Wide-Band and Wide-field Imaging - I

Urvashi Rau

National Radio Astronomy Observatory, Socorro, NM, USA

Radio Astronomy School (March 2023)

National Centre for Radio Astrophysics / TIFR, Pune, India

NRAO

Outline

Lecture 1 :

- **Measurement Equation** : What are we solving for during imaging ?
- **Wide-Field Imaging** : Primary Beams, W-term effect, Mosaics

Lecture 2 :

- **Wide-Band Imaging** : Frequency dependence of the sky and instrument
- **Algorithms** : Math to software

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iint I(l,m)\, e^{2\pi i(ul+vm)}\, dl\, dm$$

Observed visibilities (Data)

Direction Independent Gains

UV sampling pattern

Sky Brightness (Image)

Fourier transform kernel

Calibration

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} dl\, dm$$

Observed visibilities (Data)

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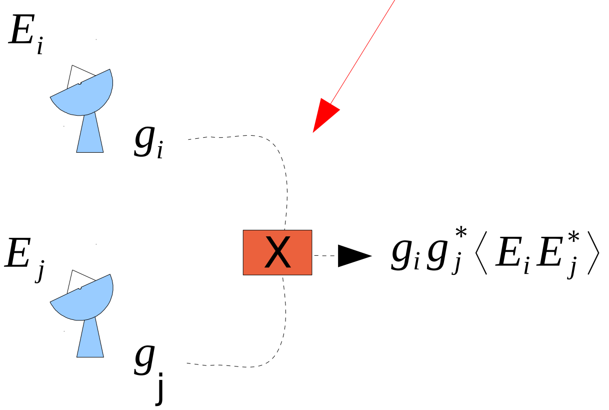


N antennas
N(N-1)/2 antenna-pairs (baselines)

Calibration

Solve for g_i and divide out $g_i g_j^*$

[Ref : J.Marvil Calibration lecture]

Imaging & Deconvolution

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} dl\, dm$$

Observed or Calibrated visibilities (Data)

Direction Independent Gains

UV sampling pattern

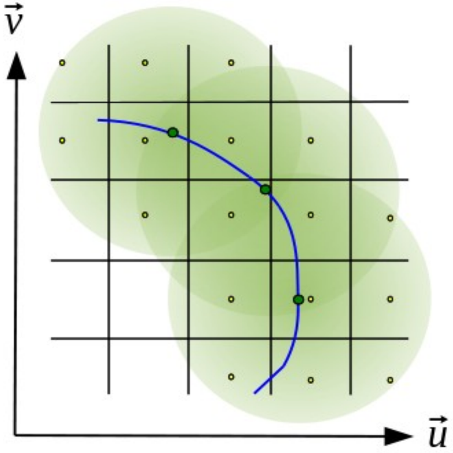
Sky Brightness (Image)

Fourier transform kernel

Imaging

Gridding
+
IFFT
+
Normalization

I^{obs}



Convolutional resampling

Accumulate measured visibilities onto a regular grid

[Ref : Imaging : D. Wilner]

Imaging & Deconvolution

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} dl\, dm$$

Observed or Calibrated visibilities (Data)

Direction Independent Gains

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Imaging

Gridding
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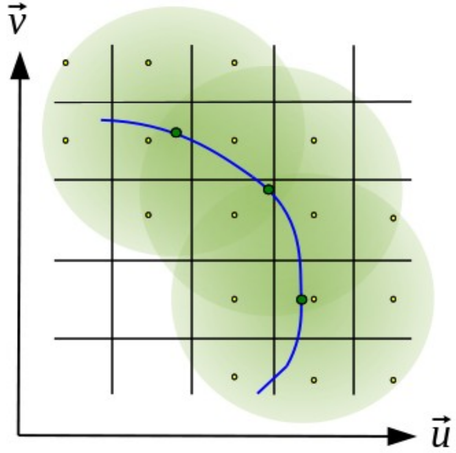
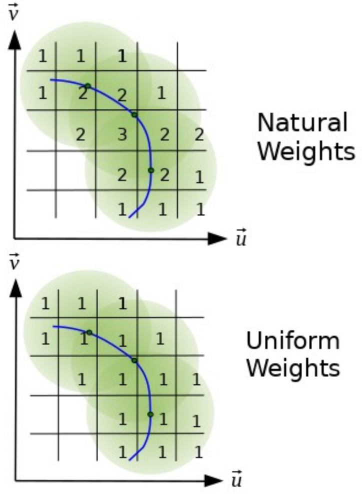
I^{obs}





Data weighting schemes during image formation

=> adjust sensitivity to different spatial scales

[Ref : Imaging : D. Wilner]

Imaging & Deconvolution

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iint I(l,m)\, e^{2\pi i(ul+vm)}\, dl\, dm$$

Observed or Calibrated visibilities (Data)

Direction Independent Gains

UV sampling pattern

Sky Brightness (Image)

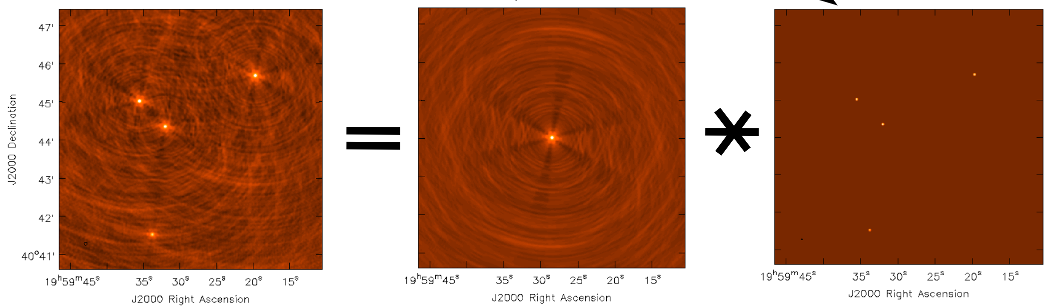
Fourier transform kernel

Gridding
+
IFFT
+
Normalization

I^{obs}

$$I^{obs}(l,m) = I^{PSF}(l,m) * I^{sky}(l,m)$$

Deconvolution



Algorithms :
Clean, MS-Clean, Asp, etc...

[Ref : Imaging : D. Wilner]

All together….

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} dl\, dm$$

- $V_{ij}^{obs}(\nu,t)$: Observed or Calibrated visibilities (Data)
- $M_{ij}(\nu,t)$: Direction Independent Gains → **Calibration**
- $S_{ij}(\nu,t)$: UV sampling pattern
- $I(l,m)$: Sky Brightness (Image)
- $e^{2\pi i(ul+vm)}$: Fourier transform kernel

UV sampling pattern, Sky Brightness and Fourier transform kernel → **Imaging + Deconvolution**

Self-Calibration

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iint I(l,m)\, e^{2\pi i(ul+vm)}\, dl\, dm$$

Observed or Calibrated visibilities (Data)

Direction Independent Gains

UV sampling pattern

Sky Brightness (Image)

Fourier transform kernel

Calibration

Keep $I(l,m)$ fixed

Imaging + Deconvolution

Keep $M_{ij}(\nu,t)$ fixed

Self-Calibration

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} \, dl \, dm$$

Observed or Calibrated visibilities (Data)

Direction Independent Gains

UV sampling pattern

Sky Brightness (Image)

Fourier transform kernel

This is only an approximation…..

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} dl\, dm$$

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t) S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t) I(l,m,\nu,t) e^{2\pi i(ul+vm+w(n-1))} dl\, dm\, dn$$

Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency (time)

W-Term

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)}\, dl\, dm$$

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Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency (time)

W-Term

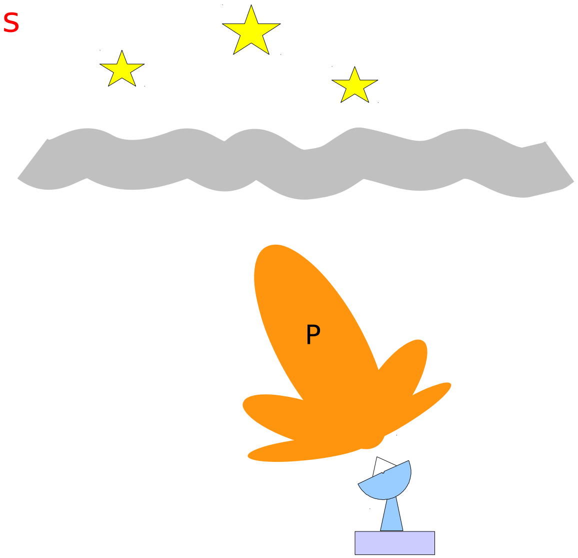
Primary beams and signal propagation effects...

Wide-Field Imaging : Direction dependent effects

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t) S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t) I(l,m,\nu,t) e^{2\pi i (ul+vm+w(n-1))} dl\, dm\, dn$$

Direction Independent Gains

Direction Dependent Effects



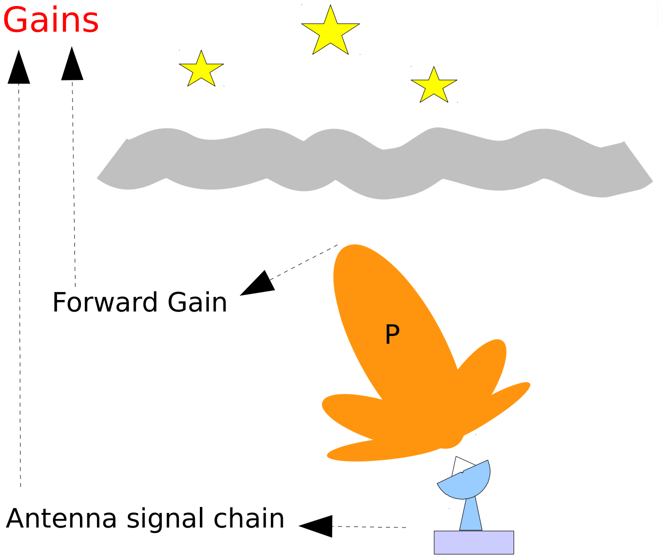
[Ref : Antennas : R.Selina
Calibration : J.Marvil]

Wide-Field Imaging : Direction dependent effects

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

Direction Independent Gains

Direction Dependent Effects



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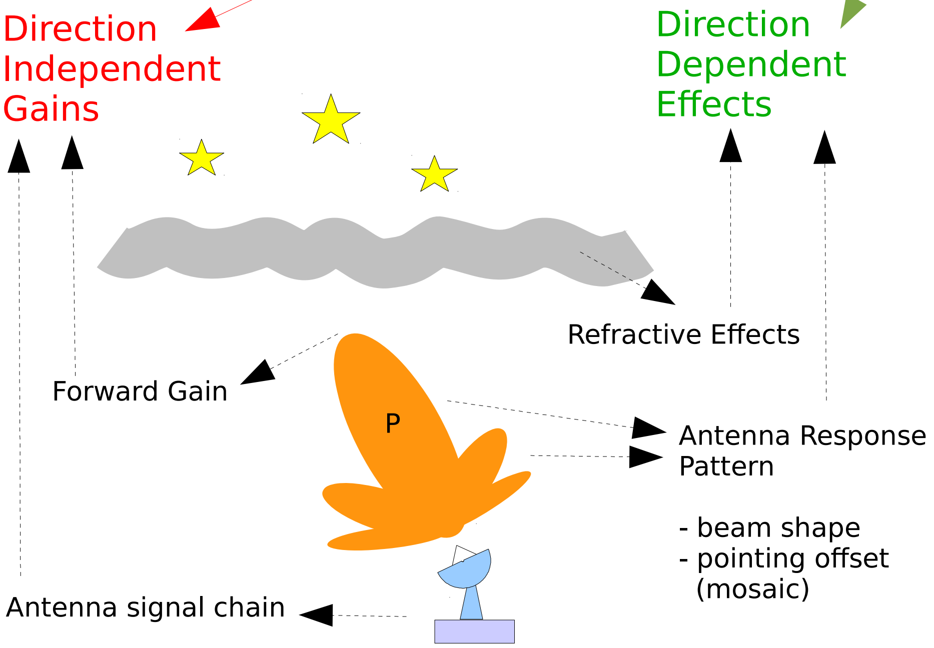
Wide-Field Imaging : Direction dependent effects

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t) S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t) I(l,m,\nu,t) e^{2\pi i (ul+vm+w(n-1))} dl\, dm\, dn$$



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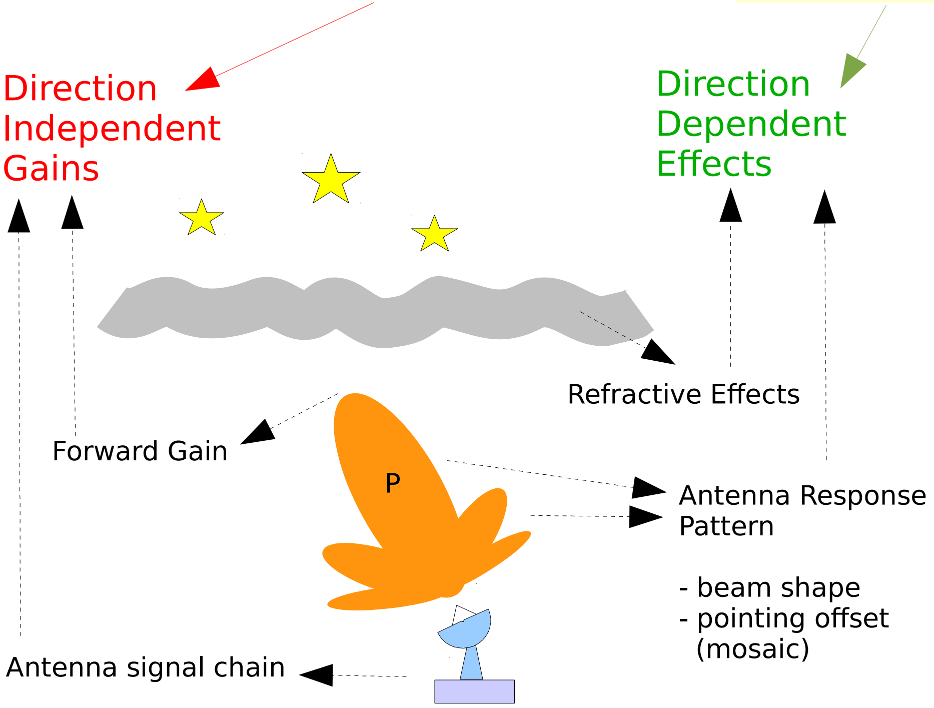
Wide-Field Imaging : Calibration + Imaging

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$



On-axis gains : **Calibration**

Off-axis effects : Use primary-beam or aperture models in **Imaging**

What models ?

– Theoretical, or measured+fitted (ray-traced, holography, ionosphere maps)

(or)

– Solve for DD gains or PB model params from the data themselves (direction-dependent calibration)

[Ref : Antennas : R.Selina
Calibration : J.Marvil]

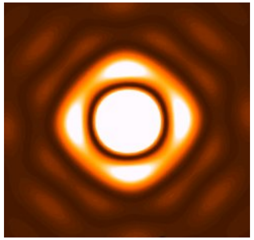
Primary Beams

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Direction
Dependent
Effects

$$M_{ij}=P_{ij}$$

Primary Beam
for baseline ij



Primary Beams

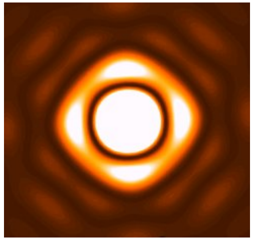
$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t) S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t) I(l,m,\nu,t) e^{2\pi i(ul+vm+w(n-1))} dl\,dm\,dn$$

Direction
Dependent
Effects

$$M_{ij} = P_{ij}$$

The Sky is multiplied by a Primary Beam,
before being sampled by each baseline

Primary Beam
for baseline ij



$$I^{obs}(l,m) = \sum_{ij,t,\nu} I_{ij}^{PSF}(l,m,t,\nu) * \left[P_{ij}(l,m,t,\nu) \cdot I^{sky}(l,m)\right]$$

=> No longer a simple convolution equation

Image-domain Primary Beam Correction (pbcor)

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i (ul+vm+w(n-1))}\, dl\, dm\, dn$$

Direction
Dependent
Effects

Assume identical and invariant primary beams.

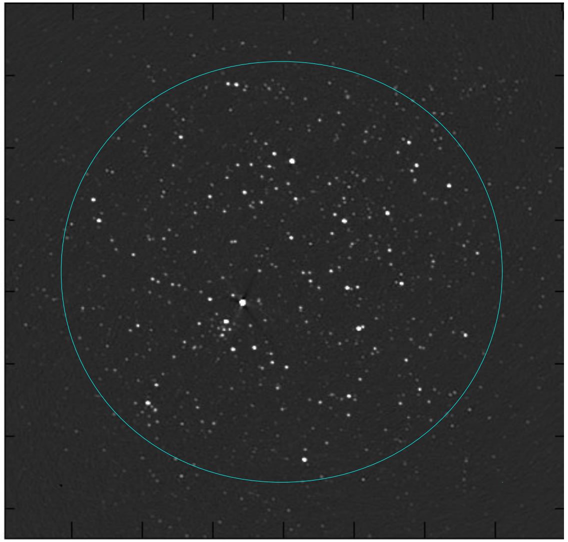
$$I^{obs} = I^{psf} * [P \,.\, I^{sky}]$$

Divide out an average primary beam model after deconvolution

This is approximate

=> Dynamic range limits...

Output Image = PB x Sky



PB-corrected Image : Sky

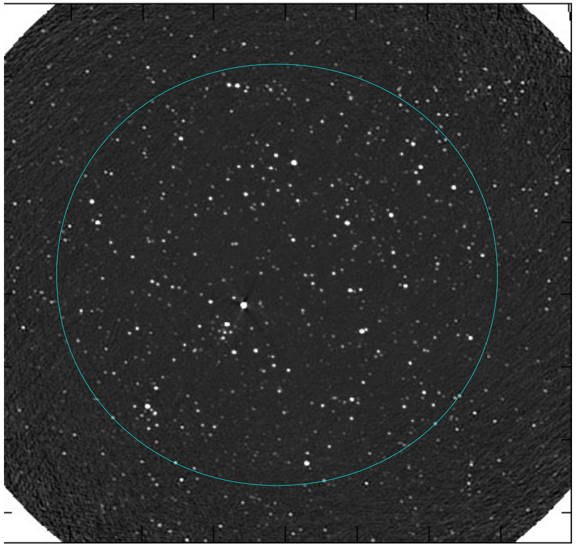


Image-domain Primary Beam Correction (pbcor)

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Direction Dependent Effects

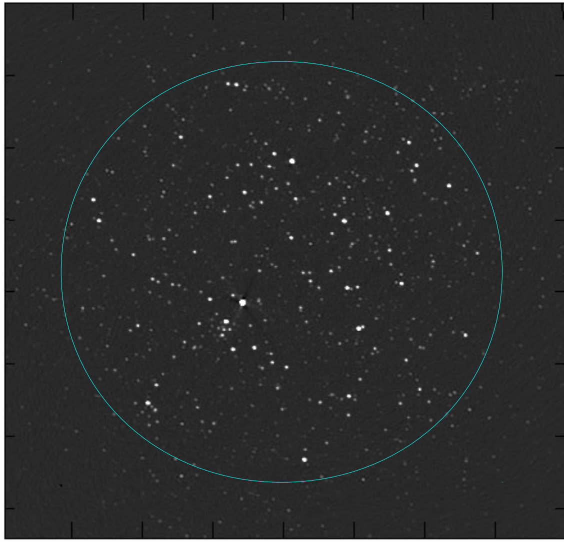
CASA

impbcor()

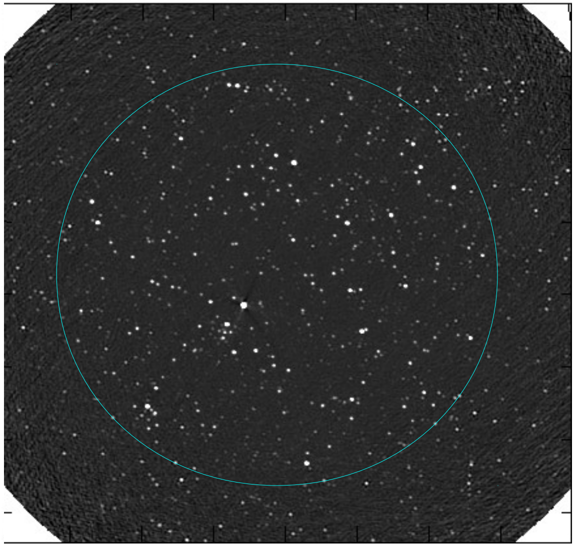
(or)

pbcor=True in tclean()

Output Image = PB x Sky



PB-corrected Image : Sky



A-Projection : Apply correction in UV-domain

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t) S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t) I(l,m,\nu,t) e^{2\pi i(ul+vm+w(n-1))} dl\, dm\, dn$$

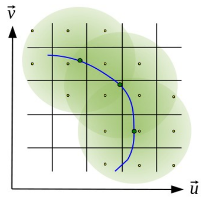
Direction Dependent Effects

When primary beams change across baseline, time, freq....

$$I_{ij}^{obs} = I_{ij}^{psf} * [P_{ij} . I^{sky}] \quad \longleftrightarrow \quad V_{ij}^{obs} = S_{ij} . [A_{ij} * V^{sky}]$$

For each visibility, apply $A_{ij}^{-1} \approx \frac{A_{ij}^{T}}{A_{ij}^{T} * A_{ij}}$

Use A_{ij}^{T} as the convolution function during gridding



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Direction Dependent Effects

When primary beams change across baseline, time, freq....

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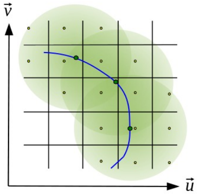
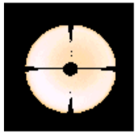
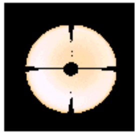


Aperture Illumination for antennas i and j :

$$P_{ij} = E_i . E_j^{*} = FT[A_i * A_j^{*}] = FT[A_{ij}]$$

$A_{ij} =$  $*$ 

Ray-traced Model
(or) Zernicke fits to
holography data

A-Projection : Apply correction in UV-domain

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i (ul + vm + w(n-1))}\, dl\, dm\, dn$$

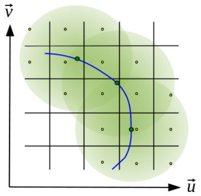
Direction Dependent Effects

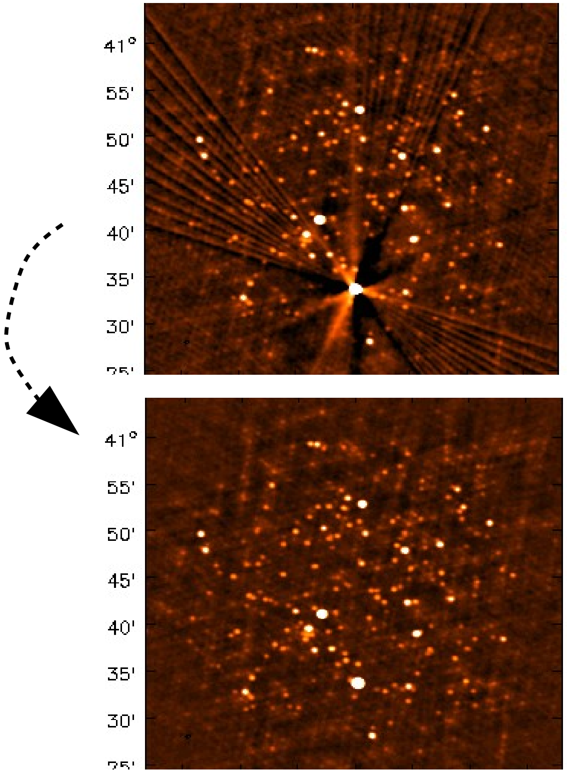
When primary beams change across baseline, time, freq....

$$I_{ij}^{obs} = I_{ij}^{psf} * \left[P_{ij} . I^{sky} \right] \quad \longleftrightarrow \quad V_{ij}^{obs} = S_{ij} . \left[A_{ij} * V^{sky} \right]$$

For each visibility, apply $A_{ij}^{-1} \approx \dfrac{A_{ij}^{T}}{A_{ij}^{T} * A_{ij}}$

Use A_{ij}^{T} as the convolution function during gridding





A-Projection : Apply correction in UV-domain

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CASA

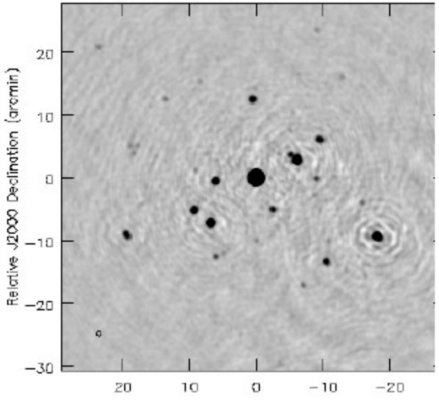
tclean()

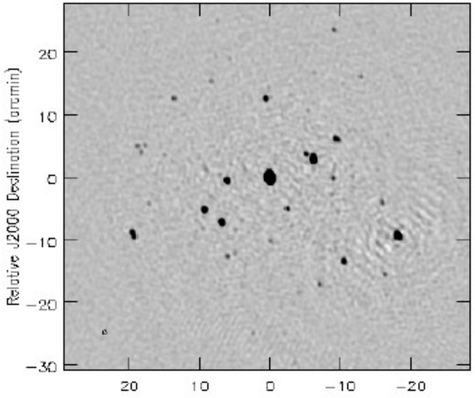
- gridder='awproject'
  - aterm=True
  - psterm=True
  - cfcache='...'
  - pblimit=0.1
  - compute/rotate pa-step = 360.0

Before | **After**

Stokes I

Artifacts around all sources away from the pointing center



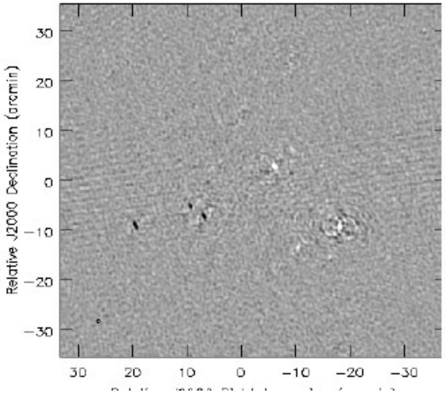


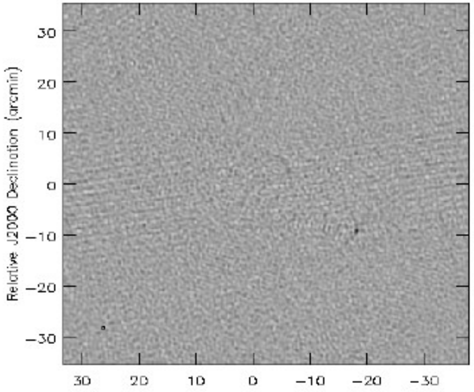
Stokes I

Artifacts removed or reduced within the main lobe

Stokes V

Artificial signals around bright sources due to beam squint





Stokes V

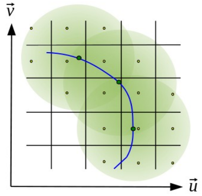
Instrumental Stokes V removed within the main lobe

Mosaicking via A-Projection

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

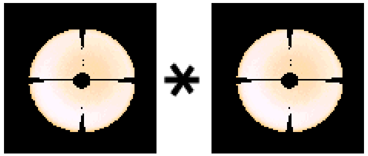
Direction
Dependent
Effects

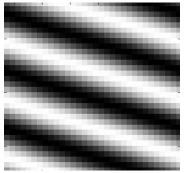
A-Projection : Use aperture illumination functions to construct gridding convolution functions.



Use A_{ij}^{T} as the convolution function

Add a phase gradient





FT => Primary Beam

FT => Shift the Primary Beam

[Ref : Mosaic : B.Mason \

Mosaicking via A-Projection

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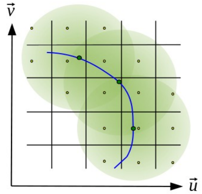
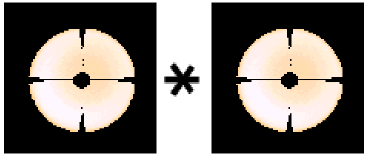
Direction Dependent Effects

Each pointing is gridded with a *different* phase gradient => shift the PB



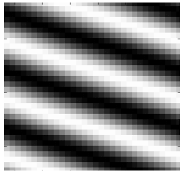
A-Projection : Use aperture illumination functions to construct gridding convolution functions.

Use A_{ij}^{T} as the convolution function

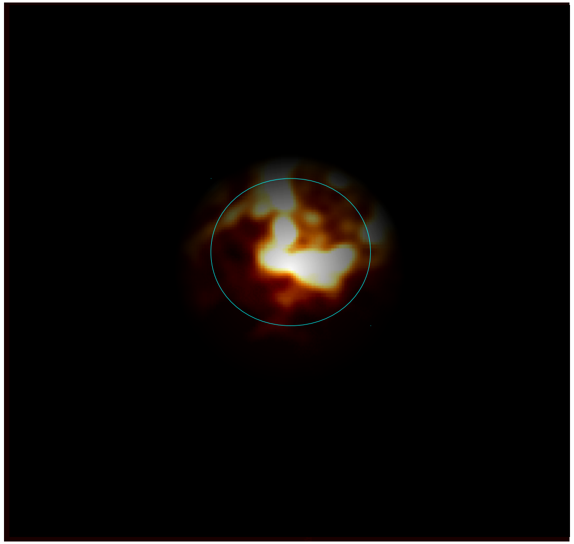


FT => Primary Beam

Add a phase gradient



FT => Shift the Primary Beam



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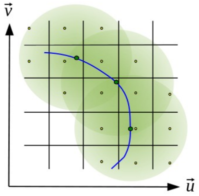
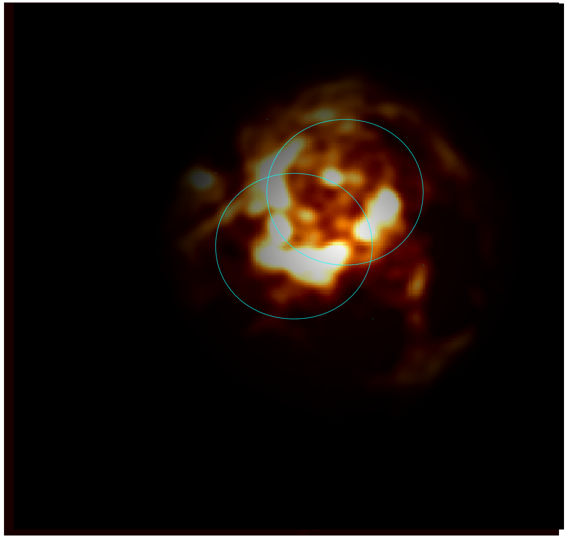
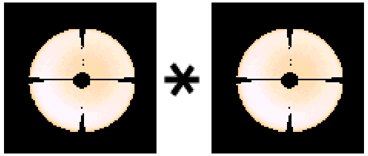


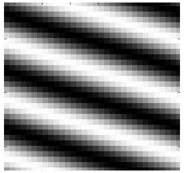


Use A_{ij}^{T} as the convolution function

Add a phase gradient



FT => Primary Beam



FT => Shift the Primary Beam

[Ref : Mosaic : B.Mason \

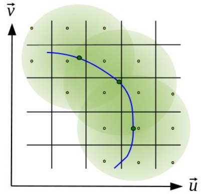
Mosaicking via A-Projection

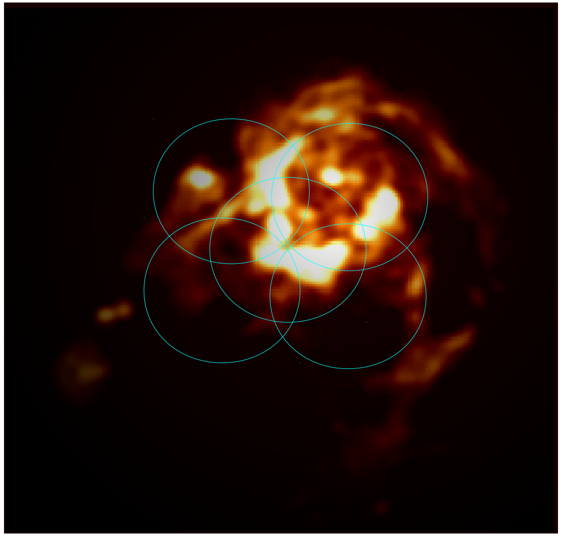
$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Direction
Dependent
Effects

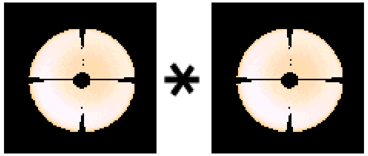
Each pointing is gridded with a *different* phase gradient => shift the PB

A-Projection : Use aperture illumination functions to construct gridding convolution functions.



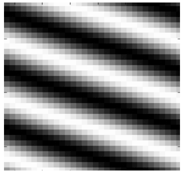


Use A_{ij}^{T} as the convolution function



FT => Primary Beam

Add a phase gradient



FT => Shift the Primary Beam

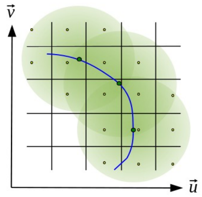
*[Ref : Mosaic : B.Mason *

Mosaicking via A-Projection

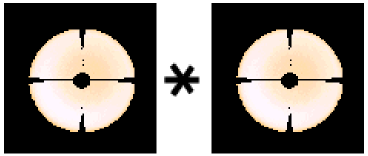
$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i (ul+vm+w(n-1))}\, dl\, dm\, dn$$

Direction Dependent Effects

A-Projection : Use aperture illumination functions to construct gridding convolution functions.

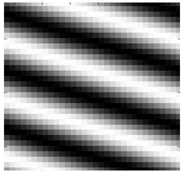


Use A_{ij}^{T} as the convolution function



FT => Primary Beam

Add a phase gradient



FT => Shift the Primary Beam

CASA

Tclean() : Select one or more pointings of data

gridder='mosaic'

- Round Pbs and no Squint corr
- No squint correction
- Heterogeneous PBs
- Known PBs : VLA, ALMA, etc
- Supply PB models via a "vptable" parameter

gridder='awproject'

- Rotating PBs with Squint corr.
- Homogeneous Pbs
- Known Pbs : VLA
- Supply PB models via an external "cfcache" (R&D)

Polarization : Full-Mueller A-Projection

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Direction
Dependent
Effects

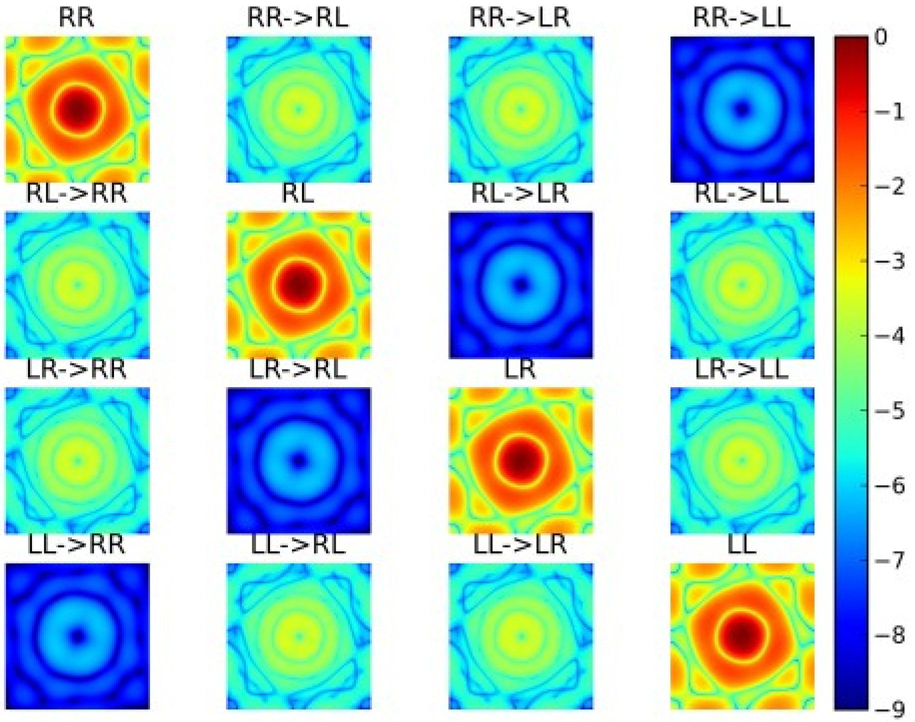
Full polarization primary beams

- Needed for IQUV imaging over the full PB

- Account for direction-dependent Polarization leakage.

Full-Mueller A-Projection applies the conjugate transpose of the full matrix

=> A_{ij}^{T} is a 4x4 matrix application



Images from P.Jagannathan

Primary Beam Models

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t) S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t) I(l,m,\nu,t) e^{2\pi i (ul+vm+w(n-1))} dl\, dm\, dn$$

Direction Dependent Effects

Accuracy of PB-correction depends on the quality of the PB model

(1) Modified Airy Disk :

Fourier transform of the autocorrelation of a tapered circular aperture

(2) Ray-traced model :

EM wave propagation with a parameterized dish optics (Brisken, 2011)

(3) Models derived from PB measurements

(a) 1D polynomial fits to azimuthally averaged primary beams (Perley, 2017)
(b) Solve for 2D Zernicke polynomial coefficients (Jagannathan et al, 2017)

CASA

To supply external PBs for non-NRAO telescopes

For ‘standard’ and ‘mosaic’ gridders only

(1) Make a vptable

- Use vpmanager tool

(2) Supply to tclean()

- vptable parameter

Direction-dependent Self-Calibration

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

Direction Independent Gains

UV sampling function

Direction Dependent Effects

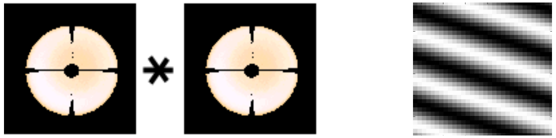
Sky-brightness varies with frequency and time

W-Term

Model the sky and instrument separately

Parameterize the aperture illumination function

- Solve for pointing offsets
- Solve for ionospheric propagation effects



Algorithms : Pointing Self-Cal (with A-Proj)

Model the sky and instrument together

Joint solutions for gains in multiple directions

- Does not use a known PB model
- Difficult to get accurate source flux
- Good approach for source-subtraction

Algorithms : DD-facets, etc....

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t) S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t) I(l,m,\nu,t) e^{2\pi i(ul+vm+w(n-1))} dl\,dm\,dn$$

Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency and time

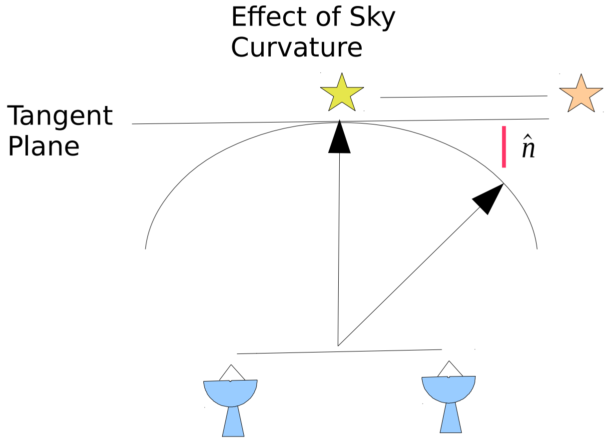
W-Term

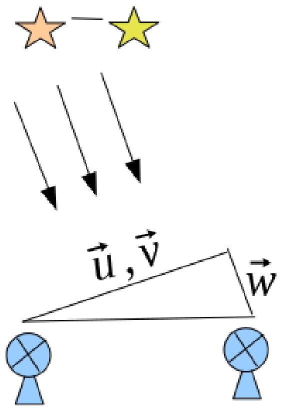
W-Term effect...

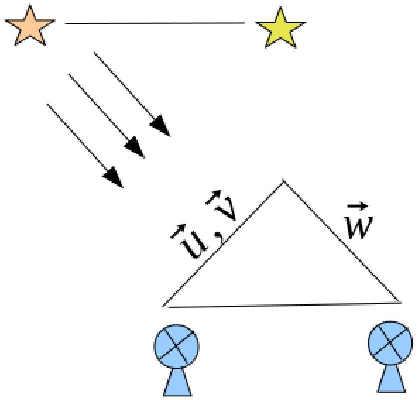
Wide-Field Imaging : Non-coplanar baselines / W-term

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

W-Term







A known geometric effect

Algorithms

3D Imaging,
W-stacking,
Faceting,
W-Projection

For a field-of-view given by the Primary Beam of an antenna of diameter D, at wavelength λ and with a maximum baseline length of B....

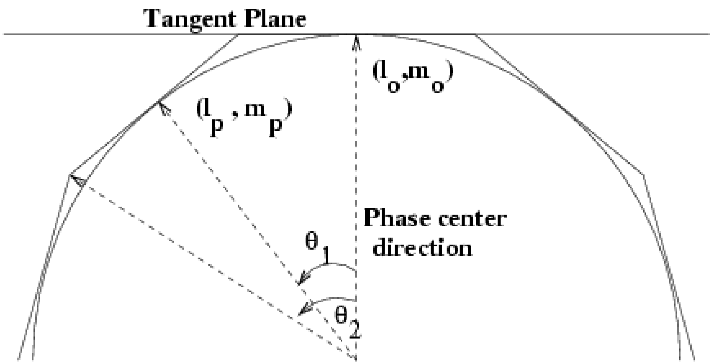
The w-term becomes relevant if $\frac{\lambda B}{D^2}>1$

See R.Perley's talks

W-term : Faceting

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t) S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t) I(l,m,\nu,t) e^{2\pi i(ul+vm+w(n-1))} dl\, dm\, dn$$

W-Term



- Approximate the celestial sphere by a set of tangent planes (facets) such that 2D geometry is valid per facet

- Image each facet with its own phase reference center and re-project to the tangent plane

Algorithm Variants:

Deconvolve facets separately before re-projecting and stitching

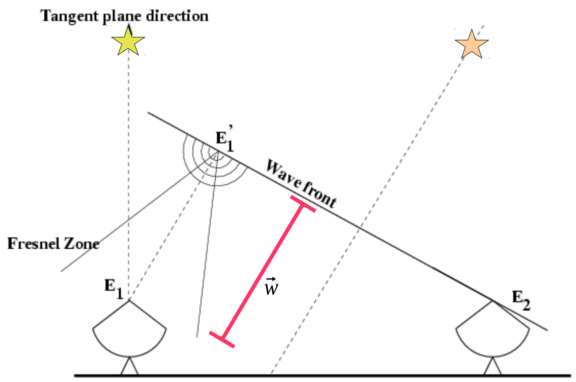
(or)

Image all facets onto the same tangent plane grid and perform a joint deconvolution.

W-term : W-Projection

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

W-Term



For ideal 2D imaging we need to measure E_1
Instead, we measure E_1'

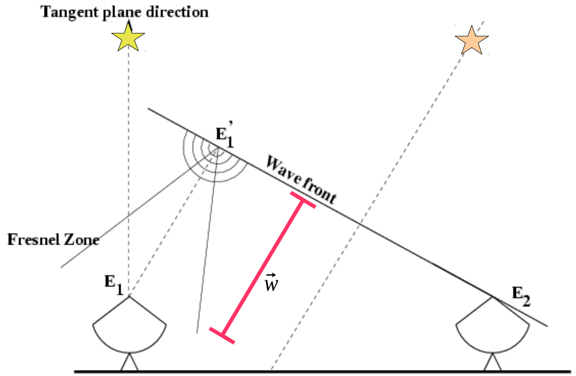
E_1' and E_1 are related by a Fresnel diffraction/propagation kernel.

$$G(u,v,w)=FT\left[e^{2\pi\iota w\sqrt{1-l^2-m^2}}\right]$$

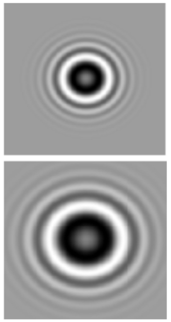
W-term : W-Projection

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t) S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t) I(l,m,\nu,t) e^{2\pi i (ul+vm+w(n-1))} dl\, dm\, dn$$

W-Term



W-term appears as a convolution in the UV-domain



$$V^{o}(u,v,w) = V(u,v,w=0) * G(u,v,w)$$

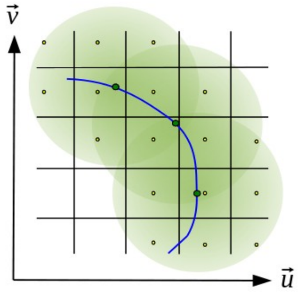
For ideal 2D imaging we need to measure E_1
Instead, we measure E_1'

E_1' and E_1 are related by a Fresnel diffraction/propagation kernel.

$$G(u,v,w) = FT\left[e^{2\pi\iota w \sqrt{1-l^2-m^2}}\right]$$

=> Correct it by another convolution with the inverse/conjugate kernel (during the gridding step)

=> Use different kernels for different W values (appropriately quantized)

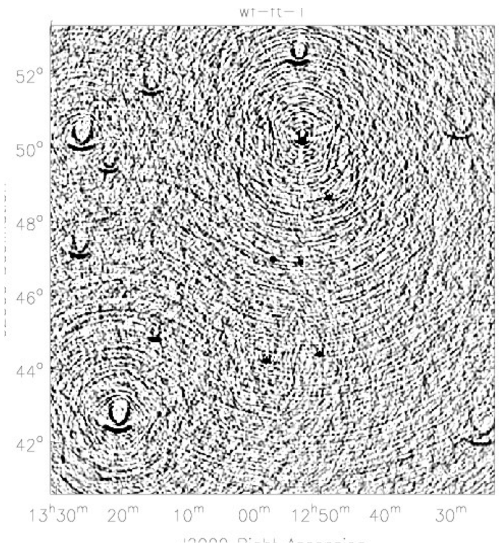


W-Term : Algorithm Comparison

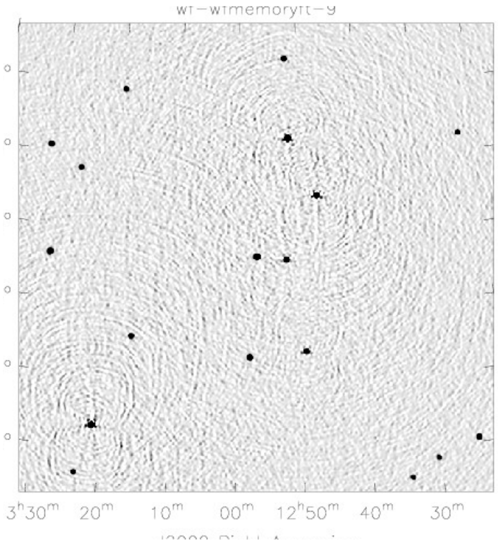
$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

W-Term

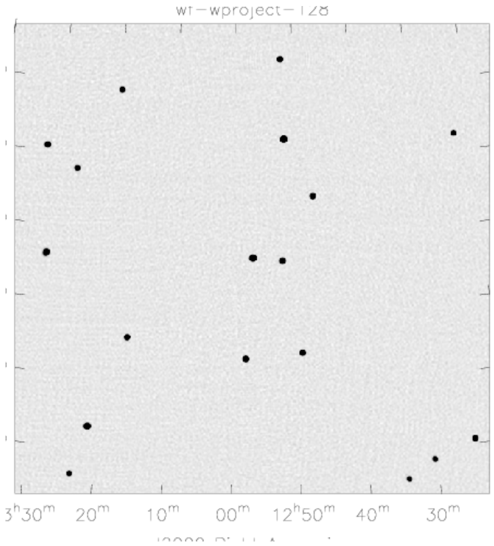
2D Imaging



Facet Imaging



W-Projection



W-Term : Algorithm Comparison

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i (ul+vm+w(n-1))}\, dl\, dm\, dn$$

W-Term

CASA

gridder='wproject'

- wprojplanes = 16

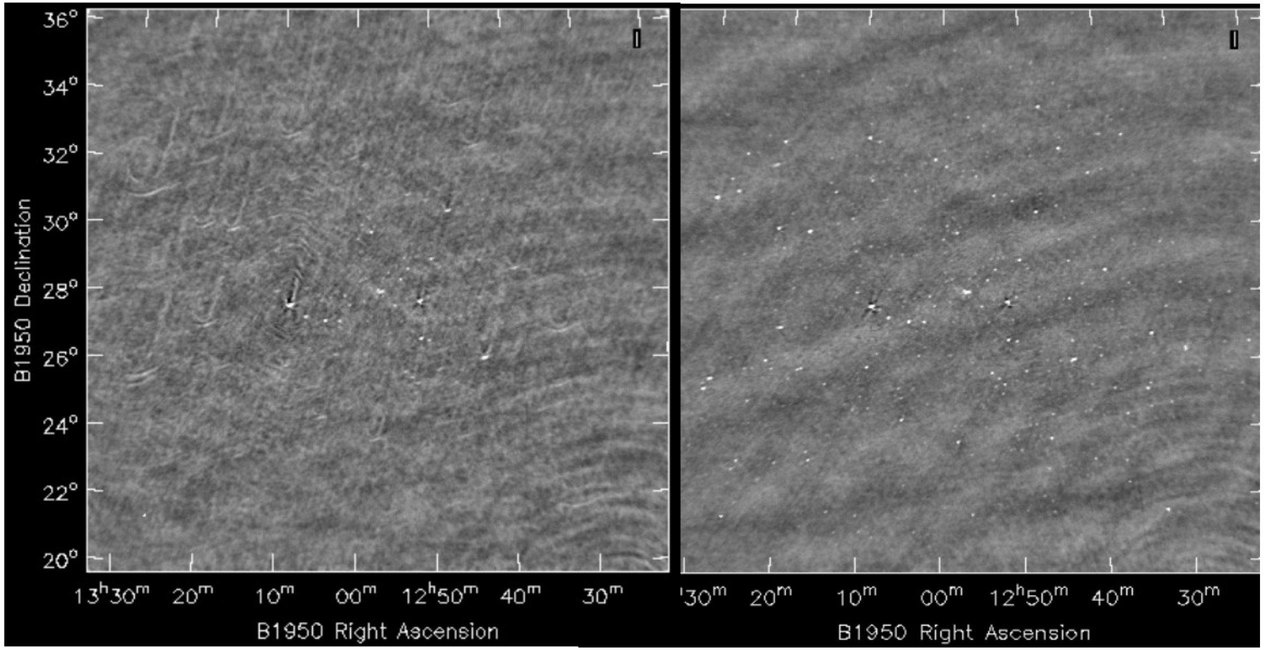
gridder='widefield'

- Faceting and/or W-project
- facets = 3
- wprojplanes = 16 (or 1)

gridder='awproject'

- wprojplanes = 16

Before After

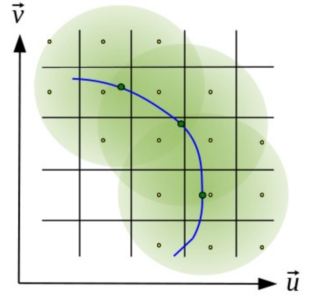


Correcting all Direction Dependent effects together……

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

Direction Dependent Effects

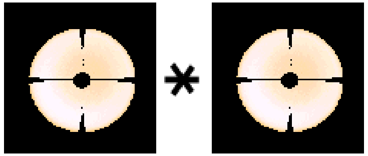
W-Term



A-W-Projection : Correct multiple Direction Dependent effects together, during gridding

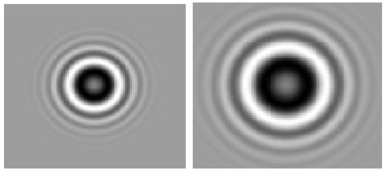
The gridding convolution function (per visibility) is a combination of the following……

Aperture illumination functions (ij, time, freq, pol)



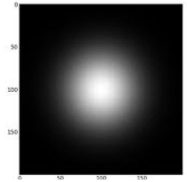
Primary Beams

FT of Fresnel kernels



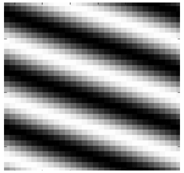
W-term effects

Prolate Spheroidal



Anti-Aliasing Filter

Phase gradient



Mosaic

Correcting all Direction Dependent effects together……

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

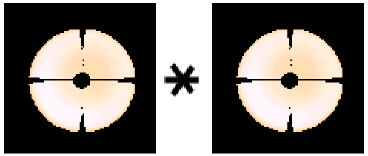
Direction Dependent Effects

W-Term

CASA

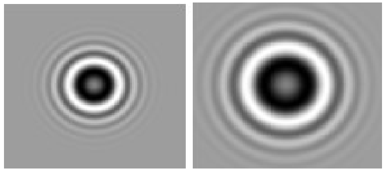
- gridder='standard' : Can use external PBs. No A-Projection. No W-term. Only single-pointing.
  For a mosaic, use 'linearmosaic' tool.
- gridder='mosaic' : Can use external Pbs. Approx A-Projection. No W-term. Single-pointing and mosaics
- gridder='awproject' : Only VLA PBs. Accurate A-Projection. Has W-term. Single-pointing and mosaic

Aperture illumination functions (ij, time, freq, pol)



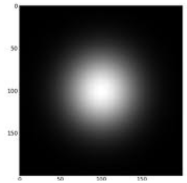
Primary Beams

FT of Fresnel kernels



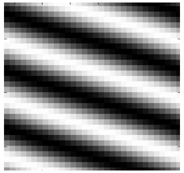
W-term effects

Prolate Spheroidal



Anti-Aliasing Filter

Phase gradient



Mosaic

Outline

Lecture 1 :

- **Measurement Equation** : What are we solving for during imaging ?
- **Wide-Field Imaging** : Primary Beams, W-term effect, Mosaics

Lecture 2 :

- **Wide-Band Imaging** : Frequency dependence of the sky and instrument
- **Algorithms** : Math to software