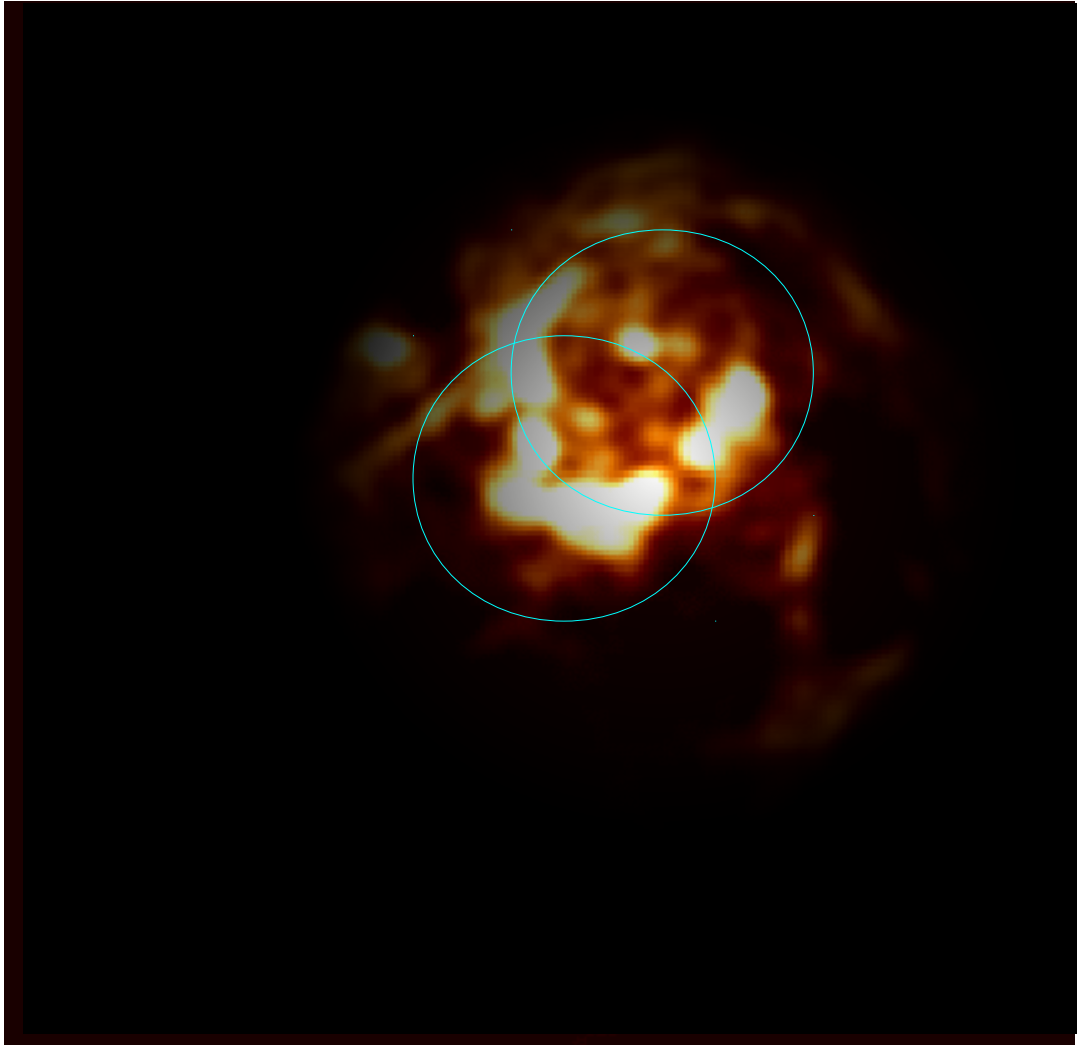
- Deconvolve each pointing separately
- Divide each image by PB
- Combine as a weighted avg

## **Joint mosaic :**

- Combine observed images as a weighted average  
(or)  
Grid all data onto one UV-grid,  
and then iFFT
- Deconvolve as one large image

# Wide-field Imaging -- Mosaics

Combine data from multiple pointings to form one large image.



Two Pointings see more.....

Combine pointings either before or after deconvolution.

## **Stitched mosaic :**

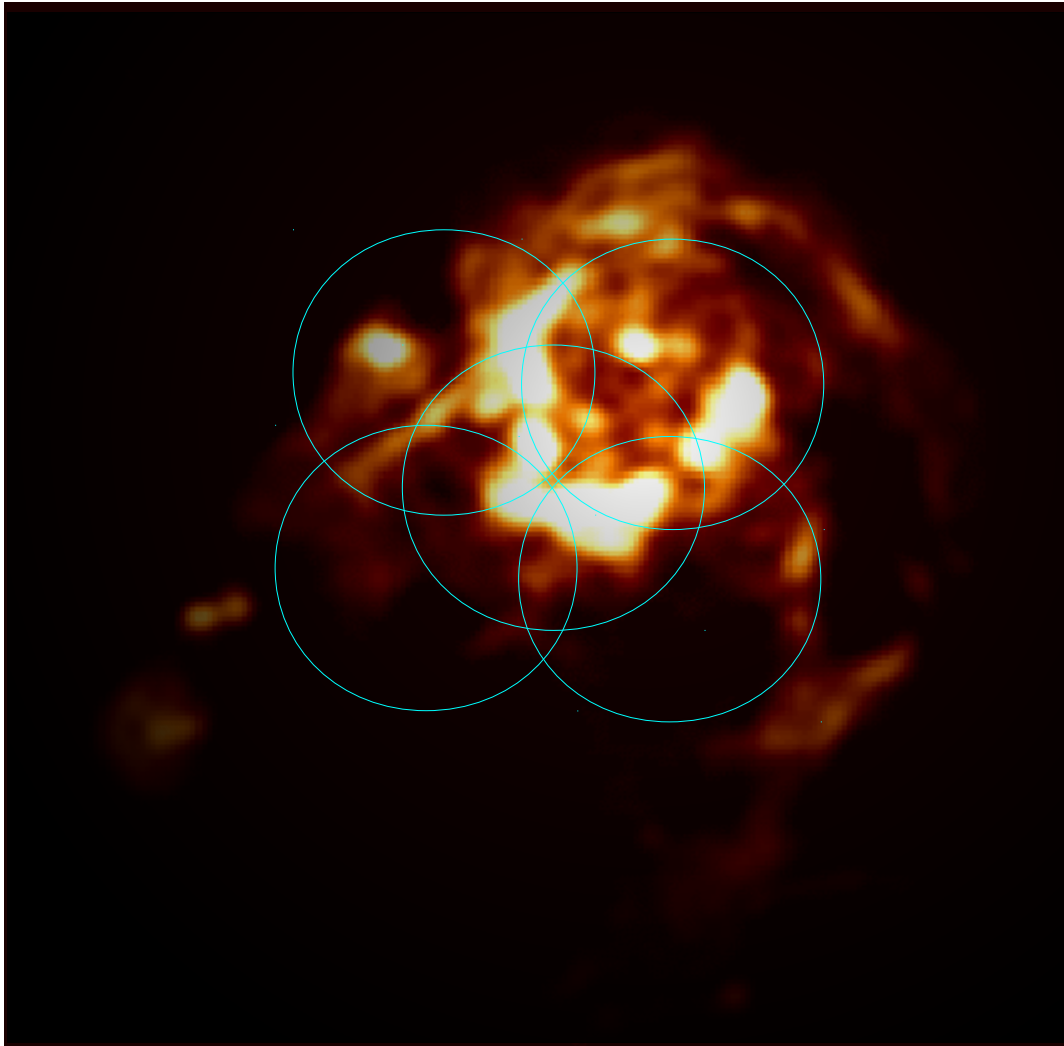
- Deconvolve each pointing separately
- Divide each image by PB
- Combine as a weighted avg

## **Joint mosaic :**

- Combine observed images as a weighted average  
(or)  
Grid all data onto one UV-grid,  
and then iFFT
- Deconvolve as one large image

# Wide-field Imaging -- Mosaics

Combine data from multiple pointings to form one large image.



Use many pointings to cover the source with approximately uniform sensitivity

Combine pointings either before or after deconvolution.

## **Stitched mosaic :**

- Deconvolve each pointing separately
- Divide each image by PB
- Combine as a weighted avg

## **Joint mosaic :**

- Combine observed images as a weighted average  
(or)  
Grid all data onto one UV-grid, and then iFFT
- Deconvolve as one large image

## Some points to remember ...

---

How does an interferometer form an image ?

- Each antenna pair measures one 2D fringe.  
Many antenna pairs => Fourier series

How do you make a raw image from interferometer data ?

- Assign weights to visibilities, grid them, take a Fourier transform

How do you choose the cell-size and image size for imaging ?

- Cell size = ( Resolution / 3 ).      Image size = field-of-view / cell size

What does the raw observed image represent ?

- Observed Sky is the convolution of the true sky and the PSF

How do you get a model of the sky ?

- Solve the convolution equation via algorithms like Clean, MS-Clean, MT-Clean...

## Some points to remember ...

---

What is calibration ?

- Use calibrator data to solve for antenna gains, apply them to target data

How does wide-band data affect the imaging process ?

- Increased sensitivity, but the imaging properties and sky change with frequency

How do you image wide-band data ?

- Make a Cube of images, or Multi-Frequency-Synthesis with a spectral fit.

What is an antenna primary beam and what is its effect on an image ?

- Antenna power pattern. It multiplies with the sky, before convolution with the PSF

What is the w-term problem ?

- 2D Fourier transform approximations are invalid far away from the image center