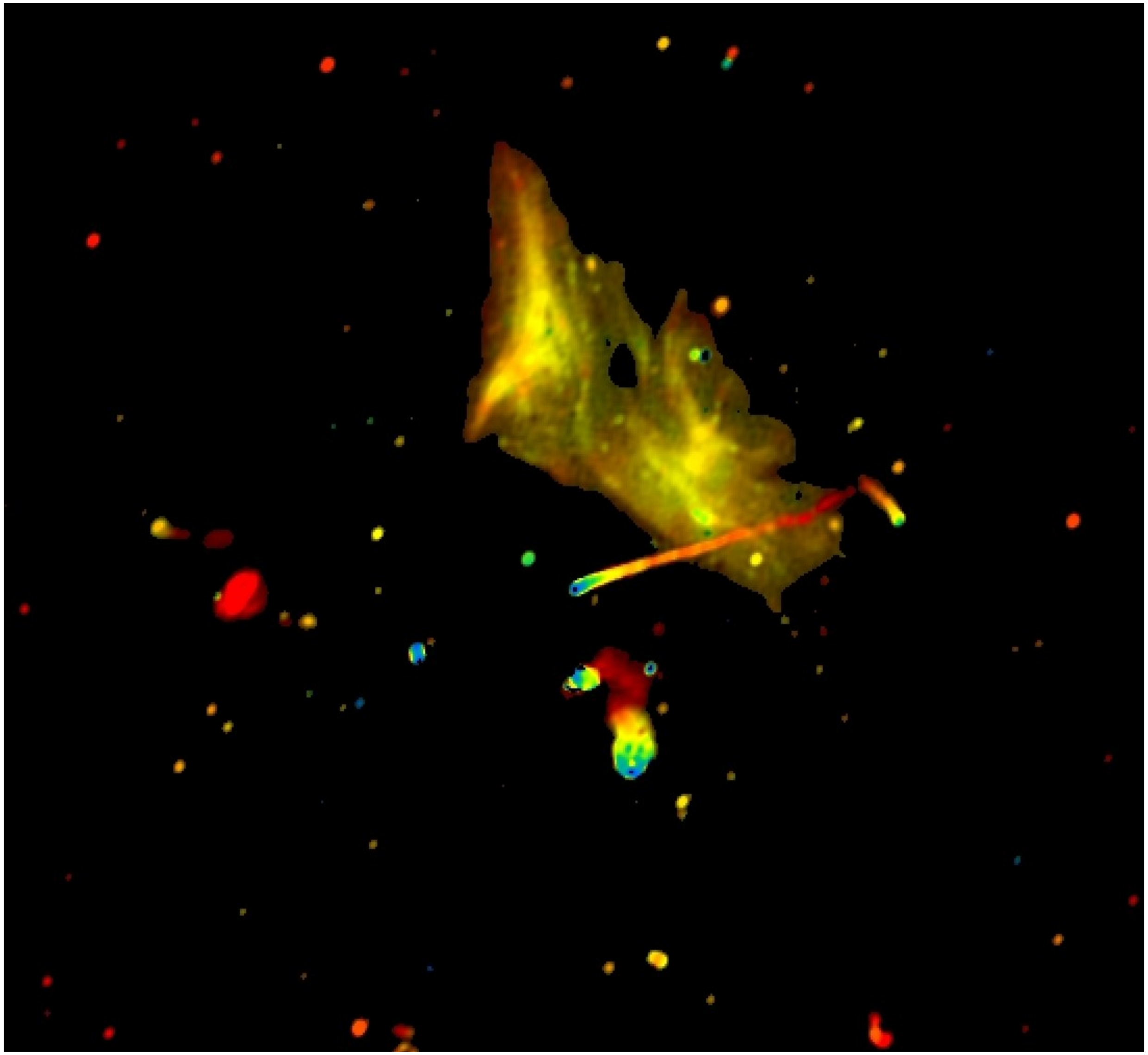
# Sky-domain algorithms to reconstruct spatial, spectral and time-variable structure of the sky-brightness distribution



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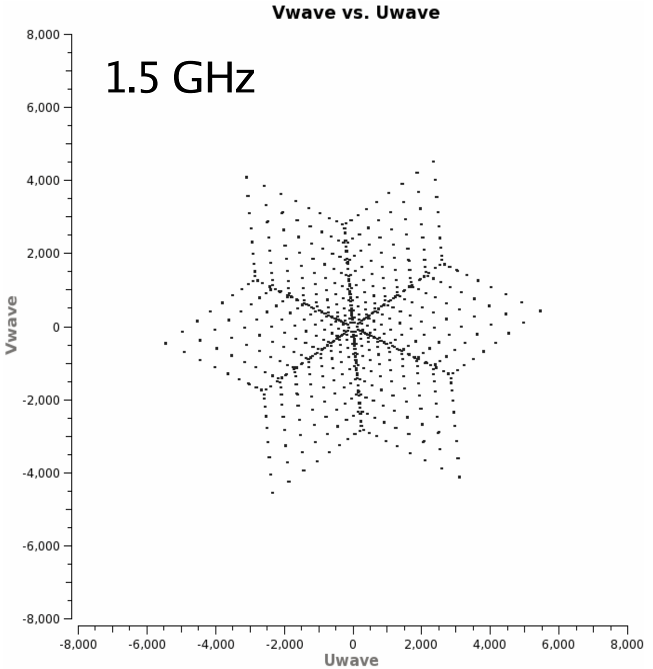
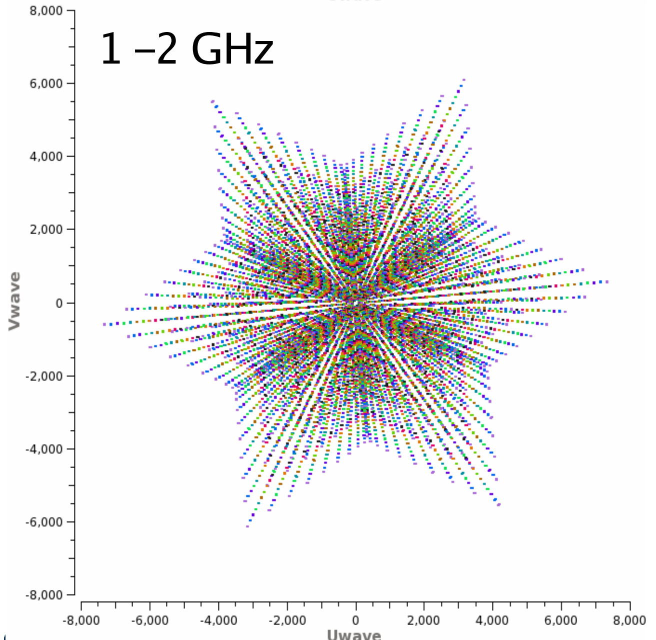
Outline :

- Overview of image reconstruction advances in the past decade.
- Image-reconstruction as a numerical optimization problem.
- Multi-term methods for sky-domain reconstruction
- Need for data-domain methods to correct for instrumental effects
- EVLA Imaging examples

Image from F.Owen, NRAO
Intensity-weighted Spectral-index of Abell2256 (EVLA-LBand)

# 'Standard' Aperture Synthesis + Imaging + Deconvolution




$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\int I^{sky}(l,m)e^{2\pi i(ul+vm)}dldm$$

– Generate visibilities by binning correlations in time and frequency (Earth-rotation and multi-frequency synthesis)

Choose $dt, d\nu$ such that $du, dv < \frac{1}{FOV}$

– Imaging / Gridding + FT

- Convolve : $V_{ij}\delta(u-u_{ij}) \ast \boxed{G(u)}$
- Resample on a regular grid ( $du, dv$ )
- Inverse FFT
- Divide by the inverse FT of $G(u)$

Projection Algorithms

– Deconvolution / Model-fitting

- Fit a sky-model via iterative $\chi^2$ minimization

$$I^{obs}(l,m) = \boxed{I^{sky}(l,m)} \ast \int S(u,v)e^{-2\pi i(ul+vm)}$$

Deconvolution algorithms

# Assumptions vs Reality

Assumptions : $V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t) S_{ij}(\nu,t) \int I(l,m) e^{2\pi i(ul+vm)} dl\, dm$

- sky brightness distribution is constant w.r.to frequency (and time)
- calibration of time and freq-dependent instrumental effects at the phase center

Ok for imaging dynamic-range < few 1000, narrow bandwidths and small fields-of-view.

Reality : $V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t) S_{ij}(\nu,t) \int \underbrace{\boxed{M_{ij}^{s}(l,m,\nu,t)}}_{\text{PB effects}} \underbrace{\boxed{I(l,m,\nu)}}_{\text{Sky brightness}} e^{2\pi i(ul+vm+\underbrace{\boxed{w(n-1)}}_{\text{W-Term}})} dl\, dm\, dn$

- Increased receiver sensitivity (lower Tsys, larger bandwidth, larger collecting-area)

  => Artifacts that were earlier below the noise, are visible in the image.
  - direction, frequency and time-dependence of the sky and instrument.

- Desire to image wide fields-of-view

  - Pressure on telescope time => wider fields-of-view from a single observation
  - Increase image fidelity for image-domain mosaics/facets.

# Imaging and Deconvolution Algorithms

## Deconvolution algorithms

Clean — Hogbom 1974, Clark 1980, Schwab & Cotton 1983; Schwarz, 1978

Maximum-Entropy Clean — Cornwell & Evans 1985; Narayan & Nityananda 1986

Multi-Frequency-Clean — Conway et al 1991, Sault & Wieringa 1994

Adaptive-Scale Pixel Clean — Bhatnagar & Cornwell, 2004

Multi-Scale-Clean — Cornwell, 2008; Greisen, 2008

Multi-Scale,Multi-Frequency Clean — Rau & Cornwell, 2011

Other experimental ideas :

- Multi-Frequency Time-variable Clean
  Stewart et al, 2011
- Ideas from compressed-sensing methods : minimize L1 instead of L2
  Wenger et al, 2010, Li & Cornwell 2011, 2012

## Wide-field Imaging Algorithms

Post-deconvolution PB-correction

Multi-facet imaging
(independent and joint deconvolution)

Mosaicing (independent deconvolution)

W-Projection — Bhatnagar, Cornwell, Golap, 2004

A-Projection — Bhatnagar, Cornwell, Golap, Uson, 2004

Projection-Mosaicing — Golap et al. ~ 2004 related to Ekers/Rots 1979

WideBand-A-Projection — Bhatnagar & Rau, 2012 (in prep)

Other ideas :

- Peeling — LOFAR group, ~ 2010
- Differential-gain calibration
  Smirnov 2011, Yatawatta 2011

Image Reconstruction : Iteratively fit a sky-model to the observed visibilities.

Measurement Equation : $[A] I^m = V^{obs}$

- The operator $[A]=[S][F]$ includes the UV-coverage and FT
- The vector $I^m$ is the sky model ( e.g. image-pixels, Gaussian set )

Fit the parameters of $I^m$ via a weighted least-squares optimization :

- Minimize $\chi^2 = [V^{obs} - A I^m]^T W [V^{obs} - A I^m]$ ==> $\frac{\delta \chi^2}{\delta I^m} = 0$

Normal Equations : $[A^T W A] I^m = [A^T W] V^{obs}$

- This describes an image-domain convolution $I^{psf} * I^m = I^{dirty}$

Iterative Solution : $\underbrace{I^m_{i+1} = I^m_i + g [A^T W A]^+}_{\text{Deconvolution}} \underbrace{\left( A^T W \left( V^{obs} - A I^m_i \right) \right)}$

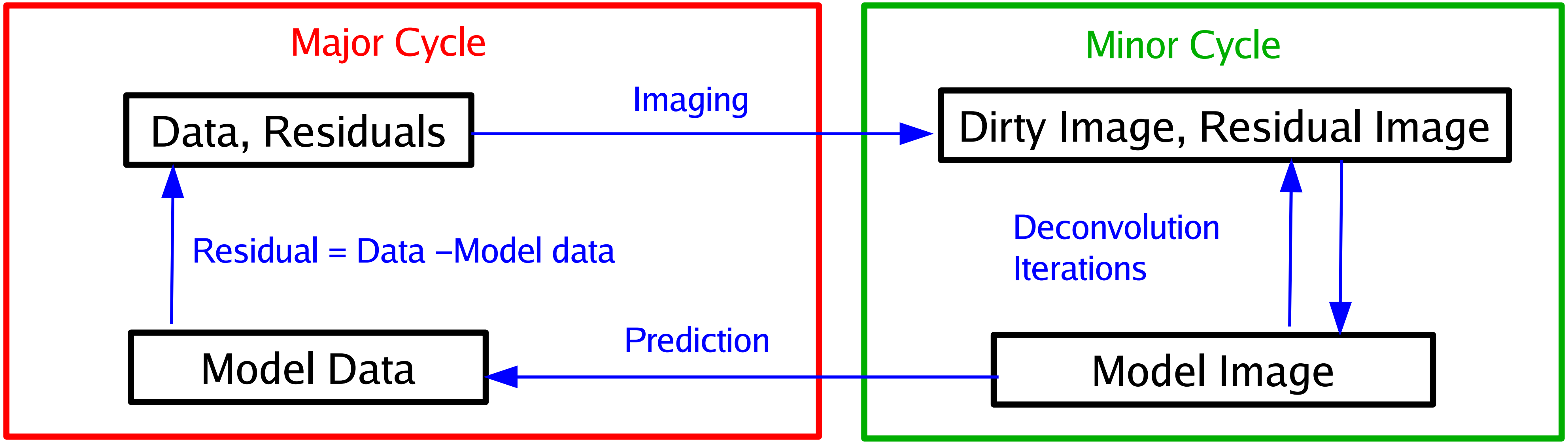
Deconvolution

Imaging (Gridding + iFT)

Prediction (FT + de-Gridding)

# Basic Imaging and Deconvolution

Image Reconstruction : Iteratively fit a sky-model to the observed visibilities.

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Deconvolution

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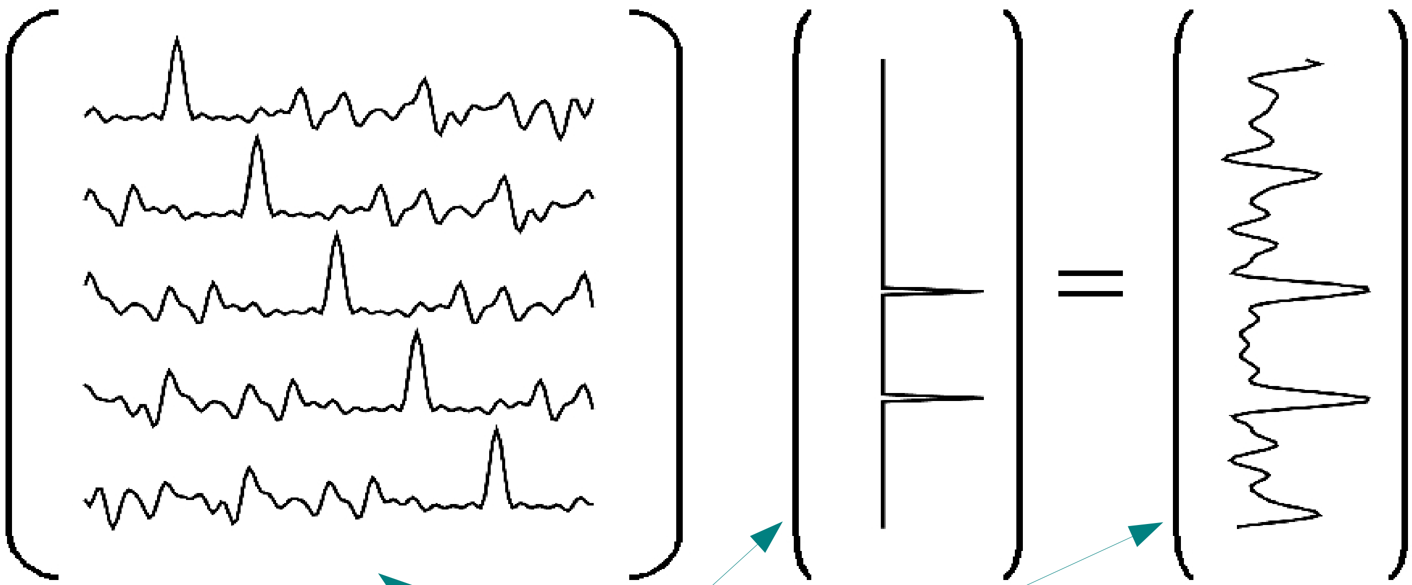
Prediction (FT + de-Gridding)

# Basic Imaging and Deconvolution



CLEAN

- Calculate dirty image
- Normalize by sum of weights
- Add location/amp of peak to model
- Update residuals

Normal Equations : $[A^T W A] I^m = [A^T W] V^{obs}$ where $[A] = [S][F]$

– This describes an image-domain convolution $I^{psf} * I^m = I^{dirty}$

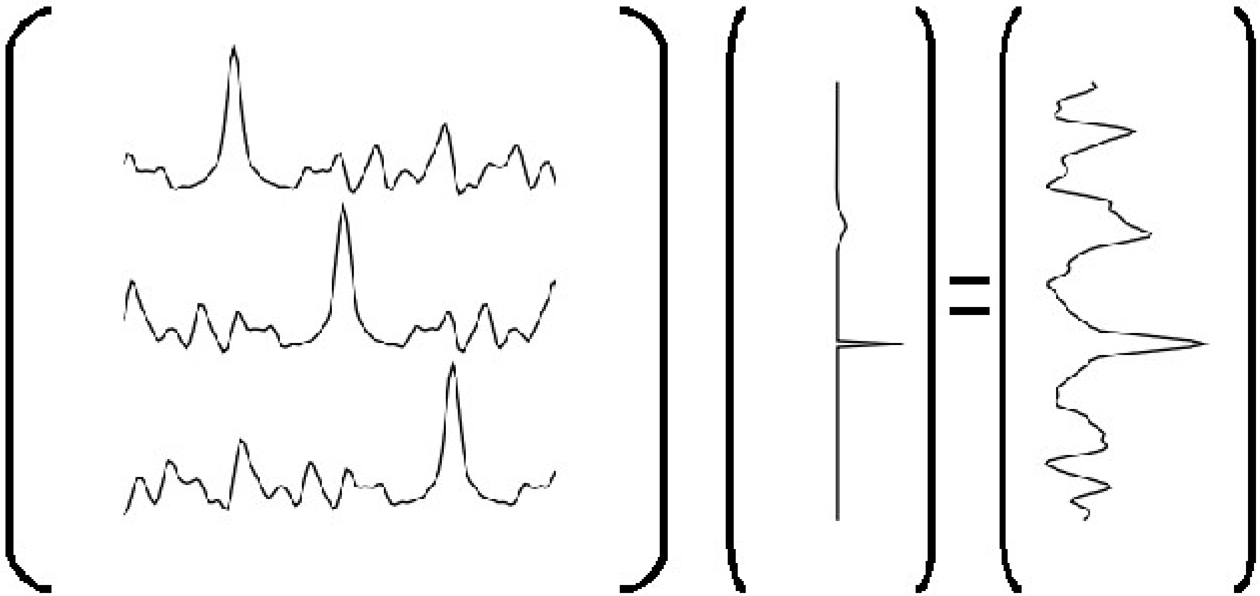
Iterative Solution : $\underbrace{I^m_{i+1} = I^m_i + g[A^T W A]^+}_{\text{Deconvolution}} \underbrace{\left(A^T W \left(V^{obs} - A I^m_i\right)\right)}$

Deconvolution

Imaging (Gridding + iFT)

Prediction (FT + de-Gridding)

# Extended emission : Standard CLEAN



CLEAN

- Calculate dirty image
- Normalize by sum of weights
- Add location/amp of peak to model
- Update residuals

Delta-functions are an inefficient way of modeling extended emission. $I^m = \sum_i a_i \delta(l - l_i)$

Pick basis functions that represent shapes in which most of the signal power is concentrated.

=> Use a basis in which the signal is ' sparse ' (described by a few parameters)

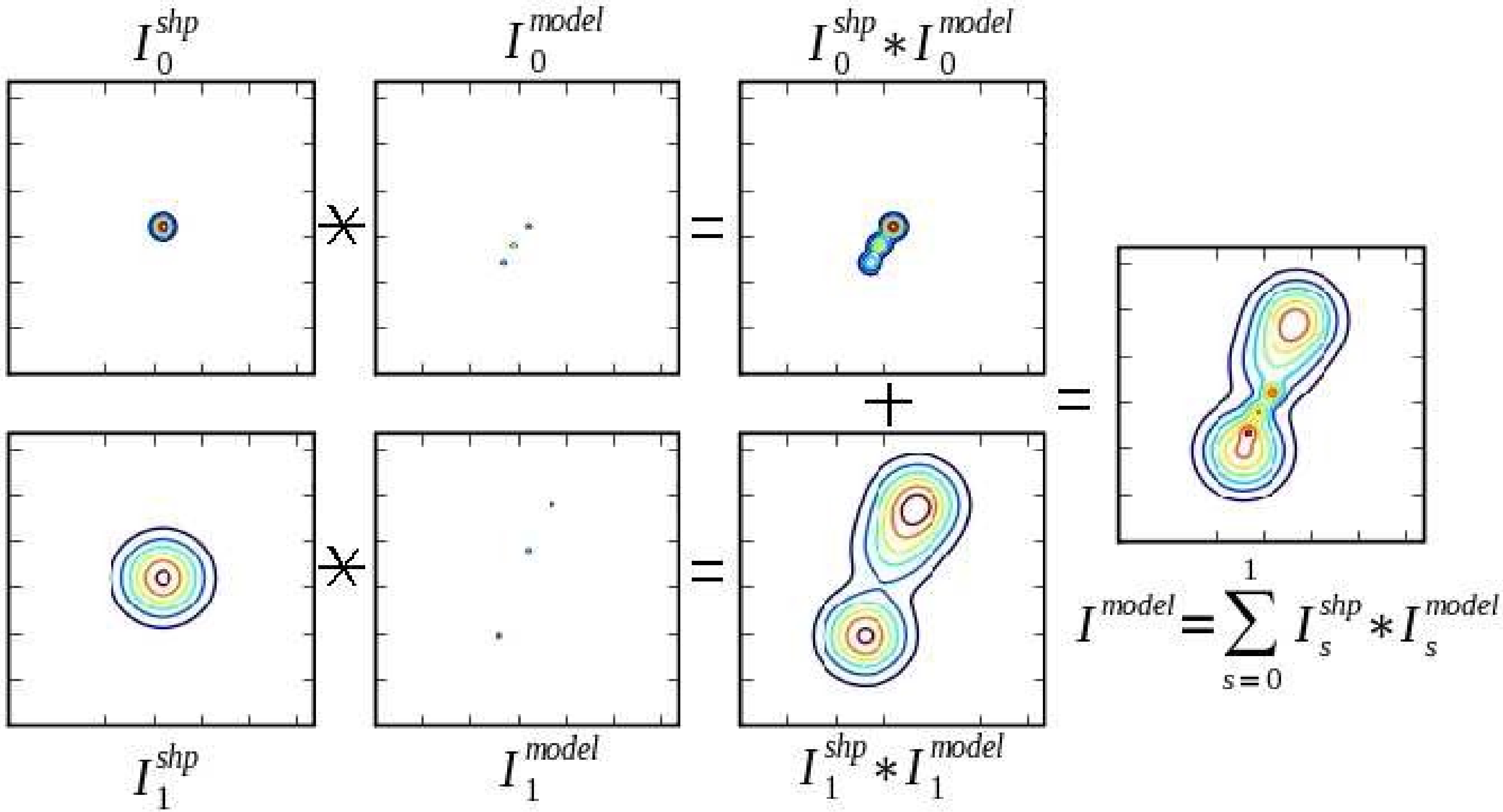
These concepts are gaining popularity as 'compressive-sampling' or 'compressed-sensing' methods in signal/image-processing

+ some new ideas on the optimization strategy (minimize L1 instead of L2).

# Extended emission : Multi-Scale CLEAN

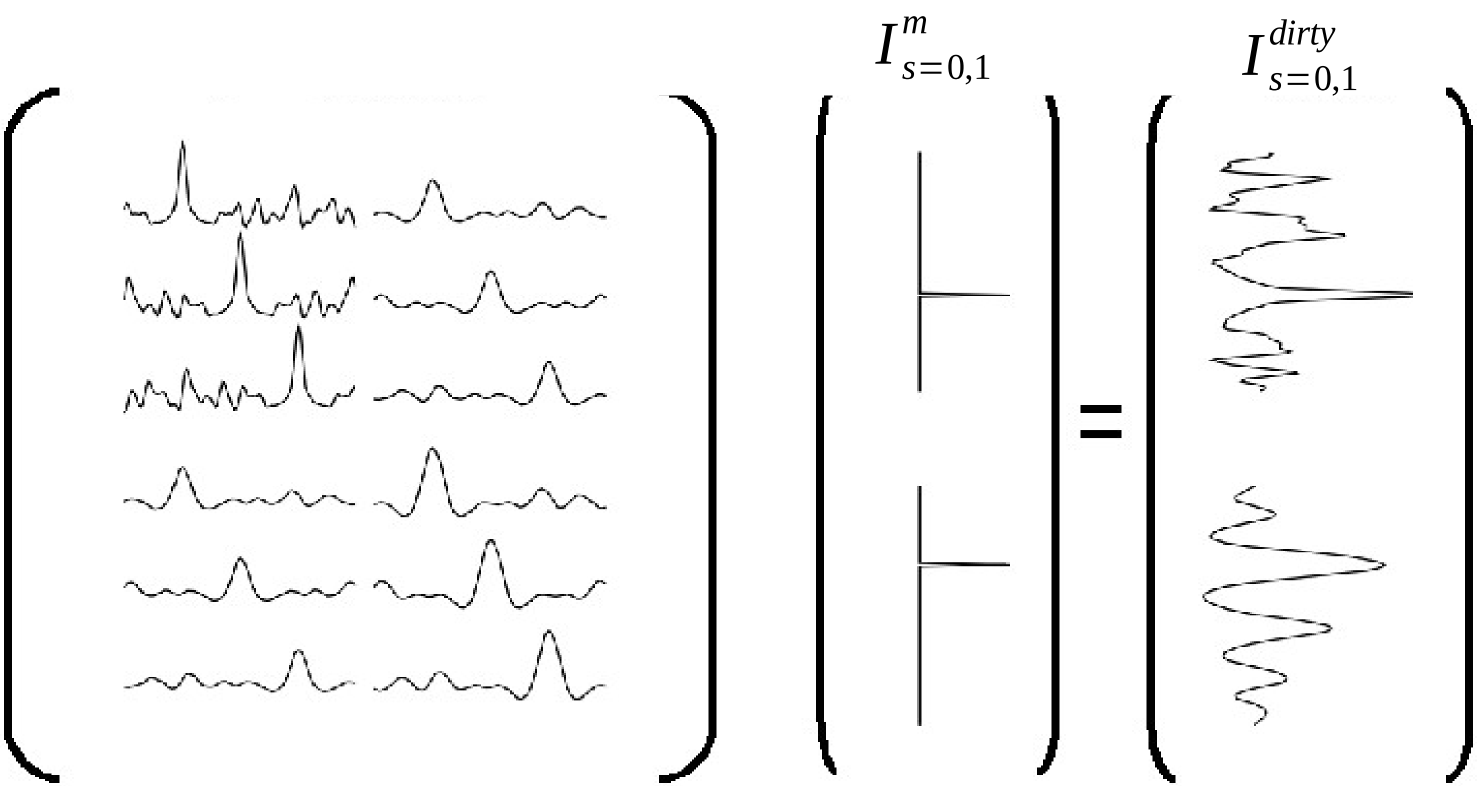
Multi-Scale Sky Model : Linear combination of 'blobs' of different scale sizes

$I^{sky} = \sum_s [I_s^{shp} * I_s^m]$ where $I_s^{shp}$ is a blob of size 's' and $I_s^m = \sum_i a_{s,i} \delta(l - l_{s,i})$



# Extended emission : Multi-Scale CLEAN

Imaging Equations : Multiple 'dirty' images, each a linear-combination of convolutions




MS-CLEAN

- Smooth residual by different scales (calc RHS)

- Calculate principal solution (2x2 matrix-multiplication per pixel)

- Find peak flux and scale. Add to model

- Subtract response from all residuals (calc LHS)

Other multi-scale algorithms :

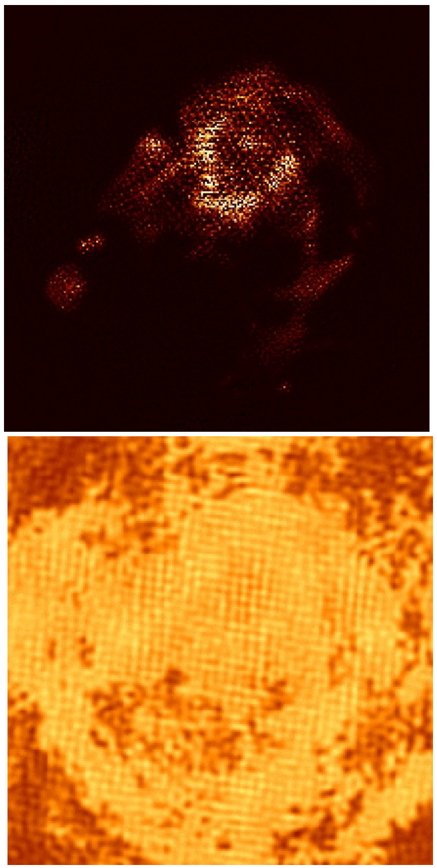
MS-Clean (Cornwell), MS-Clean (Greisen) are variants of the above.

ASP-Clean (model sky as a set of Gaussians –solve for parameters)

# Imaging examples with extended emission

## CLEAN

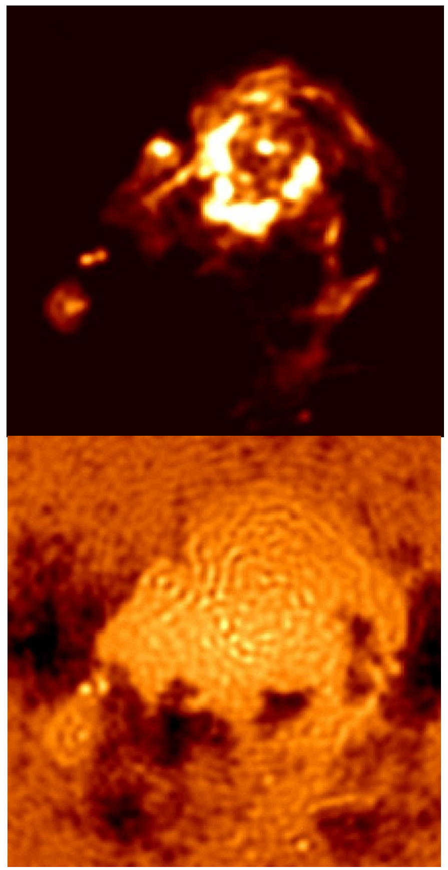
Minimize L2
(assume sparsity
in the image)



(Hogbom 1974, Clark 1980,
Schwab & Cotton 1983 )

## MEM

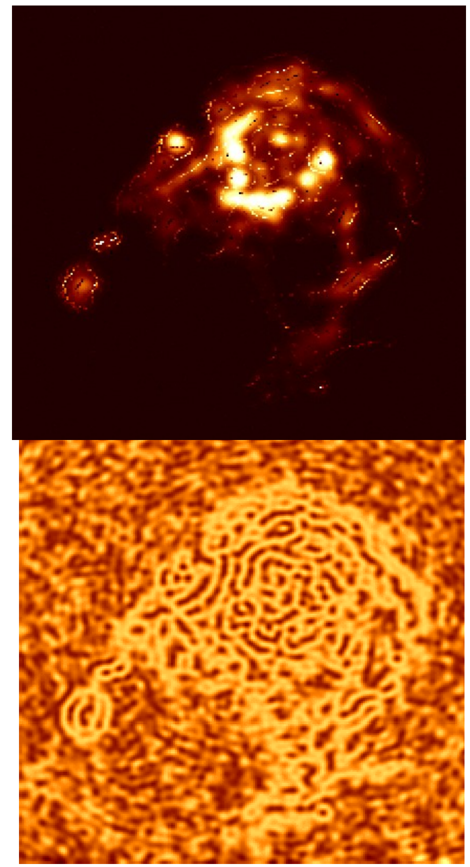
Minimize L2 subject to
an entropy-based prior
(e.g. smoothness)



( Cornwell &
Evans, 1985)

## MS-CLEAN

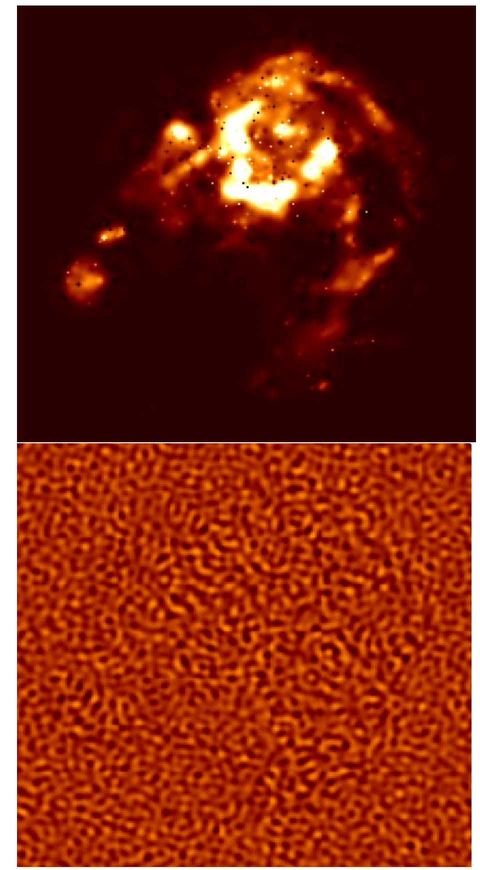
Minimize L2
(assume a set of
spatial scales)



(Cornwell, 2008)

## ASP

Minimize L2 with
TV-based subspace
searches

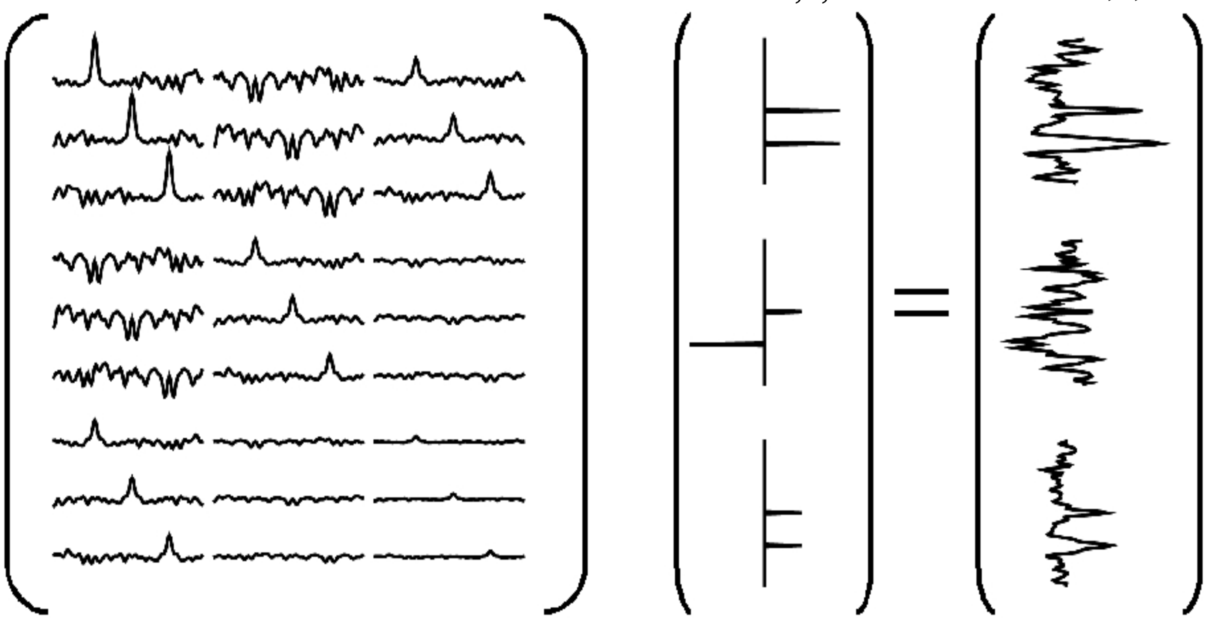


(Bhatnagar &
Cornwell 2004)

# Wide-Band Imaging : Multi-Frequency CLEAN

Model the sky spectrum with polynomial coefficients : $I_\nu^{sky} = \sum_t I_t^m \left( \frac{\nu - \nu_0}{\nu_0} \right)^t$
(sparse representation of a smooth spectrum)

$I^m_{t=0,1,2}$ $I^{dirty}_{t=0,1,2}$



**MF-CLEAN** (and variants)

- Construct dirty-images as weighted averages in frequency

- Calculate principal solution
( 3x3 matrix mult. per pixel )

- Pick Taylor-coefficient set that gives the biggest change in $\chi^2$

- Subtract response from all residuals (evaluate LHS)

Data Products : $I^m_{0,} I^m_{1,} I^m_{2,} \ldots$

Power Law model

$$I_\nu = I_{\nu_0} \left( \frac{\nu}{\nu_0} \right)^{\alpha + \beta \log(\nu / \nu_0)}$$

Interpret Taylor-coefficients as a Power-Law :

$$I^m_0 = I_{\nu_0} \qquad I^m_1 = I_{\nu_0} \alpha \qquad I^m_2 = I_{\nu_0} \left( \frac{\alpha(\alpha - 1)}{2} + \beta \right)$$

# Multi-Scale Multi-Frequency Clean

Sky Model : Collection of 'blobs' whose amplitudes follow a polynomial in frequency.

Multi-Frequency : $I_{\nu}^{sky} = \sum_t I_t \left( \frac{\nu - \nu_0}{\nu_0} \right)^t$ Multi-Scale : $I_t = \sum_s [ I_s^{shp} * I_{s,t} ]$

$$\left[ \begin{array}{cccccc} [H_{\substack{s=0,p=0\\t=0,q=0}}] & [H_{\substack{s=0,p=0\\t=0,q=1}}] & [H_{\substack{s=0,p=0\\t=0,q=2}}] & [H_{\substack{s=0,p=1\\t=0,q=0}}] & [H_{\substack{s=0,p=1\\t=0,q=1}}] & [H_{\substack{s=0,p=1\\t=0,q=2}}] \\ [H_{\substack{s=0,p=0\\t=1,q=0}}] & [H_{\substack{s=0,p=0\\t=1,q=1}}] & [H_{\substack{s=0,p=0\\t=1,q=2}}] & [H_{\substack{s=0,p=1\\t=1,q=0}}] & [H_{\substack{s=0,p=1\\t=1,q=1}}] & [H_{\substack{s=0,p=1\\t=1,q=2}}] \\ [H_{\substack{s=0,p=0\\t=2,q=0}}] & [H_{\substack{s=0,p=0\\t=2,q=1}}] & [H_{\substack{s=0,p=0\\t=2,q=2}}] & [H_{\substack{s=0,p=1\\t=2,q=0}}] & [H_{\substack{s=0,p=1\\t=2,q=1}}] & [H_{\substack{s=0,p=1\\t=2,q=2}}] \\ [H_{\substack{s=1,p=0\\t=0,q=0}}] & [H_{\substack{s=1,p=0\\t=0,q=1}}] & [H_{\substack{s=1,p=0\\t=0,q=2}}] & [H_{\substack{s=1,p=1\\t=0,q=0}}] & [H_{\substack{s=1,p=1\\t=0,q=1}}] & [H_{\substack{s=1,p=1\\t=0,q=2}}] \\ [H_{\substack{s=1,p=0\\t=1,q=0}}] & [H_{\substack{s=1,p=0\\t=1,q=1}}] & [H_{\substack{s=1,p=0\\t=1,q=2}}] & [H_{\substack{s=1,p=1\\t=1,q=0}}] & [H_{\substack{s=1,p=1\\t=1,q=1}}] & [H_{\substack{s=1,p=1\\t=1,q=2}}] \\ [H_{\substack{s=1,p=0\\t=2,q=0}}] & [H_{\substack{s=1,p=0\\t=2,q=1}}] & [H_{\substack{s=1,p=0\\t=2,q=2}}] & [H_{\substack{s=1,p=1\\t=2,q=0}}] & [H_{\substack{s=1,p=1\\t=2,q=1}}] & [H_{\substack{s=1,p=1\\t=2,q=2}}] \end{array} \right] \left[ \begin{array}{c} \vec{I}^{sky}_{\substack{p=0\\q=0}} \\ \vec{I}^{sky}_{\substack{p=0\\q=1}} \\ \vec{I}^{sky}_{\substack{p=0\\q=2}} \\ \vec{I}^{sky}_{\substack{p=1\\q=0}} \\ \vec{I}^{sky}_{\substack{p=1\\q=1}} \\ \vec{I}^{sky}_{\substack{p=1\\q=2}} \end{array} \right] = \left[ \begin{array}{c} \vec{I}^{dirty}_{\substack{s=0\\t=0}} \\ \vec{I}^{dirty}_{\substack{s=0\\t=1}} \\ \vec{I}^{dirty}_{\substack{s=0\\t=2}} \\ \vec{I}^{dirty}_{\substack{s=1\\t=0}} \\ \vec{I}^{dirty}_{\substack{s=1\\t=1}} \\ \vec{I}^{dirty}_{\substack{s=1\\t=2}} \end{array} \right]$$

## MS-MFS

- Construct dirty-images as weighted averages in frequency + smooth to different scales.

- Block-diagonal approximation for scales, principal solution for Taylor-coefficients

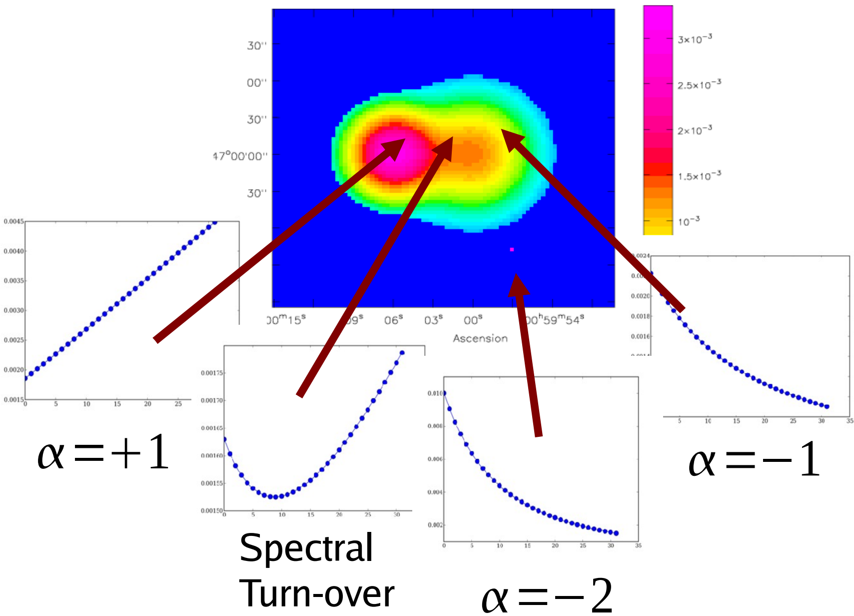
- Pick components and add to models.

- Update all residual images

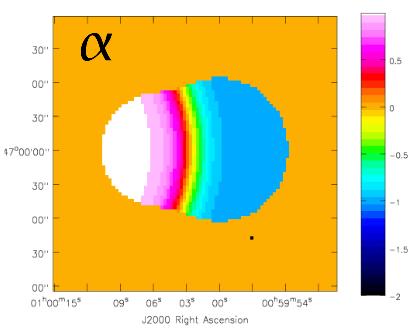
Data products are Taylor-coefficient-images (combined across scales) : $I_{0,}^m I_{1,}^m I_{2,}^m \ldots$

# Example of wideband-imaging on extended-emission

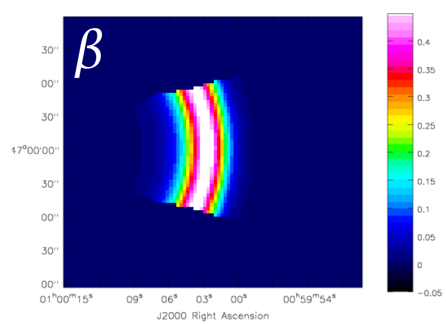
Intensity Image




Average Spectral Index

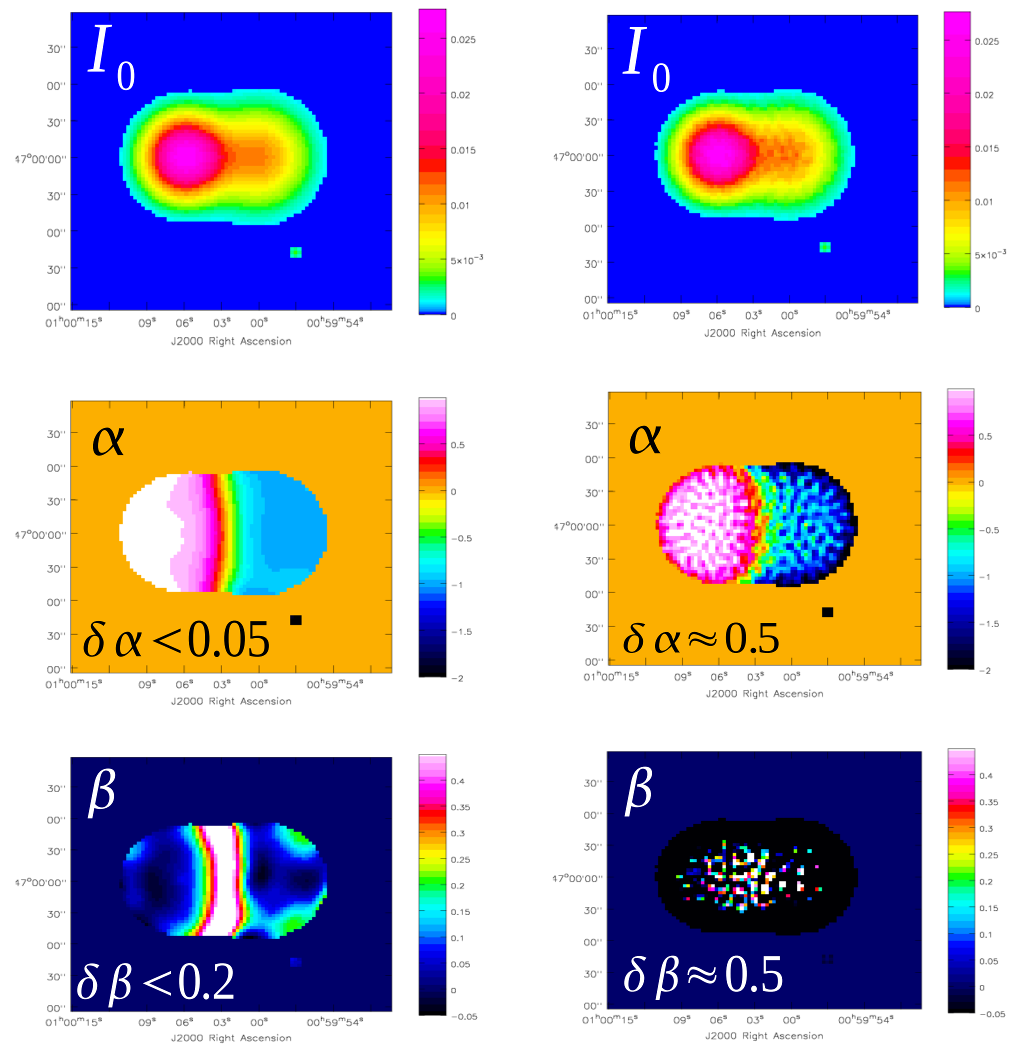



Gradient in Spectral Index




**MFS**

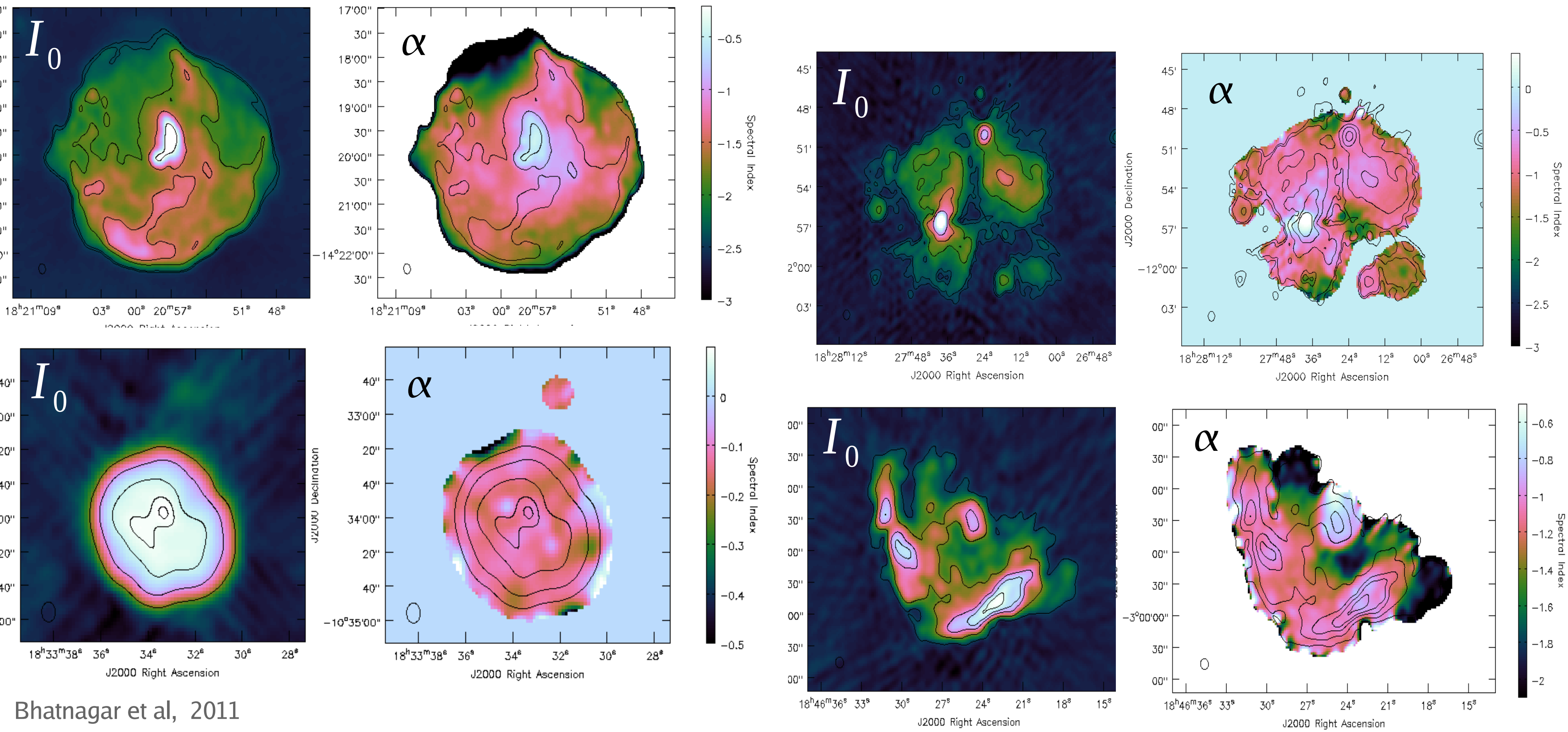
**multi-scale** **point-source**




=> For extended emission......

......... a multi-scale model gives better spectral index and curvature maps

# Extended emission – SNR example (a realistic expectation)



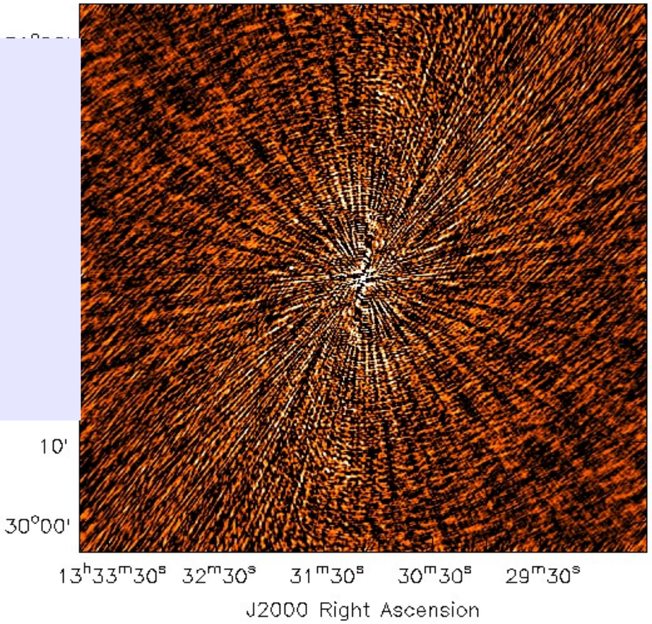

Bhatnagar et al, 2011

Wide-band Galactic-Plane SNR survey (pilot) [ MS-MFS with nterms=2, and about 5 scales ]

=> Within L-band and C-band, can tell-apart regions by their spectral-index ( +/- 0.2 ) if SNR>100.

=> These images have a dynamic-range limit of few x 1000

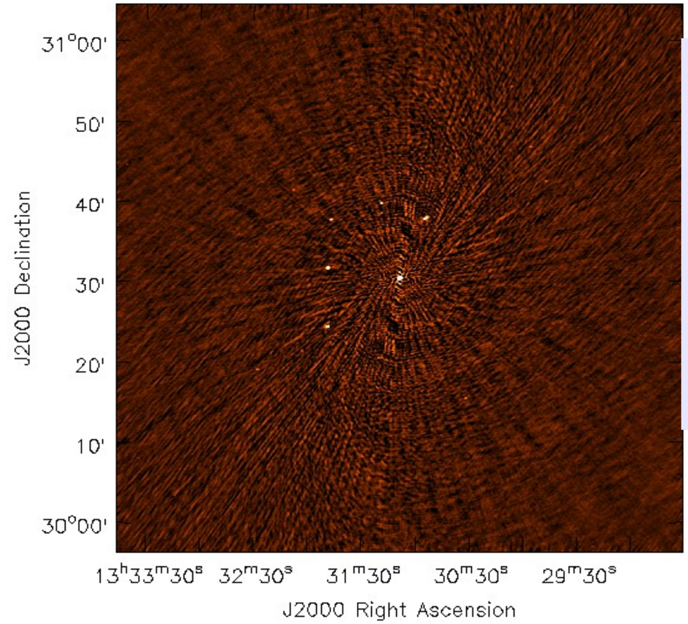
# Dynamic-range with MS-MFS : 3C286 example : Nt=1,2,3,4




**NTERMS = 1**

Rms :
9 mJy -- 1 mJy

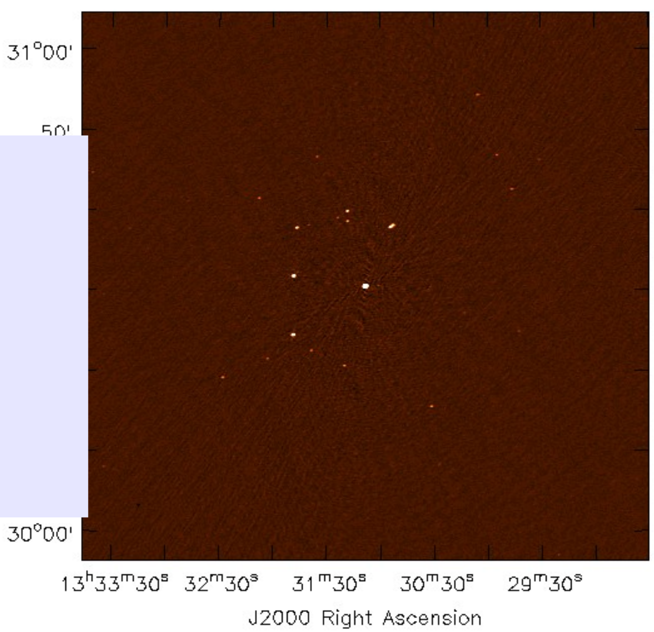
DR :
1600 -- 13000




**NTERMS = 2**

Rms :
1 mJy -- 0.2 mJy

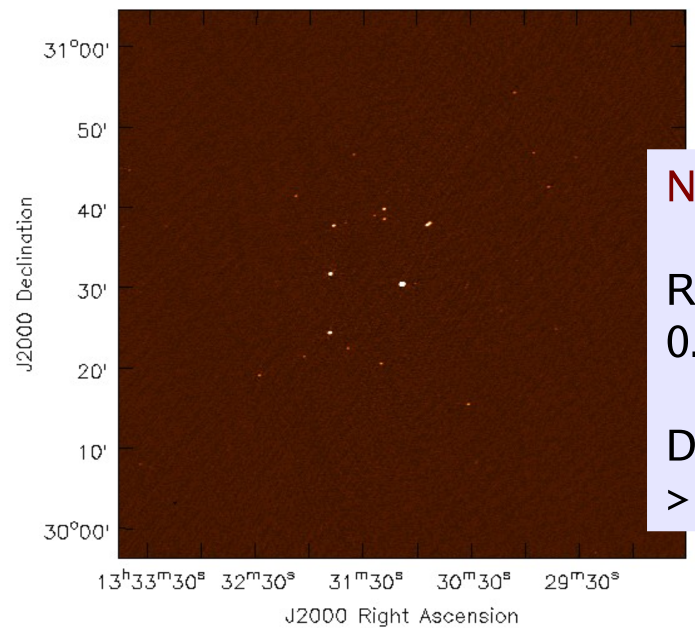
DR :
10,000 -- 17,000




**NTERMS = 3**

Rms :
0.2 mJy -- 85 uJy

DR :
65,000 -- 170,000



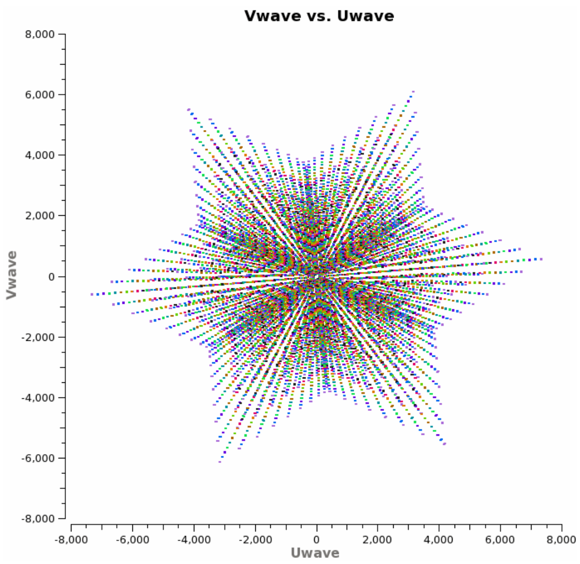

**NTERMS = 4**

Rms
0.14 mJy -- 80 uJy

DR :
>110,000 -- 180,000

# Other methods based on linear-combinations

Multi-Frequency-Synthesis <==> Earth-rotation synthesis




- Source structure that varies smoothly with time :

  Taylor-polynomial in time : $I_t^{sky} = \sum_i I_i^m \left( \frac{t - t_{ref}}{t_{ref}} \right)^i$

- Frequency-dependence of Stokes Q,U,V.

- Source structure that varies with (erratically) with time and (smoothly) with frequency

  => need different basis functions : Fourier sinusoids and Chebyshev polynomials

  Stewart et al, 2011, based on Sault-Wieringa 1994

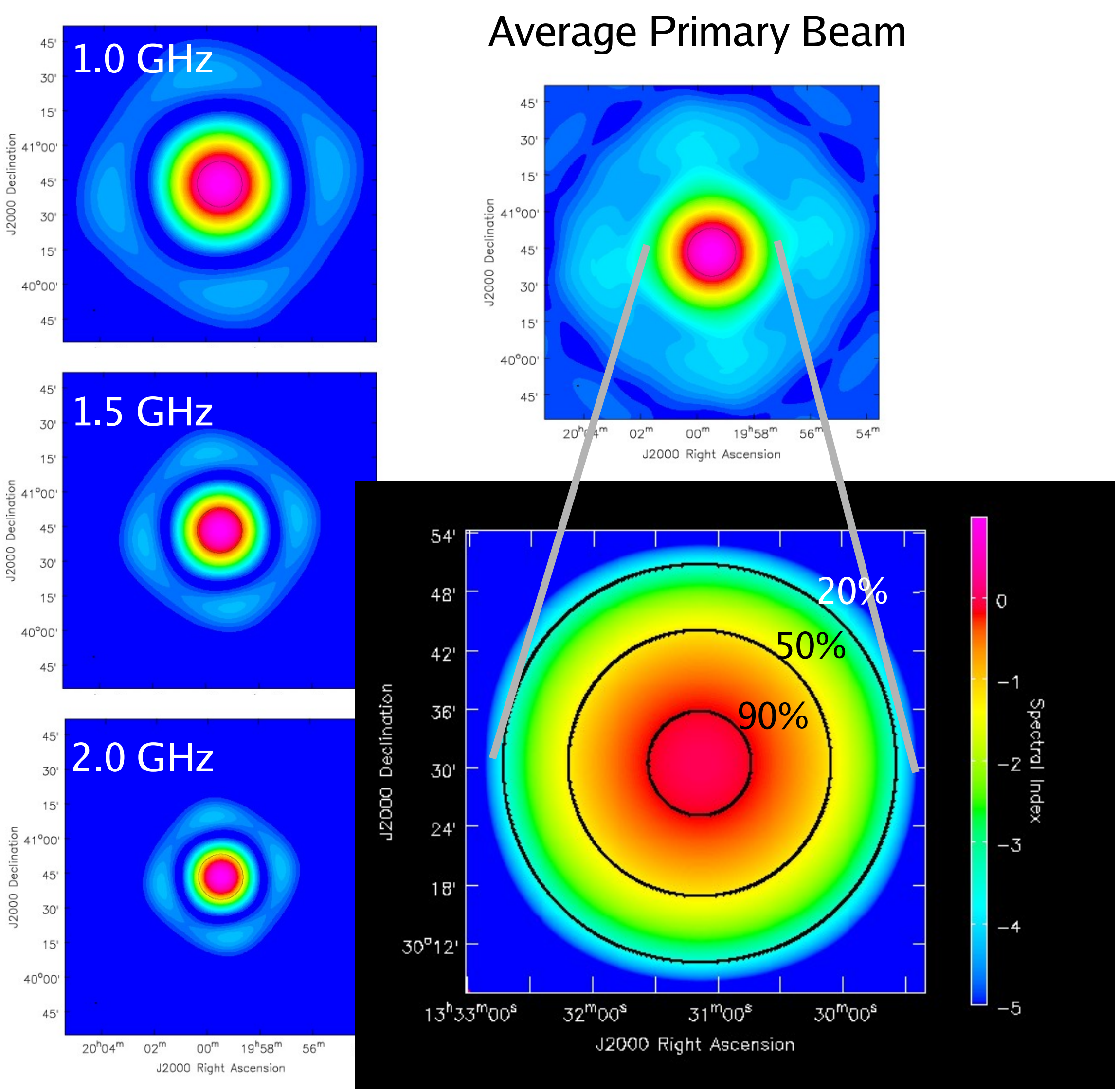
- Can add in multi-scale support to both of the above ideas (same ideas as MS-MFS)

- Stokes parameters : I = RR+LL, V = RR-LL

  => Another set of linear equations : Can do a joint I and V deconvolution.

# Handling Instrumental effects in the sky-domain

## Multi-Frequency Primary Beams




Spectral Index of PB

The sky model can absorb some time-invariant and real-valued instrumental effects

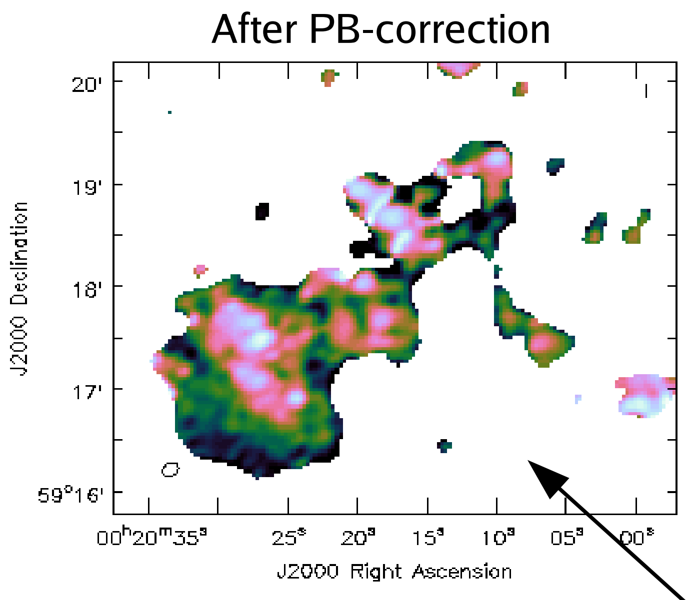
e.g. frequency-dependent PB gain for a non-squinted and non-rotating antennas.

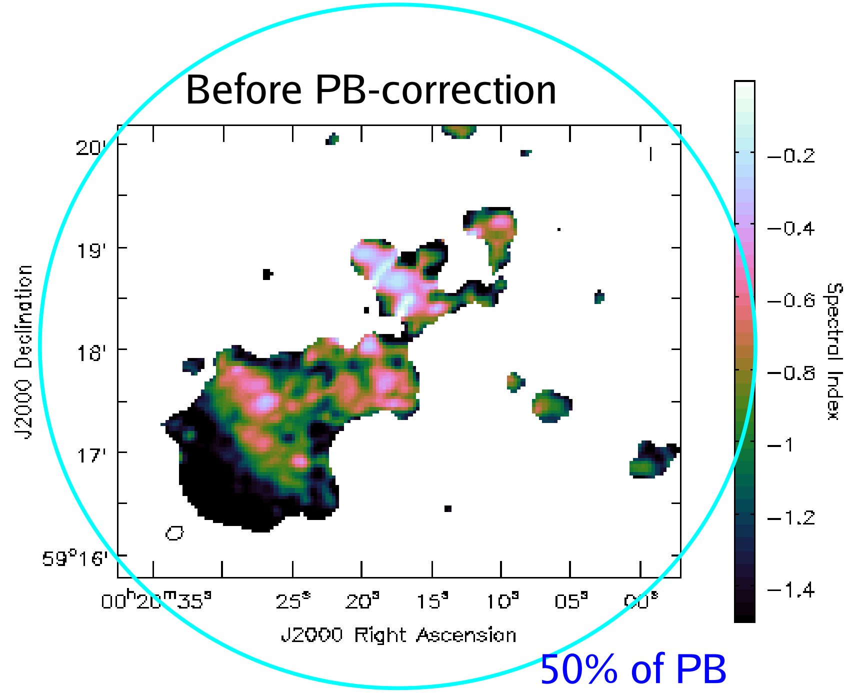
MS-MFS Taylor-coefficients represent the product of the sky and PB spectra.

=> Divide out the PB-spectrum, post-deconvolution.

$$\frac{(I_{0,}^{m} I_{1,}^{m} I_{2,}^{m} \dots)}{(P_{0,} P_{1,} P_{2,} \dots)} = (I_{0,}^{sky} I_{1,}^{sky} I_{2}^{sky} \dots)$$

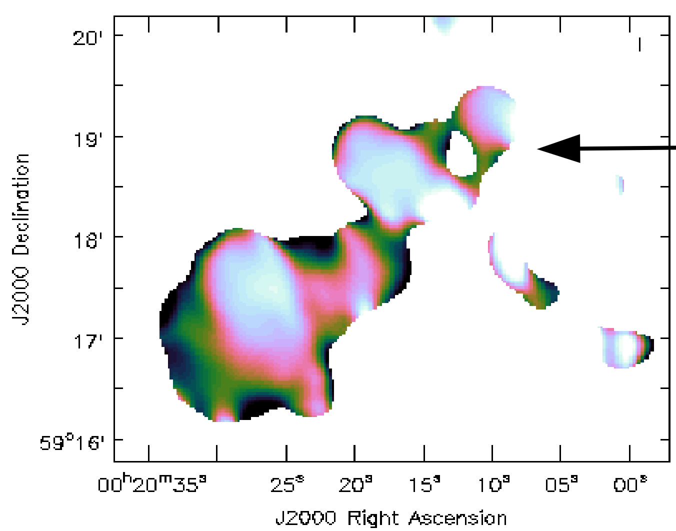
# Continuum (MS-MFS) vs Cube Imaging (with PB-correction)




IC10 Dwarf Galaxy :

Spectral Index across C-Band.



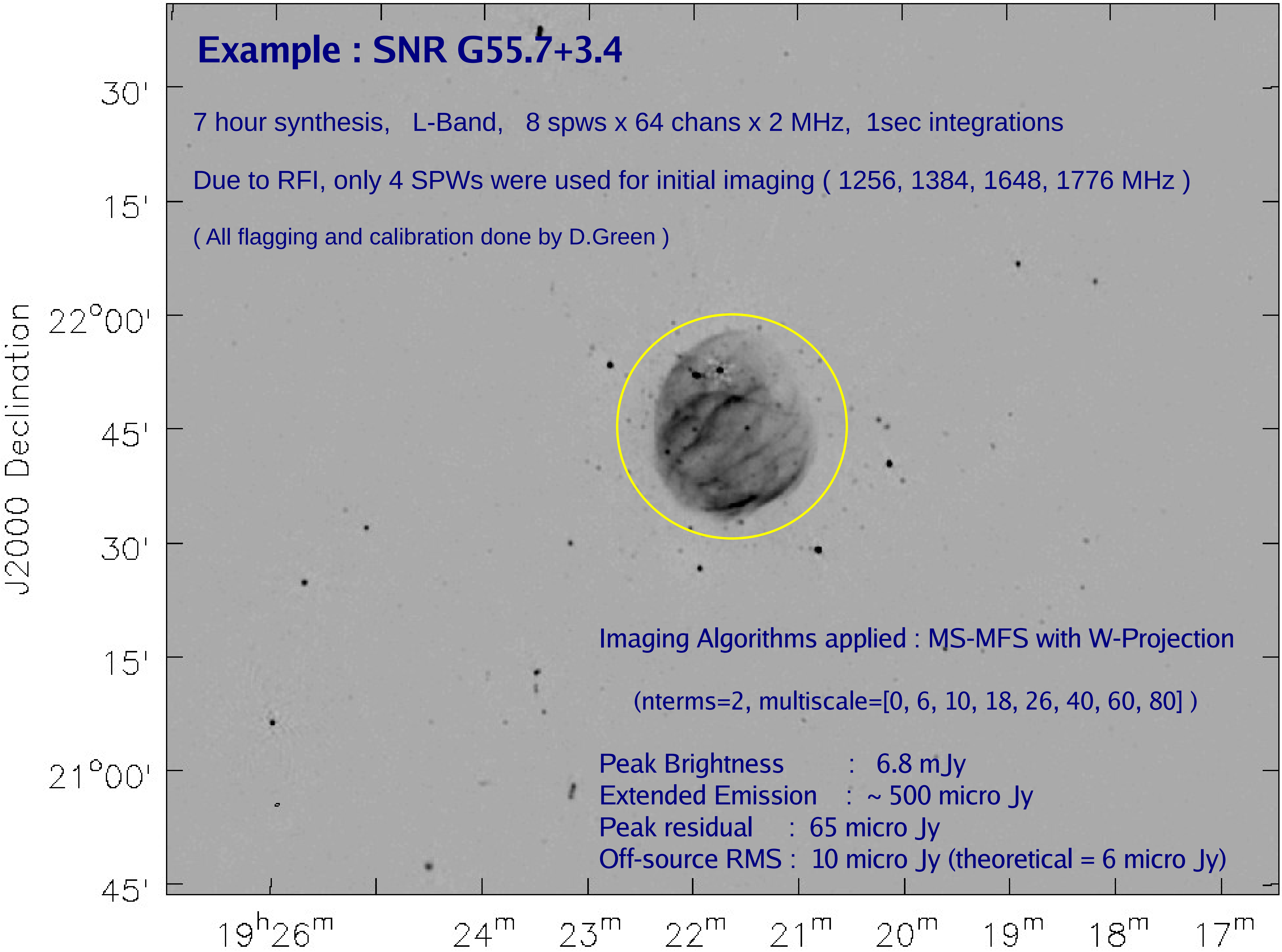

Heesen et al, 2011

MS-MFS : Result of wide-band PB-correction after MT-MS-MFS.

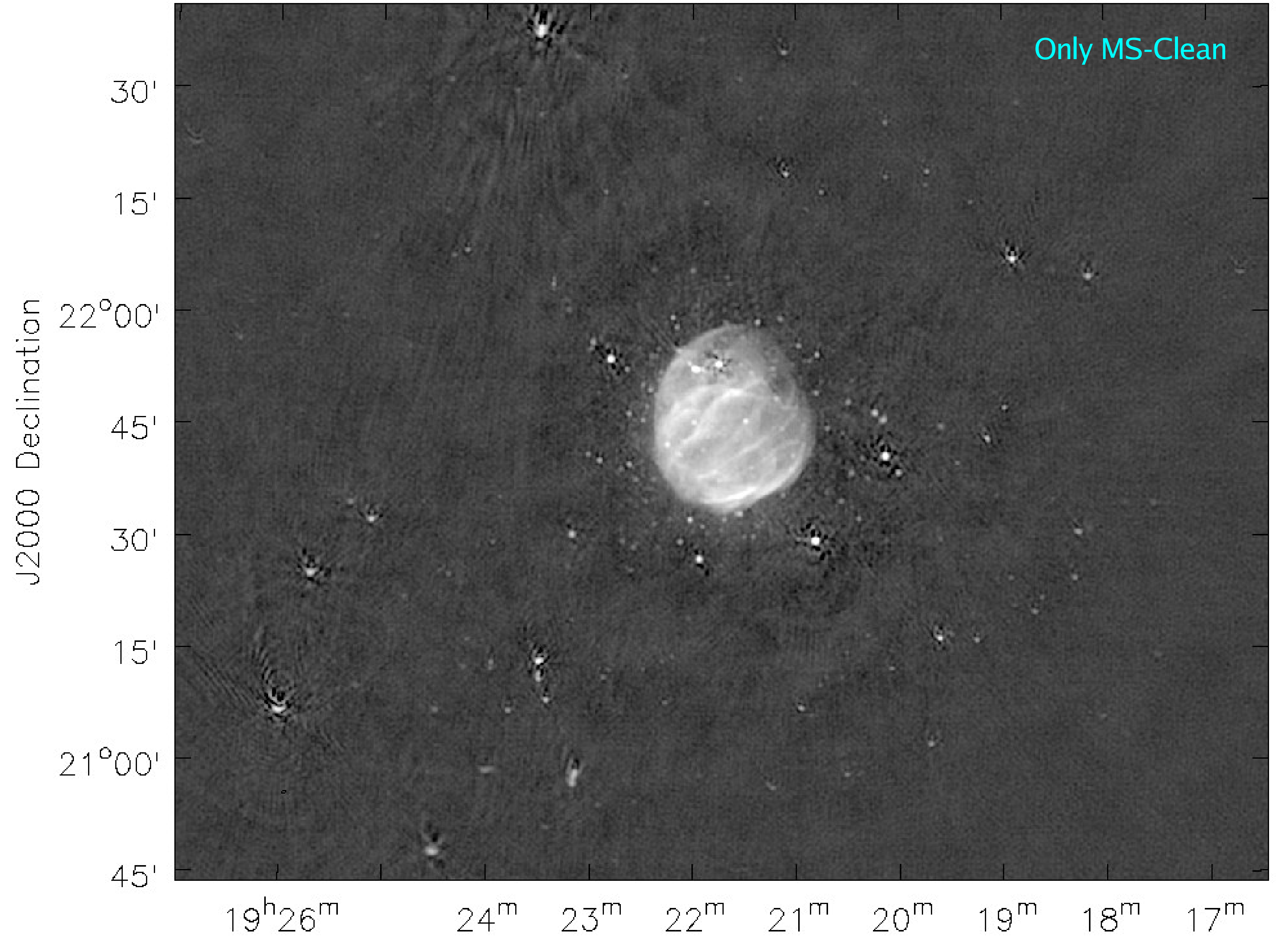
Cube : Spectral-index map made by PB-correcting single-SPW images smoothed to the lowest resolution.

Any post-deconvolution PB-correction assumes that the primary-beam does not vary / rotate during the observation.

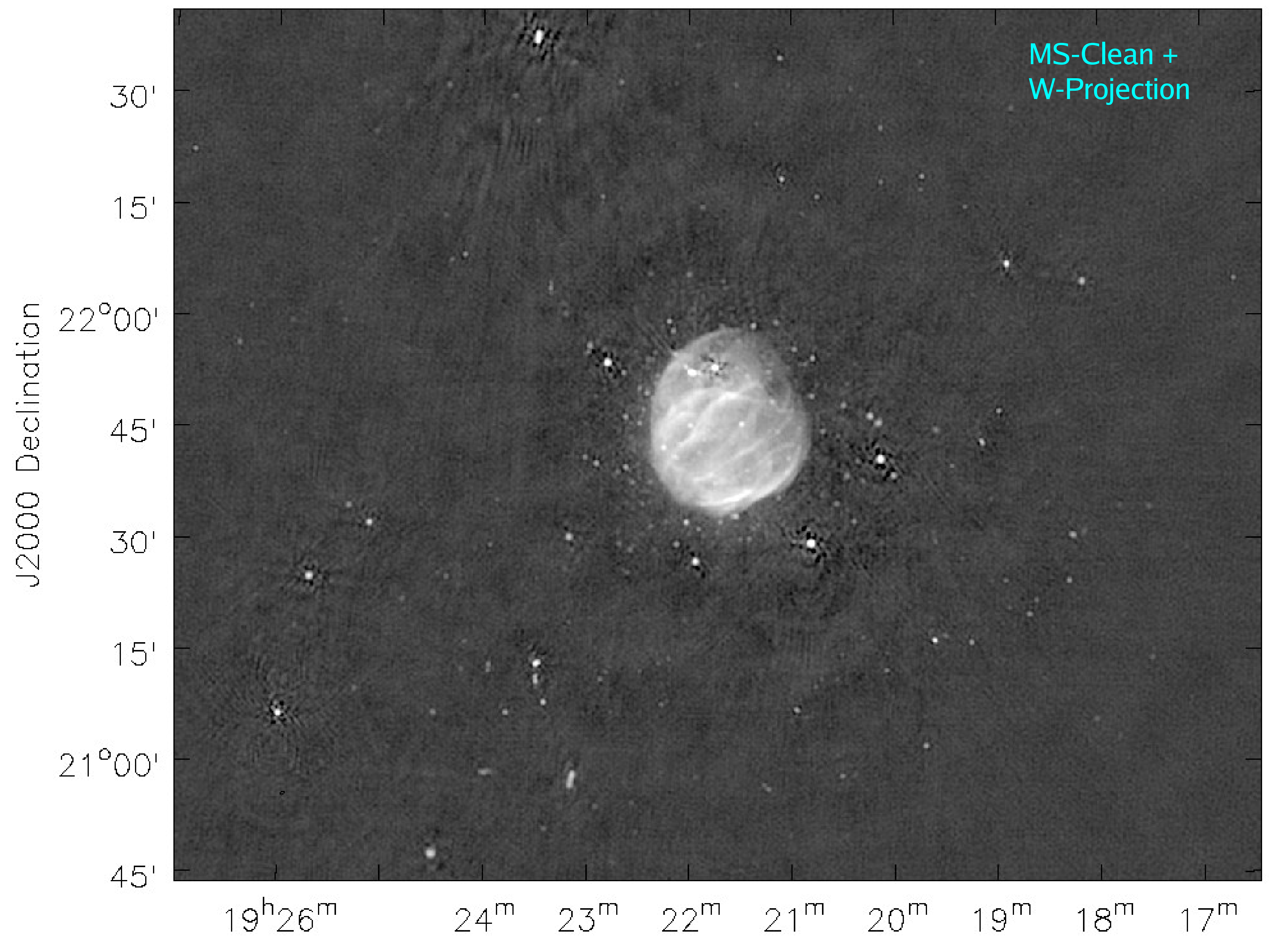
Projection Methods : W-term, PB gain, PB offsets ( pointing, squint, mosaics ), PB-frequency-dependence.......
--- Careful choices of Gridding Convolution functions.



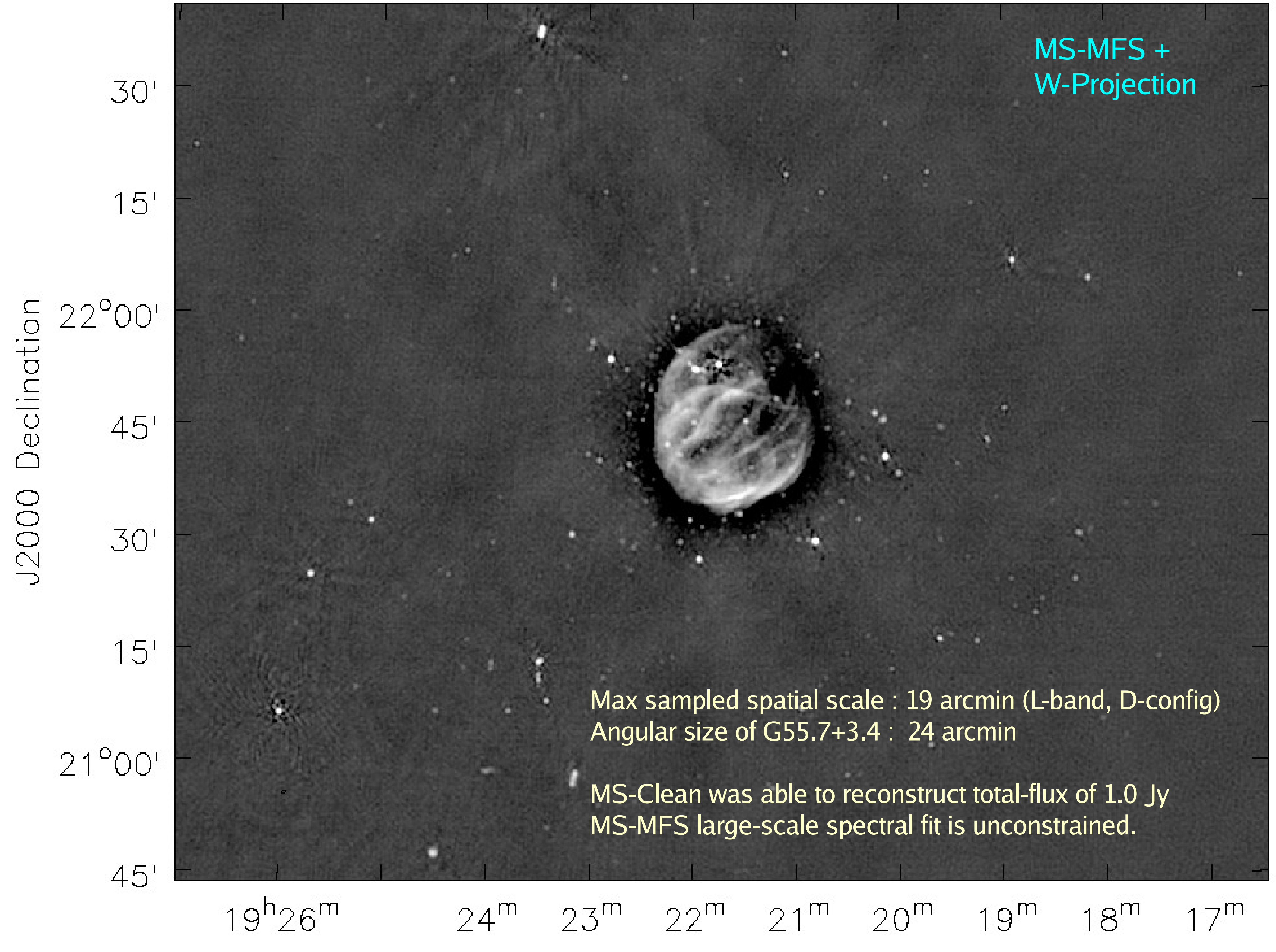
Example : SNR G55.7+3.4
7 hour synthesis, L-Band, 8 spws x 64 chans x 2 MHz, 1sec integrations
Due to RFI, only 4 SPWs were used for initial imaging ( 1256, 1384, 1648, 1776 MHz )
( All flagging and calibration done by D.Green )
Imaging Algorithms applied : MS-MFS with W-Projection
(nterms=2, multiscale=[0, 6, 10, 18, 26, 40, 60, 80] )
Peak Brightness : 6.8 mJy
Extended Emission : ~ 500 micro Jy
Peak residual : 65 micro Jy
Off-source RMS : 10 micro Jy (theoretical = 6 micro Jy)
J2000 Declination
30'
15'
22°00'
45'
30'
15'
21°00'
45'
19h26m
24m
23m
22m
21m
20m
19m
18m
17m

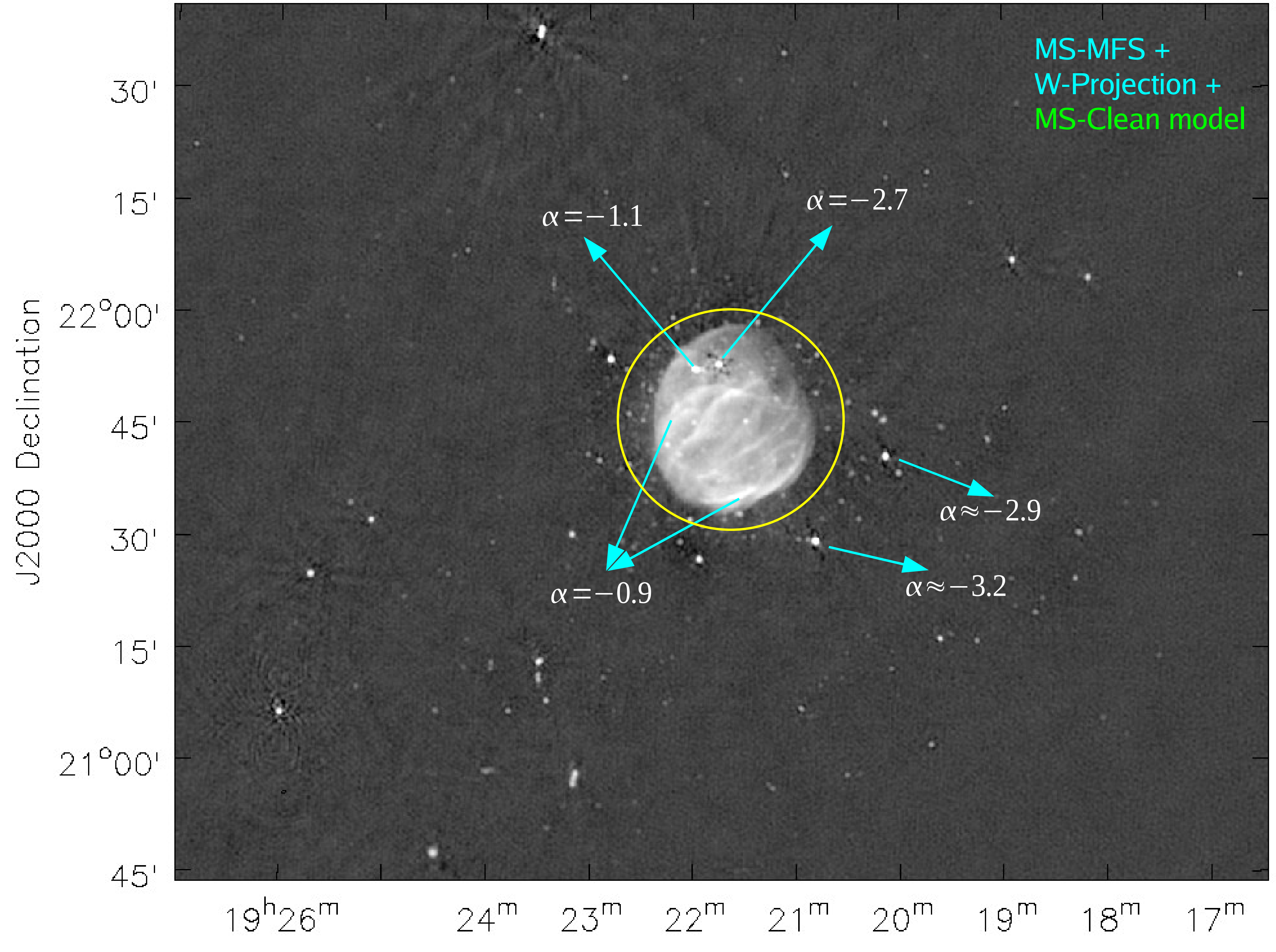


Only MS-Clean
30'
15'
22°00'
45'
30'
15'
21°00'
45'
J2000 Declination
19h26m
24m
23m
22m
21m
20m
19m
18m
17m



MS-Clean +
W-Projection
J2000 Declination
30'
15'
22°00'
45'
30'
15'
21°00'
45'
19h26m
24m
23m
22m
21m
20m
19m
18m
17m


MS-MFS +
W-Projection
Max sampled spatial scale : 19 arcmin (L-band, D-config)
Angular size of G55.7+3.4 : 24 arcmin
MS-Clean was able to reconstruct total-flux of 1.0 Jy
MS-MFS large-scale spectral fit is unconstrained.
J2000 Declination
30'
15'
22°00'
45'
30'
15'
21°00'
45'
19h26m
24m
23m
22m
21m
20m
19m
18m
17m



MS-MFS +
W-Projection +
MS-Clean model
$\alpha = -1.1$
$\alpha = -2.7$
$\alpha \approx -2.9$
$\alpha = -0.9$
$\alpha \approx -3.2$
J2000 Declination
30'
15'
22°00'
45'
30'
15'
21°00'
45'
19h26m
24m
23m
22m
21m
20m
19m
18m
17m

# Example of wide-field sensitivity, because of wide-bandwidths

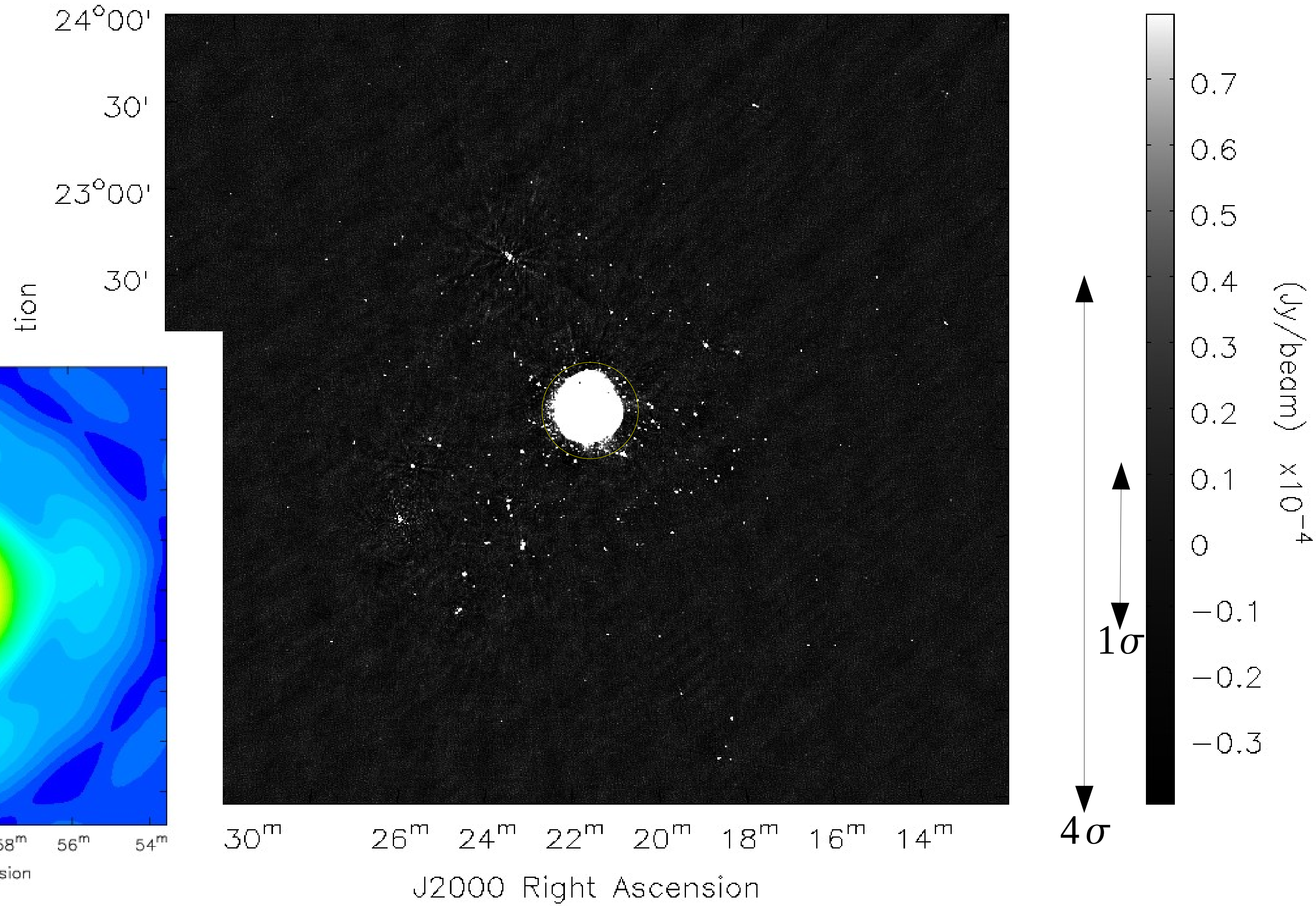
G55.7+3.4 : Galactic supernova remnant : 4 x 4 degree field-of-view from one EVLA pointing

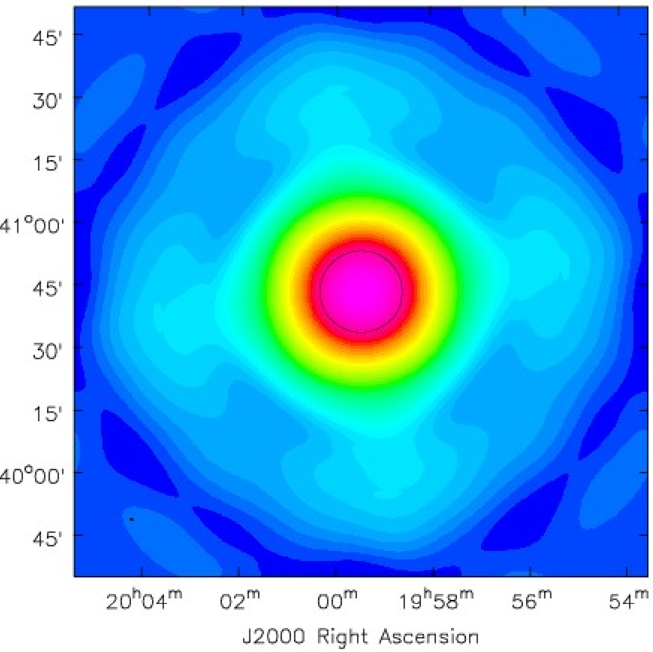
1 Jy total flux

24 arcmin

(PB: 30 arcmin)

10 micro Jy RMS



# Summary

Image reconstruction is done via an iterative least-squares minimization.

Sky domain : natural domain to parameterize the sky signal (non-closing). Sees only averages.

Minor Cycle : Construct a model from the 'dirty' image(s). Algorithms differ in choice of model parameters, objective function, optimization strategy.

→ Clean (hogbom, clark), mem, msclean, asp, mfclean, ms-mfs, …)

Data domain : model instrumental effects and correct them ( 'calibration' )

Major Cycle : Model prediction (FT+de-gridding), Residual calculation, Imaging (gridding+IFT). Algorithms differ in the choice of gridding-convolution function (Sanjay's talk : July 9)

W-Projection, A-Projection, WB-A-Projection, Pointing Offsets and Mosaicing.

=> A combination of all these methods is required for HDR imaging. LDR imaging (few 1000) is possible through various approximate methods (as has always been the case).

==> Already getting EVLA proposals for low-freq wide-band mosaics with A-config