Wide-Band and Wide-field Imaging - II

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Outline

Lecture 1 :

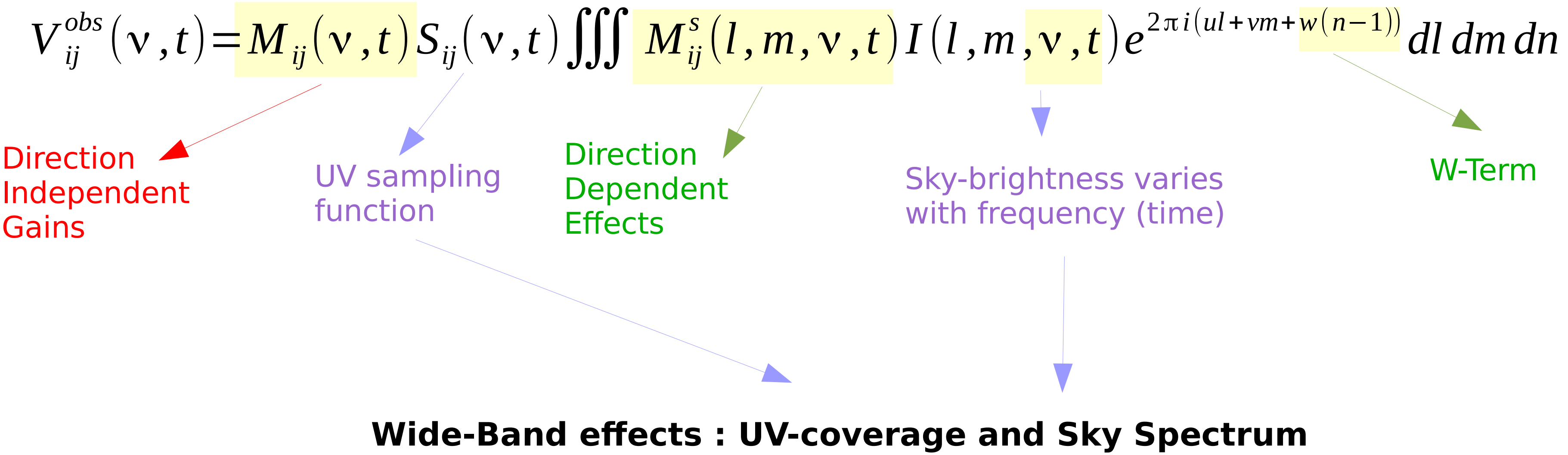
- **Measurement Equation** : What are we solving for during imaging ?
- **Wide-Field Imaging** : Primary Beams, W-term effect, Mosaics

Lecture 2 :

- **Wide-Band Imaging** : Frequency dependence of the sky and instrument
- **Algorithms** : Math to software

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iint I(l,m)\, e^{2\pi i(ul+vm)}\, dl\, dm$$



$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency (time)

W-Term

Wide-Band effects : UV-coverage and Sky Spectrum

Wide-Band Imaging

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

UV sampling function

Primary Beam varies with frequency

Sky-brightness varies with frequency

Frequency Range :	(1 – 2 GHz)	(4 – 8 GHz)	(8 – 12 GHz)
Bandwidth : $\nu_{max}-\nu_{min}$	1 GHz	4 GHz	4 GHz
Bandwidth Ratio : $\nu_{max}:\nu_{min}$	2 : 1	2 : 1	1.5 : 1
Fractional Bandwidth : $(\nu_{max}-\nu_{min})/\nu_{mid}$	66%	66%	40%

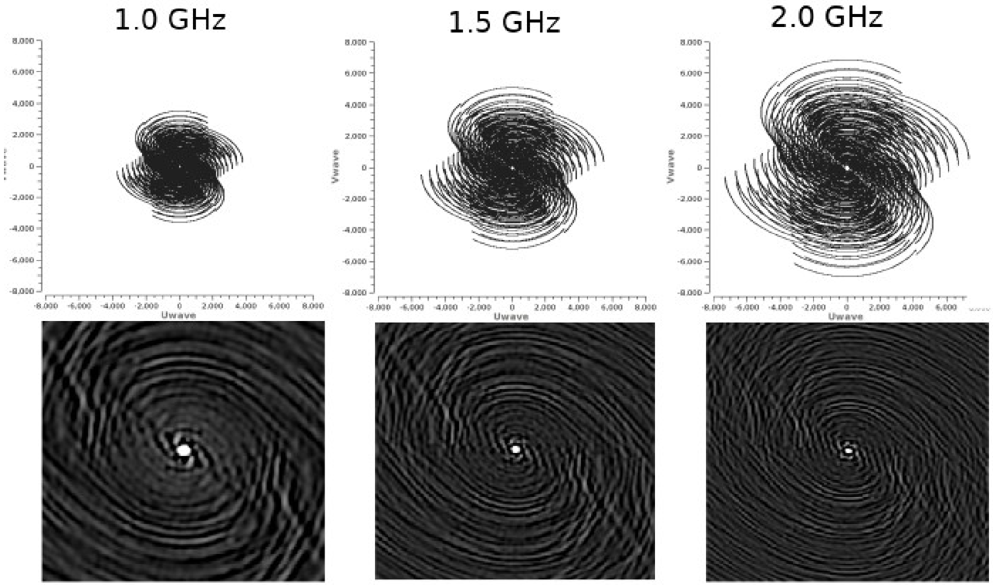
+ Imaging sensitivity improves with increased bandwidth

– Frequency dependent effects (sky and instrument) are stronger with increased bandwidth

Multi-Frequency Synthesis

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

UV sampling function

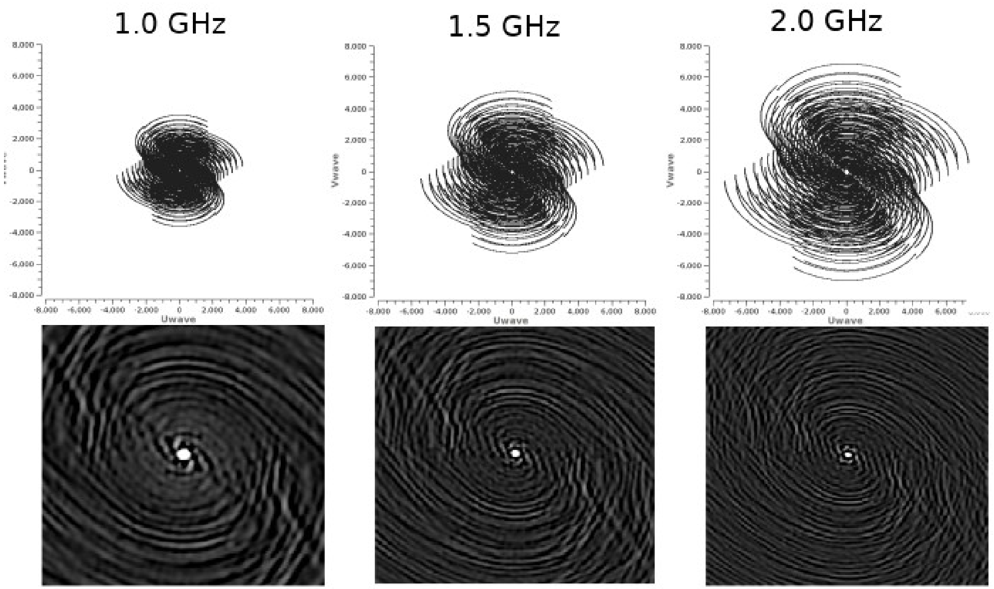


Observed image : $I_{\nu}^{obs} = I_{\nu}^{sky} * PSF_{\nu}$

Multi-Frequency Synthesis

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

UV sampling function

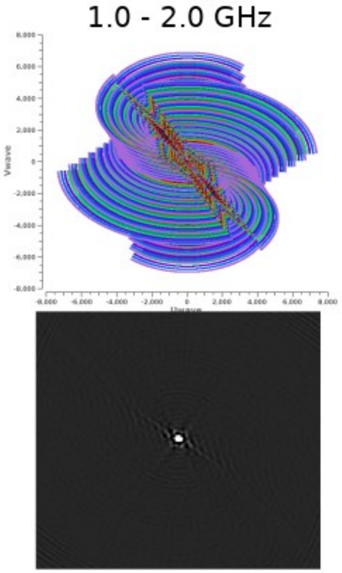


Observed image : $I_{\nu}^{obs} = I_{\nu}^{sky} * PSF_{\nu}$

Multi-Frequency Synthesis

Combine data from multiple channels

- Improve PSF
- Improve SNR



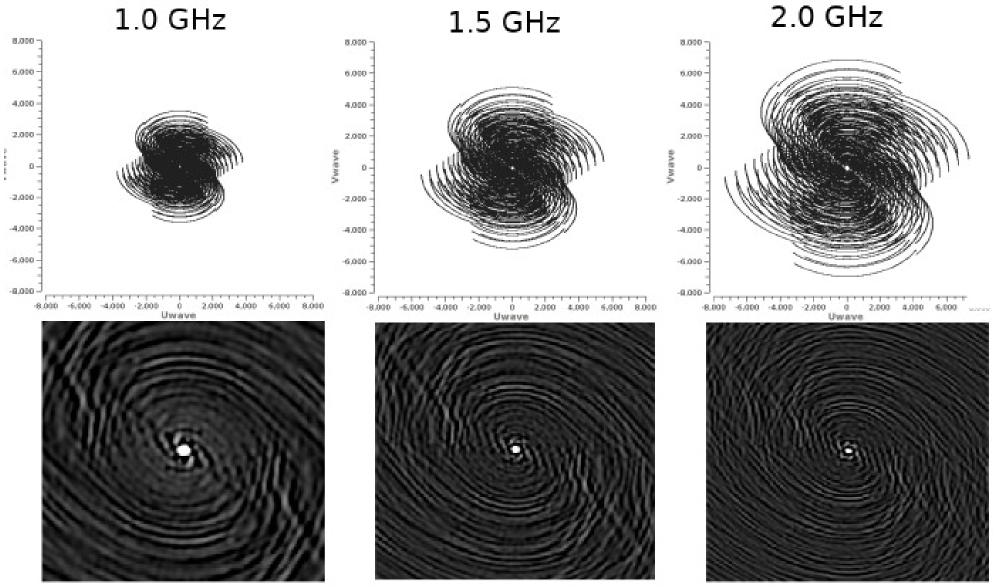
$$I_{wb}^{obs} = \sum_{\nu} \left[I_{\nu}^{sky} * PSF_{\nu} \right]$$

Multi-Frequency Synthesis

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

UV sampling function

Sky-brightness varies with frequency



Observed image : $I_\nu^{obs} = I_\nu^{sky} * PSF_\nu$

Multi-Frequency Synthesis

Combine data from multiple channels

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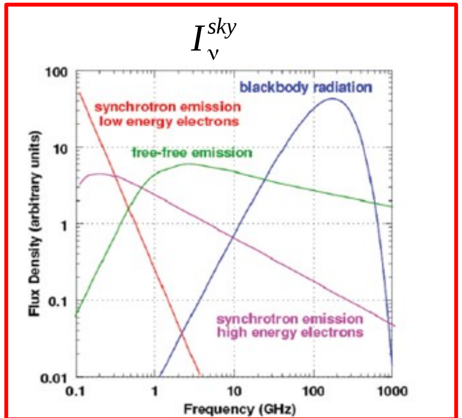
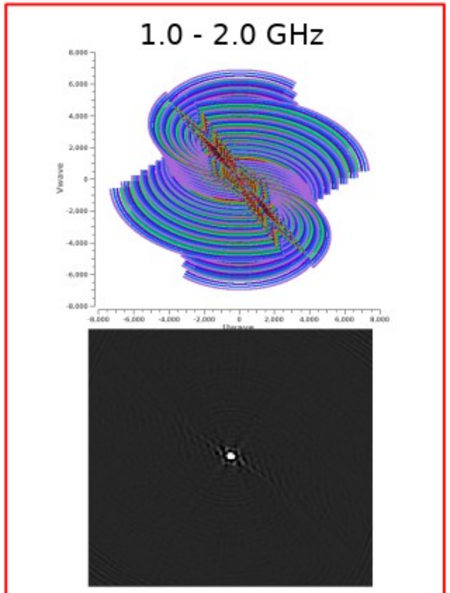




$I_{wb}^{obs} = \sum_\nu \left[I_\nu^{sky} * PSF_\nu \right]$

Wide-band Deconvolution

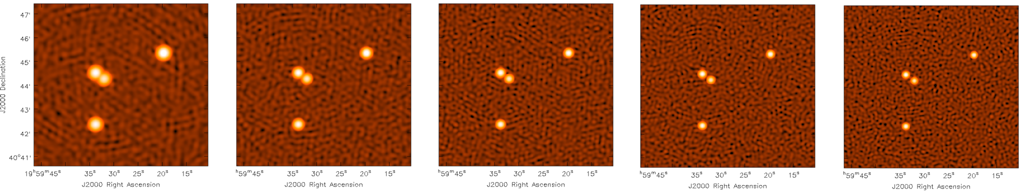
$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

UV sampling function

Sky-brightness varies with frequency

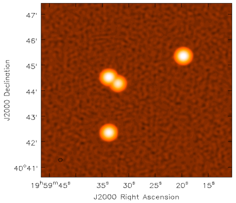
Cube Imaging :

(1) Reconstruct each chan/spw separately
(2) Smooth to the lowest available resolution
(3) Combine to calculate continuum and spectra



Continuum image

(sum of smoothed
channel maps)



Wide-band Deconvolution

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

UV sampling function

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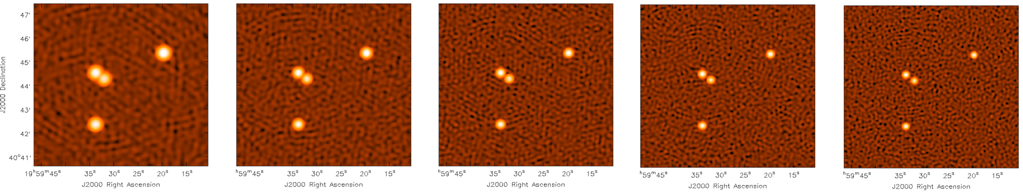

Continuum image

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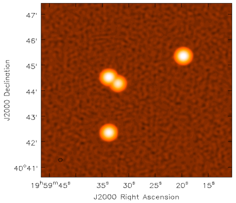
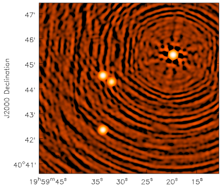


Multi-Frequency-Synthesis (MFS) :

Combine data from all frequencies and do a joint reconstruction

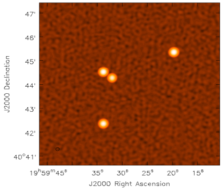
Flat-spectrum assumption



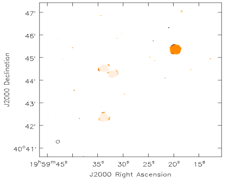
Intensity

Model intensity and spectrum together :

$$I_{\nu}^{sky}=\sum_{t} I_{t}^{m}\left(\frac{\nu-\nu_0}{\nu_0}\right)^{t}$$



Intensity



Spectral Index

Wide-band Deconvolution

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i (ul+vm+w(n-1))}\, dl\, dm\, dn$$

UV sampling
function

Sky-brightness varies
with frequency

Cube Imaging :

(1) Reconstruct each chan/spw separately
(2) Smooth to the lowest available resolution
(3) Combine to calculate continuum and spectra

CASA

tclean()

specmode='cube'
- start, width, nchan….

deconvolver='hogbom','multiscale','asp'

Multi-Frequency-Synthesis (MFS) :

Combine data from all frequencies and do a joint reconstruction

CASA

tclean()

specmode='cont'
deconvolver='mtmfs'
nterms=2
scales=[...]

Wide-band Deconvolution

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

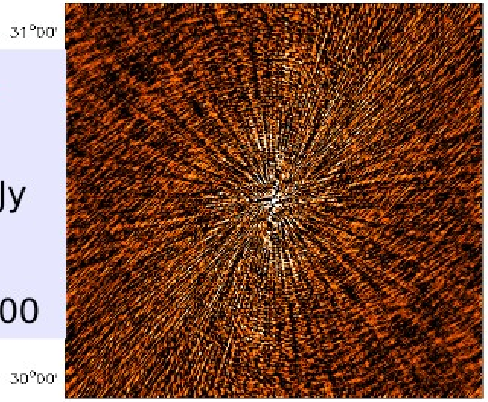
Strong Sources with thermal/nonthernal spectra

+ High BWR

=> More terms in the spectral model

VLA data with 3C286 across L-Band (1-2 GHz)

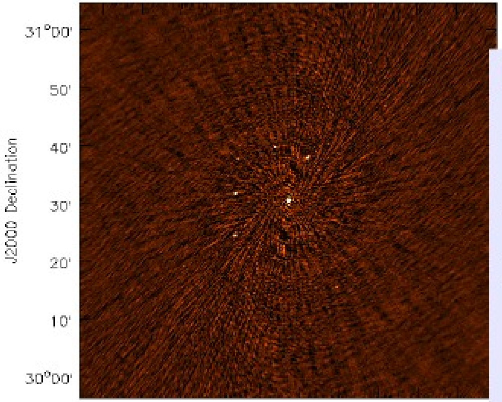
Nterms=1,2,3,4



NTERMS = 1

Rms :
9 mJy -- 1 mJy

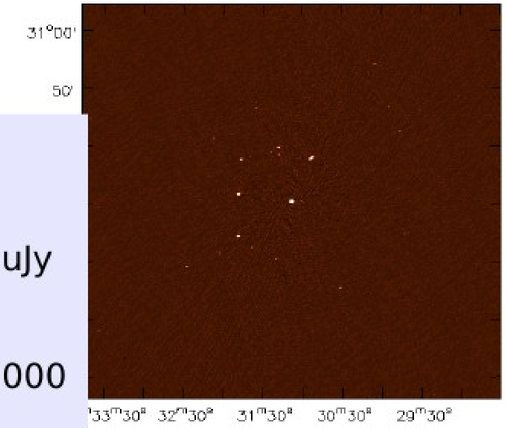
DR :
1600 - 13000



NTERMS = 2

Rms :
1 mJy -- 0.2 mJy

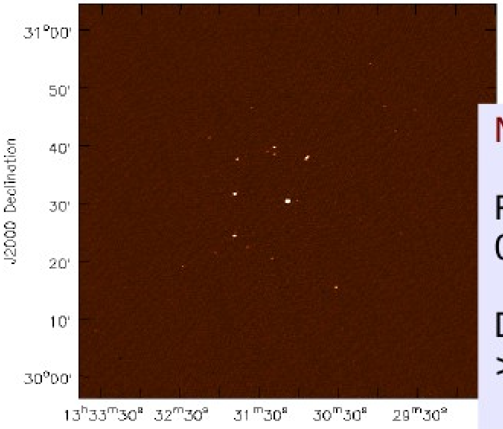
DR :
10,000 - 17,000



NTERMS = 3

Rms :
0.2 mJy -- 85 uJy

DR :
65,000 - 170,000



NTERMS = 4

Rms
0.14 mJy -- 80 uJy

DR :
>110,000
- 180,000

Wide-band Deconvolution

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

UV sampling function

Sky-brightness varies with frequency

Differences

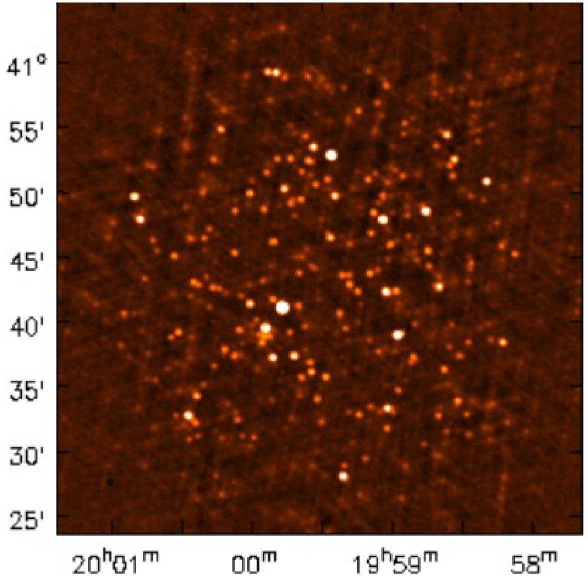
Angular Resolution

Deconvolution depth

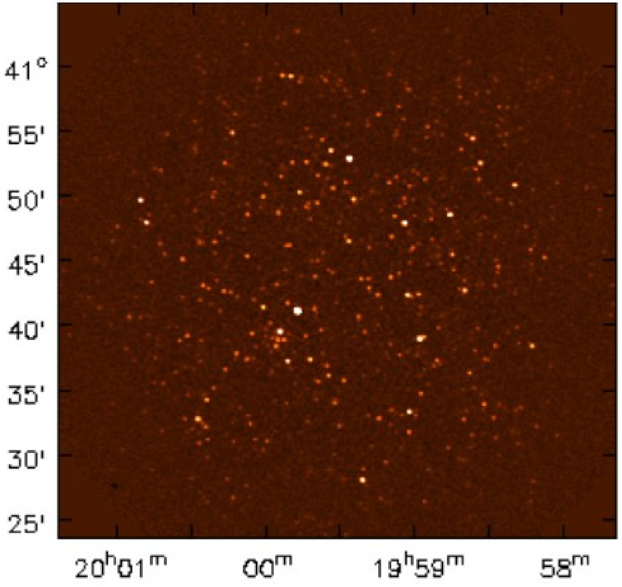
PSF Sidelobe levels and Clean Bias

Dependence on Spectral Model

Cube



(MT) MFS



Wide-band Deconvolution

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

UV sampling
function

Sky-brightness varies
with frequency

Angular Resolution of Spectral Modeling.

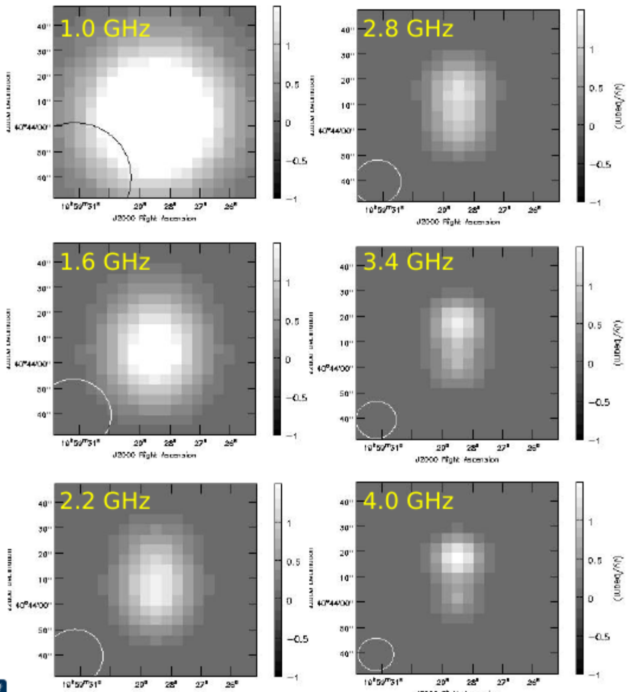
Low-frequency : Unresolved

High-frequency : Resolved

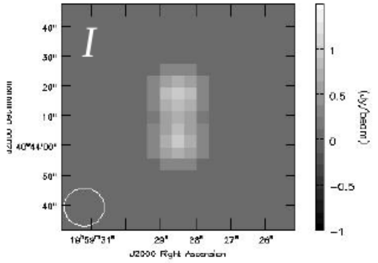
MFS angular resolution is close to
the high-frequency resolution

Compact emission has a signature across the
uv-plane

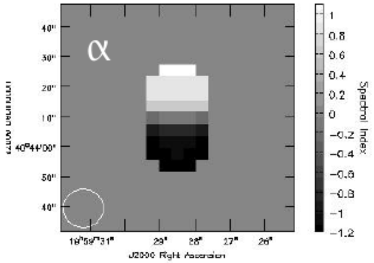
=> enough data constraints to model the
spectrum of compact emission.



Restored Intensity image



Spectral Index map



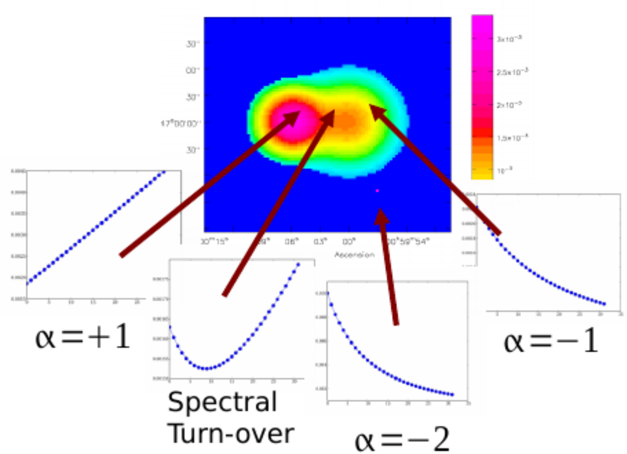
Wide-band Deconvolution

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}dl\,dm\,dn$$

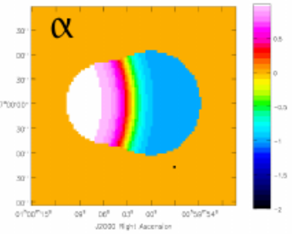
A good multi-scale model gives accurate spectral index and curvature.

=> Pick multi-scale scales carefully

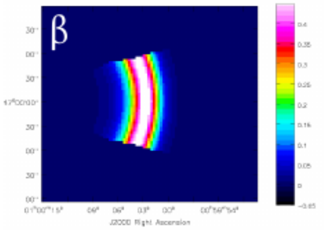
Intensity Image



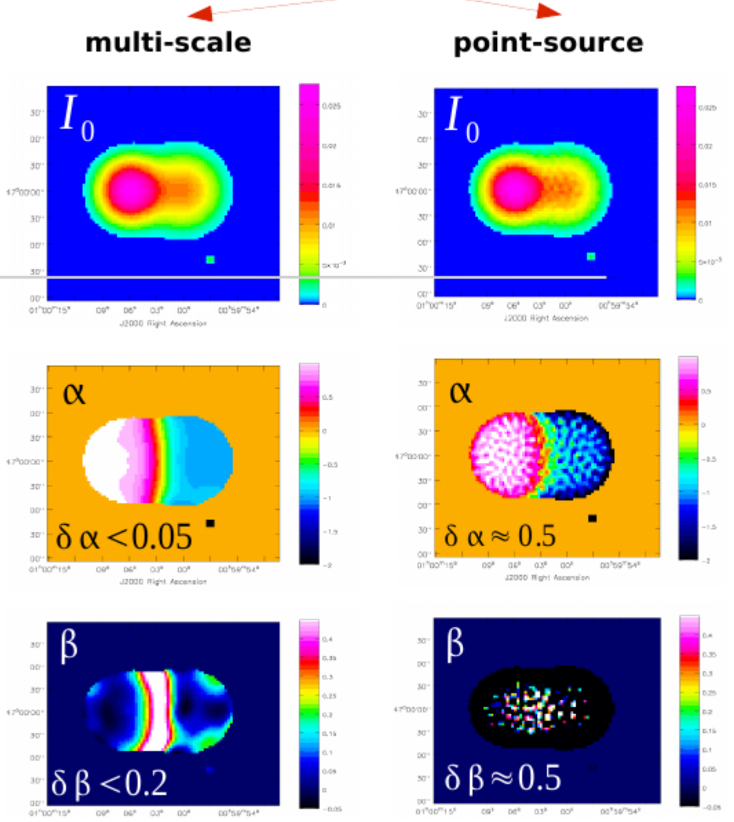
Average Spectral Index



Gradient in Spectral Index



MT-MFS



Wide-band Deconvolution

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

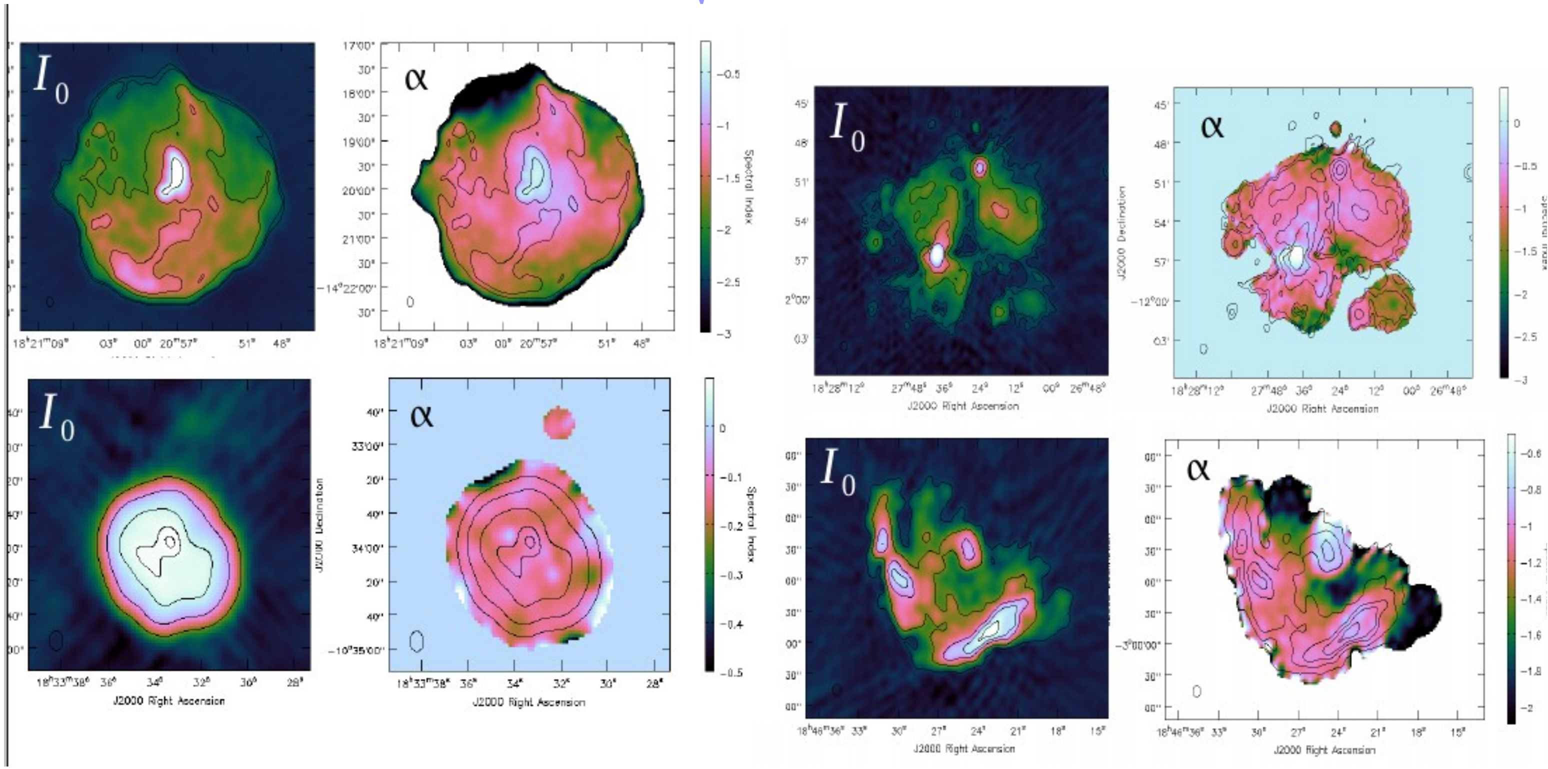
Examples of typical accuracy of spectral index

Within 2:1 BWR,

Spectral index error < 0.2 for SNR > 100.

Better multi-scale models will give better spectral index accuracy

(e.g. WB-ASP (tbd in casa), Resolve, etc..)



Wide-band Deconvolution

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

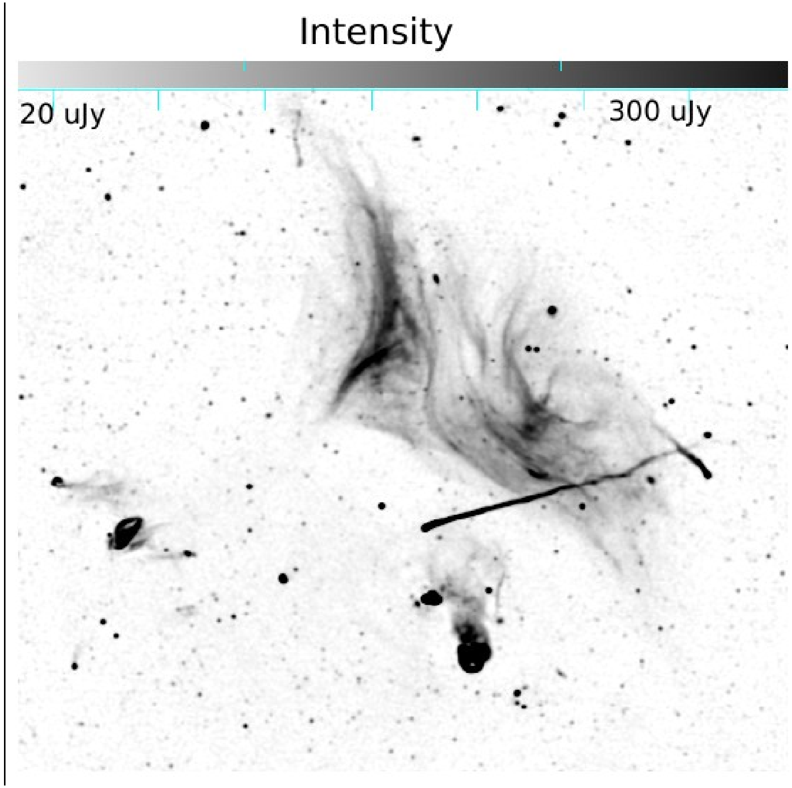
Example :
Abell 2256

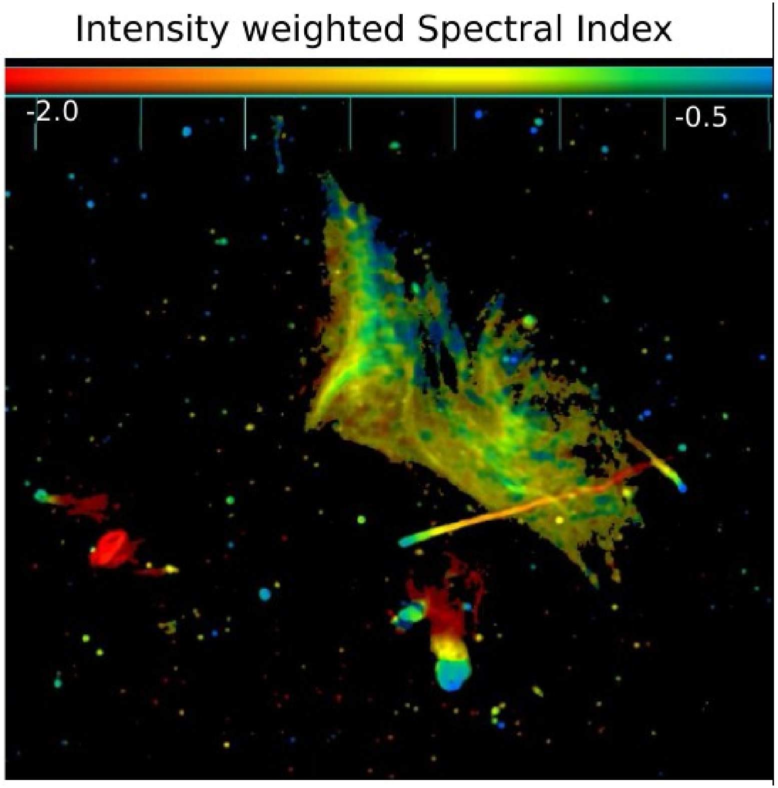
VLA A,B,C,D
at L-Band

VLA A at S&C bands
(2-4, 4-6, 6-8 GHz)

Calibration and Auto-flagging in AIPS.
Intensity/Spectral index Imaging in CASA.

(Owen et al, 2014)


Intensity


Intensity weighted Spectral Index

Radio Interferometry – Measurement Equations

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$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency (time)

W-Term

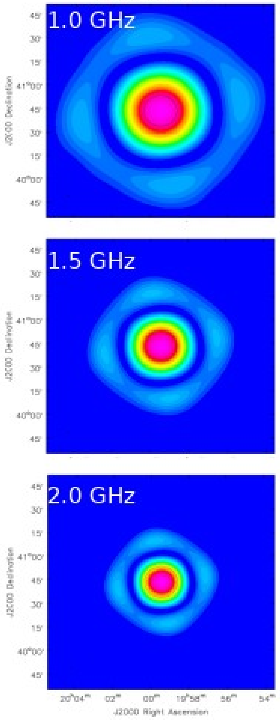
Wide-Band effects : Primary Beam changes with Frequency

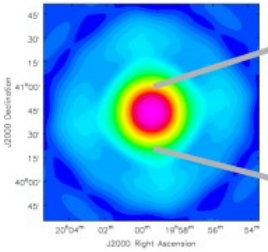
Wide Band + Wide Field : Frequency-dependent Primary Beams

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

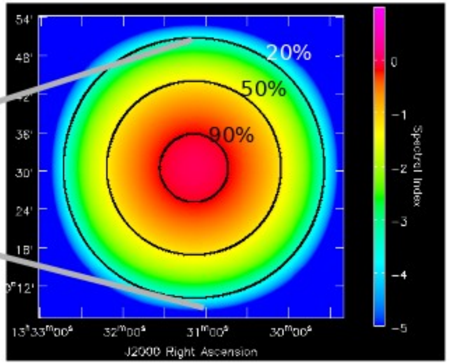
Primary Beam varies with frequency

VLA PBs





Average PBs



Spectral Index of PB

(About -1.4 at the HPBW)

$$I^{obs}_{wf,wb} = \sum_{\nu} \left[\left(P_{\nu} \cdot I^{sky}_{\nu} \right) * PSF_{\nu} \right]$$

Wide Band + Wide Field : Frequency-dependent Primary Beams

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i (ul+vm+w(n-1))}\, dl\, dm\, dn$$

Primary Beam varies with frequency

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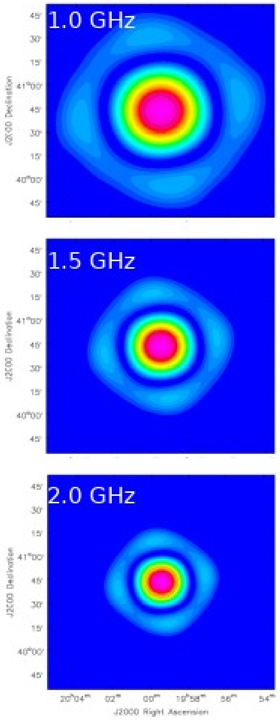
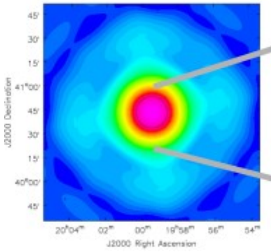
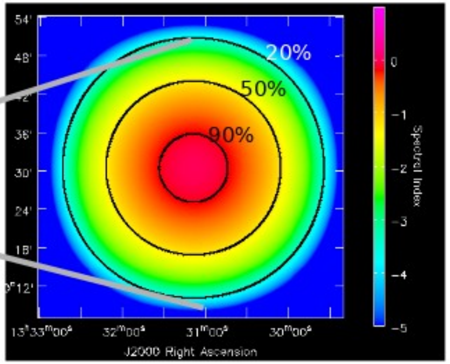




Average PBs



Spectral Index of PB

(About -1.4 at the HPBW)

$$I^{obs}_{wf,wb} = \sum_{\nu} \left[\left(P_\nu \cdot I_\nu^{sky} \right) * PSF_\nu \right]$$

Cube Imaging

-- Sky model represents $I(\nu) P(\nu)$

-- Divide the output image at each frequency by $P(\nu)$

Multi-Term MFS + Wideband-PBcor

-- Taylor coefficients represent $I(\nu) P(\nu)$

-- Polynomial division by PB Taylor coefficients $\frac{(I_{0,}^{m} I_{1,}^{m} I_{2,}^{m} \ldots)}{(P_{0,} P_{1,} P_{2,} \ldots)} = (I_{0,}^{sky} I_{1,}^{sky} I_{2}^{sky} \ldots)$

Wideband A-Projection

-- Remove $P(\nu)$ during gridding (before model fitting)

$$A_\nu^{-1} \approx \frac{A_{\nu_c}^{T}}{A_{\nu_c}^{T} * A_\nu} \quad \text{where} \quad P_\nu . P_{\nu_c} \approx P^2_{\nu_{mid}}$$

-- Output spectral index image represents only the sky

Wide Band + Wide Field : Frequency-dependent Primary Beams

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}dl\,dm\,dn$$

Primary Beam varies with frequency

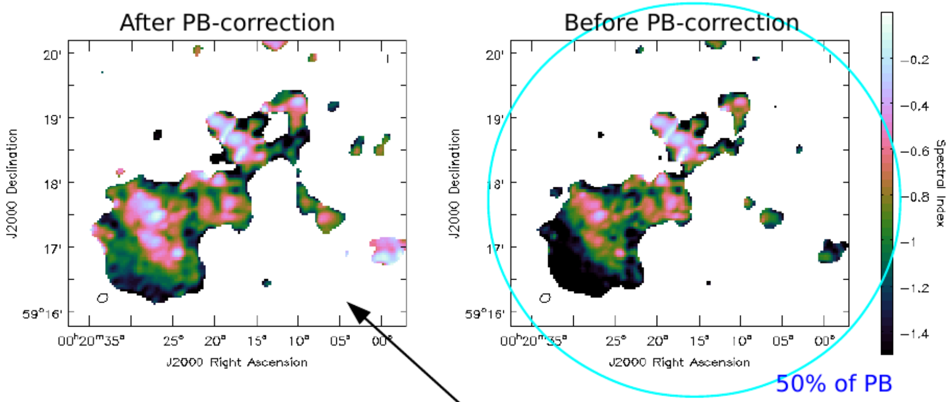
(Heesen et al, 2011)

CASA

gridder='standard'
- tclean() with pbcor=False
- widebandpbcor()

gridder='mosaic', 'awproject'
- tclean() with conjbeams=True
- pbcor=True/False

In a future CASA release :
specmode='mtmfs_via_cube'

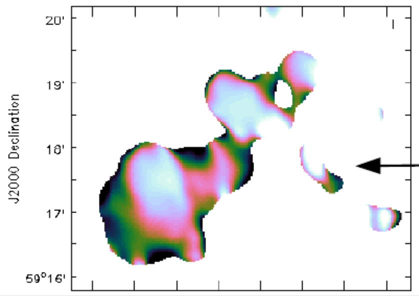


IC10 Dwarf Galaxy :

Spectral Index across C-Band.

Dynamic-range ~ 2000

MT-MFS : Wide-band PB-correction after multi-term multi-scale MFS.

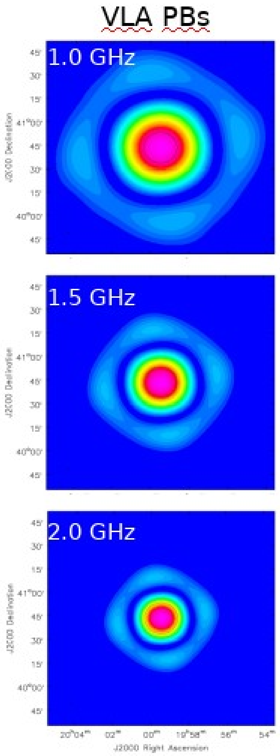


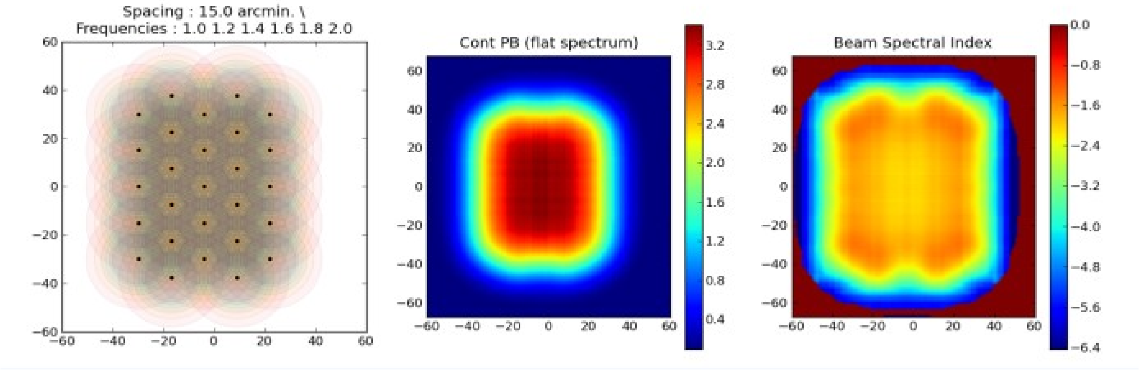
Cube : Spectral-index map made by cube imaging, smoothing to lowest resolution, and spectral fitting.

Wide Band Mosaics

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Primary Beam varies
with frequency





The mosaic primary beam has an artificial spectral index all over the F-O-V

=> There is no part of the mosaic that is free
from wideband primary-beam effects.

Algorithms :

Combinations of.....

- Stitched vs
  Joint Mosaics
- Cube vs MFS
- PBCor vs A-Projection

Different numerical accuracies and compute costs.

Depends on the sky.

Wide Band Mosaics

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Primary Beam varies
with frequency

When multiple pointings overlap on the region being imaged.....

Joint solutions use more data constraints

→ Generally, better images

Example : Simulation
WB-AWP + MT-MFS

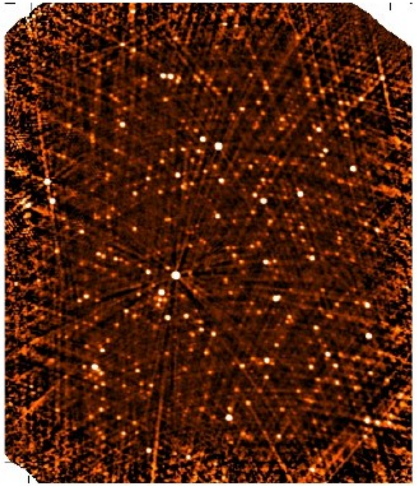
Alpha error :

> 50 uJy : 0.2 → 0.05

10-50 uJy : 0.5 → 0.2

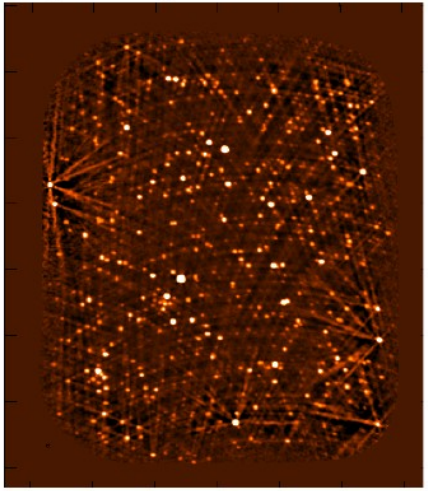
Cube + Joint Mosaic
(with static Primary Beams)

Dyn.Range = 5000:1



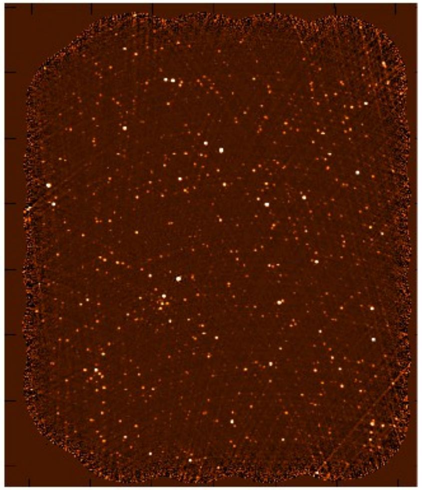
Cube + A-Projection + Joint Mosaic

Dyn.Range = 10000:1



Wideband A-Proj + Joint Mosaic + Multi-term MFS

Dyn.Range = 40000:1



[Rau et al, 2016]

Wide Band Mosaics

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i (ul+vm+w(n-1))}\, dl\, dm\, dn$$

CASA

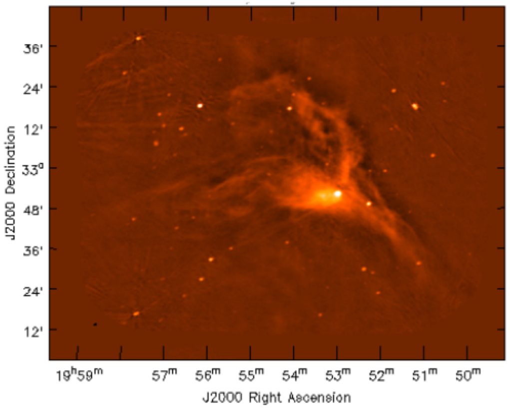
gridder='mosaic' or
'awproject'

conjbeams=True (for wbpbcor)

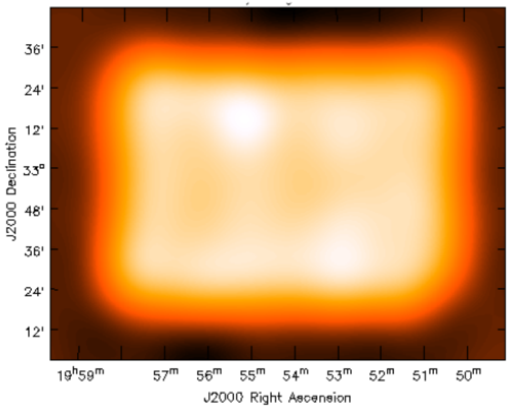
Data selection : Select multiple pointings and specify mosaic phasecenter

(With gridder='standard', make images separately per pointing, and then use the linearmosaic tool)

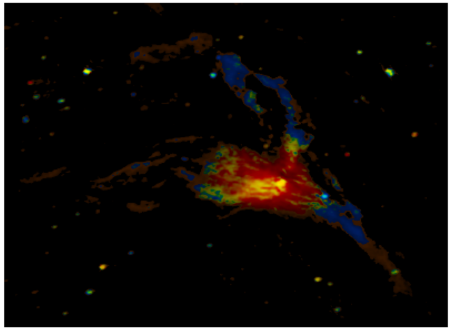
Intensity



Mosaic Primary Beam



Intensity-weighted Spectral Index

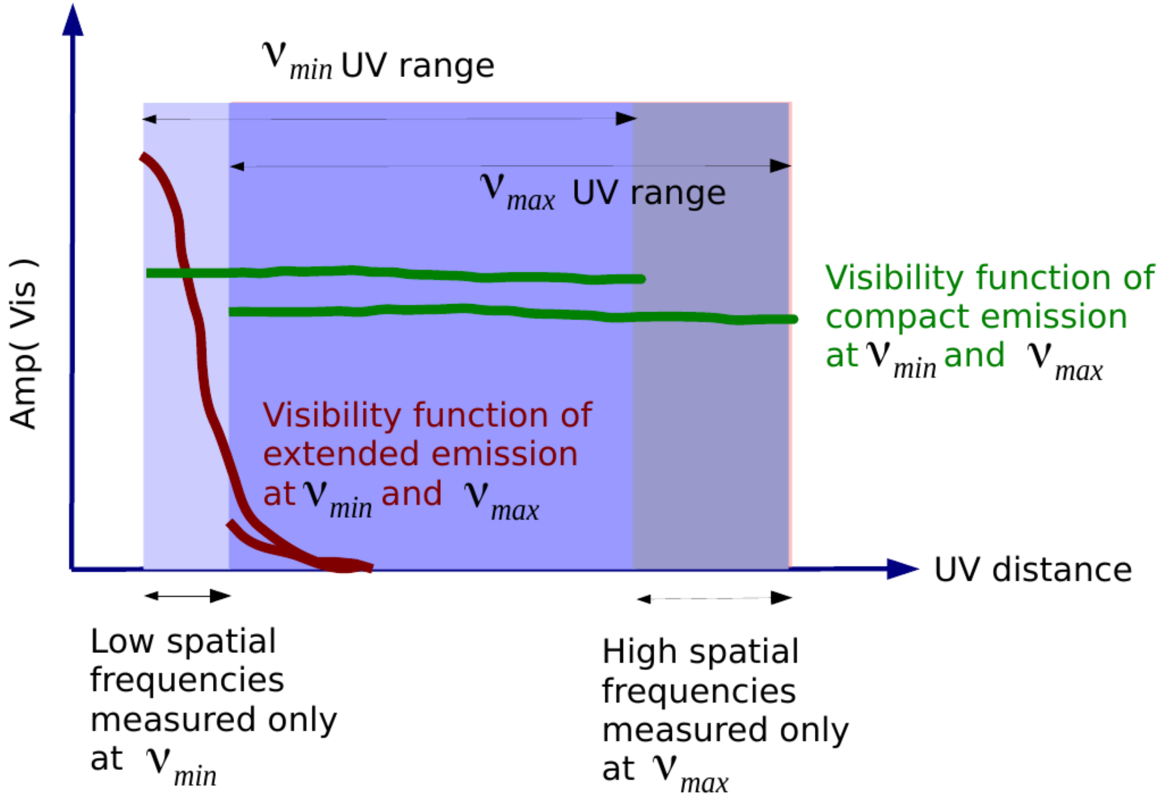


300GB calibrated dataset, 106 pointings over 1.5x2 deg, imaged with Multi-Scale Multi-Term MFS, Joint Mosaic and WB-A-Projection.

=> Mosaic primary beam spectral index of ~ -1.5 has been removed prior to the wideband sky model fitting.

Single-Dish and Interferometer Combination

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$



Interferometer UV-coverage

– Very poor sampling of spectral structure for extended emission.

– INT-only imaging can get the spectral index very wrong (but still fit the data)

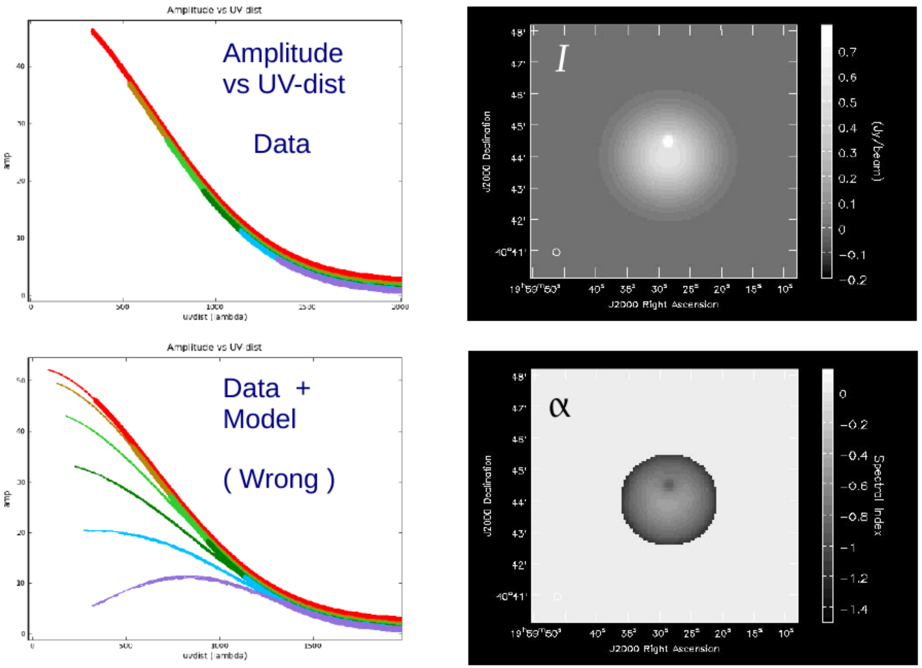
Add constraints from Single Dish data

- **- Joint imaging**
- **- Feathering**
- **- StartModel**

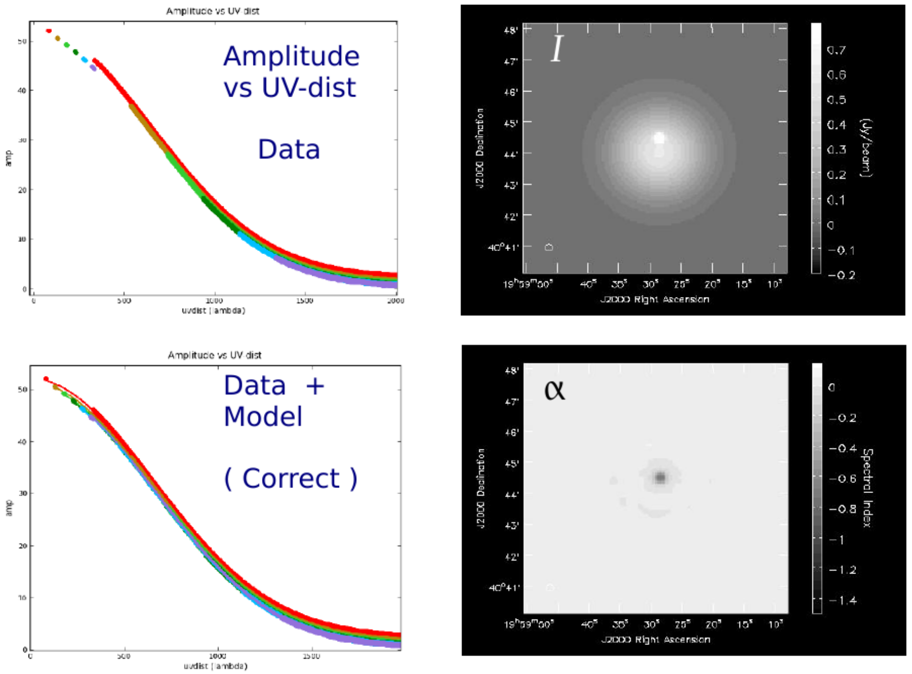
Single-Dish and Interferometer Combination

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

INT-only imaging



SD+INT imaging



CASA : sdintimaging(), feather() , startmodel In tclean()

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency and time

W-Term

Put it all together…..

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Direction
Independent
Gains

UV sampling
function

Direction
Dependent
Effects

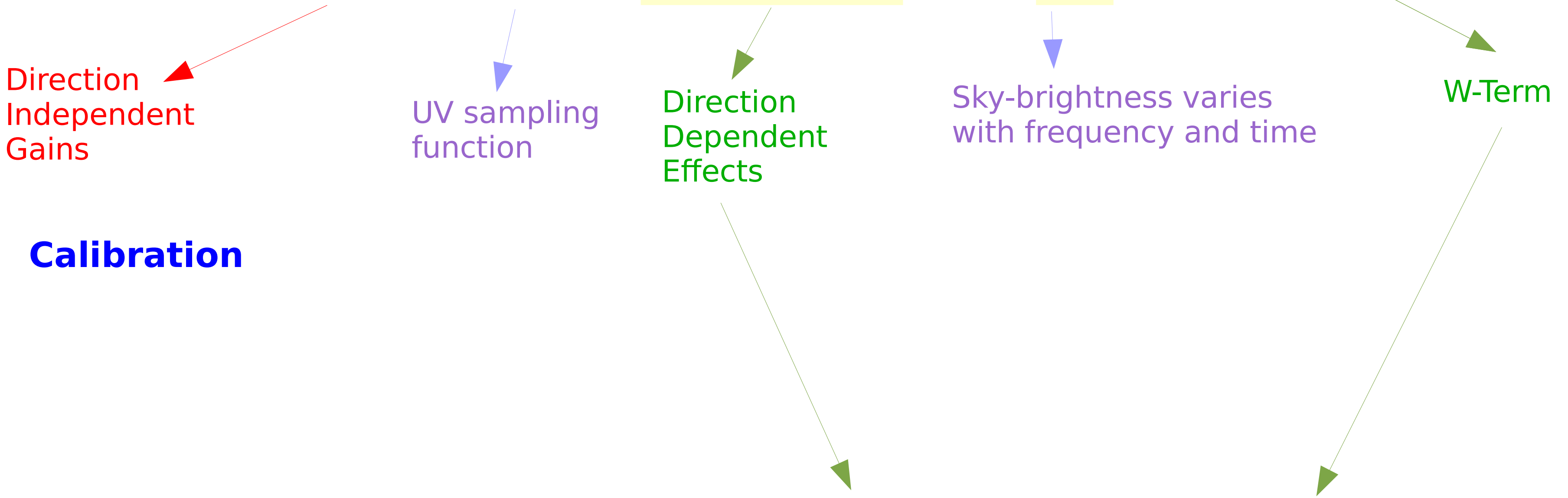
Sky-brightness varies
with frequency and time

W-Term

Calibration

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$



Direction
Independent
Gains

UV sampling
function

Direction
Dependent
Effects

Sky-brightness varies
with frequency and time

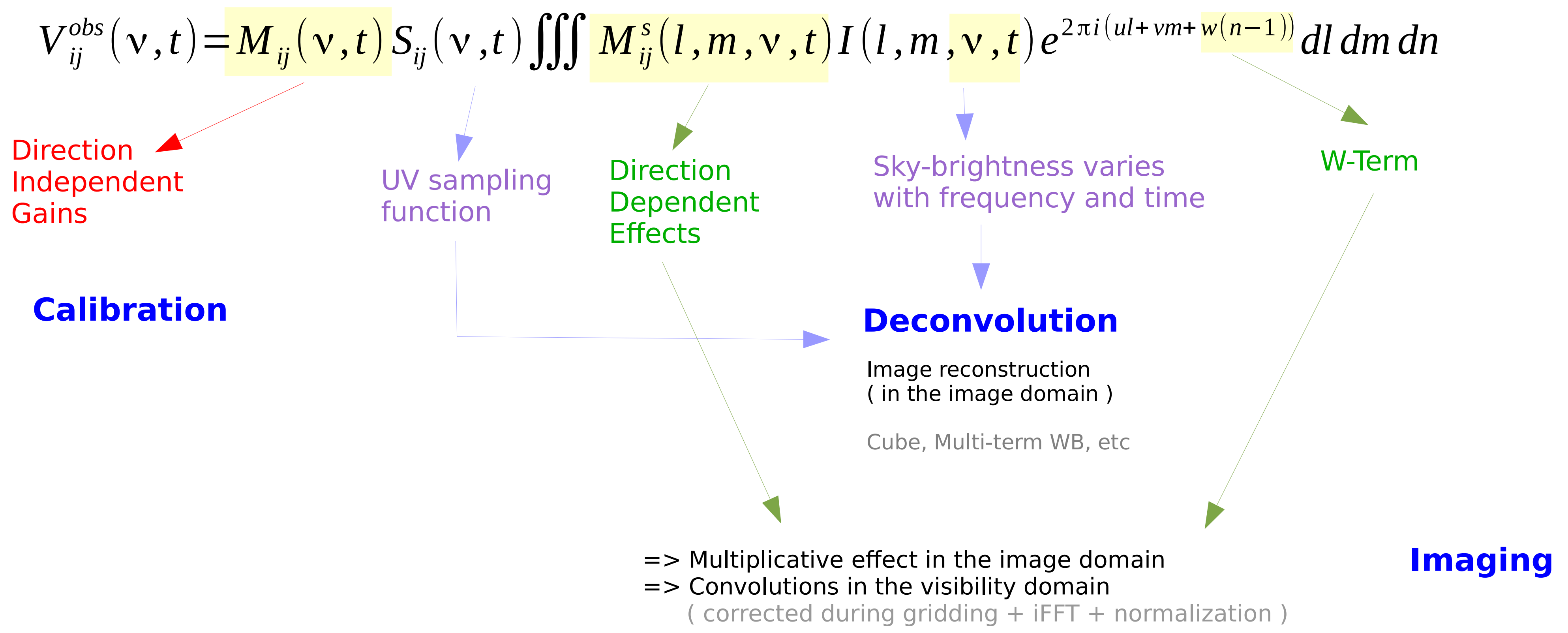
W-Term

Calibration

=> Multiplicative effect in the image domain
=> Convolutions in the visibility domain
(corrected during gridding + iFFT + normalization)

Imaging

Radio Interferometry – Measurement Equations



$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Direction
Independent
Gains

UV sampling
function

Direction
Dependent
Effects

Sky-brightness varies
with frequency and time

W-Term

Calibration

Deconvolution

Image reconstruction
(in the image domain)

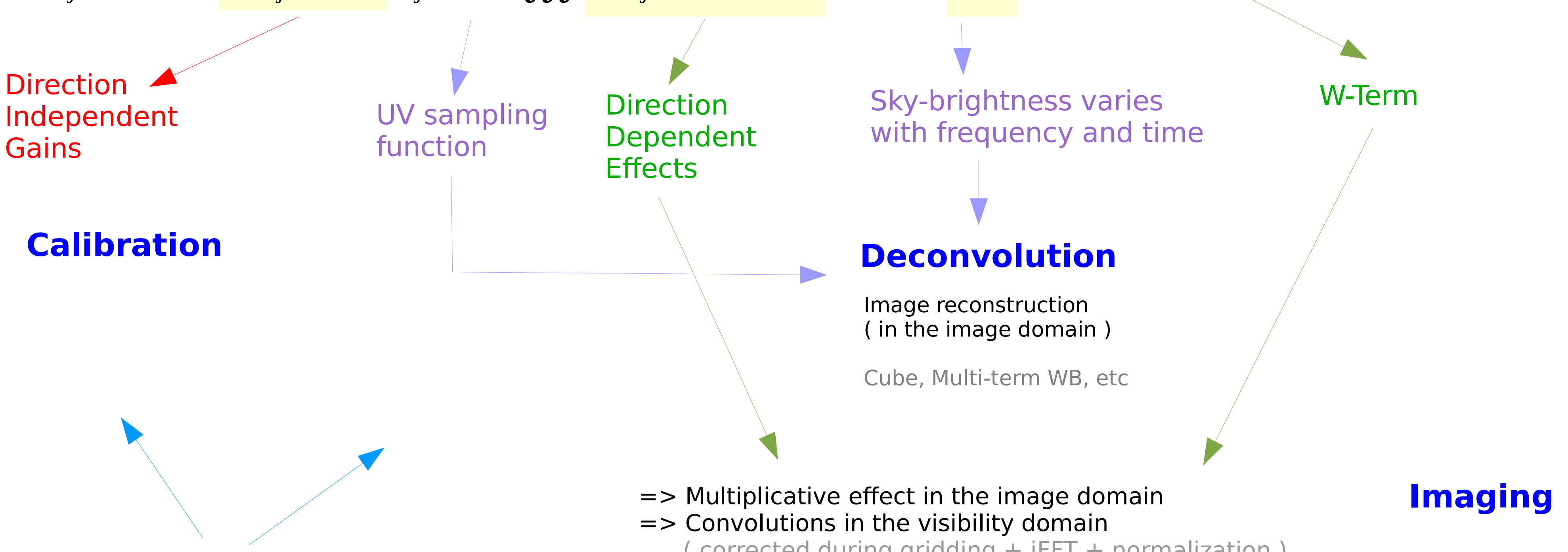
Cube, Multi-term WB, etc

=> Multiplicative effect in the image domain
=> Convolutions in the visibility domain
(corrected during gridding + iFFT + normalization)

Imaging

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i (ul+vm+w(n-1))}\, dl\, dm\, dn$$



Direction
Independent
Gains

UV sampling
function

Direction
Dependent
Effects

Sky-brightness varies
with frequency and time

W-Term

Calibration

Deconvolution

Image reconstruction
(in the image domain)

Cube, Multi-term WB, etc

=> Multiplicative effect in the image domain
=> Convolutions in the visibility domain
(corrected during gridding + iFFT + normalization)

Imaging

Self-Calibration

Direction-dependent Self-Cal

Wideband Self-Cal

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency and time

W-Term

How are these algorithms realized in software…… ?

Iterative Image Reconstruction

The generalized forward problem $V^{obs} = [A] I^m + n$

The generalized inverse problem $I^m = [A]^{-1} V^{obs}$

L2 data regularization

+ Sky model (multiscale, wideband, timevar)
+ Solver/Optimizer with constraints/biases

Iterative Image Reconstruction

The generalized forward problem $V^{obs} = [A] I^{m} + n$

The generalized inverse problem $I^{m} = [A]^{-1} V^{obs}$

L2 data regularization

+ Sky model (multiscale, wideband, timevar)
+ Solver/Optimizer with constraints/biases

Forward and Reverse transforms

Calc $\frac{\delta \chi^{2}}{\delta I^{m}}$

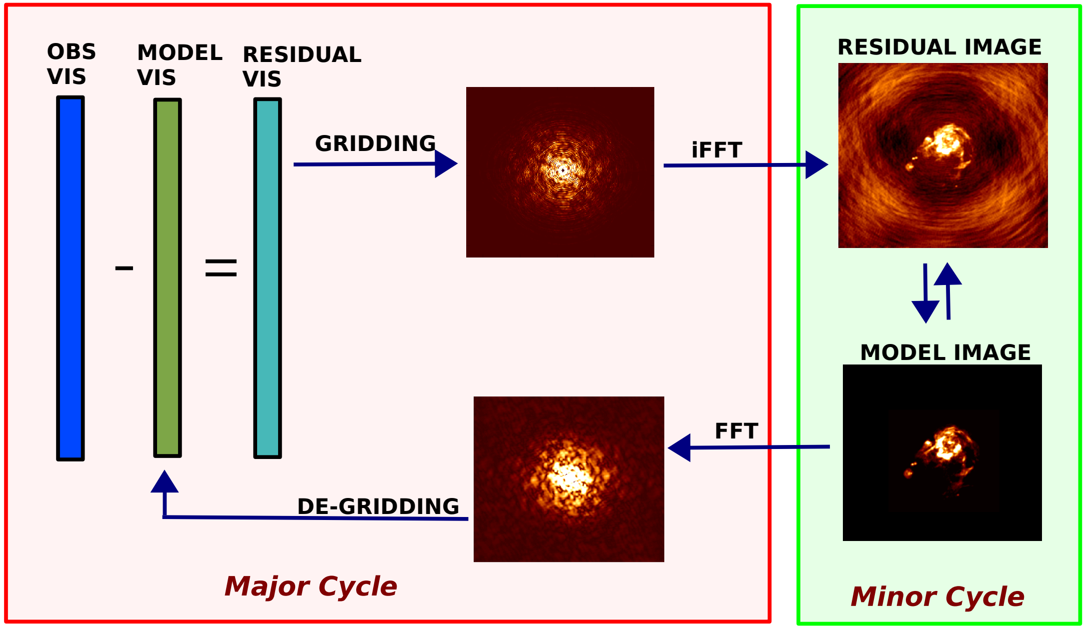


Image Reconstruction

Deconvolution Algorithms

Iterative Image Reconstruction

The generalized forward problem $V^{obs} = [A] I^m + n$

The generalized inverse problem $I^m = [A]^{-1} V^{obs}$

L2 data regularization

+ Sky model (multiscale, wideband, timevar)
+ Solver/Optimizer with constraints/biases

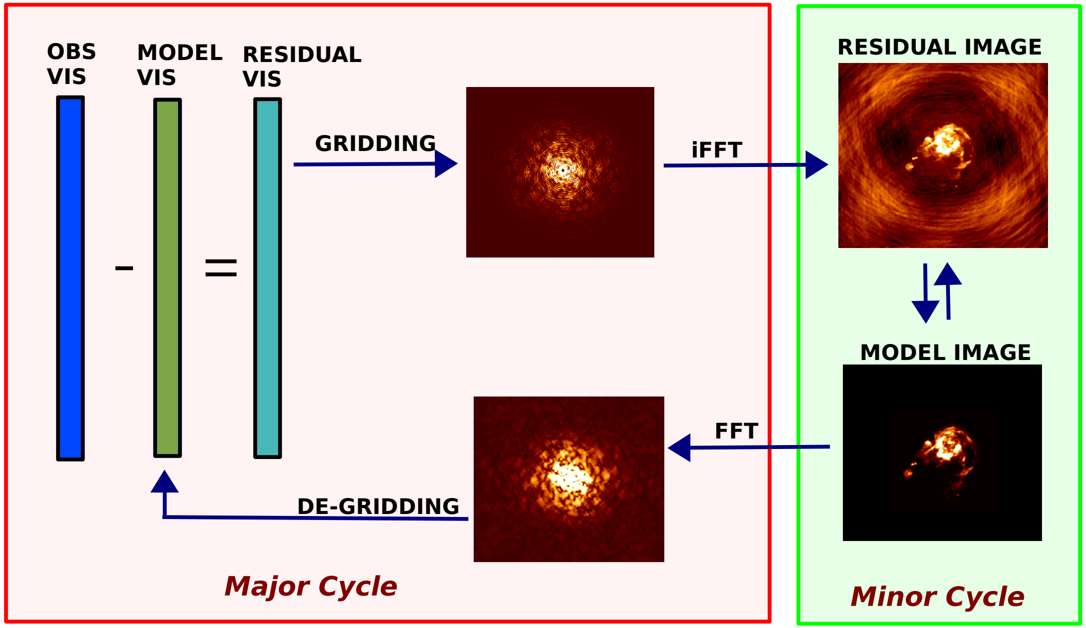
Forward and Reverse transforms

Calc $\frac{\delta \chi^2}{\delta I^m}$

Gridding Options

- Primary-Beams
  - Wideband
  - Full Pol

- W-Term
- Mosaic



Compute Cost : Data volume, Image size, N_channels
Size of gridding convolution function
Deconvolution algorithm
Sky brightness (intensity and structure)

Image Reconstruction

Deconvolution Algorithms

Sky models
- Delta function
- Gaussians
- Wideband

Algorithms
- Clean (greedy)
- Many other compressed sensing ideas

Iterative Image Reconstruction

The generalized forward problem $V^{obs} = [A] I^m + n$

The generalized inverse problem $I^m = [A]^{-1} V^{obs}$

L2 data regularization

+ Sky model (multiscale, wideband, timevar)
+ Solver/Optimizer with constraints/biases

Forward and Reverse transforms

CASA

gridder=

'standard'
'mosaic'
'awproject'
'wproject'

Weighting =

'natural',
'uniform',
'briggs'

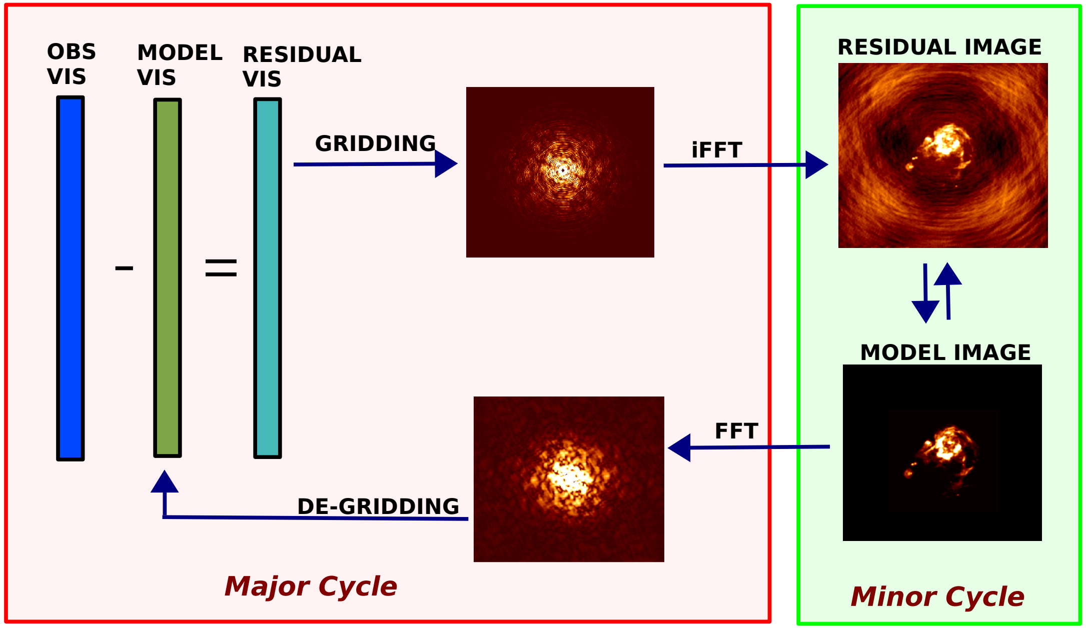


Image Reconstruction

CASA

Deconvolver
(hogbom, clark. multiscale, asp)

Masks (manual, auto, interactive)

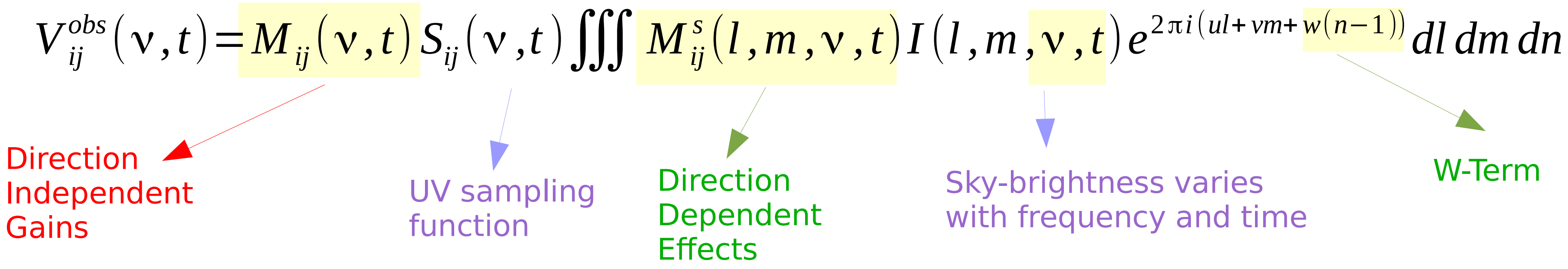
Iteration control
(niter, threshold, gain, nsigma…)

Restoration

Image Definition

CASA
specmode='cube'/'cont', stokes=IQUV
Faceting, multi-field, linearmosaic, etc..

Some References….



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Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency and time

W-Term

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