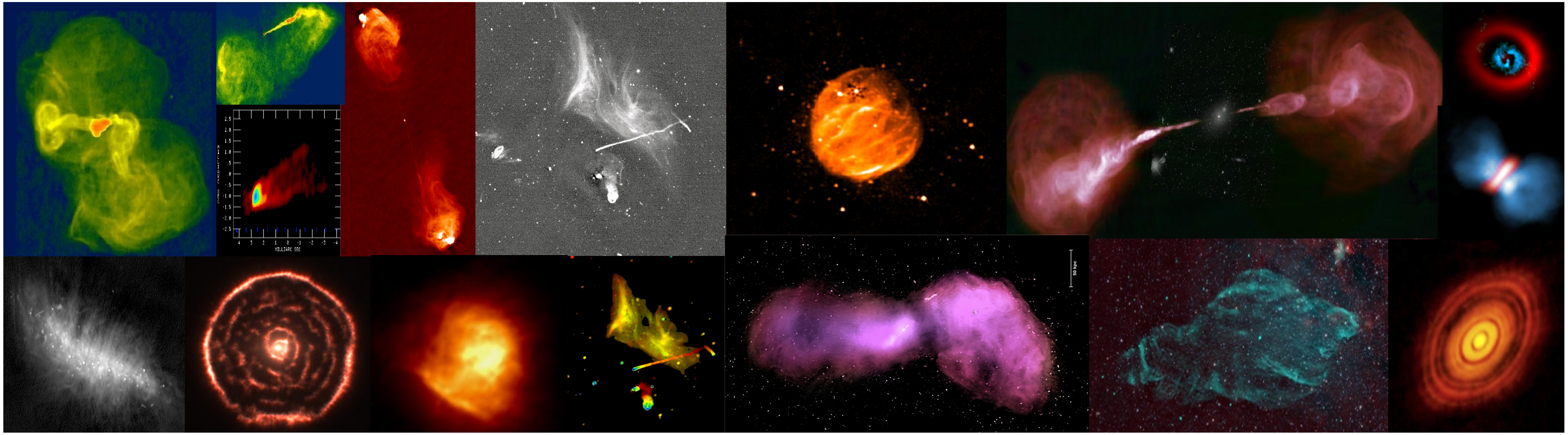
# Synergy between Radio and Optical Interferometry

## Image Reconstruction,  Calibration and Data Analysis



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National Radio Astronomy Observatory, Socorro, NM, USA

SPIE Optical and Infrared Interferometry and Imaging
13 June 2018,  Austin, TX

# Radio and Optical Interferometry

Literature :

- Most discussions focus on the measurement process and differences therein
- Pearson & Readhead 1984 : Self-calibration and hybrid mapping (estimating phases)
- Recent work by the Event Horizon Telescope : Tests with OI methods + new algorithms
- Thiebaut & Young, 2009/2017 : Imaging algorithms in both radio and optical fields

[Other related topics : Speckle imaging, Aperture masking … ]

This talk :

- Modern radio interferometric calibration and imaging
- Possible topics of synergy and shared R&D with Optical/IR
    - Sky models and solvers (direct modeling / imaging)
    - Handling narrow-field and wide-field instrumental effects (+ self-calibration)
    - Flexible algorithm frameworks, joint reconstructions, intelligent automation

[Please see paper for references]

# Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} dl\, dm$$

Observed visibilities (Data)

Direction Independent Gains

UV sampling pattern

Sky Brightness (Image )

Fourier transform kernel

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(1) External models
(2) Self-Calibration

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UV sampling pattern

Sky Brightness (Image )

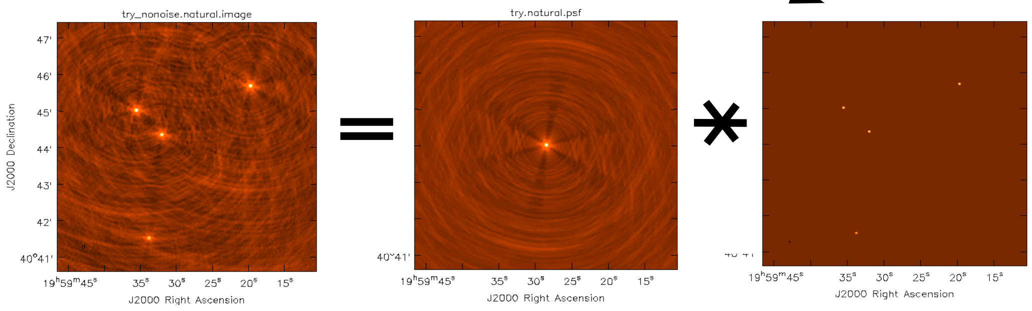
Fourier transform kernel

**Calibration**

(1) External models
(2) Self-Calibration

**Imaging + Deconvolution**

$$I^{obs}(l,m) = I^{PSF}(l,m) * I^{sky}(l,m)$$



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Direction
Independent
Gains

- feed gains,
delays,bandpass

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Direction Independent Gains

- feed gains, delays,bandpass

Antenna primary beam

- Power pattern varies with time, frequency and baseline

- Ionospheric refraction

W-Term

-Non-coplanar baselines

-Sky curvature

Direction Dependent Instrumental Effects

=> Multiplicative effect in the image domain
=> Convolutions in the visibility domain

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Sky-brightness varies with frequency (time)

- Include source spectra and time variability into the sky model and/or the regularization process.

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**W-Term**

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**Image Reconstruction**

**Calibration** (DI & DD)

(1) External models
(2) Self-Calibration

Direction Dependent Instrumental Effects

=> Multiplicative effect in the image domain
=> Convolutions in the visibility domain

# Iterative Image Reconstruction

L2 data regularization
+ Non-linear sky model
and solver/constraints.

$$[A]I^m = V^{obs} \qquad \frac{\delta\chi^2}{\delta I^m} = 0$$

Normal Equations
(convolution eqn)

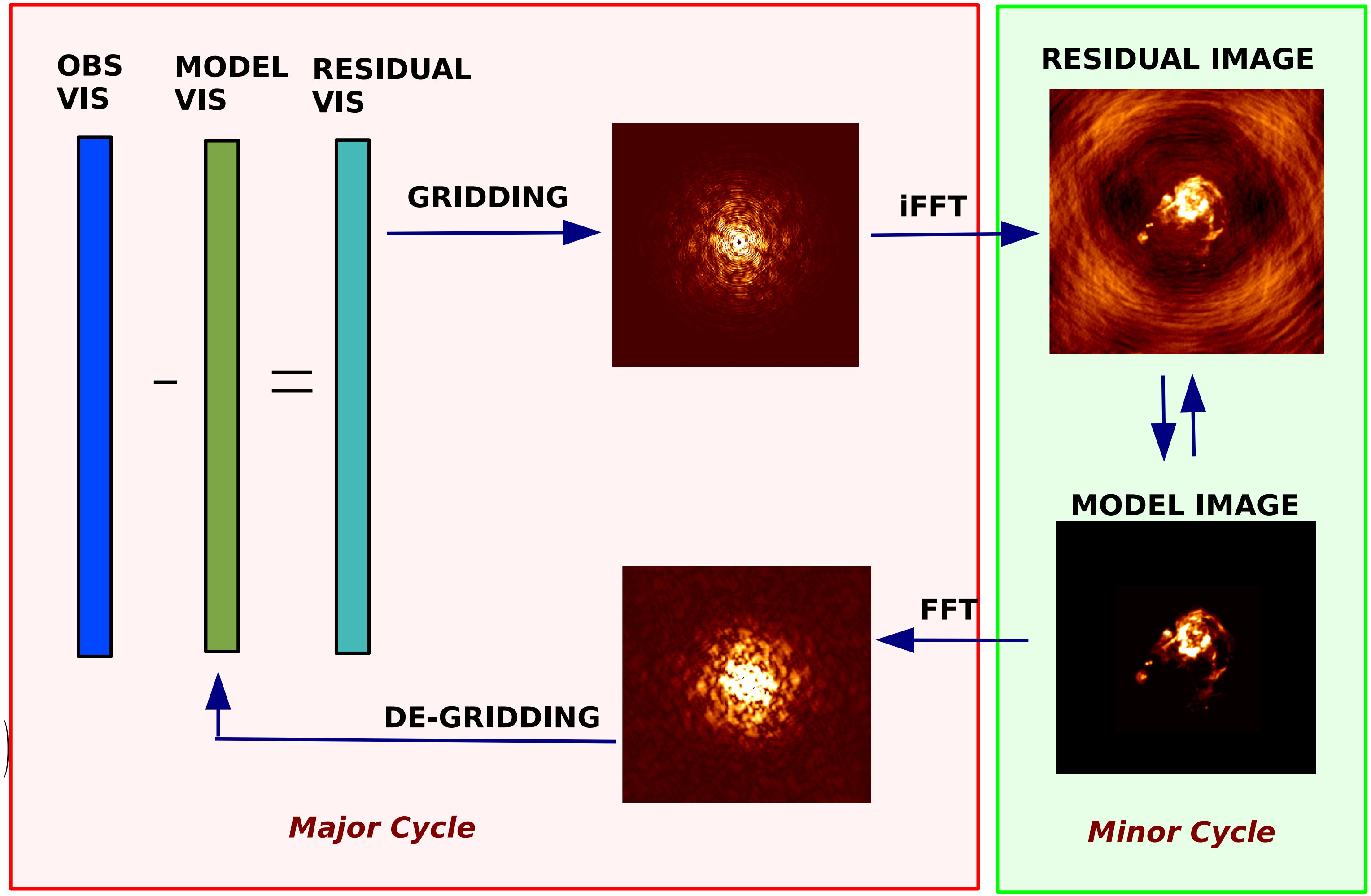
$$[A^T W A] I^m = [A^T W] V^{obs}$$

Sky model solver

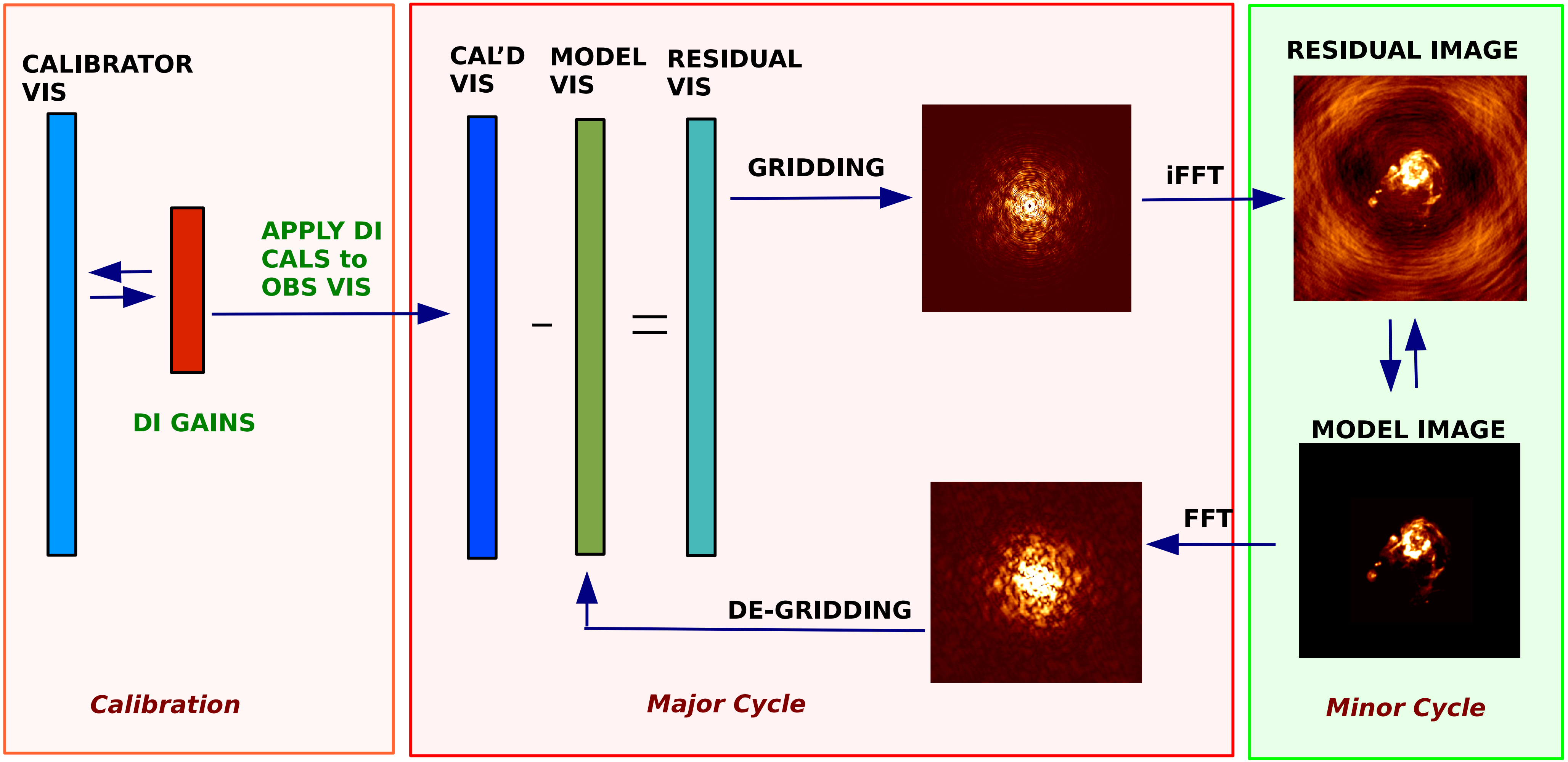
$$I^m_{i+1} = I^m_i + g[A^T W A]^+ \boxed{R}$$

$$\boxed{R} = \left( A^T W (V^{obs} - A I^m_i) \right)$$

Calculate Forward Model
& Residual Image



# Iterative Image Reconstruction + External calibration



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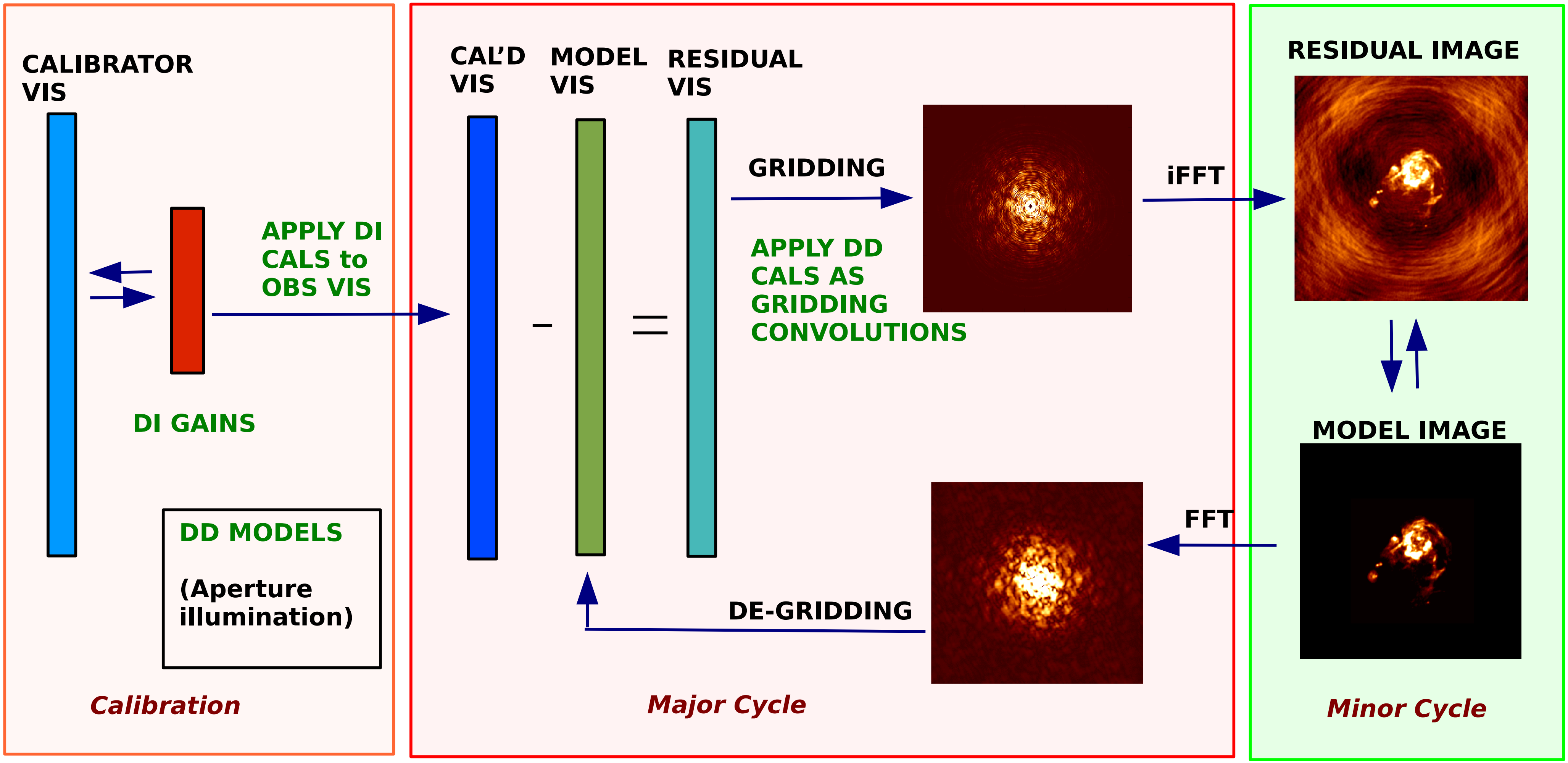
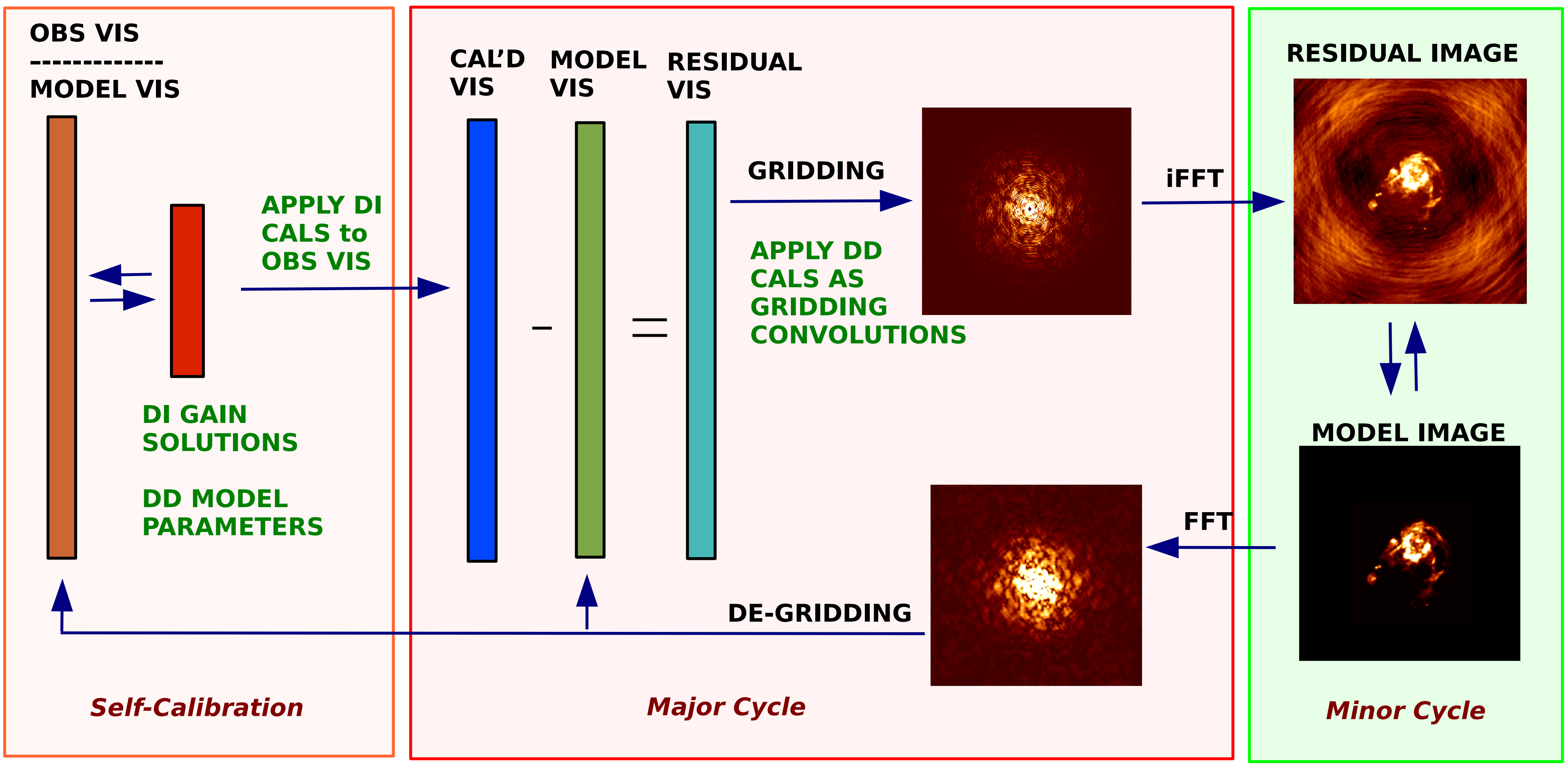
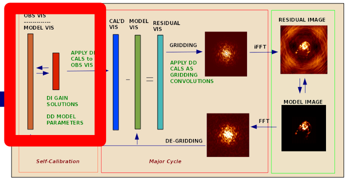
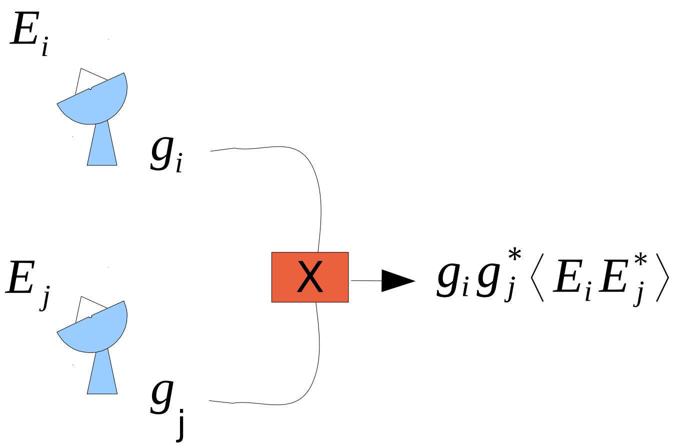


# Iterative Image Reconstruction + Self-Calibration



# Calibration – Direction Independent effects






(1) Observe a known source

$$\langle E_i E_j^* \rangle \text{ is known}$$

(2) Use data from all correlation pairs ij
Solve for complex gains $g_i$

(3) Apply corrections to target data : $$\frac{g_i g_j^* \langle E_i E_j^* \rangle}{g_i g_j^*}$$

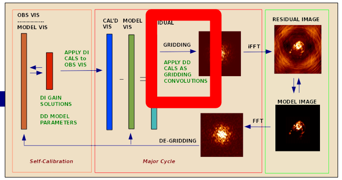
Typically, solutions are done in a sequence, with averaging to increase SNR for the solutions.

E.g. Average in time and solve for average stable bandpass
Apply bandpass solutions, average in frequency, solve for time-variable gains.

Baseline based calibration : Possible, but to be used with caution.

( Equivalence between solving for antenna-based terms and satisfying closure relations )

# Direction Dependent Corrections – Using known models




Models of DD instrumental effects can be used to correct the data

Antenna Beams, Ionospheric Phase screen, Non-coplanar Baselines, Sky curvature

$$I^{obs} = \sum_{ij} I^{psf}_{ij} * \left[ P_{ij} . I^{sky} \right] \longleftrightarrow V^{obs}_{ij} = S_{ij} . \left[ A_{ij} * V^{sky} \right]$$

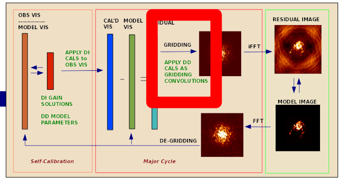
For each visibility, apply $A^{-1}_{ij} \approx \dfrac{A^T_{ij}}{A^T_{ij} * A_{ij}}$

(1) Use $A^T_{ij}$ as the convolution function during gridding

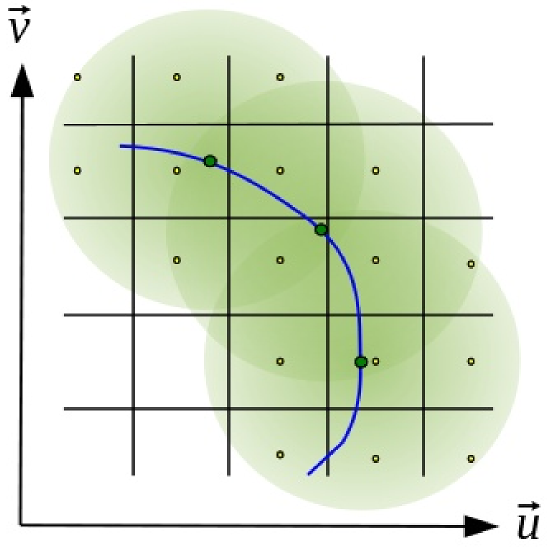
(2) Divide out $FT\left[ \sum_{ij} A^T_{ij} * A_{ij} \right]$ from the image (in stages).

- If $P_{ij}$ is invariant, an image-domain correction can be done instead (flat-fielding)
- Phase gradients across $A^T_{ij}$ can fix known pointing offsets ( tip-tilt ) and make mosaics
- $A^{-1}_{ij}$ is often only approximate => Convergence depends on an accurate forward model

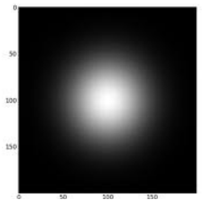
# Applying DD corrections ( $A_{ij}^{T}$ ) during gridding



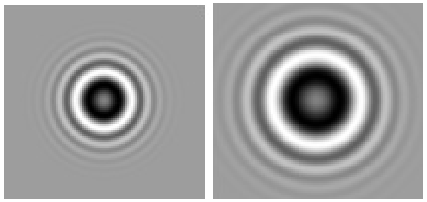

Gridding = Convolutional Resampling of visibilities to a regular grid




Standard Imaging :
Prolate Spheroidal



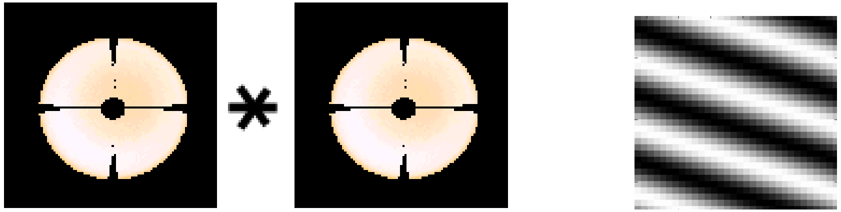
W-Projection :
FT of a Fresnel kernel



A-Projection : Baseline aperture illumination functions
+ phase gradients for pointing offsets
+ ionospheric refraction models

Convolution in UV-domain (per vis)

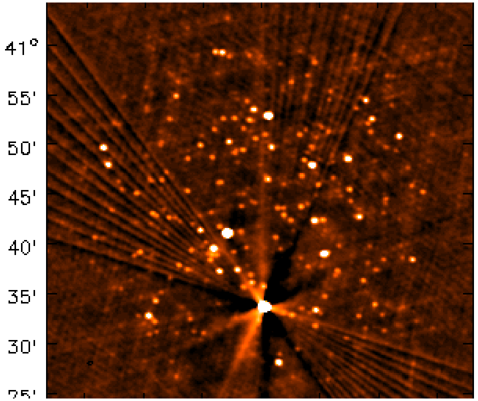
=> Handle wide-field imaging effects
before averaging in time/baseline

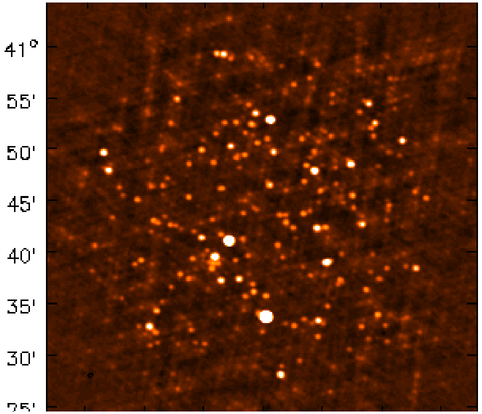


( De-Gridding : Calculate Forward Model )

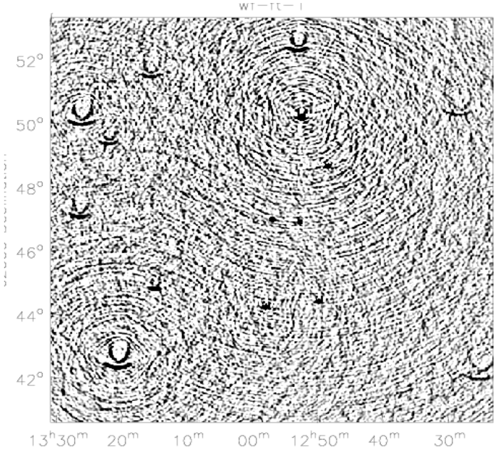
# Examples of wide-field instrumental corrections

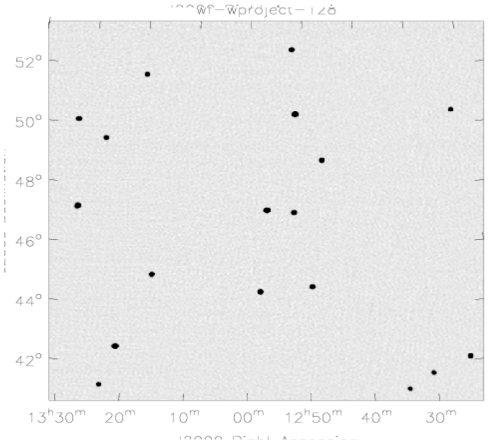
Antenna Beam rotation with parallactic angle



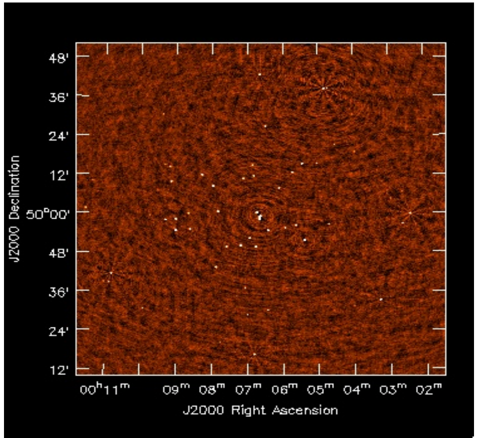



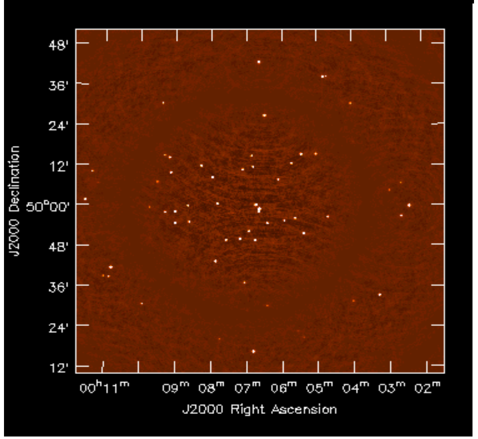

Sky curvature, non-coplanarity ( W-term )



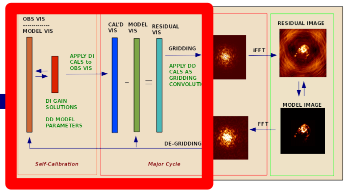




Antenna-based, time-variable pointing offsets



# Unknown instrumental models : DD Self-Calibration




(1) Solve for parameters of instrument-specific models in a self-calibration loop

E.g. Pointing Self-Cal : Solve for antenna-based phase gradients across each aperture illumination function ( Adaptive Optics ! )

=> Correction : Apply opposite phase gradient during gridding

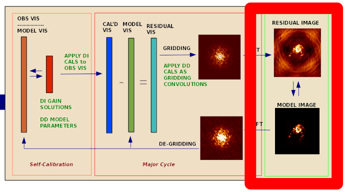
(2) Perform direction-independent (DI) calibrations for multiple directions on the sky

E.g. DD-Facets : Define regions around all bright sources.

Solve V = $g_i g_j^* \langle E_i E_j^* \rangle$ to get N complex gains for each selected direction

=> Correction : Image each region using different DI gain solutions.

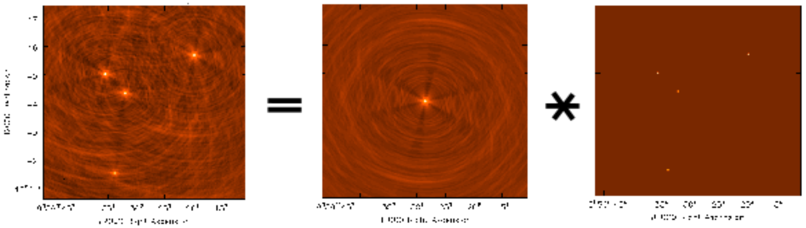
# Sky models and solvers ( image reconstruction )




**Convolution Equation ==> Deconvolution**

For Point Sources :

– Hogbom CLEAN
– Clark CLEAN



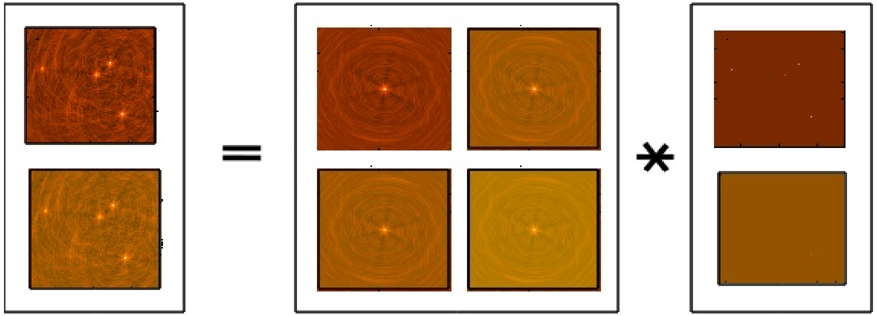
**Multi-Term Convolution Equation ==> Joint Deconvolution**

For Point/Extended Sources :

– Multi-Scale-CLEAN

For Wide-band Sky models

– Multi-Term Multi-Frequency Synthesis
( with or without Multi-Scale / Time variability )



Others : Any non-linear image-domain solver (many compressed sensing formulations )

Gaussians (ASP), Wavelets and other Atoms with Sparsity (SARA / PURIFY / MORESANE), Bayesian forms (MEM, RESOLVE), wide-band non-parametric models, etc..

# Algorithm Comparison

Algorithm choice depends on sky structure, data quality, target science.

Each algorithm needs (different) tuning for best results.

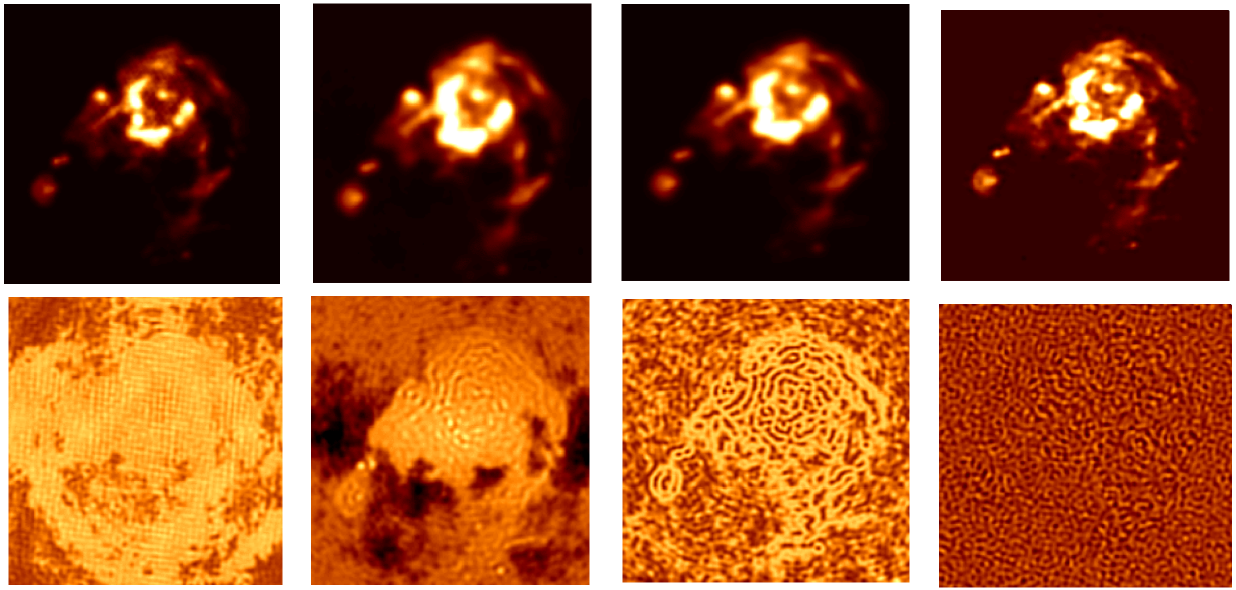



**Image Quality Metrics :**

Noise RMS,

Peak residual,

Dynamic Range

Easier to compare algorithms within a common software framework....

# Joint Reconstructions : Wide-Band (multi-spectral) Solvers

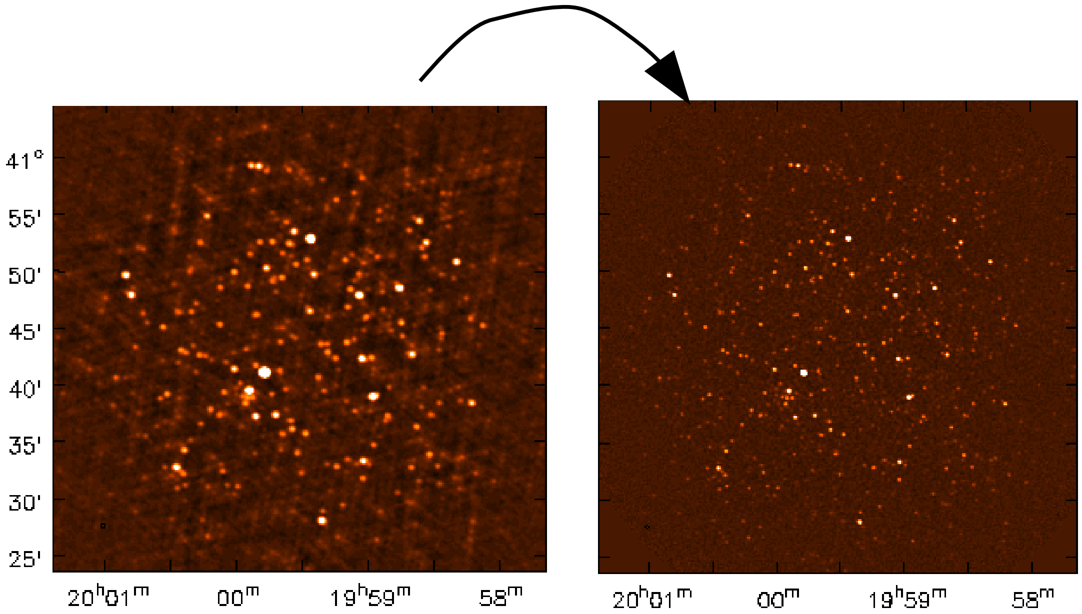
Model the spectrum per ‘atom’ as a smooth polynomial

$$I_{\nu}^{sky} = \sum_t I_t^m \left( \frac{\nu - \nu_0}{\nu_0} \right)^t$$

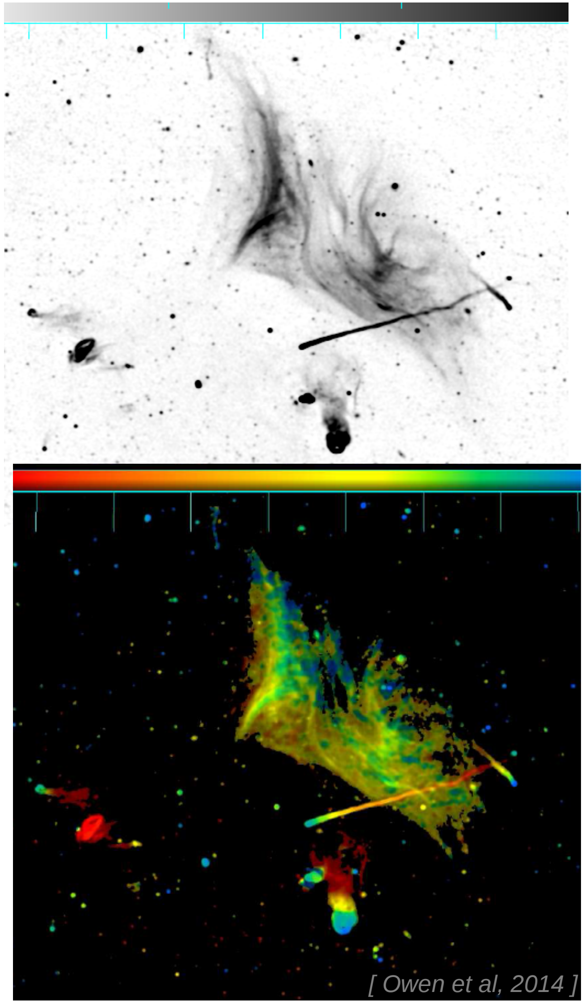
MT-MFS : Multi-term linear least squares + CLEAN-based greedy algorithm in a transformed (sparse) space.

Improve

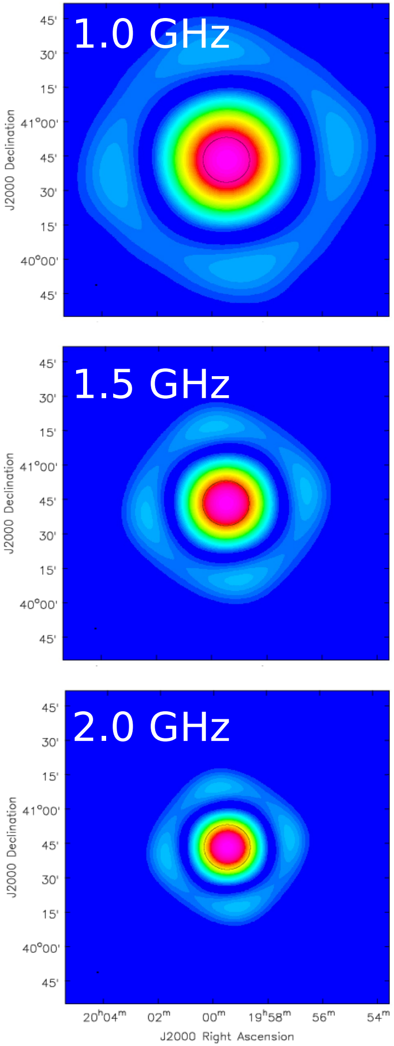
Angular Resolution & Imaging Fidelity & Dynamic Range




Intensity and Spectral Index



# Joint Reconstructions : Wide-Band+ Wide-Field

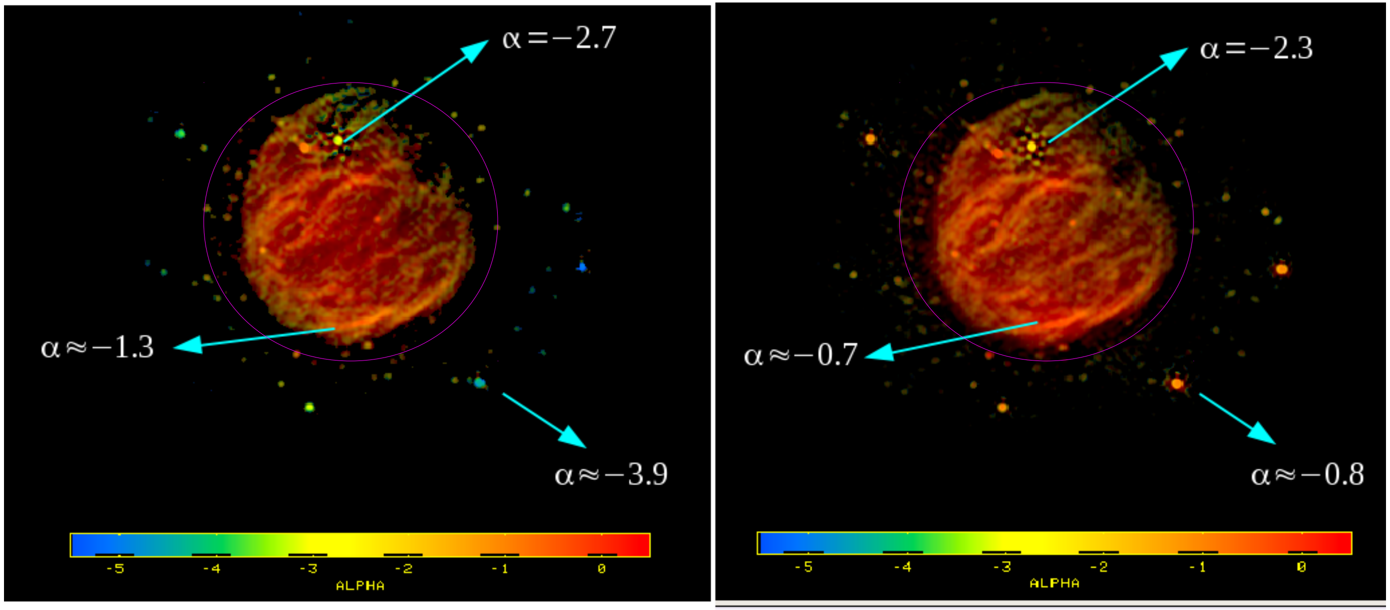



Antenna Power Patterns scale with observing frequency

=> Artificial spectral structure for all sources away from center

(1) Include PB spectrum in Sky Model. Remove post-reconstruction.

(2) Eliminate as an instrumental correction before modeling sky



# Joint Reconstructions : Adding single-dish (low resolution) data

Missing Short Spacings

=> Negative Bowls
=> Unconstrained Spectra

Use Low-Resolution information from single dish maps.

Methods :

(1) Joint Reconstructions (better!)

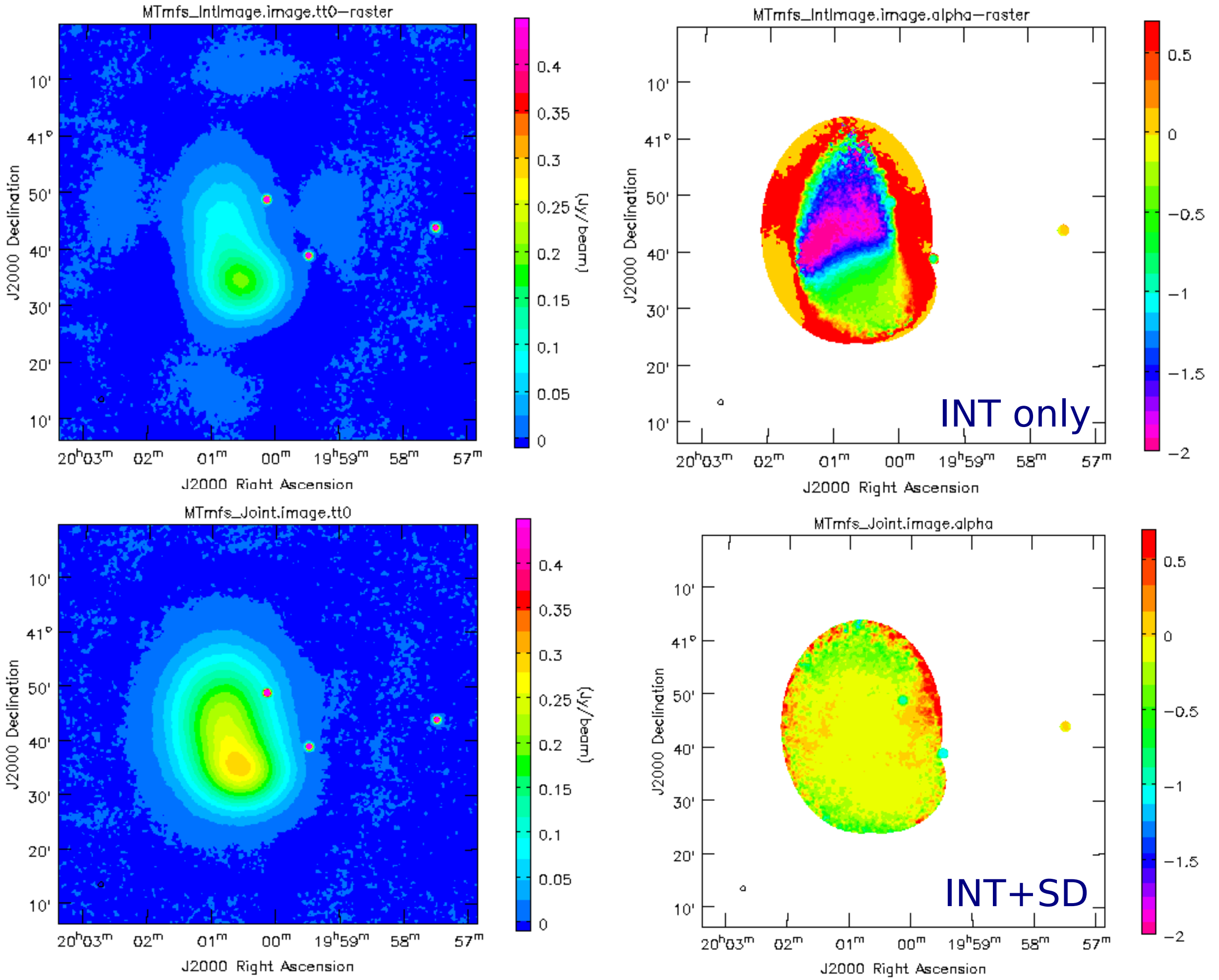
Add another data regularization term to any existing solver

E.g. Joint PSFs and Residuals

(2) Feathering

Weighted UV-domain average of INT-only and SD-only images

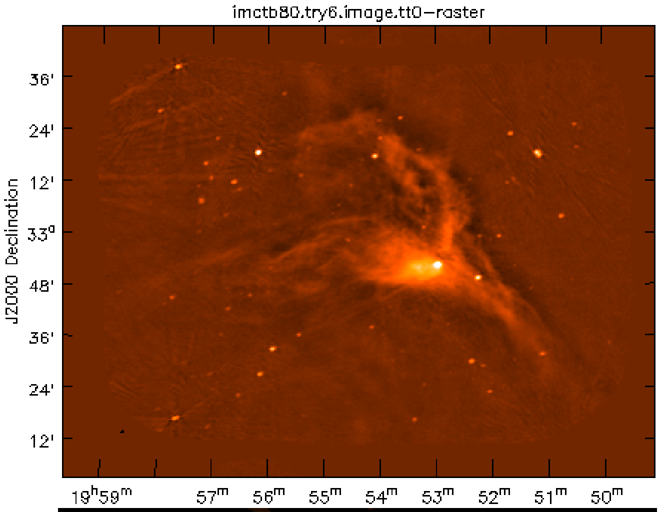
Intensity | Spectral Index

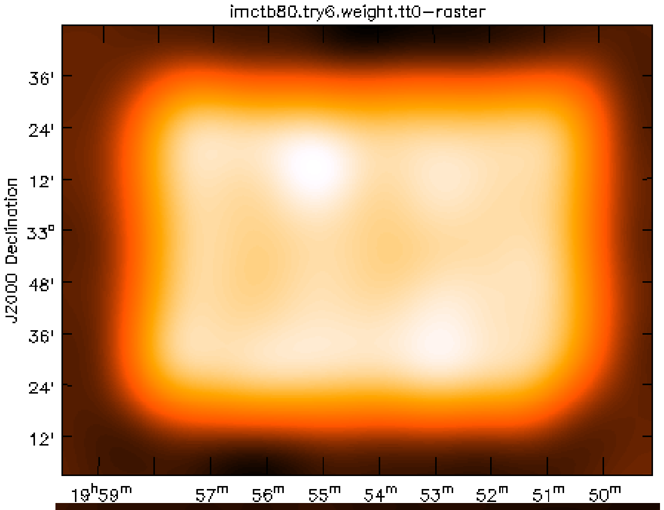


# Joint Reconstructions : Mosaics + Wide-Band + Wide-Field + Single Dish

Interferometer joint mosaic intensity

(used phase gradients across aperture functions during gridding)

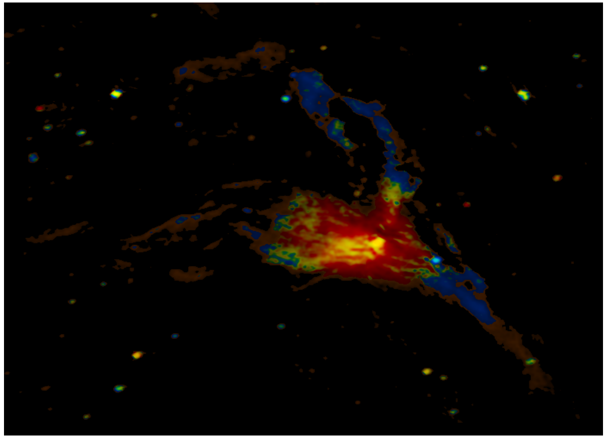


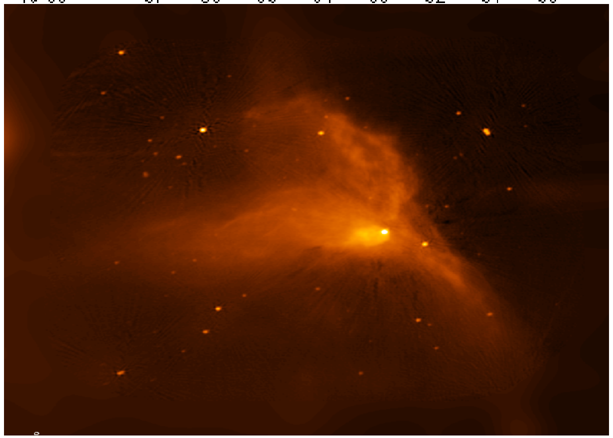




Joint mosaic primary beam from 106 VLA pointings

Wide-Field Spectral Index

(with corrections for PB spectrum)





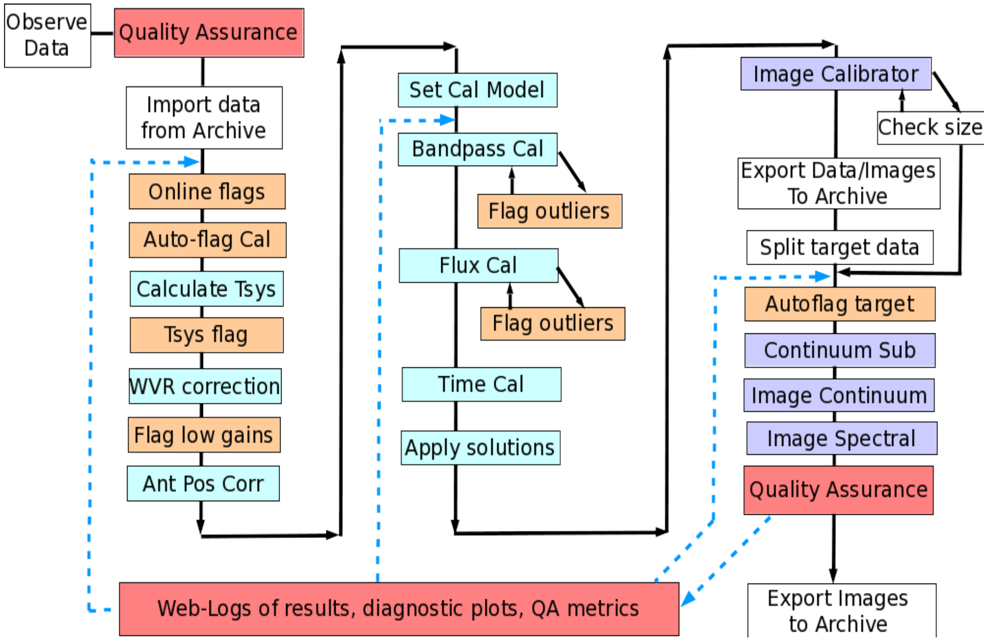
Interferometer + Single dish

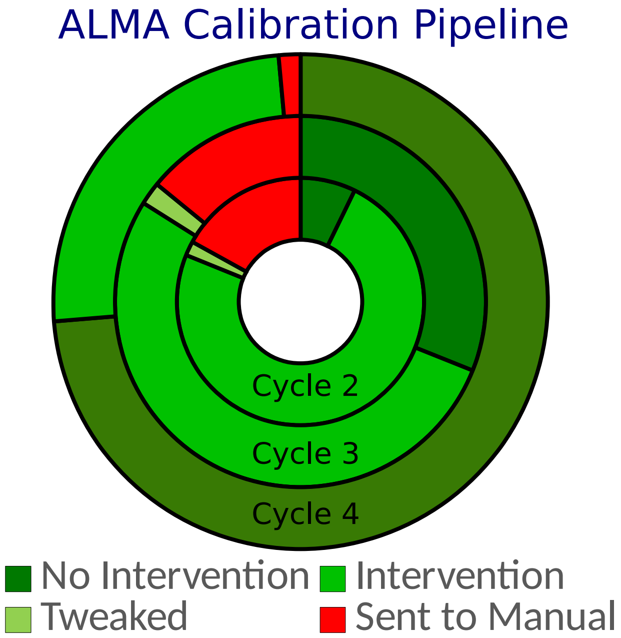
(intensity only)

An example of the current state-of-the-art ...

# Automated Data Analysis : Science Ready Data Products

Our current end-to-end pipelines are the result of hand-optimized manual tuning by a team of scientists, validated on ~100 datasets, for a few standard imaging modes.



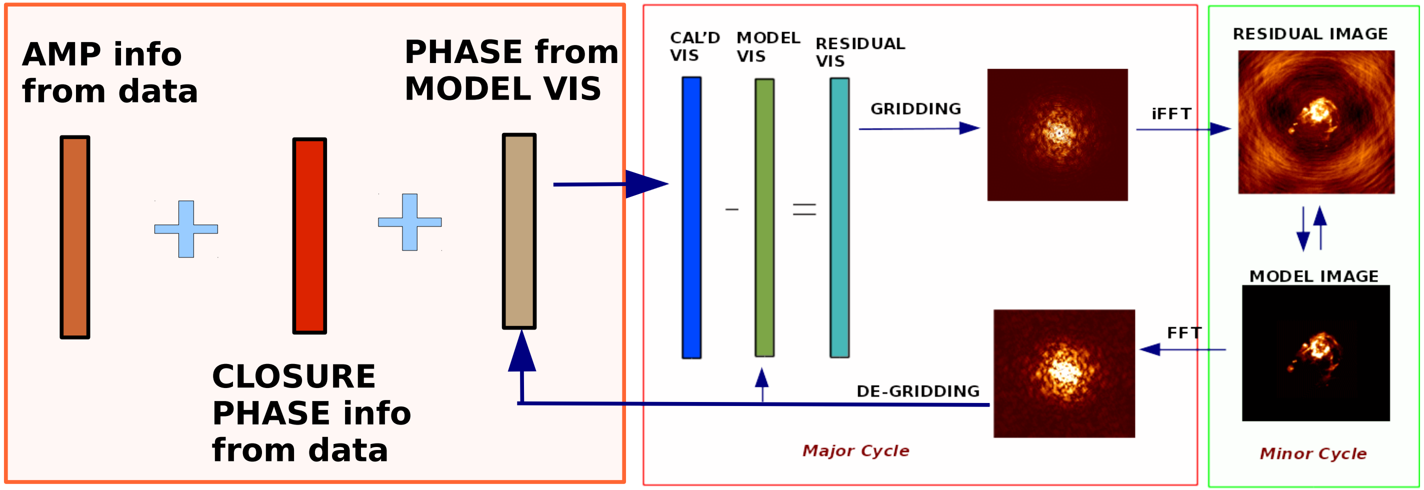

Ongoing R&D : Automate the quality checks
Automate the decision tree
Automate algorithm and parameter choices (+ more robust imaging algorithms)

# Radio vs Optical ( & radio VLBI , Infrared )

| | Radio | Optical |
|---|---|---|
| Frequency Range | 10 MHz to 1 THz | 430 – 770 THz |
| Detectors | Heterodyne systems | Intensity detectors |
| Measurables | Visibility Amplitude & Phase | Vis amplitude, Closure phase |
| UV-coverage | Dense. Easy to add more data for sensitivity.<br>Large compute cost | Sparse. Expensive to add more telescopes. |
| Calibration | External and Self-calibration<br>Instrument models for wide-field effects | Adaptive Optics in real-time |
| Imaging | L2 regularization with a variety of non-linear sky models and solvers.<br>Joint reconstructions + self-calibration. | Variety of models and solvers (mostly direct modeling solvers) |
| Current R&D | - High dynamic range imaging<br>- Detailed instrumental calibration (DI/DD)<br>- Improving imaging fidelity<br>- High Performance Computing<br>- Automated pipelines | - High fidelity imaging from very sparse data<br>- Joint reconstructions<br>- Calibration ideas<br>( - others ? ) |

# Similarities in image reconstruction algorithms

Estimating True Vis Amp/Phase : Radio (VLBI) : Hybrid methods Optical : WISARD




Radio (shorter baselines)

=> Complex Visibilities
& Self-calibration (DI/DD)
using antenna-based terms

Most algorithms follow the framework of L2 major cycles and non-linear minor cycles

Matching pursuit ( image domain ) : Radio : CLEAN and variants Optical : Building Block

Direct Modeling Approaches :

Optical : BSMEM, IRBis, MACIM, MiRA, SQUEEZE, SPARCO, PAINTER, MiRA-3D,...
Radio : ASP, MEM, RESOLVE, Fast-RESOLVE, SARA / PURIFY, MORESANE,.. VLBI : CHIRP,EHT

Practical adaptation in radio (non-VLBI) largely depends on ability to fit into standard framework

# Areas of possible synergy

**Sky Models and Solvers :**

- Algorithms with more efficient convergence and robustness to imperfect calibration
- Producing uncertainty estimates
- Joint reconstructions are generally better

**Self-Calibration :**

– Using direct model solvers (and hybrid methods) for radio interferometric data with unstable or missing phase information. [ E.g. Event Horizon Telescope, closure amp/phase, full-pol ]
– If/when vis phases are available, all DI and DD calibration algorithms from radio may apply

**Software Frameworks :** Standard data formats + shared core software
Flexible analysis frameworks with plug'n'play options are helpful
Incorporate instrumental corrections within imaging framework

**Automation :** Many algorithms/options exist. Automation required (R&D ongoing in radio).

( For all references : Please see paper )