Wide Band and Wide Field Imaging - II

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Sixteenth Synthesis Imaging Workshop
16-23 May 2018

Measurement Equation

The visibility measured by each baseline *ij* at one frequency and time

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t) S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t) I(l,m,\nu,t) e^{2\pi i(ul+vm+w(n-1))} dl\, dm\, dn$$

Direction Independent Gains

- Eliminated during calibration

Primary Beams

- Power pattern varies with time, frequency and baseline

– PBcor (post-deconvolution)
– A-Projection
- WB-A-Projection
- Mosaics

Full Beam

Sky-brightness varies with frequency (time)

- All sources have spectral structure (some vary with time)

– Cube Imaging
– Multi-Frequency Synthesis (MFS)
– Multi-Term-MFS (point source or multi-scale models)

Wide-Band

W-Term

-Non-coplanar baselines

-Sky curvature

- Faceting
- W-Projection
- 3D FT
- W-Stacking

Wide-Field

Wide Band + Full Beam/Wide-field

+ Mosaics

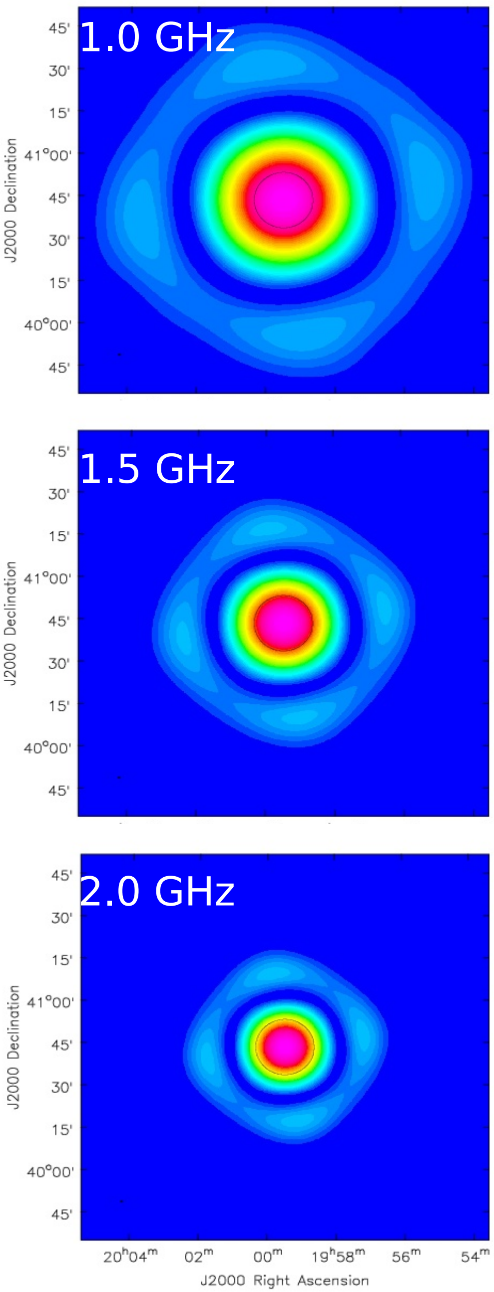
+ Single Dish

Example : Imaging the G55 supernova remnant

Imaging Framework

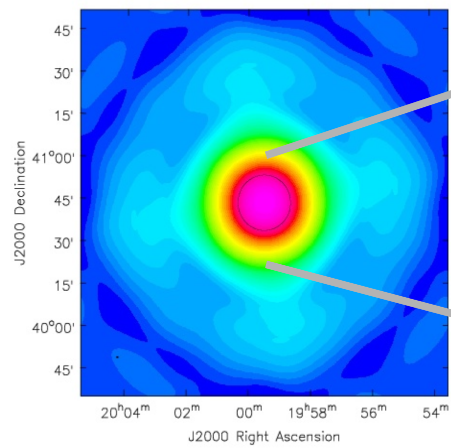
Wide-Band Wide-Field Imaging : Primary Beams

VLA PBs



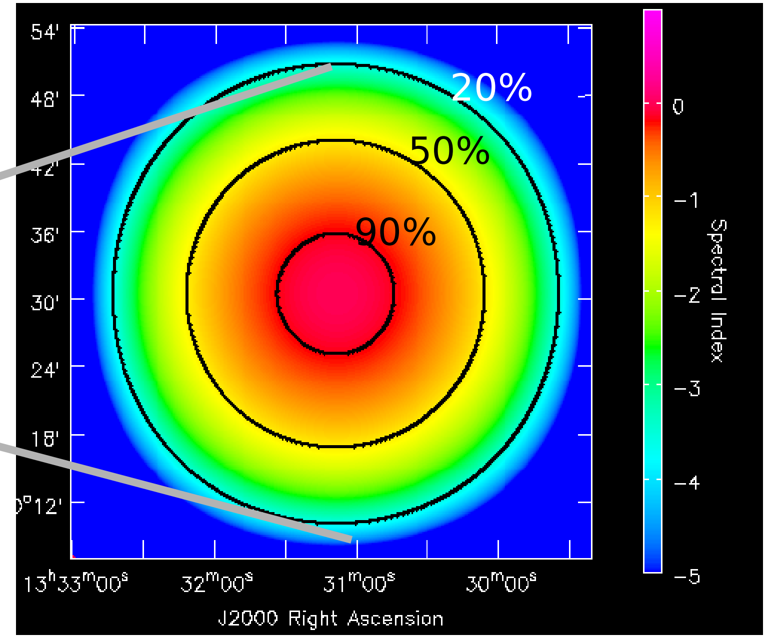
Primary beam scales (or changes) with frequency

Average Primary Beam



A very wide shelf of sensitivity outside the main lobe

Spectral Index of PB

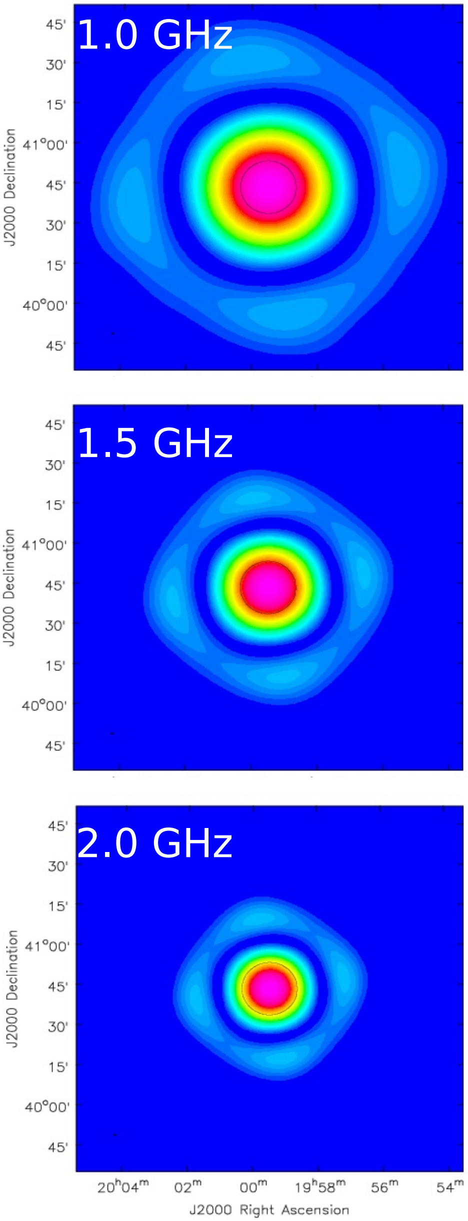


For VLA L-Band (1-2 GHz)

- About -0.4 at the PB=0.8 (6 arcmin from the center)
- About -1.4 at the HPBW (15 arcmin from the center)

$$I^{obs}_{wf,wb} = \sum_{\nu} \left[\left(P_{\nu} \cdot I^{sky}_{\nu} \right) * PSF_{\nu} \right]$$

Wide-Band Primary Beam Correction



Cube Imaging

-- Sky model represents $I(\nu)P(\nu)$

-- Divide the output image at each frequency by $P(\nu)$

Multi-Term MFS + Wideband-PBcor

-- Taylor coefficients represent $I(\nu)P(\nu)$

-- Polynomial division by PB Taylor coefficients

$$\frac{(I_{0,}^{m} I_{1,}^{m} I_{2,}^{m} \ldots)}{(P_{0,} P_{1,} P_{2,} \ldots)} = (I_{0,}^{sky} I_{1,}^{sky} I_{2}^{sky} \ldots)$$

Wideband A-Projection

-- Remove $P(\nu)$ during gridding (before model fitting)

$$A_{\nu}^{-1} \approx \frac{A_{\nu_c}^{T}}{A_{\nu_c}^{T} * A_{\nu}} \quad \text{where} \quad P_{\nu} . P_{\nu_c} \approx P_{\nu_{mid}}^{2}$$

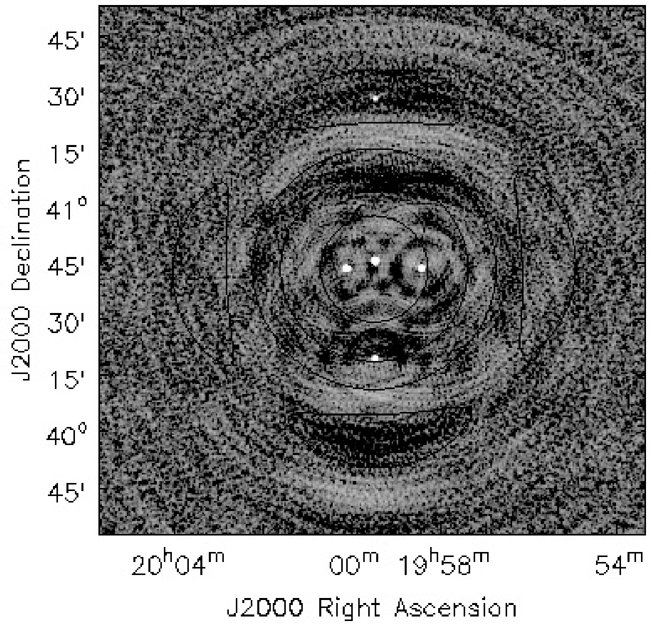
-- Output spectral index image represents only the sky

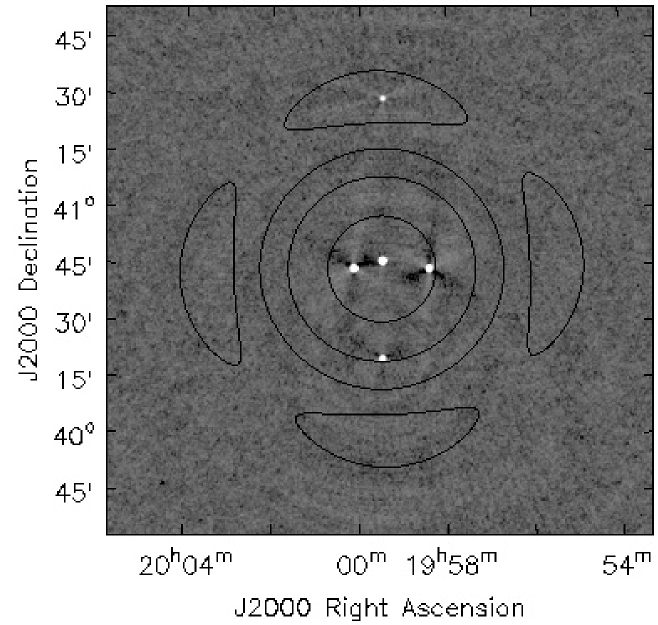
Bhatnagar et al, 2013

Wide Band Full Beam imaging – Different algorithms

Basic MFS imaging

(no WB,WF corrections)



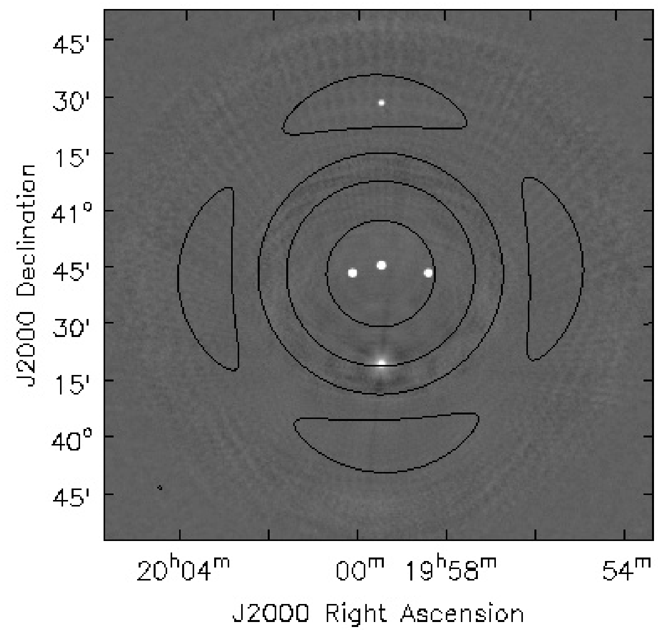


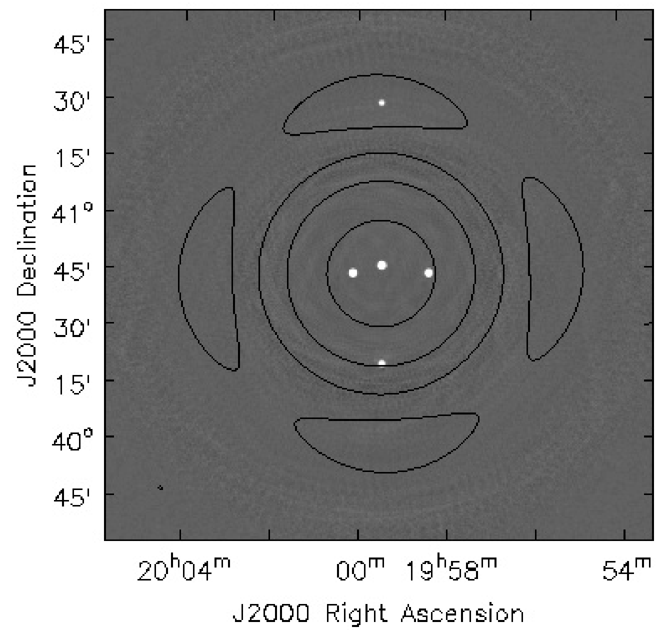
MT-MFS wideband imaging

(No WF corrections, PB freq dependence part of sky model)

MT-MFS wideband imaging + A-Proj

(PB^2 freq dependence part of sky model)



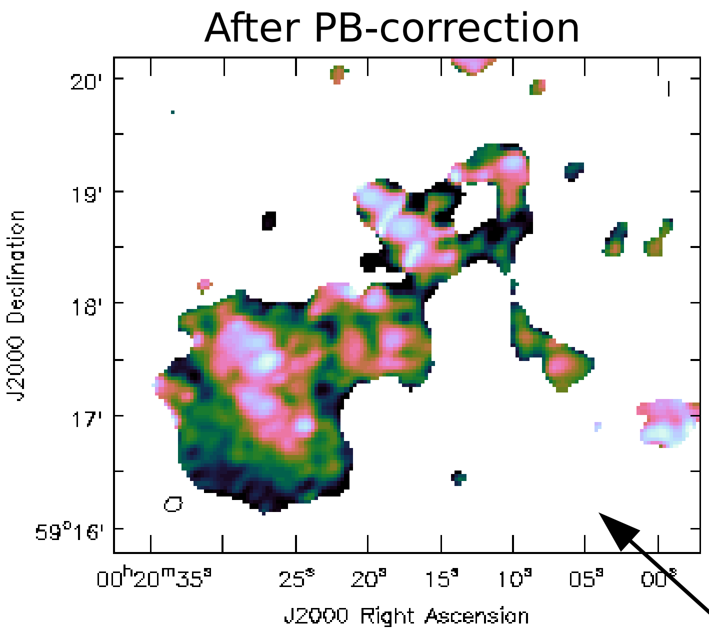


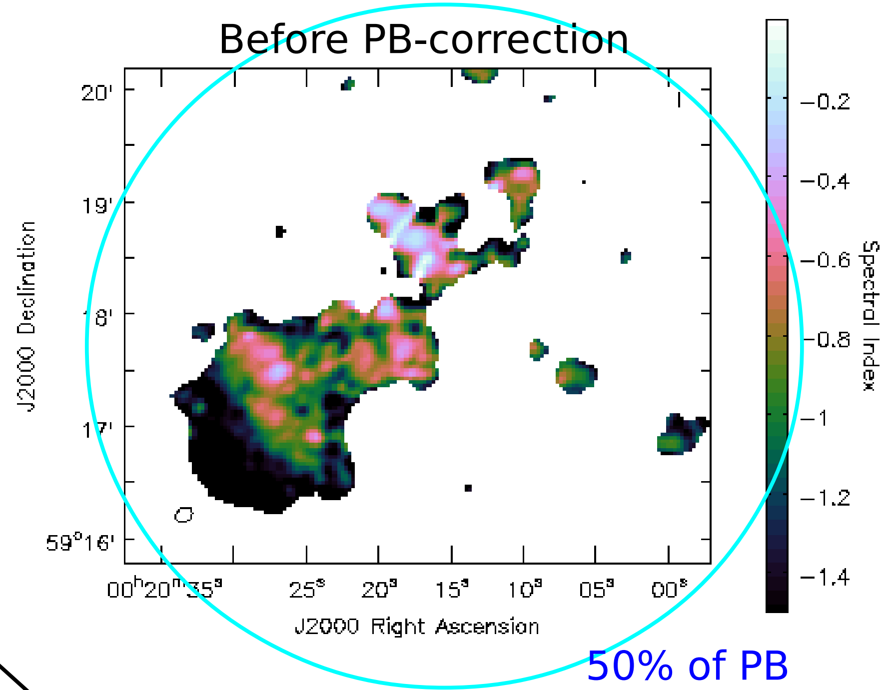
MT-MFS wideband imaging + WB-A-Proj

(PB freq dependence removed during gridding)

Wideband VLA imaging of IC10 Dwarf Galaxy

[Heesen et al, 2011]

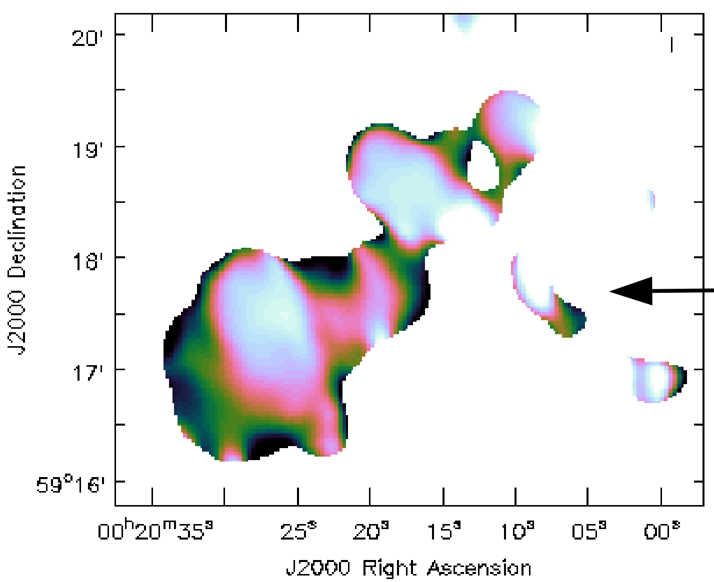




IC10 Dwarf Galaxy :

Spectral Index across C-Band.

Dynamic-range ~ 2000



MT-MFS : Wide-band PB-correction after multi-term multi-scale MFS.

Cube : Spectral-index map made by cube imaging, smoothing to lowest resolution, and spectral fitting.

Wide Band + Full Beam/Wide-field

+ Mosaics

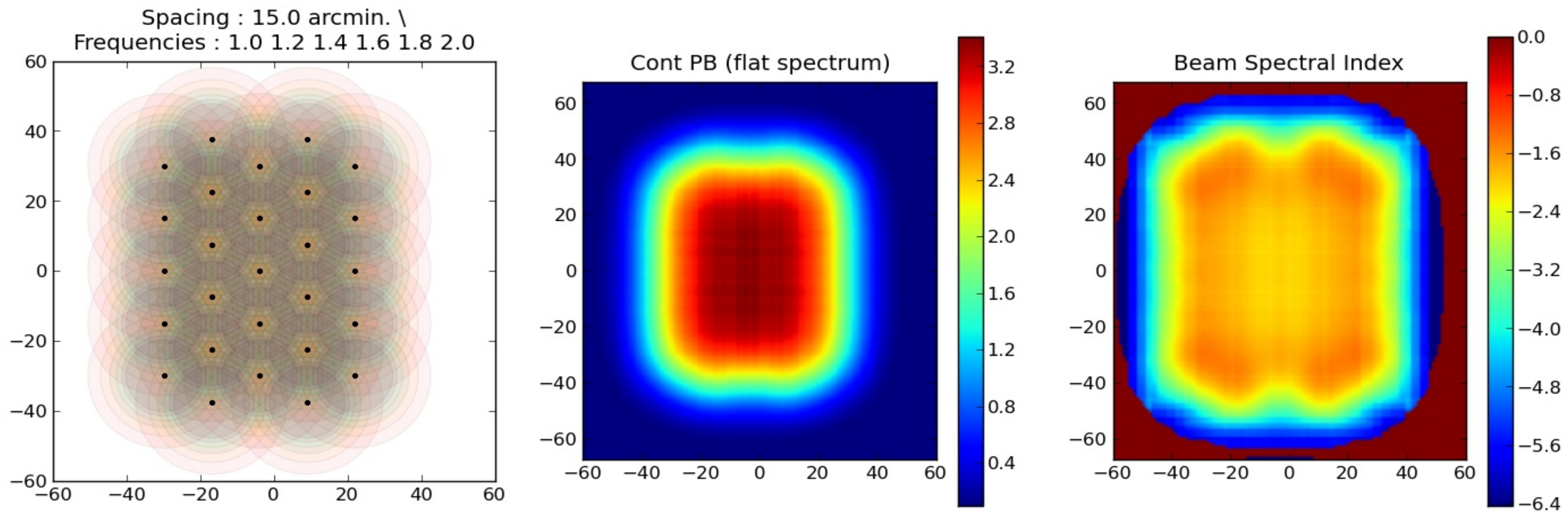
+ Single Dish

Example : Imaging the G55 supernova remnant

Imaging Framework

Wide-Band Mosaic Primary Beam

The mosaic primary beam has an artificial spectral index all over the FOV



Algorithms :

- Deconvolve Pointings separately or together (Stitched vs Joint Mosaic)
    - Impacts image fidelity, especially of common sources.

- Deconvolve Channels separately or together (Cube vs MFS)
    - Impacts imaging fidelity and sensitivity, dynamic range

- Use A-Projection or not (Accurate vs Approximate PB correction)
    - Impacts dynamic range and spectral index accuracy

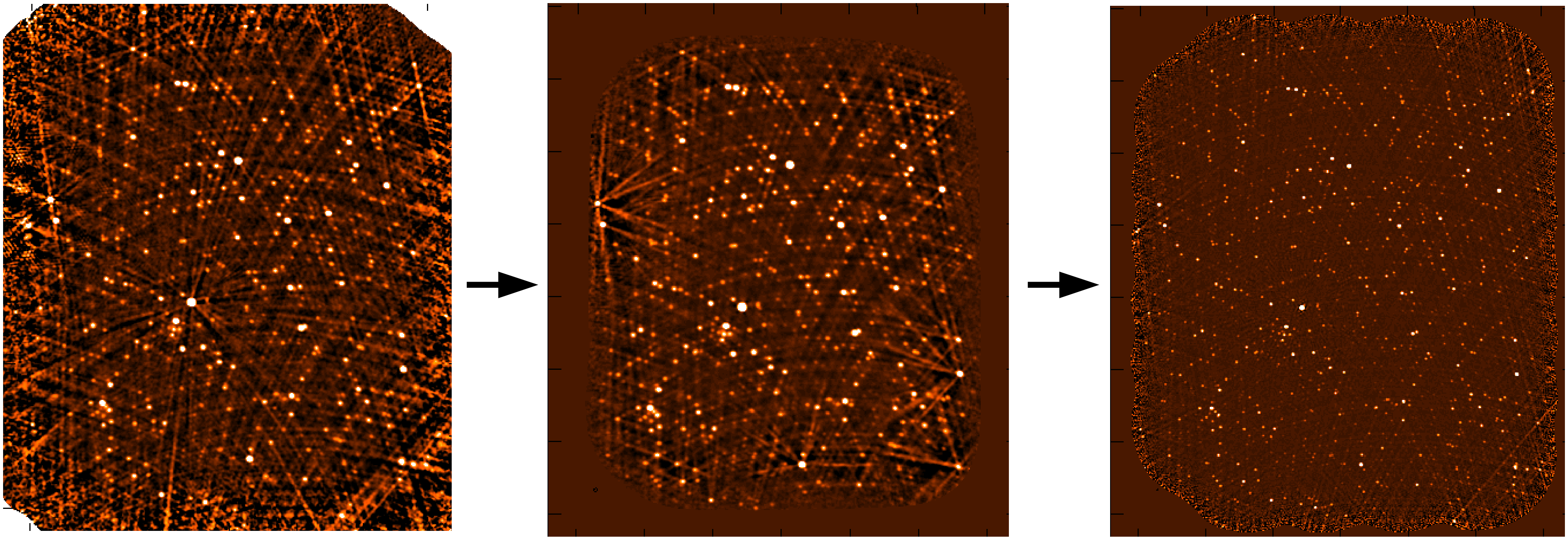
Wideband Mosaic Imaging Accuracy

[Rau et al, 2016]

Cube + Joint Mosaic (with static Primary Beams)
Dyn.Range = 5000:1

Cube + A-Projection + Joint Mosaic
Dyn.Range = 10000:1

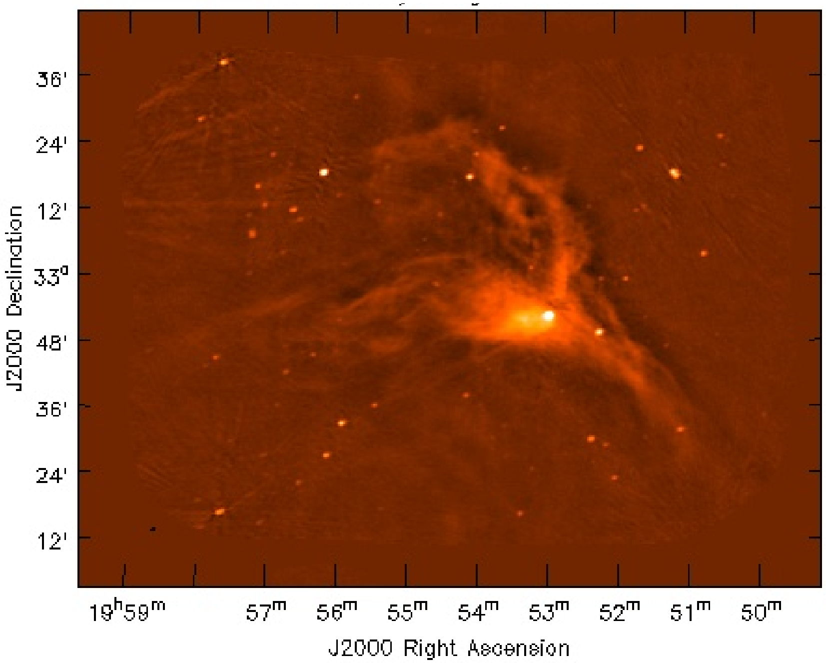
Wideband A-Proj + Joint Mosaic + Multi-term MFS
Dyn.Range = 40000:1



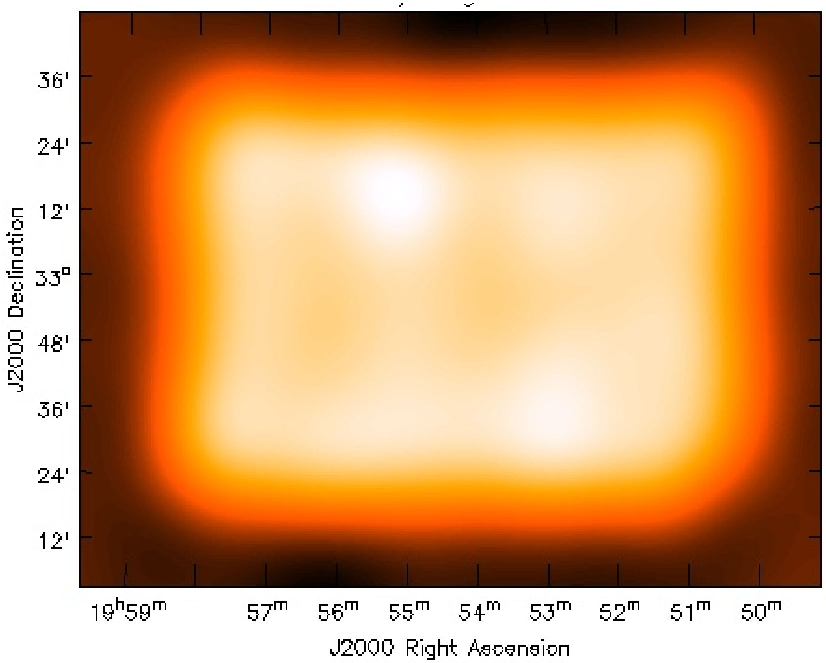
Method Intensity Range	I/I_{true} $> 20\mu Jy$	I/I_{true} $5-20\mu Jy$	I/I_{true} $< 5\mu Jy$	$\alpha-\alpha_{true}$ $> 50\mu Jy$	$\alpha-\alpha_{true}$ $10-50\mu Jy$
Cube	0.9 ± 0.1	0.9 ± 0.3	0.9 ± 0.5	-0.5 ± 0.2	-0.6 ± 0.5
Cube + AWP	1.0 ± 0.05	1.0 ± 0.2	1.0 ± 0.3	-0.15 ± 0.1	-0.1 ± 0.25
MTMFS + WB-AWP	1.0 ± 0.02	1.0 ± 0.04	1.0 ± 0.15	-0.05 ± 0.05	-0.1 ± 0.2

Wideband Mosaic of CTB80 (1-2 GHz, VLA-D config)

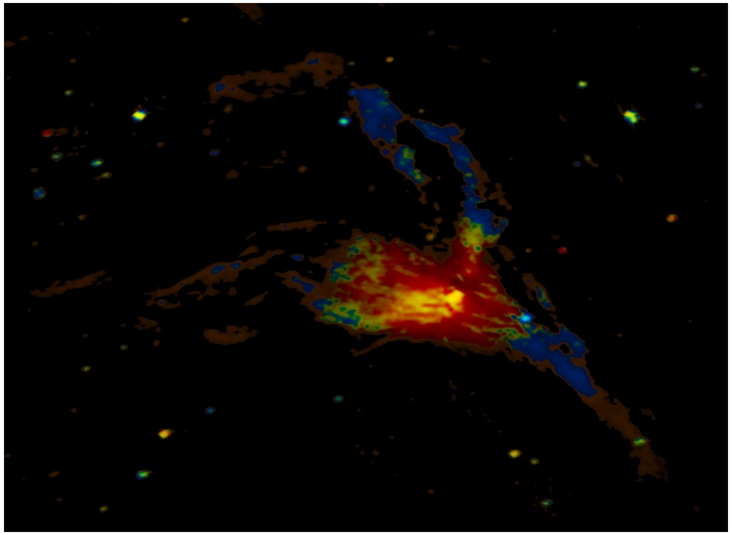
Intensity



Mosaic Primary Beam



Intensity-weighted Spectral Index



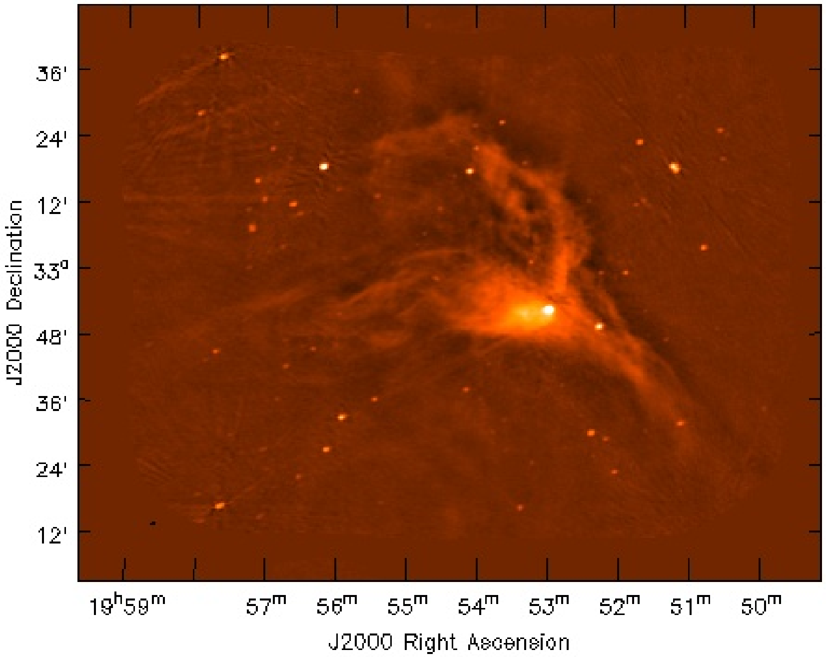
300GB calibrated dataset, 106 pointings over 1.5x2 deg, imaged with MS-MT-MFS (NT=2) + WB-A-Projection.

=> Mosaic primary beam spectral index of ~ -1.5 has been removed prior to the wideband sky model fitting.

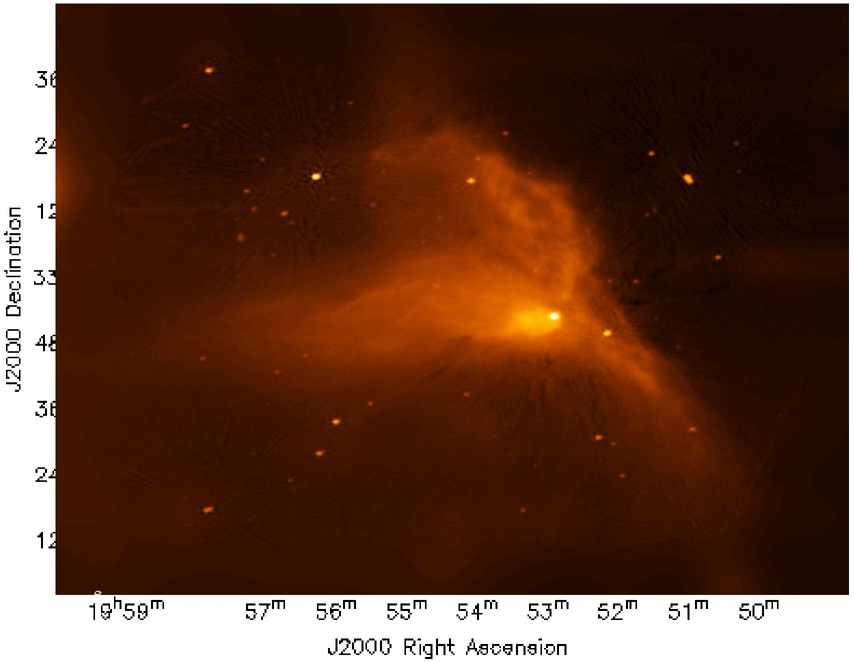
Wideband Mosaic + Single Dish data

Example : Combining Interferometer intensity image with
Single dish data at reference frequency, using Feathering.

Int WB Mosaic



Int WB Mosaic + Single Dish



Joint SD+INT Spectral Index Map => Work in progress

Algorithms needed : Multiscale, Multi-term MFS, with A-Projection,
W-Projection, form a Joint Mosaic, and Joint
deconvolution with wideband single dish data.

(Must run in finite time → robust parallelization)

Wide Band + Full Beam/Wide-field

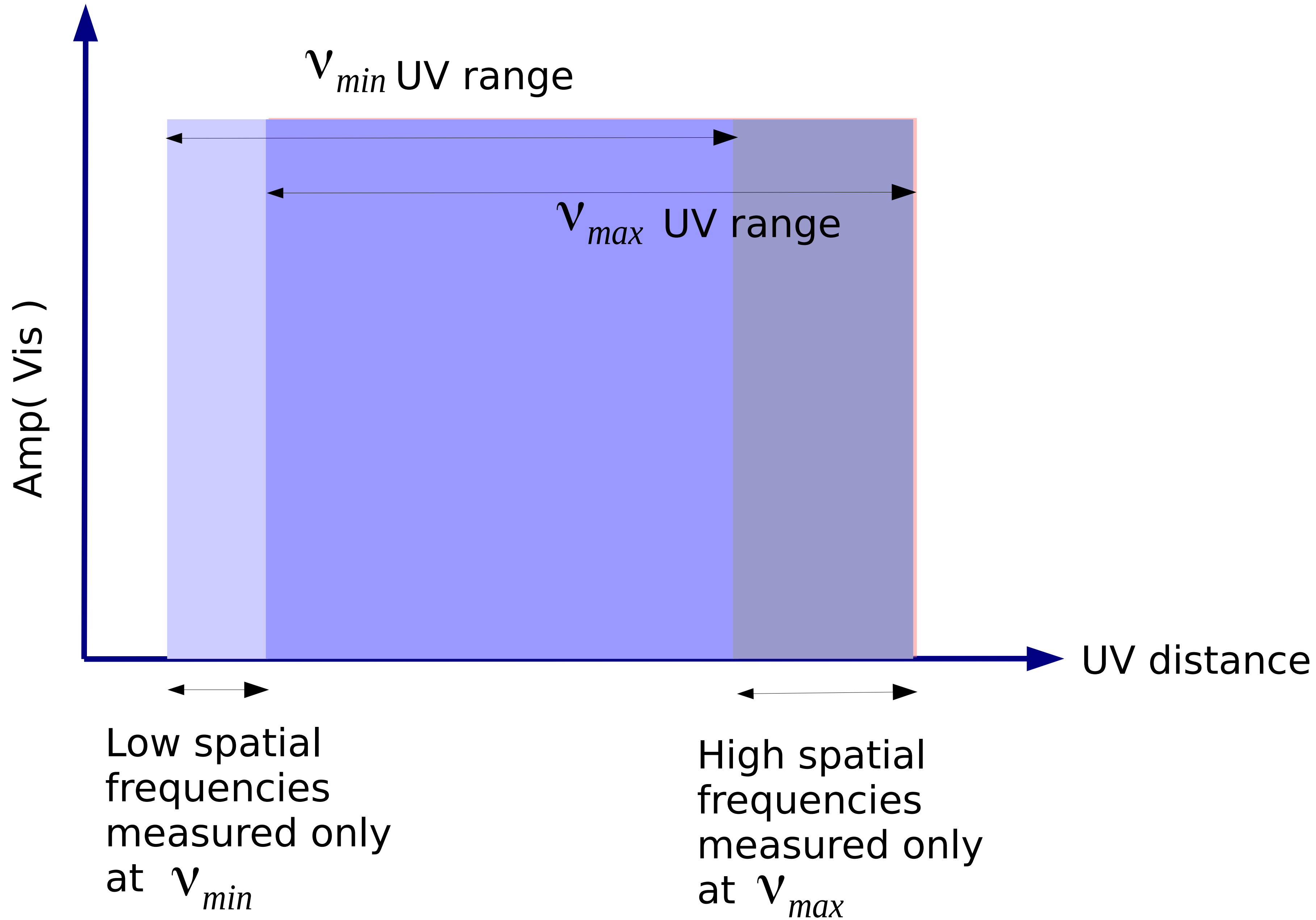
+ Mosaics

+ Single Dish

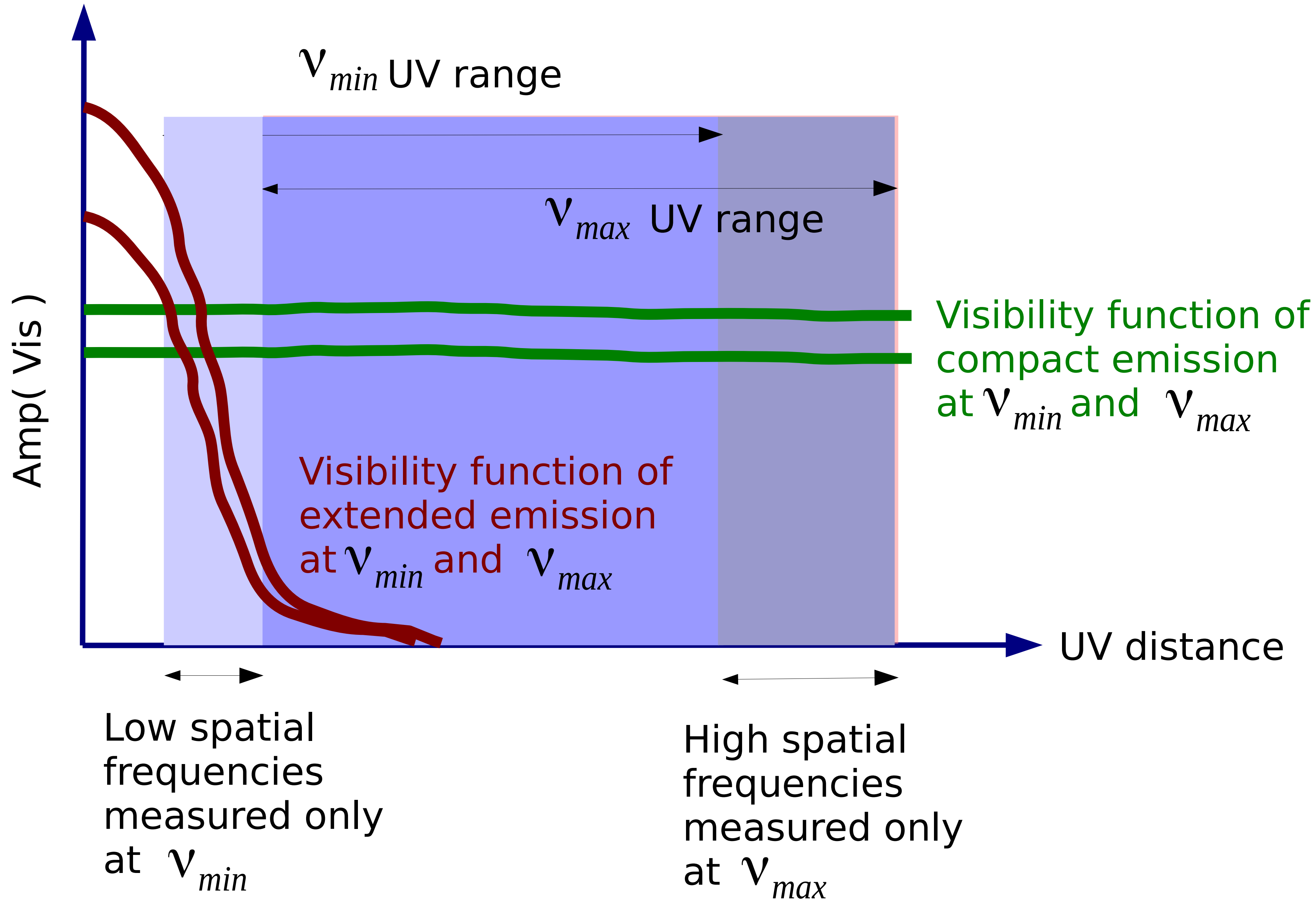
Example : Imaging the G55 supernova remnant

Imaging Framework

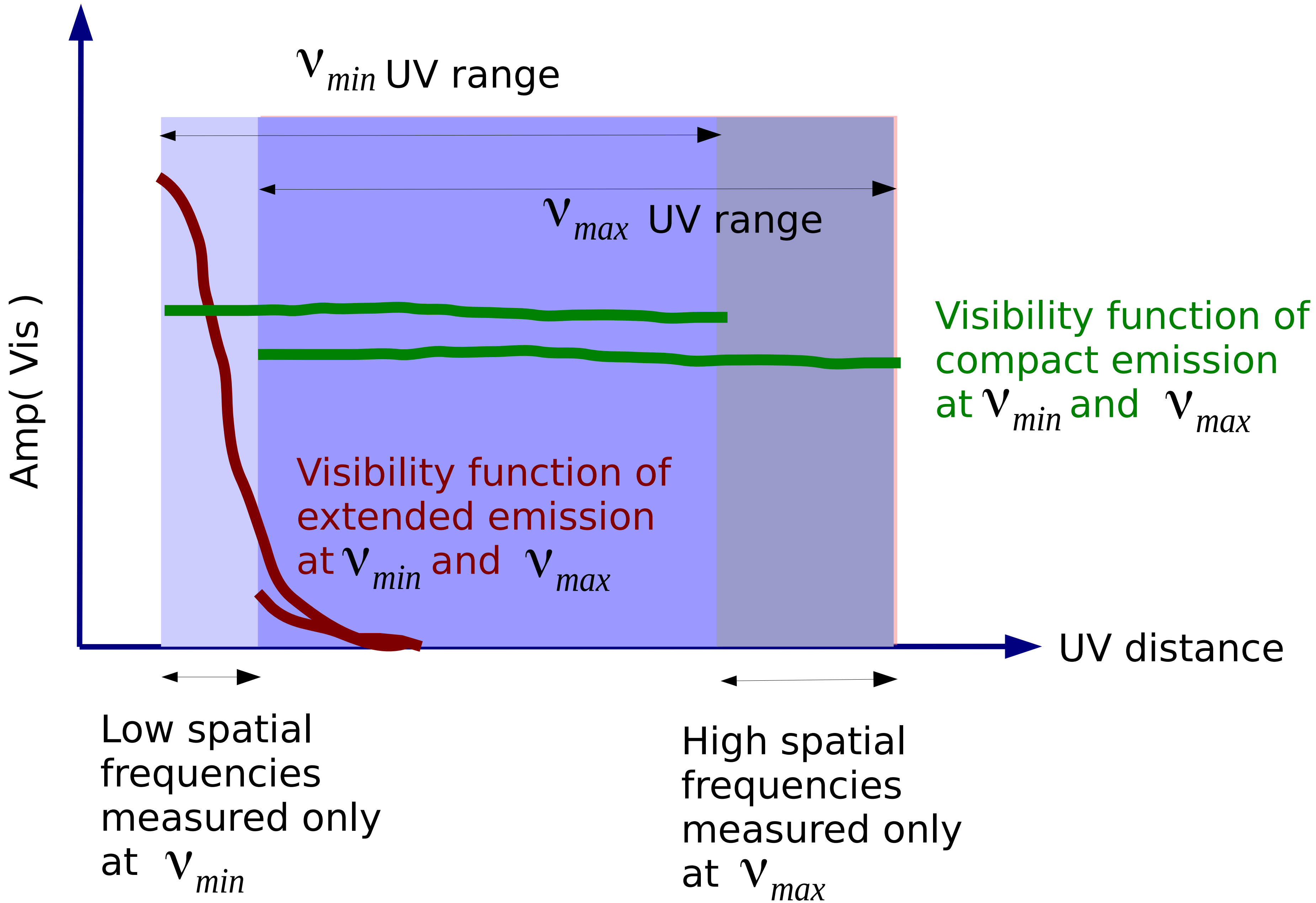
For which scales can we reconstruct the spectrum ?



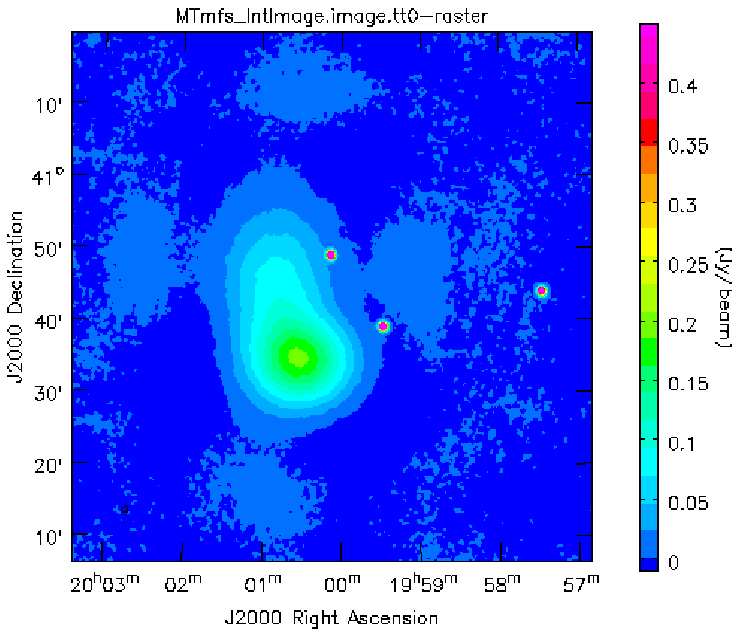
For which scales can we reconstruct the spectrum ?

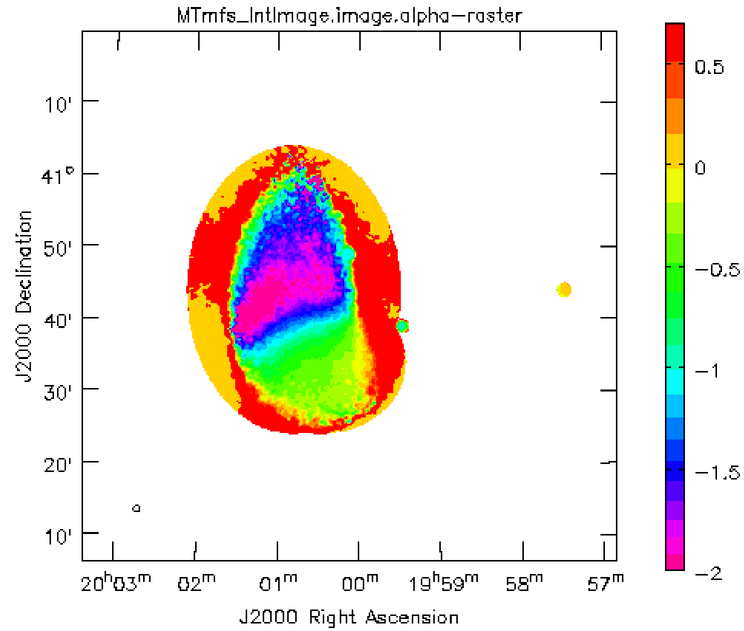


For which scales can we reconstruct the spectrum ?



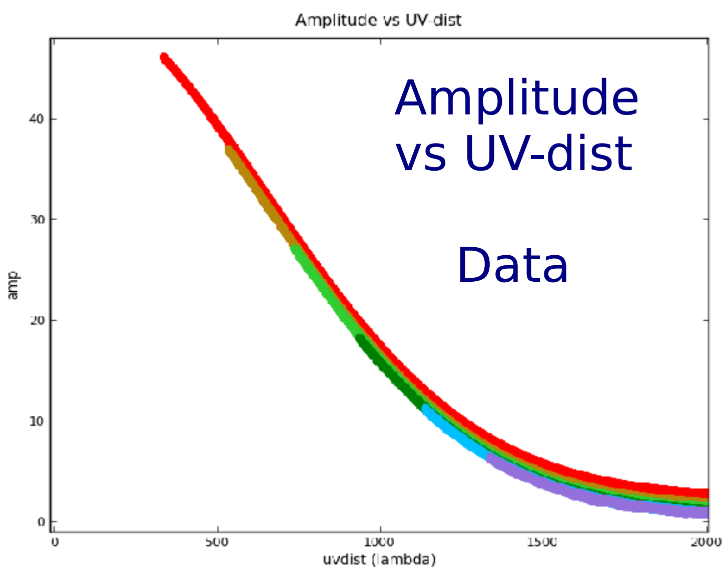
Very large spatial scales : wideband single dish data

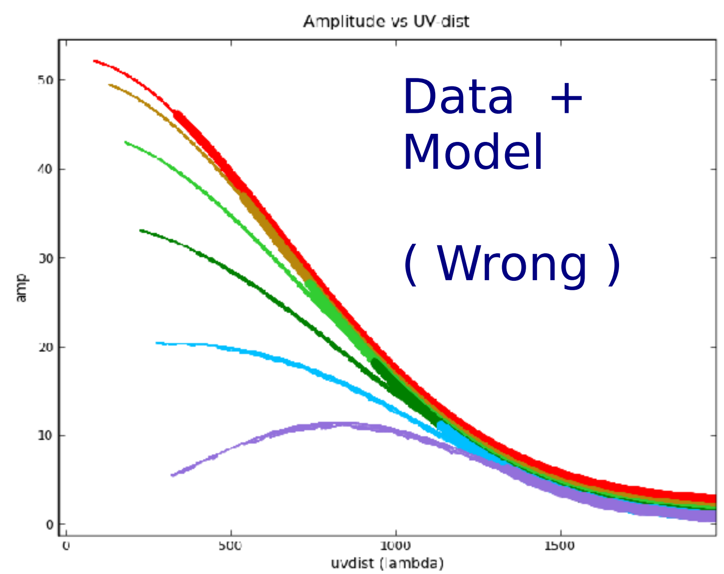




Example : Flat spectrum emission at very large scales

Top : Only interferometer data
=> Negative bowl and artificial steep spectrum





No short spacings to constrain the spectra

=> False steep spectrum reconstruction

Wideband Single Dish + Interferometer Combination

Several Algorithms can be applied to wideband data.

(1) Feathering : Image SD and INT data separately (in wideband mode)
Combine outputs using a UV-domain weighted average
Perform feather per Taylor coefficient map.

=> Works best when noise levels match, weighting choice is obvious, and no mid-scale artifacts in the INT-only reconstruction.

(2) Startmodel : Use SD images as a starting model for the INT reconstruction

=> Works if there is clear overlap in UV-range between SD and INT data.

(3) Artificial visibilities : Simulate virtual SD visibilities, combine with INT data

=> Flexible, a true joint reconstruction, relative weights handled externally.

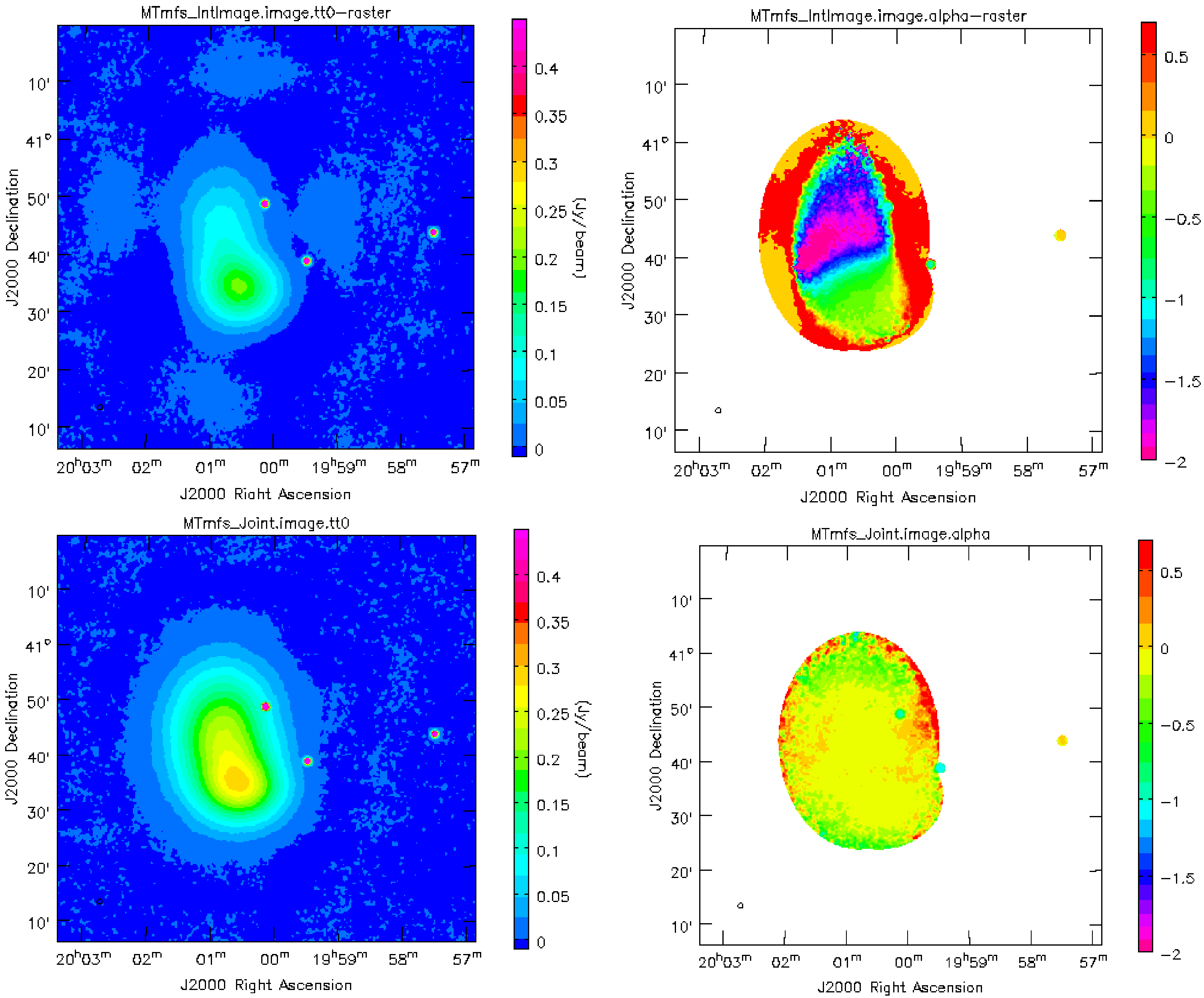
Koda et al, 2011

(4) Merge residual images and PSFs between major and minor cycle :

=> Flexible, a true joint reconstruction, weight functions part of reconstruction framework, compatible with all wide-field, wide-band algorithms.

[Rau & Naik, 2018 (in prep)]

Very large spatial scales : wideband single dish data



Example : Flat spectrum emission at very large scales

Top : Only interferometer data
=> Negative bowl and artificial steep spectrum

Bottom : Joint wideband reconstruction (4)

=> Recovers more flux and gets accurate spectrum

=> Compatible with wide-field, wideband, mosaics

Wide Band + Full Beam/Wide-field

+ Mosaics

+ Single Dish

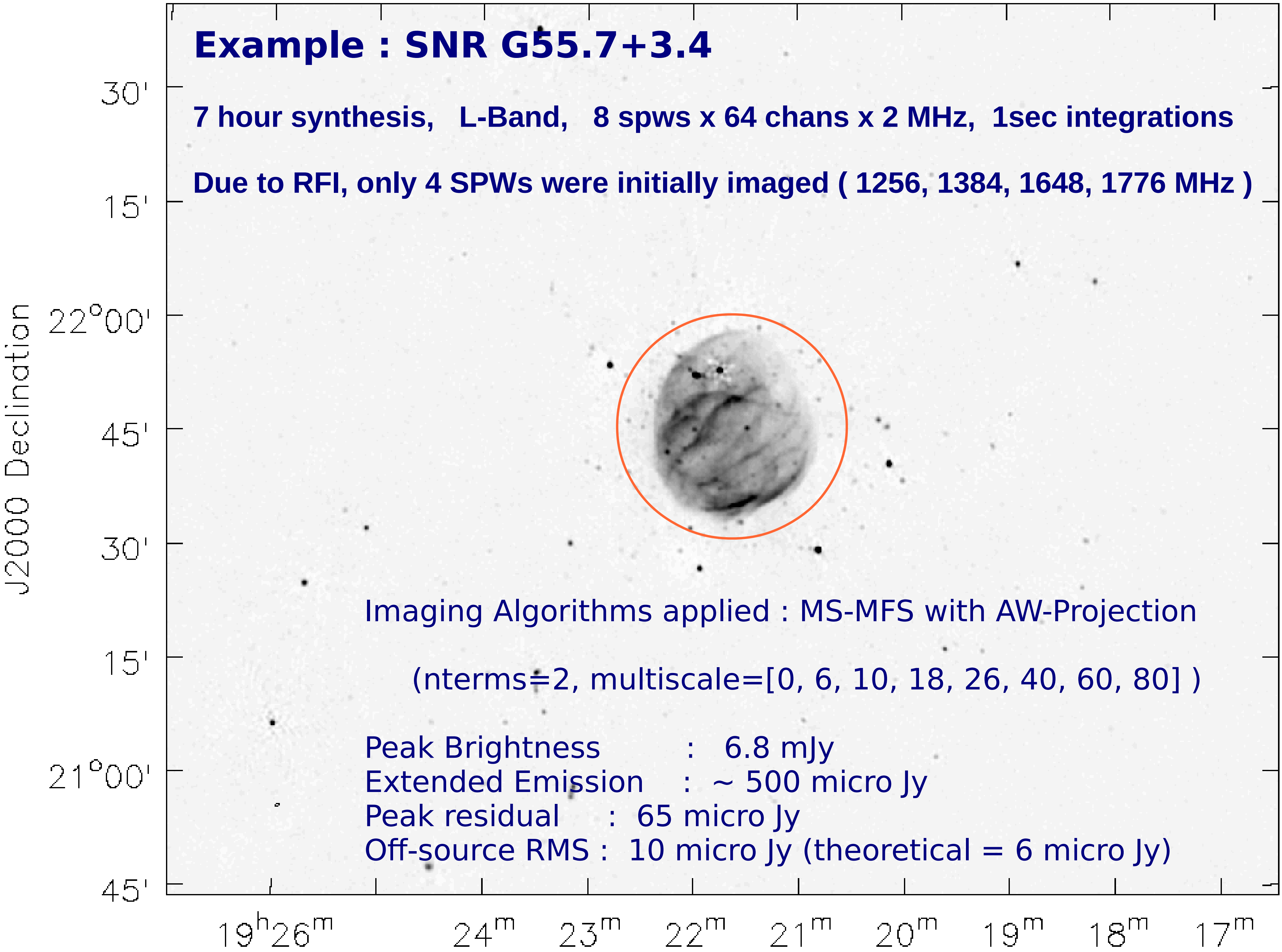
Example : Imaging the G55 supernova remnant

Imaging Framework

Example : SNR G55.7+3.4

7 hour synthesis, L-Band, 8 spws x 64 chans x 2 MHz, 1sec integrations

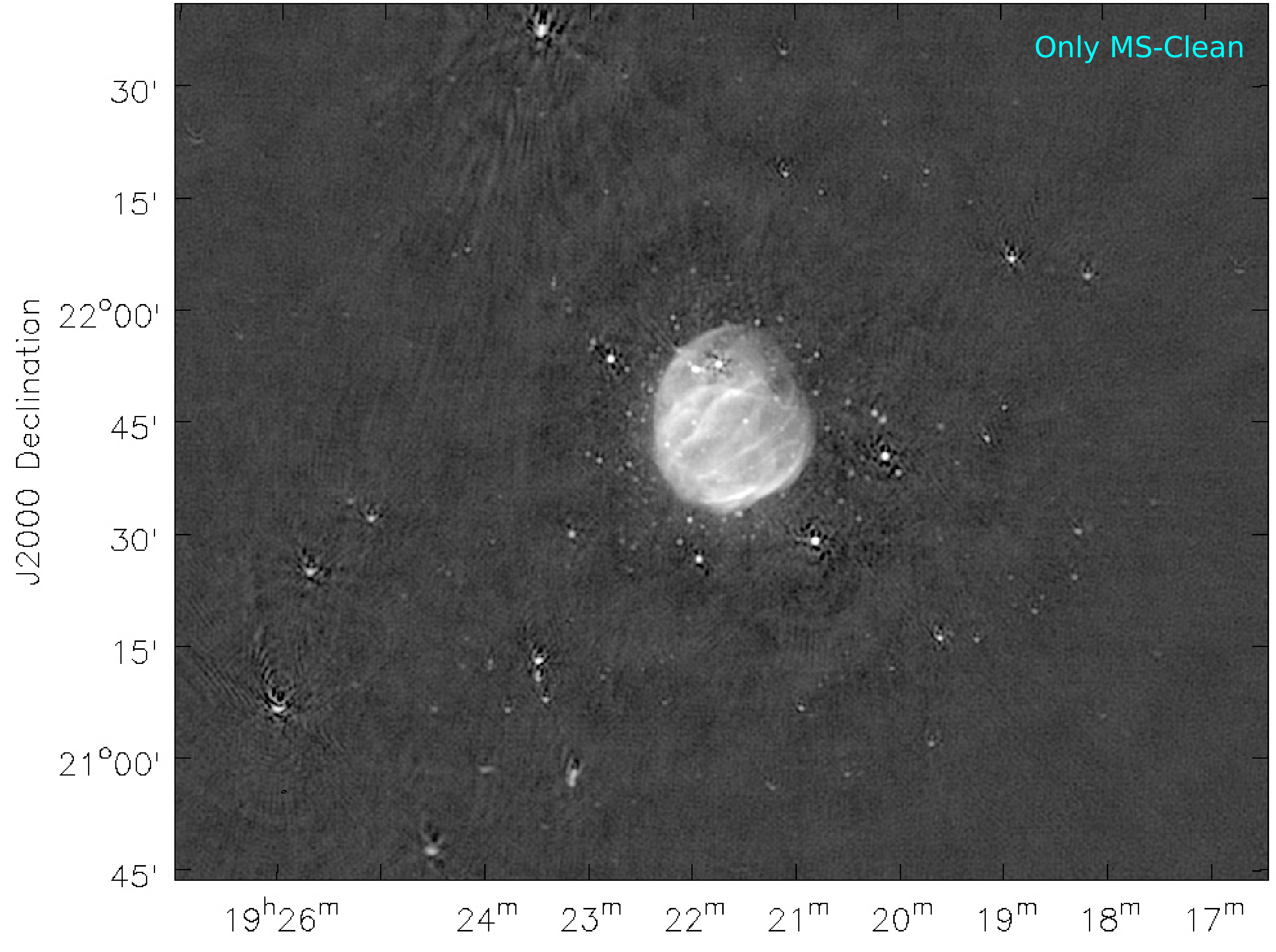
Due to RFI, only 4 SPWs were initially imaged (1256, 1384, 1648, 1776 MHz)

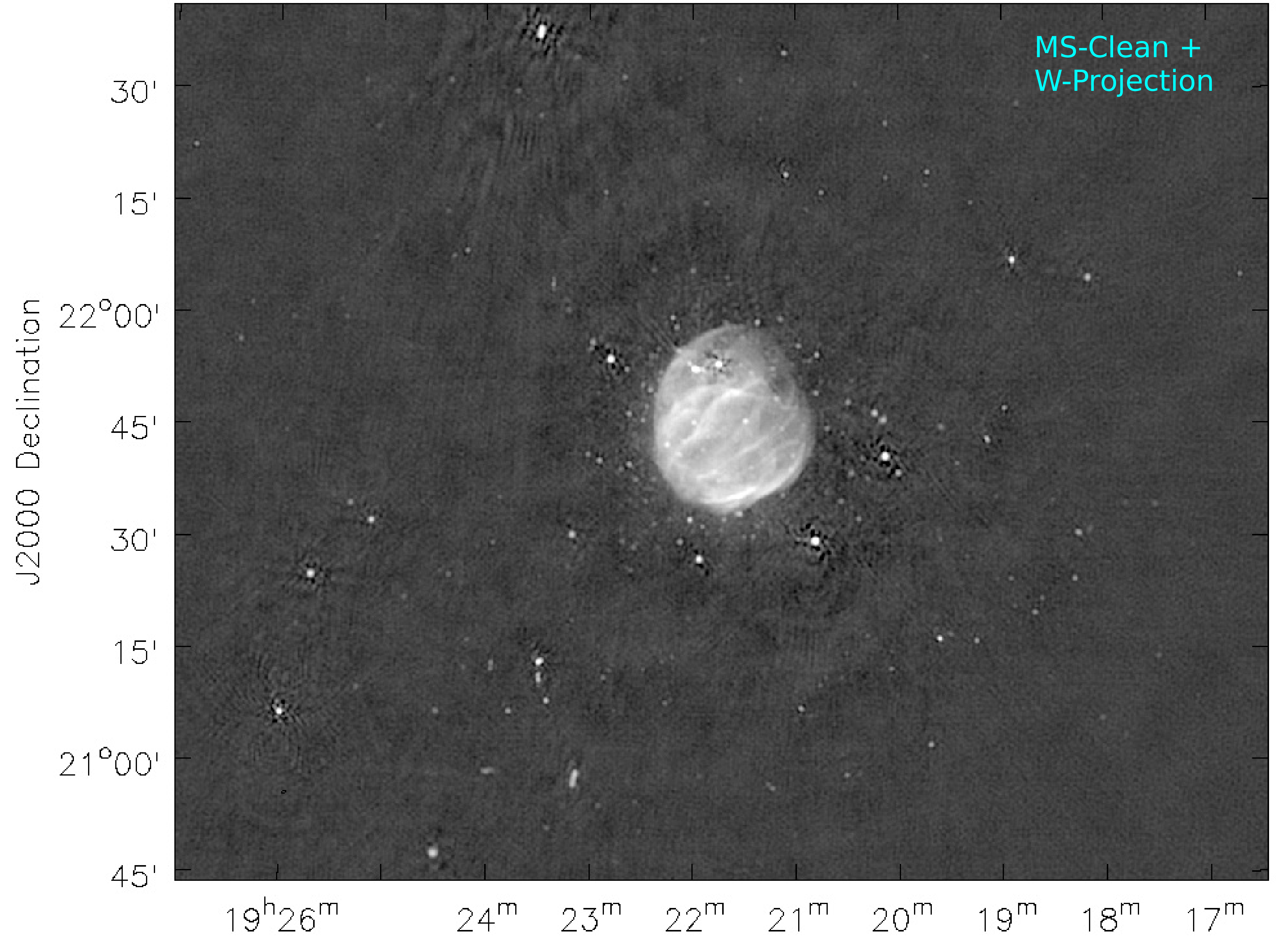


Imaging Algorithms applied : MS-MFS with AW-Projection

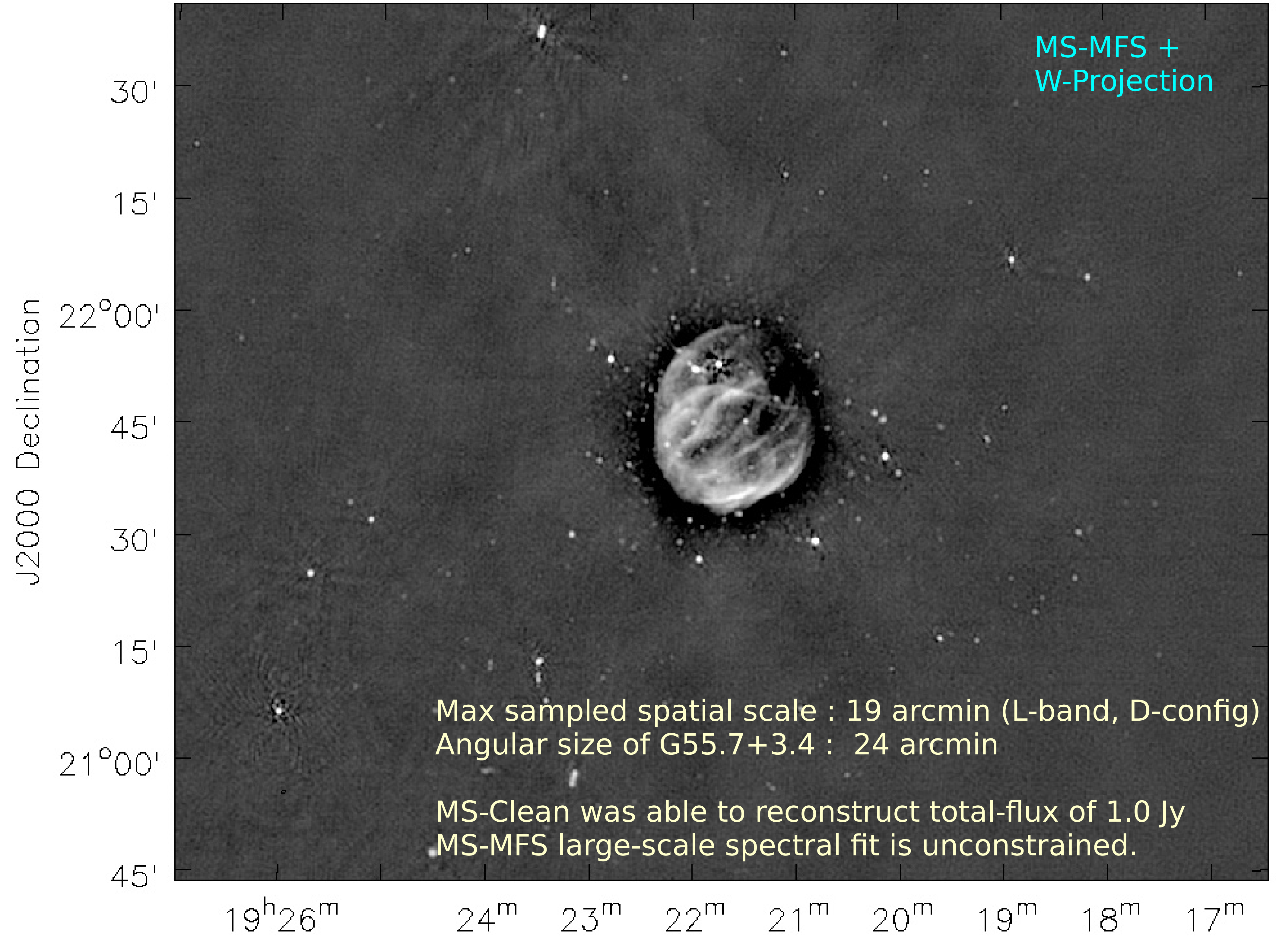
(nterms=2, multiscale=[0, 6, 10, 18, 26, 40, 60, 80])

Peak Brightness : 6.8 mJy
Extended Emission : ~ 500 micro Jy
Peak residual : 65 micro Jy
Off-source RMS : 10 micro Jy (theoretical = 6 micro Jy)

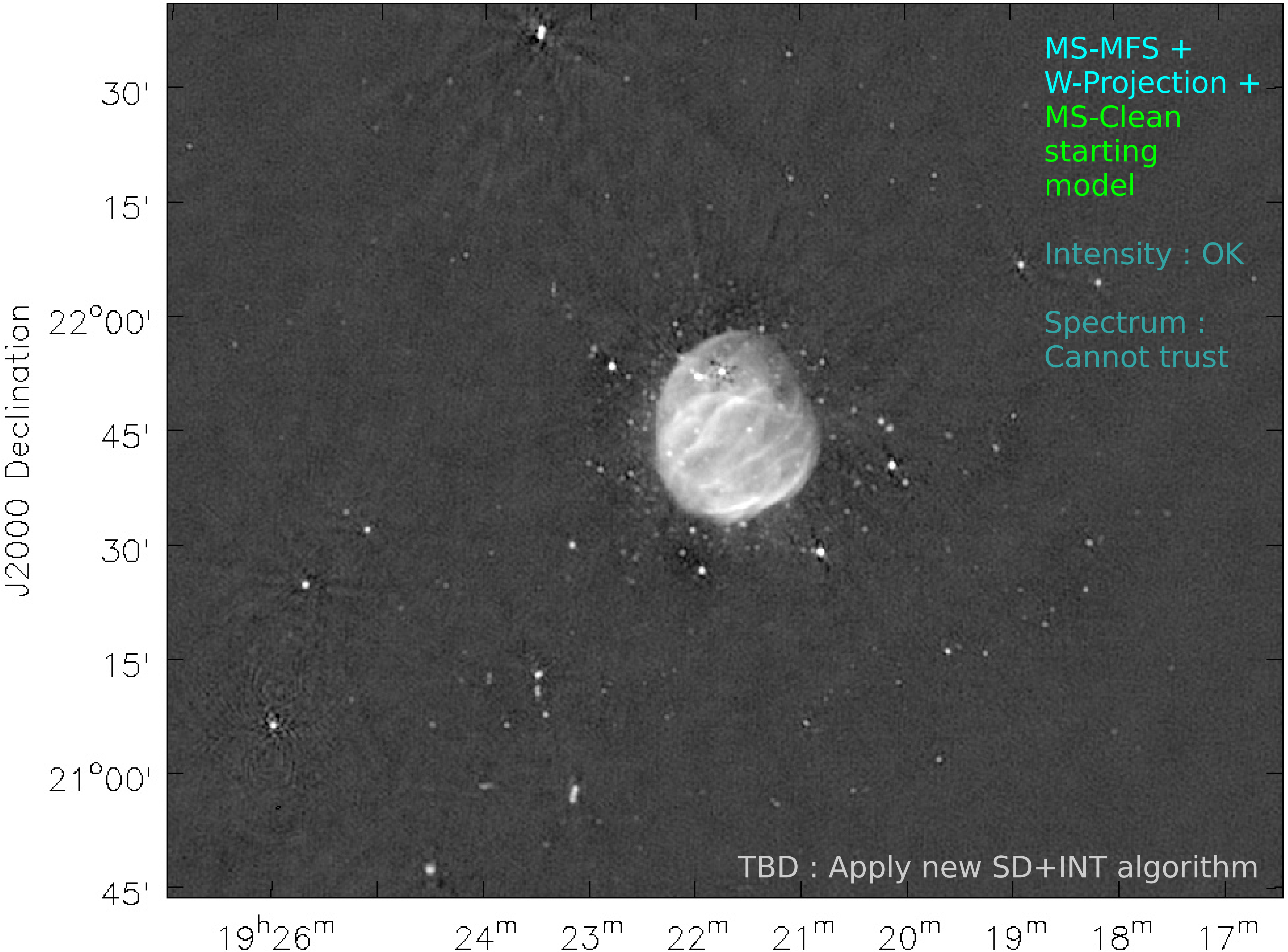

Only MS-Clean
J2000 Declination
30'
15'
22°00'
45'
30'
15'
21°00'
45'
19h26m
24m
23m
22m
21m
20m
19m
18m
17m



MS-Clean +
W-Projection
J2000 Declination
30'
15'
22°00'
45'
30'
15'
21°00'
45'
19h26m
24m
23m
22m
21m
20m
19m
18m
17m

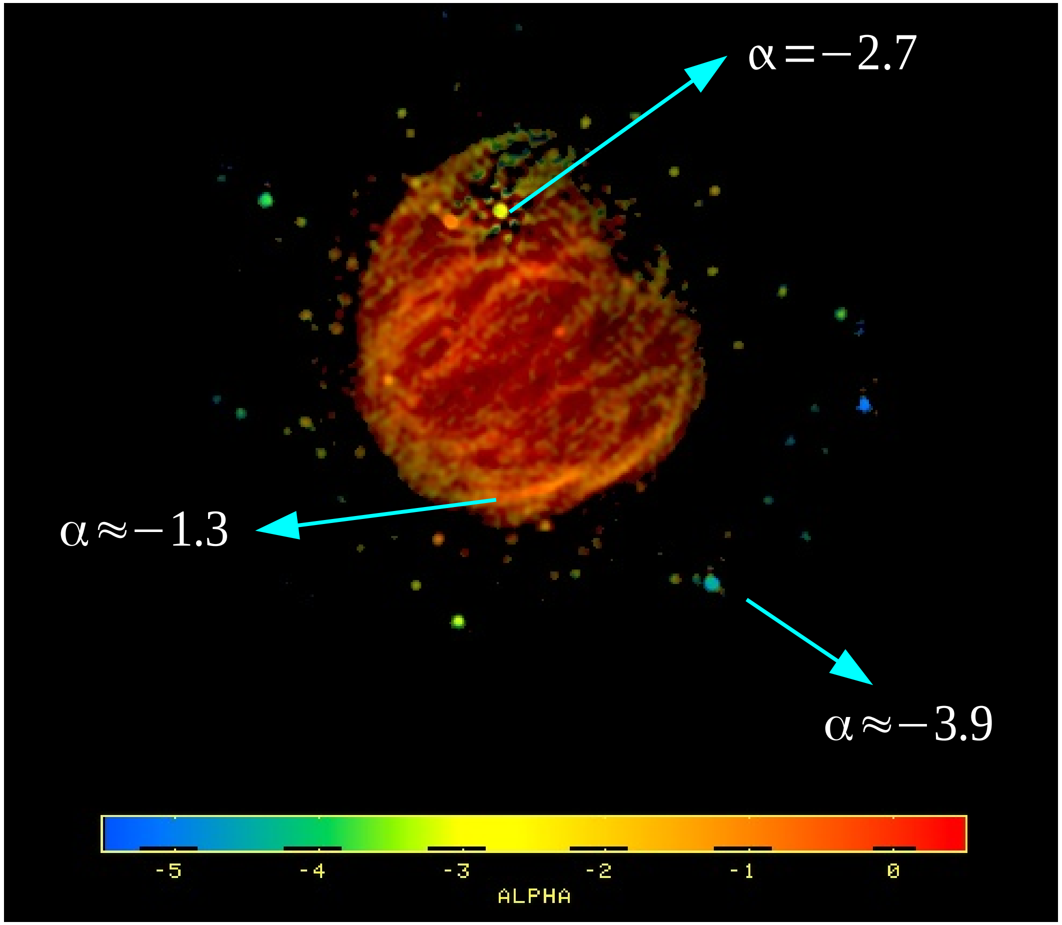


MS-MFS +
W-Projection
Max sampled spatial scale : 19 arcmin (L-band, D-config)
Angular size of G55.7+3.4 : 24 arcmin
MS-Clean was able to reconstruct total-flux of 1.0 Jy
MS-MFS large-scale spectral fit is unconstrained.
J2000 Declination
30'
15'
22°00'
45'
30'
15'
21°00'
45'
19h26m
24m
23m
22m
21m
20m
19m
18m
17m

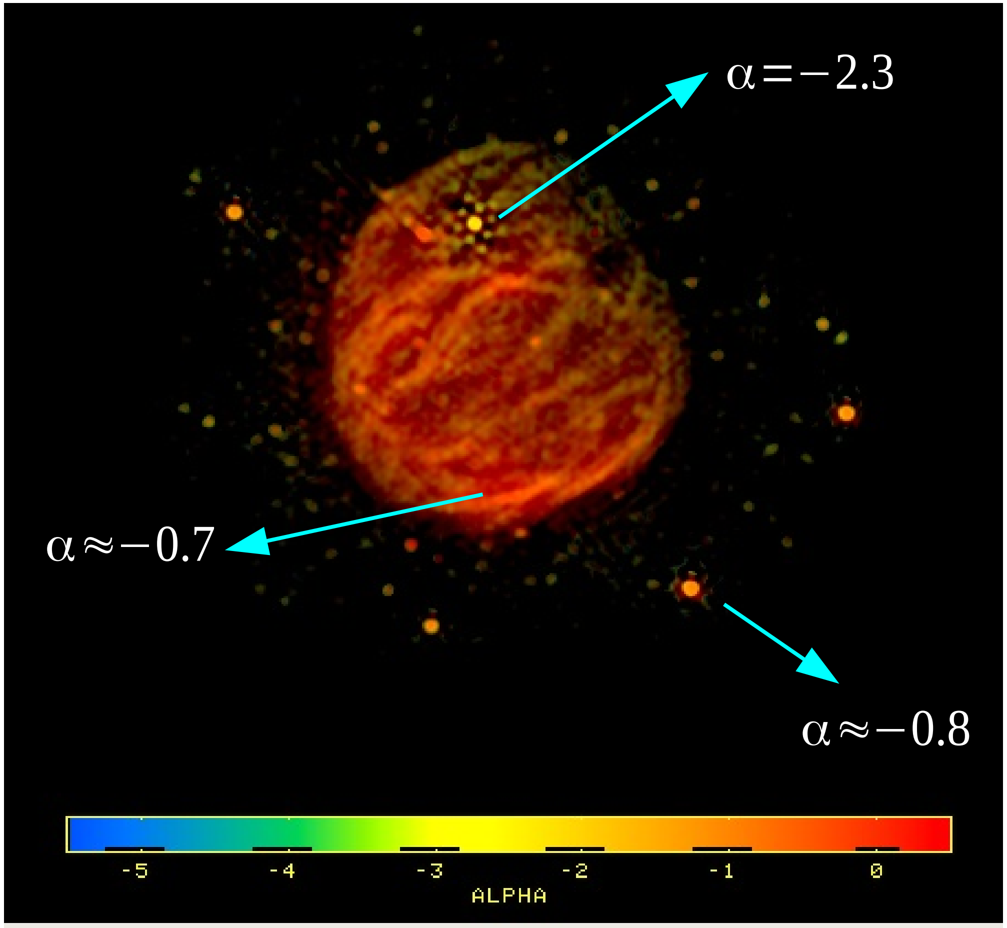

MS-MFS +
W-Projection +
MS-Clean
starting
model
Intensity : OK
Spectrum :
Cannot trust
TBD : Apply new SD+INT algorithm
J2000 Declination
30'
15'
22°00'
45'
30'
15'
21°00'
45'
19h26m
24m
23m
22m
21m
20m
19m
18m
17m

Spectral Indices before and after WB-A-Projection

Without PB correction
Outer sources are artificially steep



With PB correction (via WB-AWP)
Outer sources have correct spectra



Intensity-weighted spectral index maps (color = spectral index from -5.0 to +0.2)

Wide-field sensitivity because of wide-bandwidths

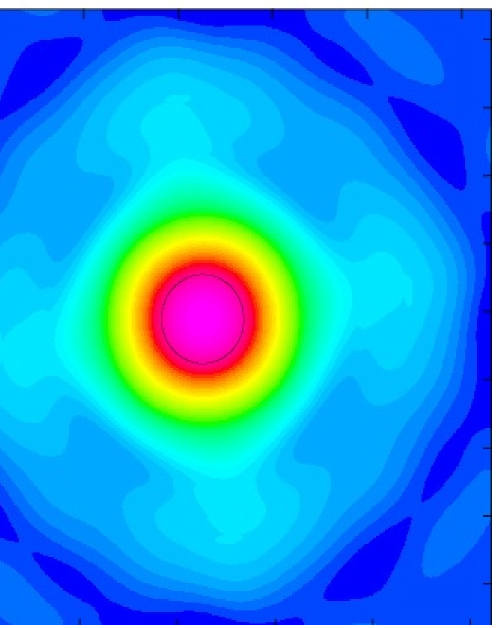
G55.7+3.4 : Field-of-view of 4x4 degrees from one EVLA pointing at 1-2 GHz

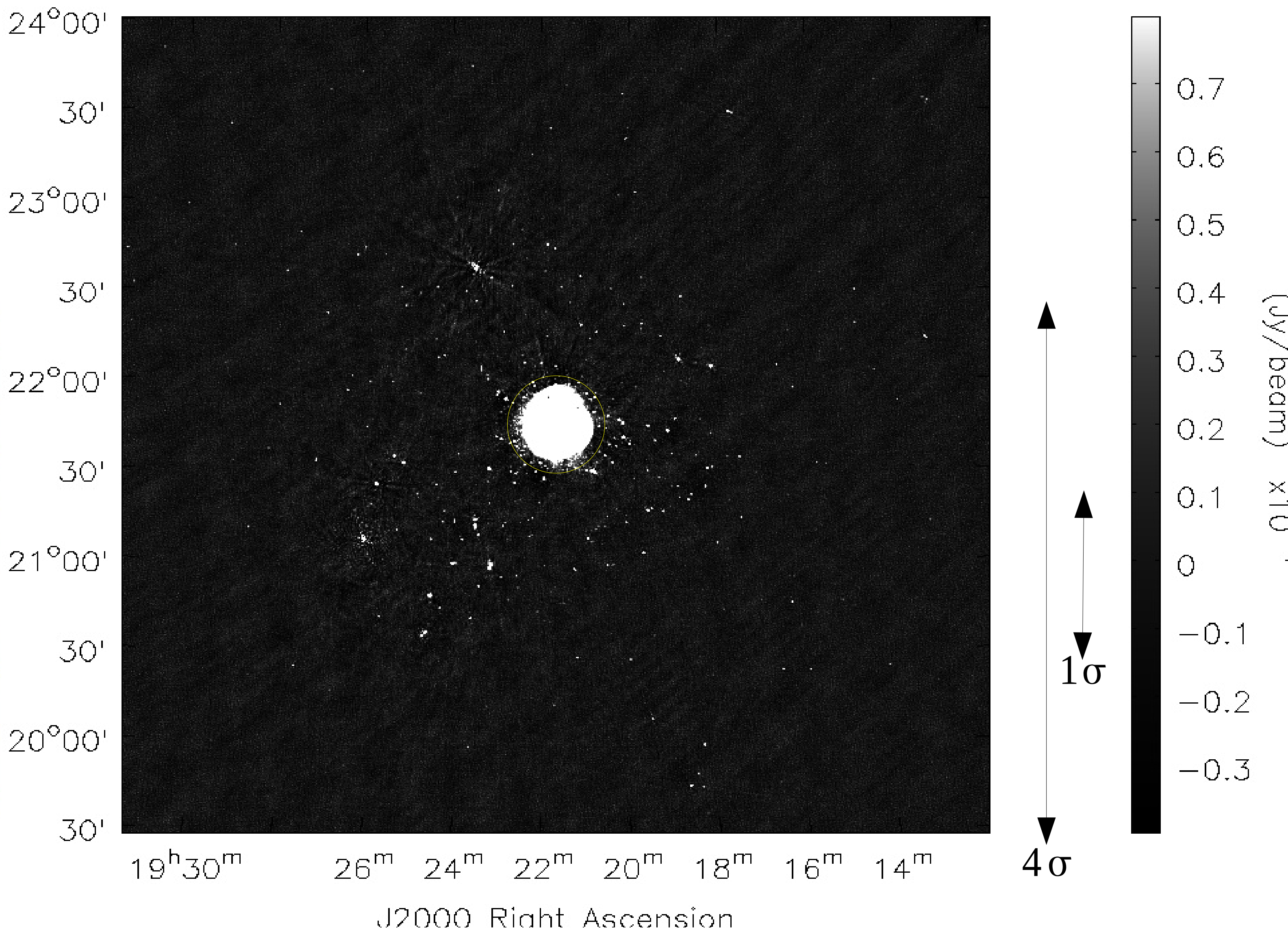
1 Jy total flux

24 arcmin

(PB: 30 arcmin)

10 micro Jy RMS





=> Wideband Imaging implies wide-field imaging

Wide Band + Full Beam/Wide-field

+ Mosaics

+ Single Dish

Example : Imaging the G55 supernova remnant

Imaging Framework

Measurement Equation

The visibility measured by each baseline *ij* at one frequency and time

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t) S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

Direction Independent Gains

- Eliminated during calibration

Primary Beams

- Power pattern varies with time, frequency and baseline

– PBcor (post-deconvolution)
– A-Projection
– WB-A-Projection
- Mosaics

Full Beam

Sky-brightness varies with frequency (time)

- All sources have spectral structure (some vary with time)

– Cube Imaging
– Multi-Frequency Synthesis (MFS)
– Multi-Term-MFS (point source or multi-scale models)

Wide-Band

W-Term

-Non-coplanar baselines

-Sky curvature

- Faceting
- W-Projection
- 3D FT
- W-Stacking

Wide-Field

Imaging Framework - Major and Minor cycles

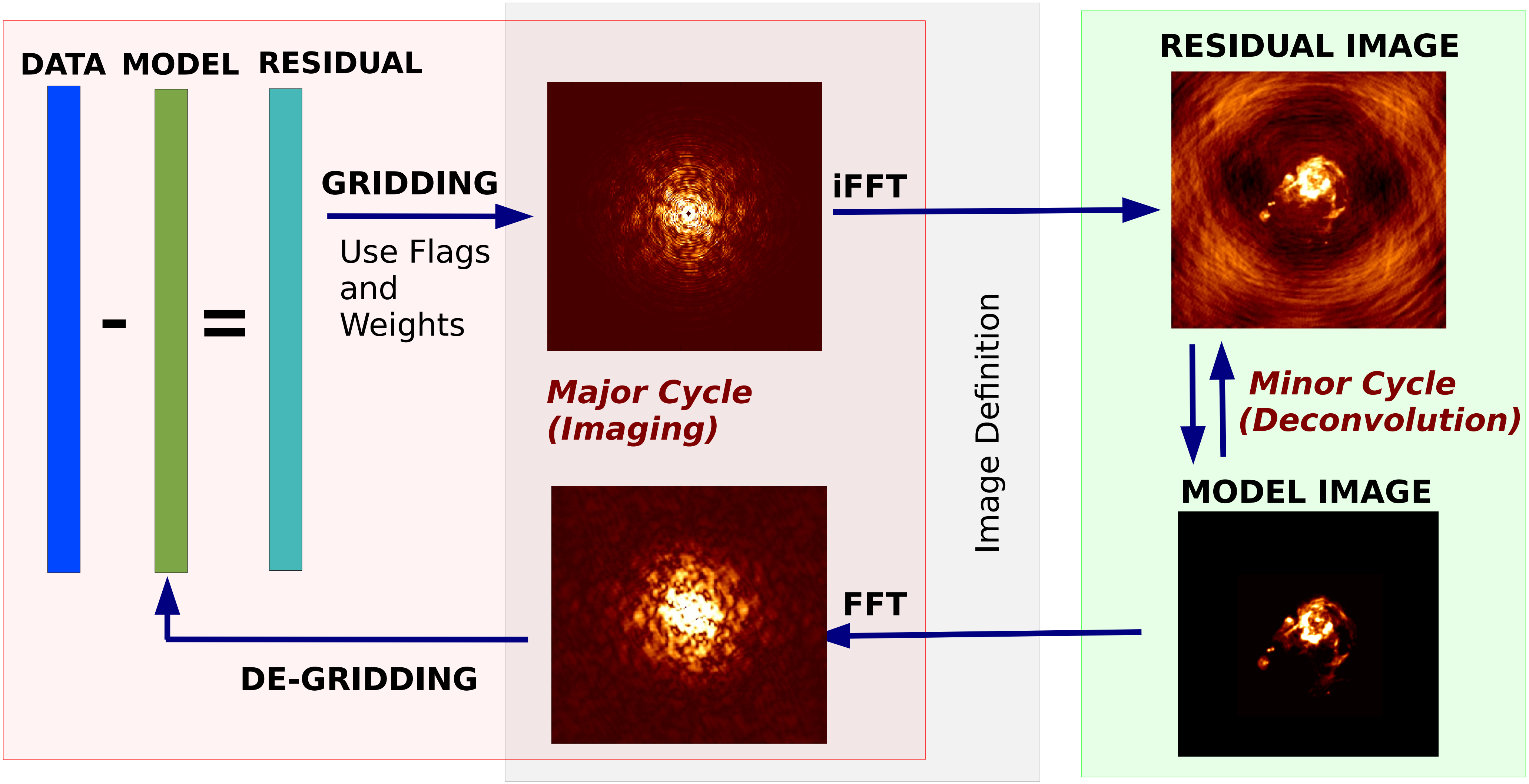
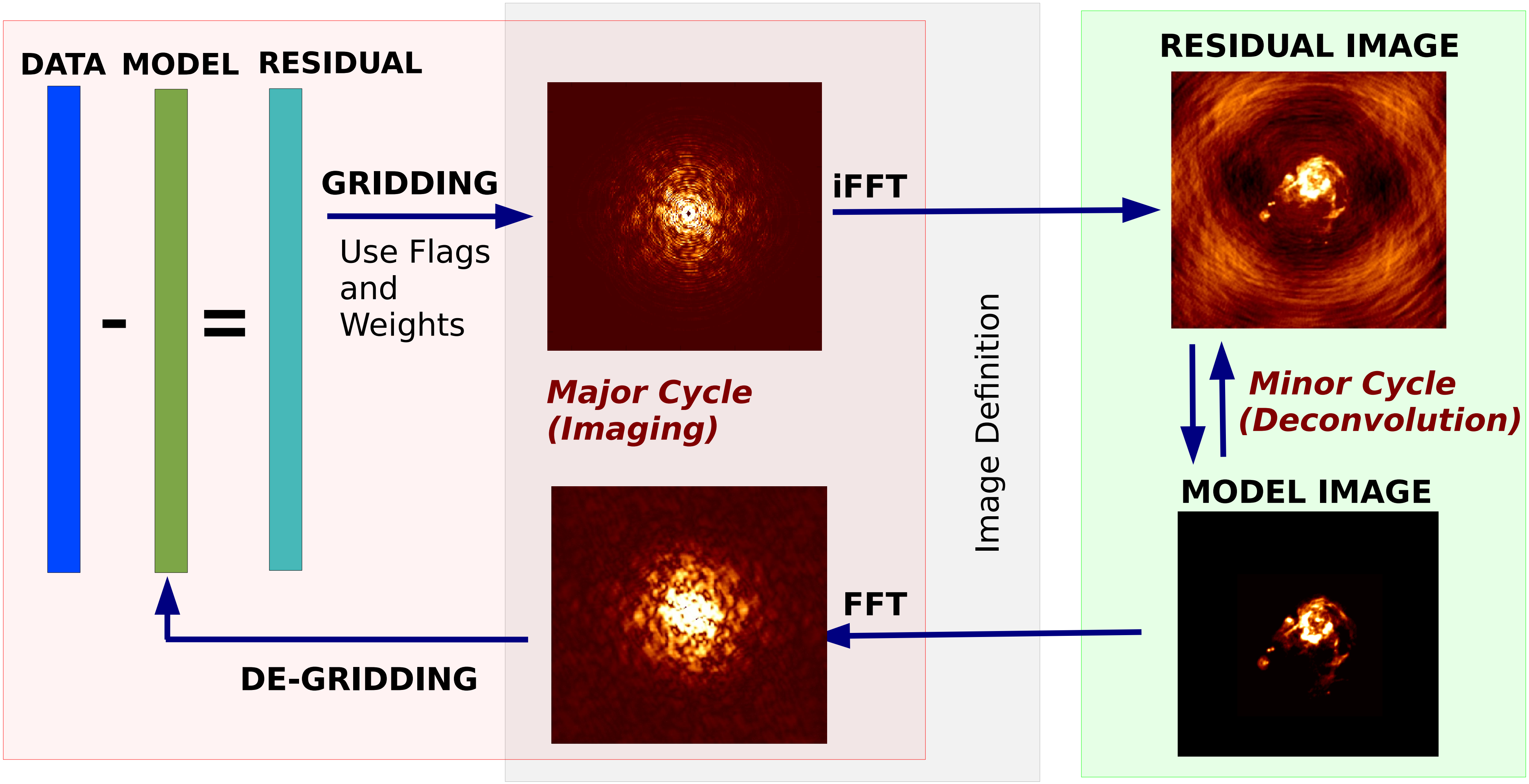


Image reconstruction is an iterative model-fitting / optimization problem

Measurement Eqn : $[A] I^m = V^{obs}$

Iterative solution : $I^m_{i+1} = I^m_i + g[A^T W A]^+ \left(A^T W \left(V^{obs} - A I^m_i \right) \right)$

Imaging & Deconvolution

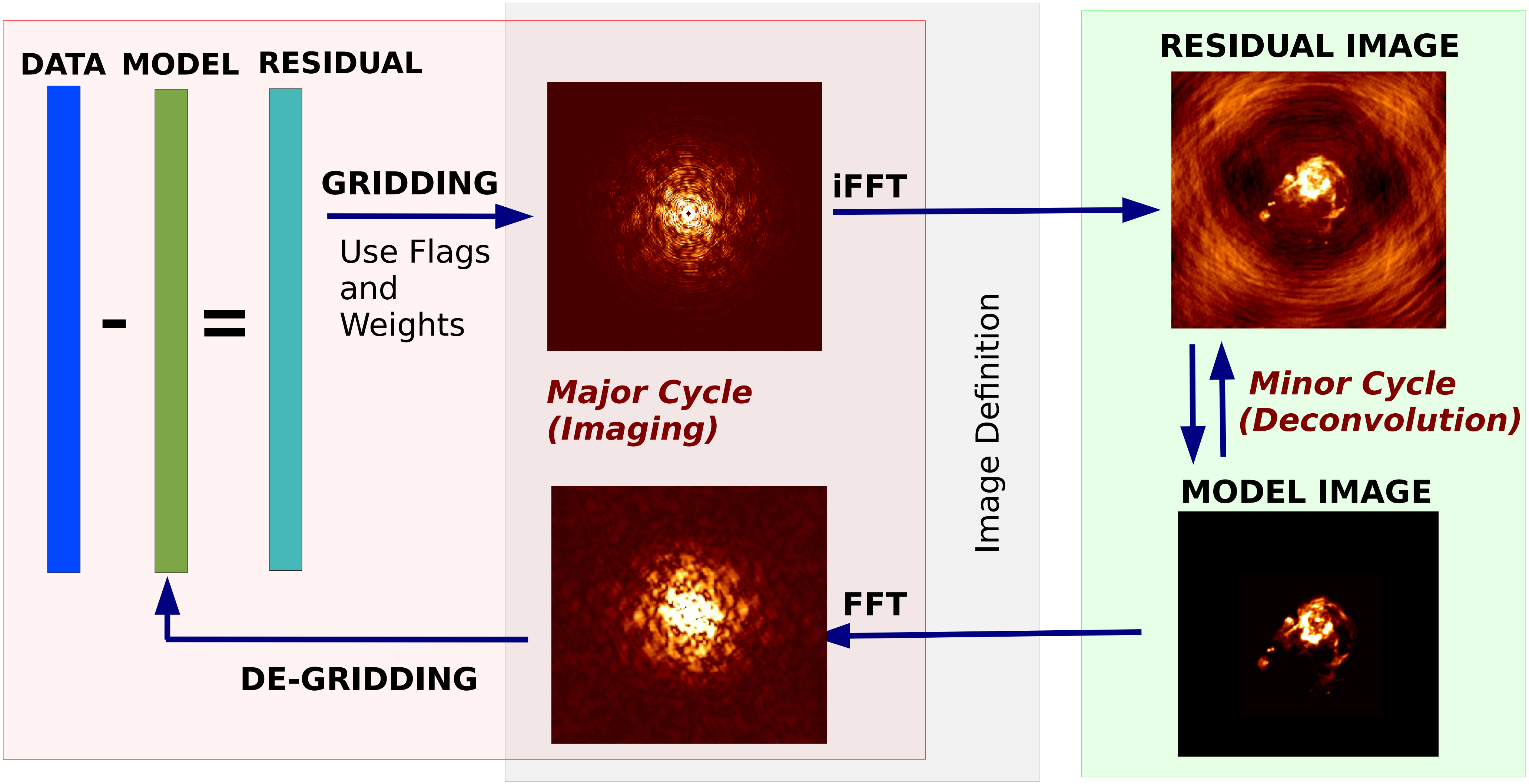


Instrumental Corrections (applied per visibility during gridding)

Mapping of data to Image Shape/Type

Solving for the sky model (non-linear optimization)

Algorithm Options

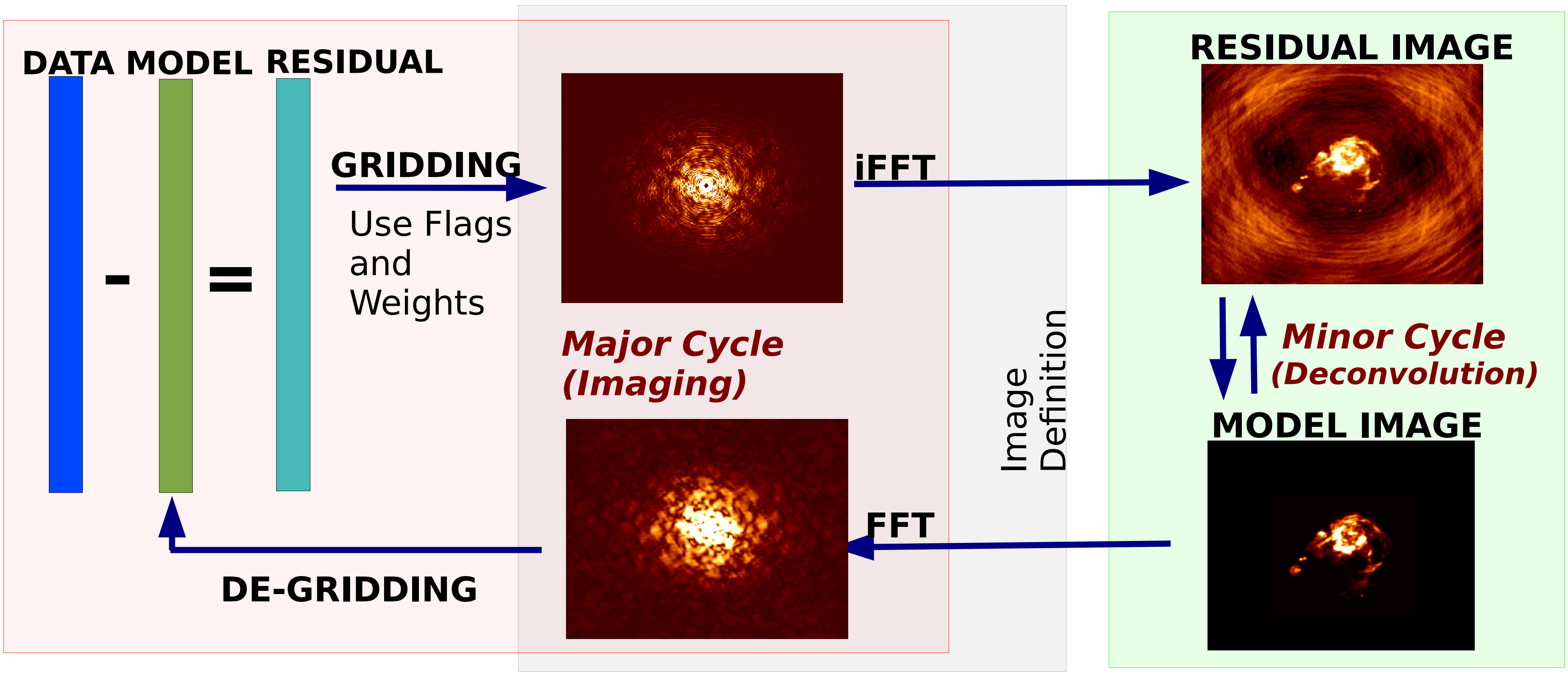


Standard gridding, W-Proj (WB)-A-Proj, Joint Mosaics, (Parallelization)

Cube, MFS, MT-MFS, Faceting,Stokes, Multi-Field, SD+INT Stitched Mosaic

Clean (Hogbom, Clark, MultiScale, MultiTerm, etc...)

Computational Cost



Runtime and computing resources depend on many factors.

=> Choose algorithms wisely....

(a) Data Volume, (b) Gridding Algorithm, (c) Joint vs Separate reconstructions, (d) Deconvolution algorithm, (e) Sky brightness structure and convergence rate (f) Dynamic range, calibration accuracy (g) Iteration Control

Summary – Lectures 1 & II

Wide Band Imaging
Sky and instrument change with frequency
=> Cube vs MFS, wideband/multiscale model, spectral index

Wide Field Imaging
non-coplanar baselines and the W-term
=> W-Projection, W-Stacking, Faceting, 3D FFTs

Full Beam Imaging
antenna primary beams
=> pbcor, A-Projection, beam models

Wide-Band + Primary-Beams + Mosaics + W-term + Single-Dish
(+ Full-Pol + Clean/MS-Clean/etc...)

Major/Minor Cycle Imaging Framework
=> Flexible imaging framework that logically organizes all the pieces

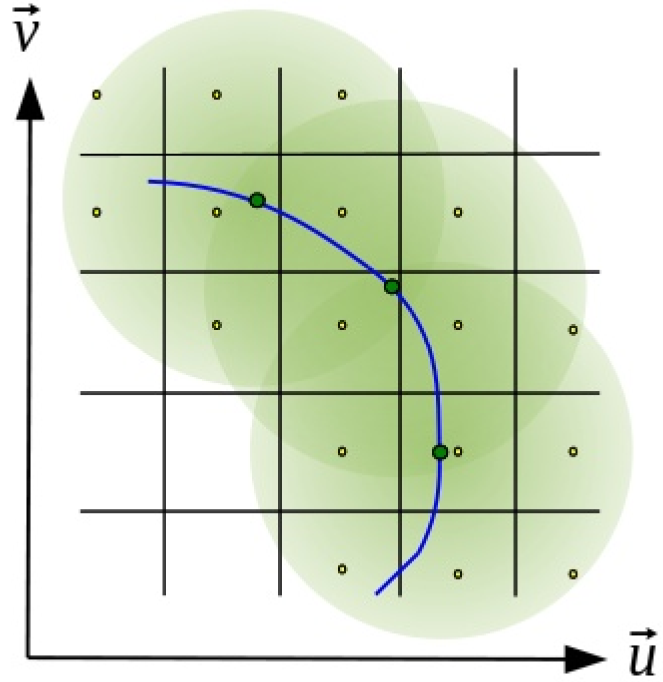
Need to choose algorithms carefully

[[Algorithm/software development is ongoing to refine all these ideas !]]

EXTRA SLIDES

Major Cycle : Data to Image, Model to Data

Gridding = Convolutional Resampling of visibilities to a regular grid



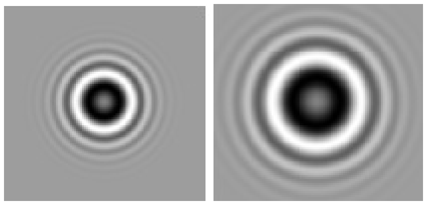
Convolution in UV-domain (per vis)
= Multiplication in Sky domain

=> Handle wide-field imaging effects

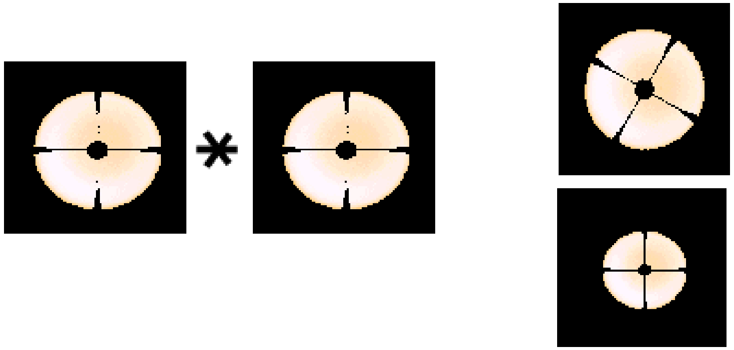
Degridding : Model → Data

Standard Imaging :
Prolate Spheroidal

W-Projection :
FT of a Fresnel kernel



A-Projection :
Baseline aperture illumination functions
+ phase gradients for joint mosaics



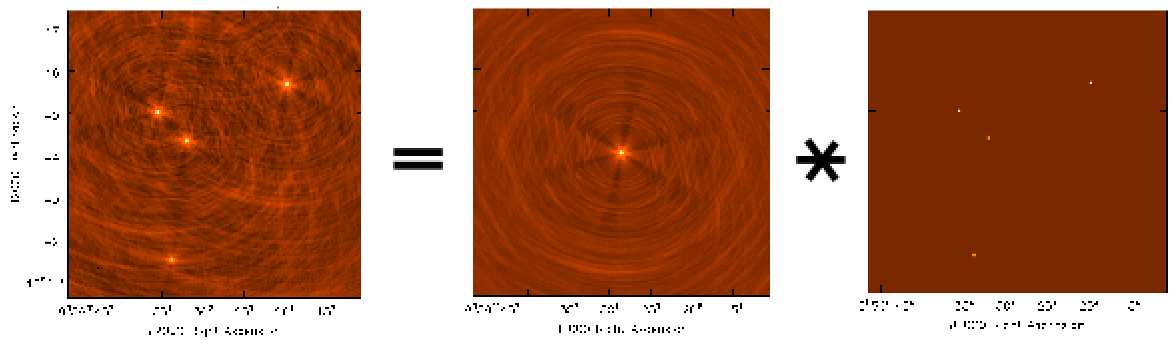
Combined algorithms :
Convolutions of different kernels

Minor Cycle : Solving for a sky model

For Point Sources :

- Hogbom Clean
- Clark Clean

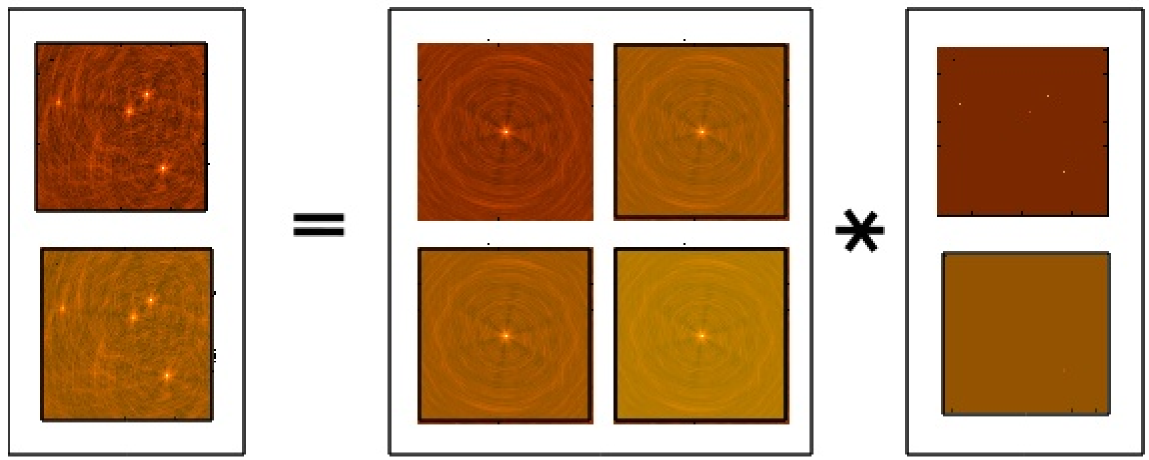
Convolution Equation ==> Deconvolution



For Point/Extended Sources :

– Multi-Scale-Clean

Multi-Term Convolution Equation
==> Joint Deconvolution



For Wide-band Sky models

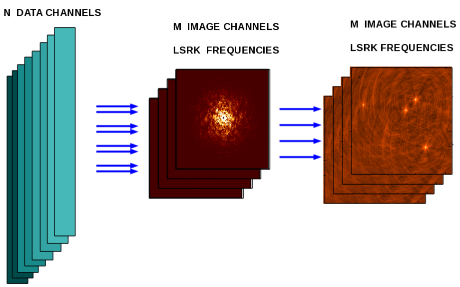
– Multi-Term MFS Clean
with or without Multi-Scale

(similar algo for time-variability)

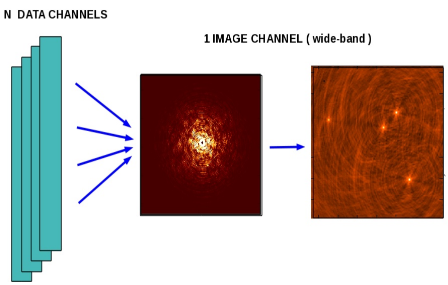
Other potential options : Any non-linear image-domain solver
E.x. Compressed sensing ideas : Gaussians (ASP), Wavelets (SARA/PURIFY),
Bayesian forms (MEM, RESOLVE, etc), wide-band non-parametric models, etc..

Mapping data to image coordinates/shapes

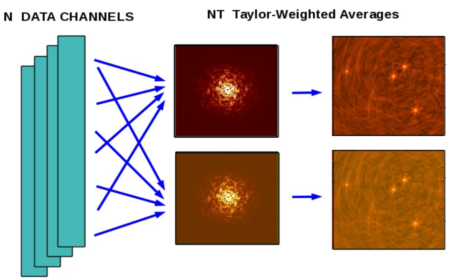
Spectral Cube



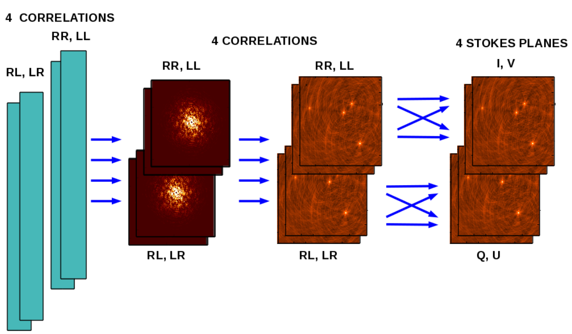
Continuum



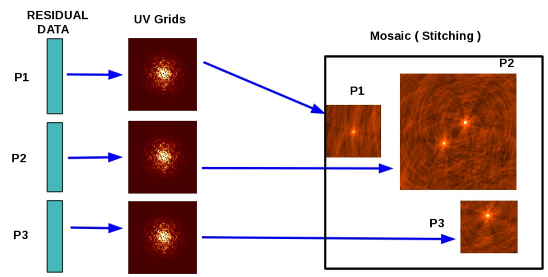
Wideband Continuum



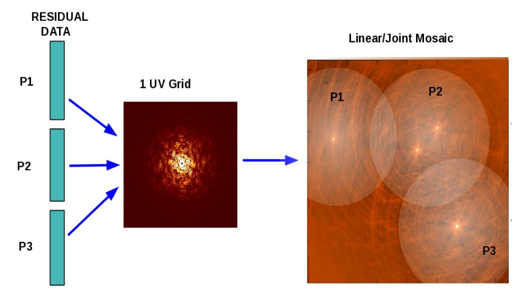
Stokes Planes



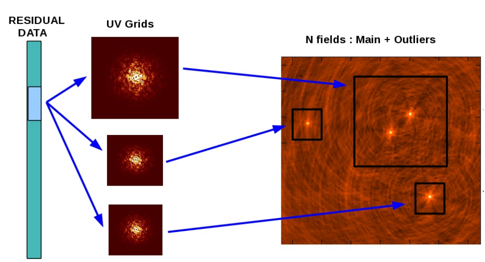
Stitched Mosaic



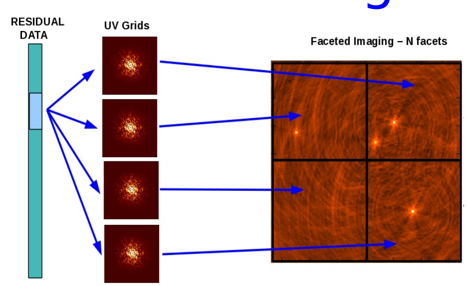
Joint Mosaic



Multi-Field



Faceting

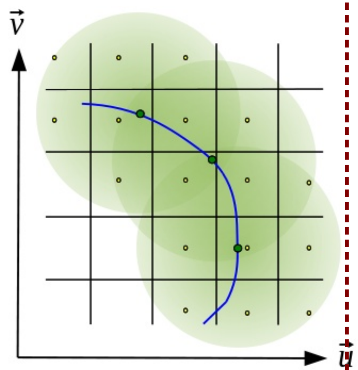


Different algorithms arise from different mappings of data to images

https://casa.nrao.edu/casadocs/casa-5.1.2/synthesis-imaging/image-definition

Computational Costs

Imaging runtime and compute resources depend on data size, sky signal, and algorithms chosen



Gridding : Convolutional resampling

O(N_data) x (nxn) complex multiply/add (n=5 - 100)
=> Compute load : O(N_data) * 10^{2-5} flops

Data parallelization, Multi-threading, GPUs, etc...

Example : Major cycle : 1hr → 10 days (Diff Algorithms)

Data volume

N_data =
N_ant^2 x
N_chan x
N_pol x
N_time

Complex numbers

Lustre I/O

Example :
8hr data
300 GB

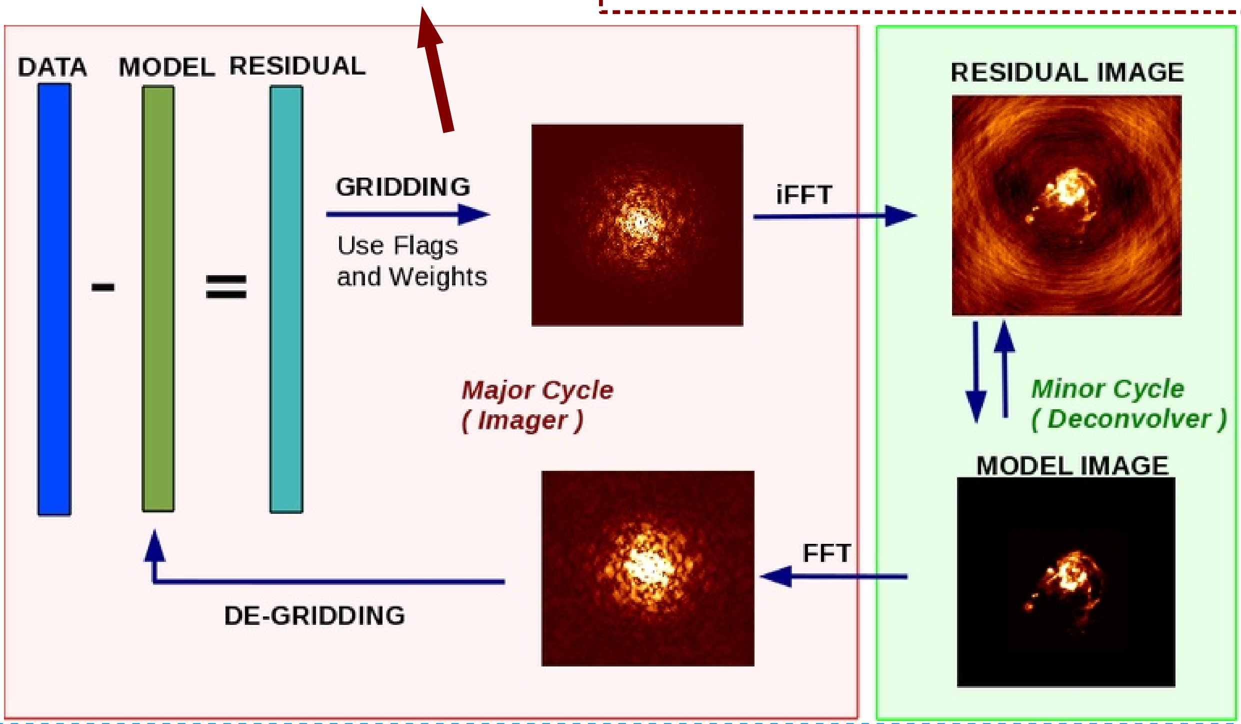


Image sizes

N_pix = Nx x Ny x N_chan x N_pol

Real / Complex
FFTs : O(NlogN)
Pixel math: O(N^2)
Mem : ~8 copies

Multi-threading
Chan parallelization

Nx : 1k → 40k
N_chan : 200 - 16K

Example :
1K x 1K x 256
~1 GB per image

Number of iterations : 5 – 10 major cycle loops
10^2 to 10^4 minor cycle steps

Runtime varies by 1-2 orders of magnitude. Depends on data.

Wide Band + Full Beam Imaging – Some guidelines

- MFS has better imaging fidelity, resolution and sensitivity than Cube

  -- For 2:1 bandwidth, the dynamic range limit with standard MFS (no spectral model) is few 100 to 1000 for a spectral index of -1.0

- MT-MFS gives HDR images when the spectral model is appropriate and there is sufficient SNR.

  -- For point sources, spectral index errors < 0.1 for SNR > 50 (2:1 bwr)
  for SNR > 10 (4:1 bwr)
  -- For extended emission, spectral index errors < 0.2 for SNR > 100

- W-Projection is more accurate and faster than Faceting

  -- For D-config,L-Band, uncorrected W errors are visible outside 1 deg

- PBcor assumes invariant beams, (WB)-A-Projection handles variability

  -- Uncorrected VLA beam squint and rotation causes DR < few x 10^4
  -- For 2:1 bwr, the PB's artificial spectral index at the HPBW is -1.4