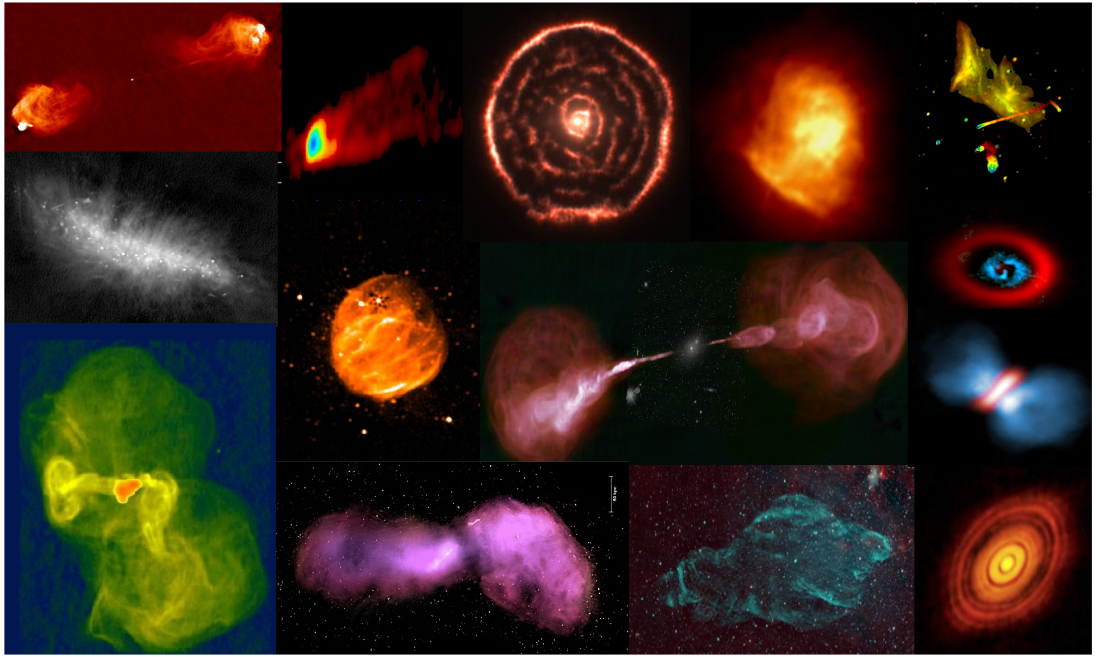
Imaging in Radio Interferometry



Urvashi Rau
National Radio Astronomy Observatory, Socorro, NM, USA

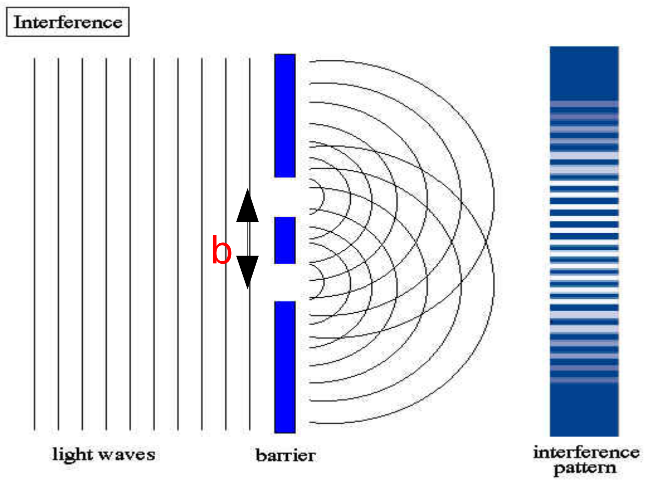
VLTI and ALMA Synthesis Imaging Workshop (9-12 January 2023)

Outline

- Imaging with a radio interferometer
    - Image formation
    - The forward and inverse problems
    - Algorithms and software
- Current limits and future trends

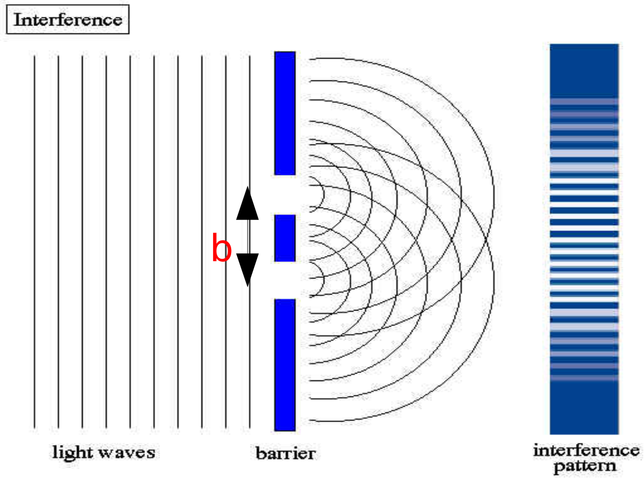
An indirect imaging device

Young's double slit experiment



An indirect imaging device

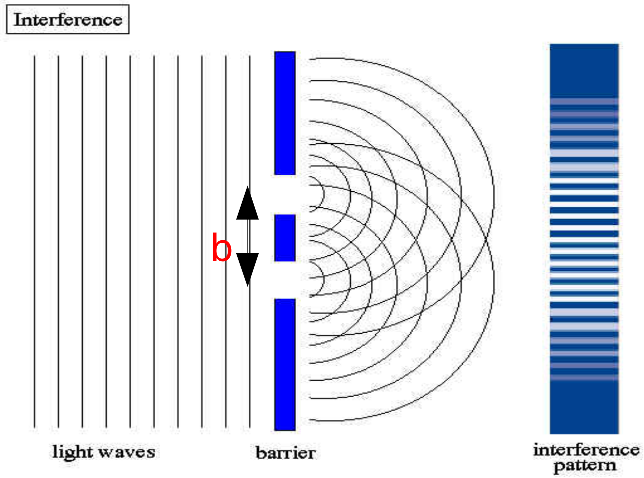
Young’s double slit experiment



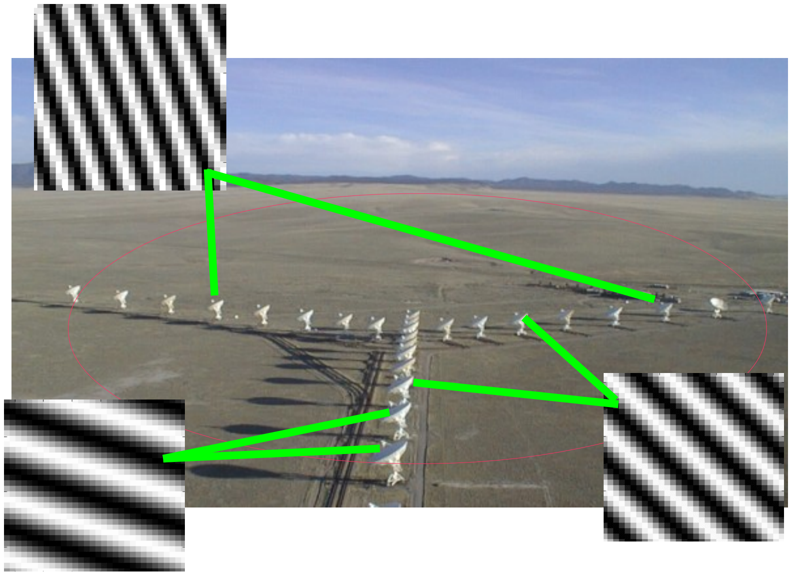
Instrument : An array of detectors

An indirect imaging device

Young's double slit experiment



Each antenna-pair measures the parameters of one 'fringe'.

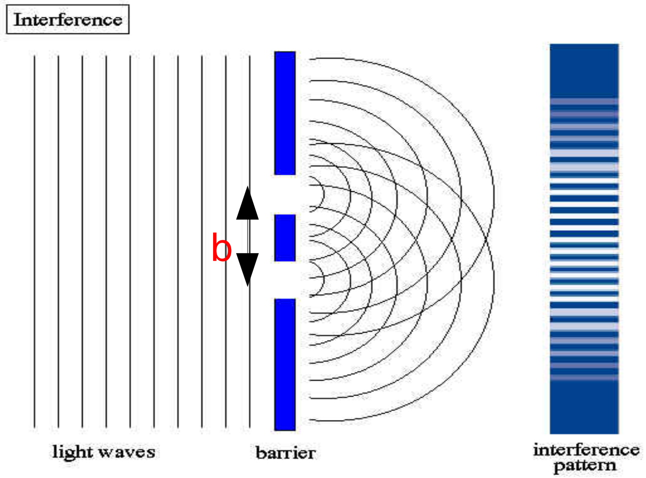


Measured Fringe Parameters :

Amplitude, Phase
Orientation, Wavelength

An indirect imaging device

Young’s double slit experiment



2D Fourier transform :

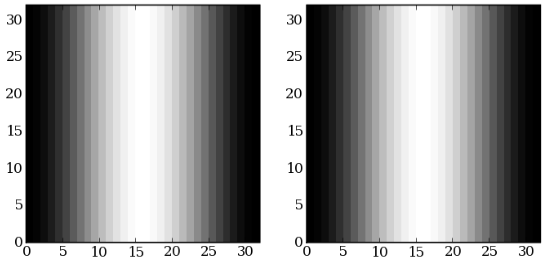
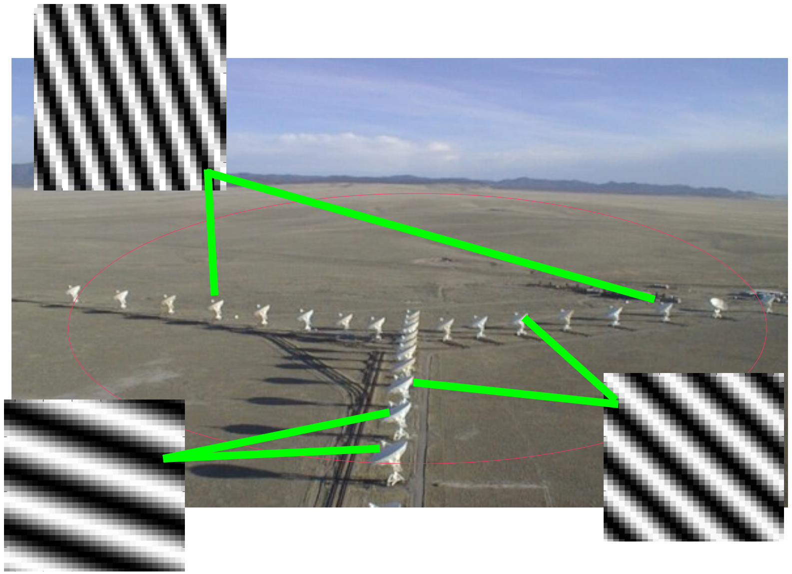


Image = sum of cosine 'fringes'.

Each antenna-pair measures the parameters of one 'fringe'.

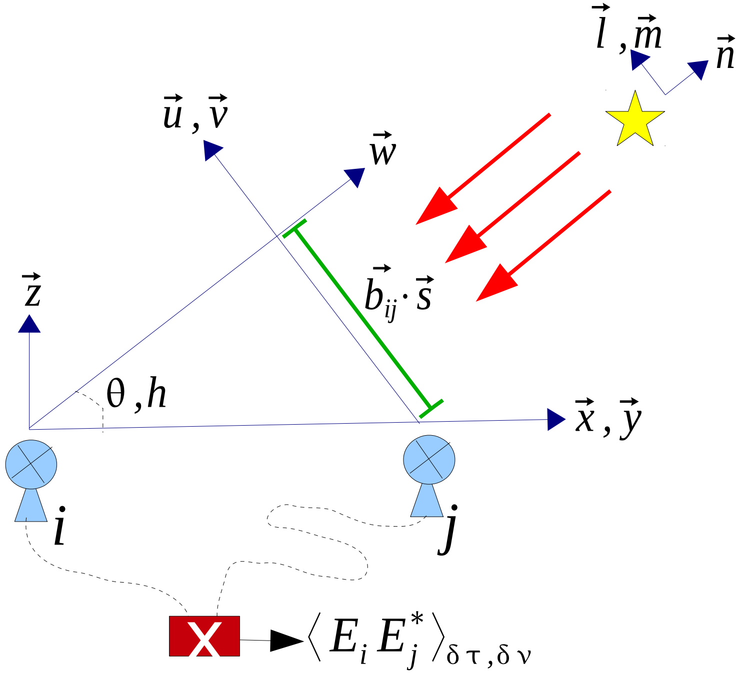


Measured Fringe Parameters :

Amplitude, Phase
Orientation, Wavelength

Measuring the visibility function

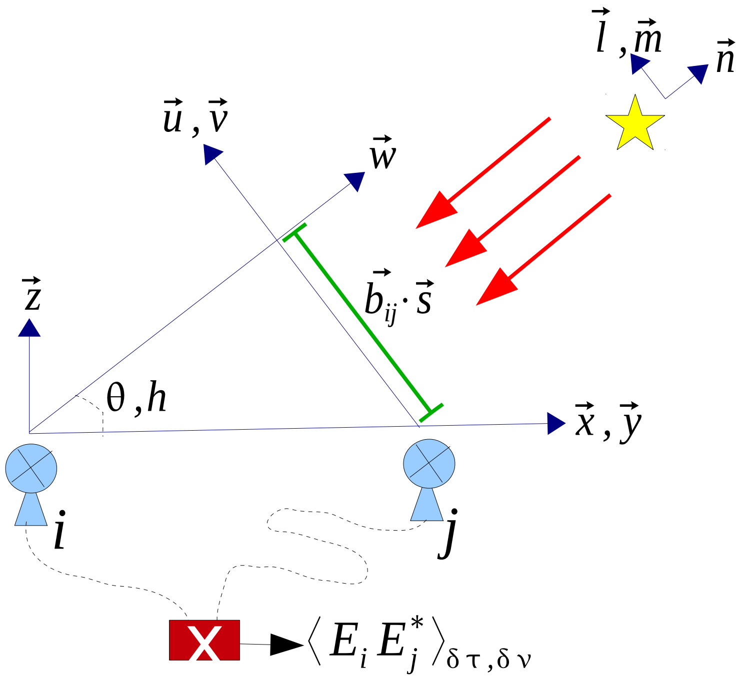
Measure the spatial correlation of the E-field
incident at each pair of antennas



N antennas
N(N-1)/2 antenna-pairs (baselines)

Measuring the visibility function

Measure the spatial correlation of the E-field incident at each pair of antennas



N antennas
N(N-1)/2 antenna-pairs (baselines)

Parameters of a Fringe :

Amplitude, Phase :

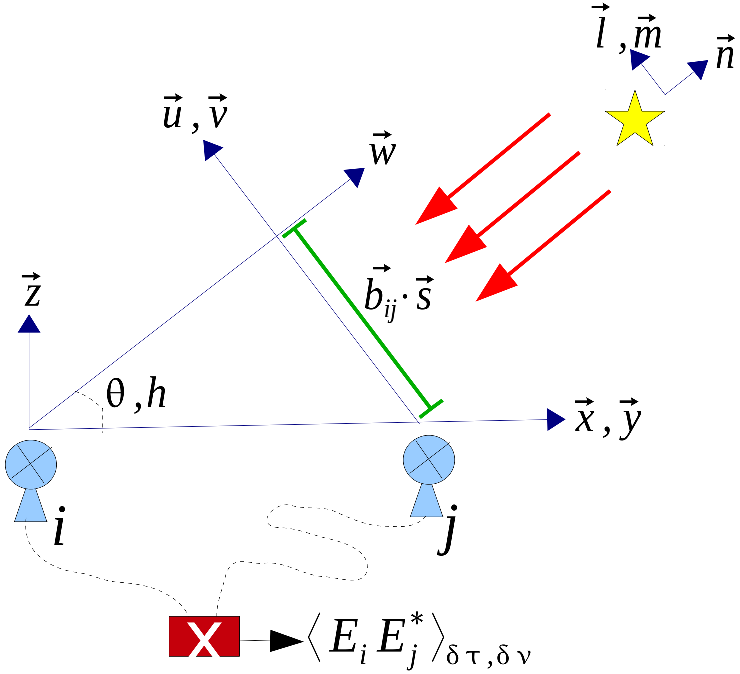
$\langle E_i E_j^* \rangle$ is a complex number.

Orientation, Wavelength :

$\vec{u}, \vec{v}, \vec{b}$ (geometry)

Measuring the visibility function

Measure the spatial correlation of the E-field
incident at each pair of antennas



N antennas
N(N-1)/2 antenna-pairs (baselines)

Parameters of a Fringe :

Amplitude, Phase :

$\langle E_i E_j^* \rangle$ is a complex number.

Orientation, Wavelength :

$\vec{u},\vec{v},\vec{b}$ (geometry)

Van Cittert Zernicke theorem (far-field)

$$\langle E_i E_j^* \rangle \propto V_{ij}(u,v) = \iint I^{sky}(l,m)\, e^{2\pi i(ul+vm)}\, dl\, dm$$

General Form : $V(\vec{b}_{ij}) = \iiint I^{sky}(\vec{s})\, e^{2\pi i(\vec{b}_{ij}\cdot\vec{s})}\, d^3\vec{s}$

Outline

- Imaging with a radio interferometer
    - Image formation
    - The forward and inverse problems
    - Algorithms and software
- Current limits and future trends

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} dl\, dm$$

The forward problem

Observed visibilities (Data)

Direction Independent Gains

UV sampling pattern

Sky Brightness (Image)

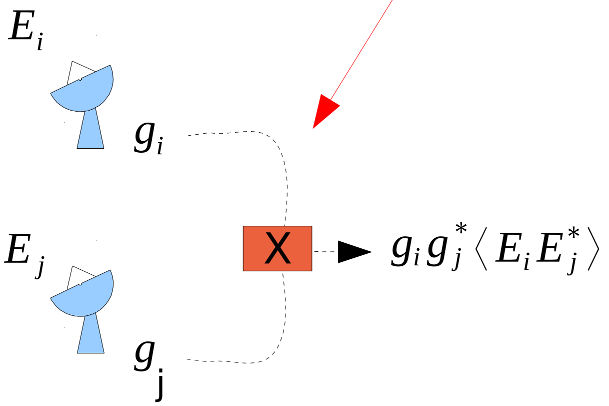
Fourier transform kernel

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iint I(l,m)\, e^{2\pi i (ul+vm)}\, dl\, dm$$

The forward problem

- $V_{ij}^{obs}(\nu,t)$ → Observed visibilities (Data)
- $M_{ij}(\nu,t)$ → Direction Independent Gains
- $S_{ij}(\nu,t)$ → UV sampling pattern
- $I(l,m)$ → Sky Brightness (Image)
- $e^{2\pi i (ul+vm)}$ → Fourier transform kernel



Calibration

Solve for g_i and divide out $g_i g_j^*$

N antennas
N(N-1)/2 antenna-pairs (baselines)

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iint I(l,m)\, e^{2\pi i(ul+vm)}\, dl\, dm$$

The forward problem

Observed or Calibrated visibilities (Data)

Direction Independent Gains

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Sky Brightness (Image)

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Direction Independent Gains

UV sampling pattern

Sky Brightness (Image)

Fourier transform kernel

Gridding
+
IFFT
+
Normalization

I^{obs}

Image Formation

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} dl\,dm$$

The forward problem

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Direction Independent Gains

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Sky Brightness (Image)

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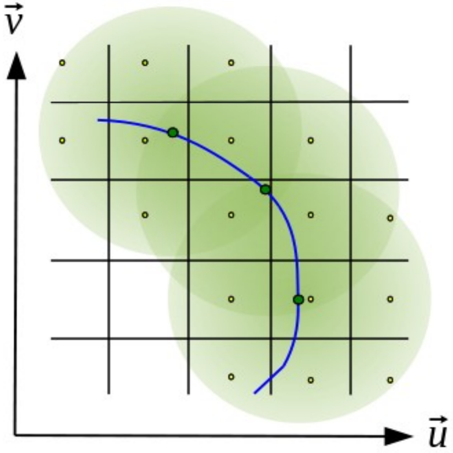


Convolutional resampling

Image Formation

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iint I(l,m)\, e^{2\pi i(ul+vm)}\, dl\, dm$$

The forward problem

Observed or Calibrated visibilities (Data)

Direction Independent Gains

UV sampling pattern

Sky Brightness (Image)

Fourier transform kernel

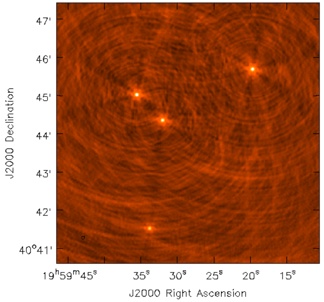
Gridding
+
IFFT
+
Normalization

I^{obs}

Image Formation

The inverse problem

$$I^{obs}(l,m) = I^{PSF}(l,m) * I^{sky}(l,m)$$



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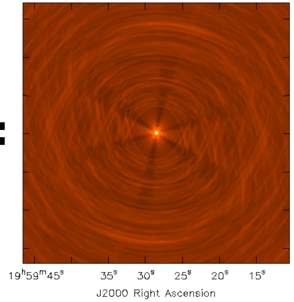

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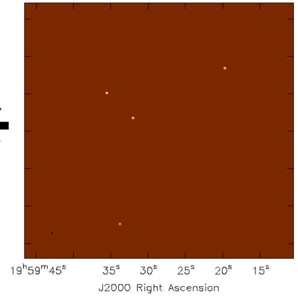

Image Reconstruction (deconvolution)

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} dl\, dm$$

The forward problem

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} dl\, dm$$

The generalized forward problem

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t) S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t) I(l,m,\nu,t) e^{2\pi i(ul+vm+w(n-1))} dl\, dm\, dn$$

Radio Interferometry – Measurement Equations

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General Form : $V(\vec{b}_{ij}) = \iiint I^{sky}(\vec{s}) e^{2\pi i(\vec{b}_{ij}\cdot\vec{s})} d^3\vec{s}$

$$V(\vec{b}_{ij}) = \iiint M(\vec{s},t) I^{sky}(\vec{s},t) e^{2\pi i(\vec{b}_{ij}\cdot\vec{s})+\phi(\vec{s},t)} d^3\vec{s}$$

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} dl\,dm$$

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Direction Independent Gains

Direction Dependent Effects

Sky-brightness varies with frequency (time)

W-Term

Radio Interferometry – Measurement Equations

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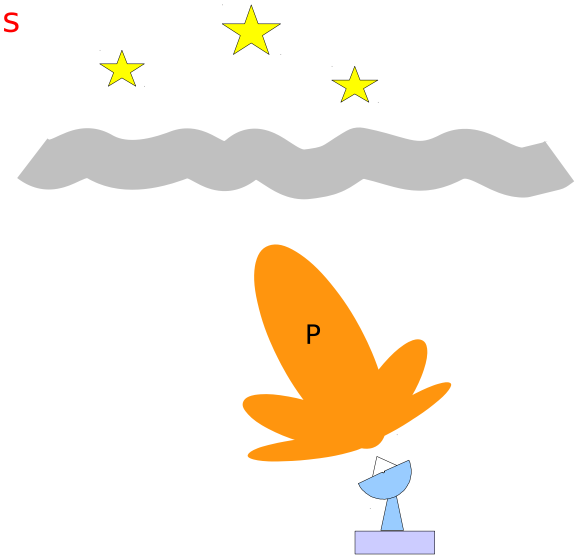
Direction Independent Gains

Direction Dependent Effects



Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} \, dl \, dm$$

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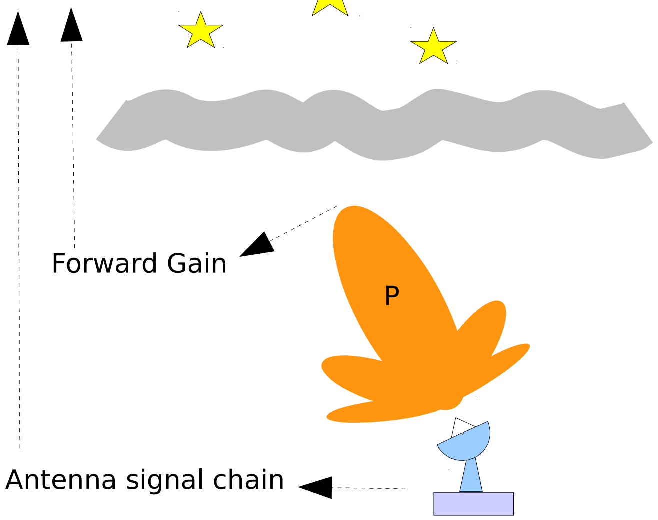

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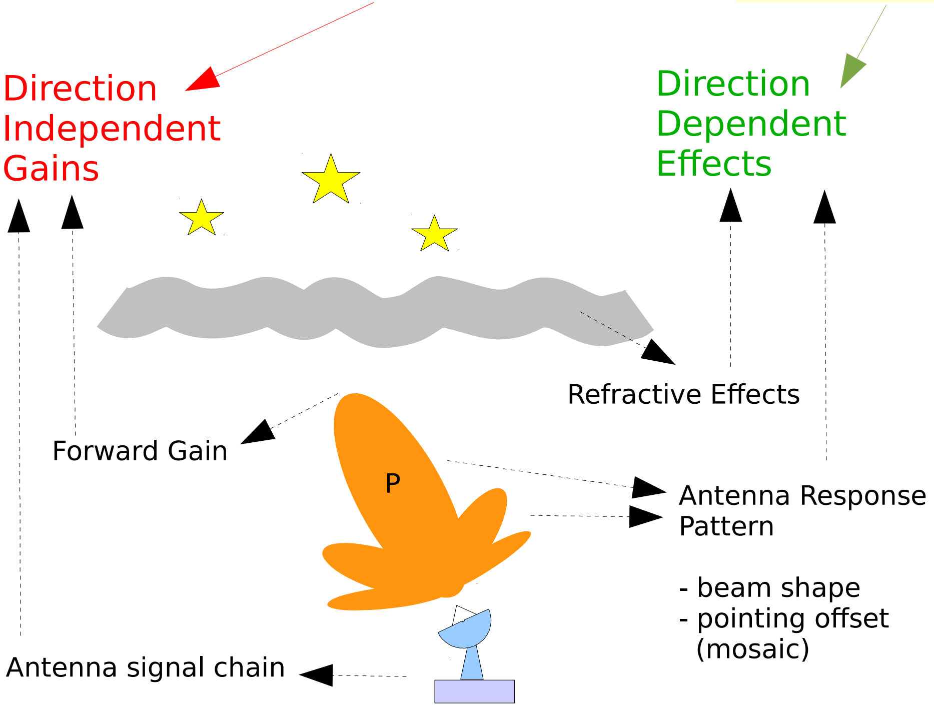

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Direction Independent Gains

Direction Dependent Effects

Sky-brightness varies with frequency and time

W-Term

Radio Interferometry – Measurement Equations

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Sky-brightness varies with frequency

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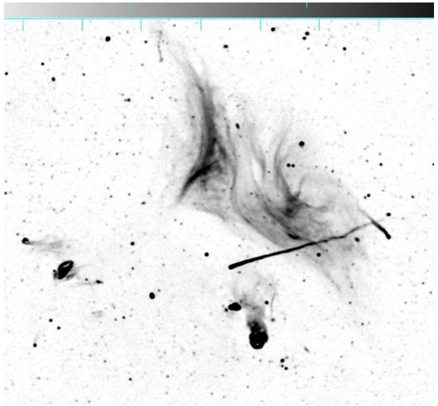
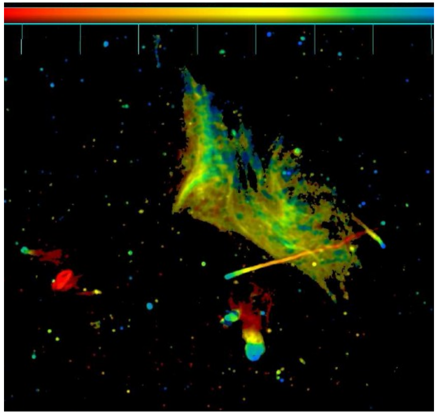
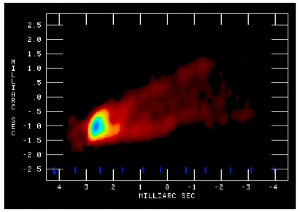


Intensity



Intensity weighted Spectral Index



Radio Interferometry – Measurement Equations

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Direction Independent Gains

Direction Dependent Effects

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W-Term

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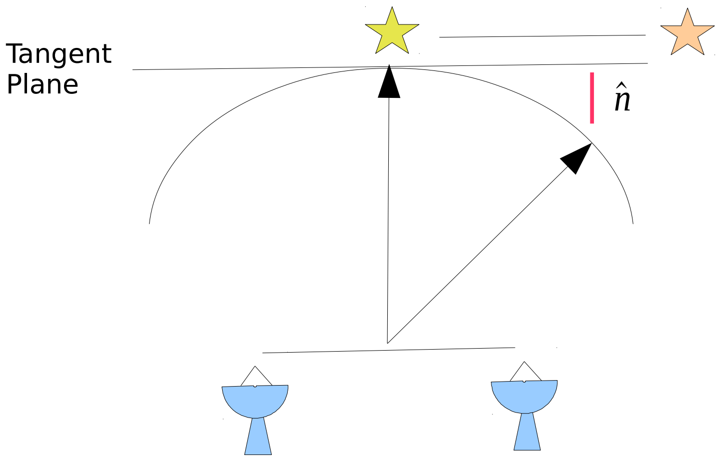
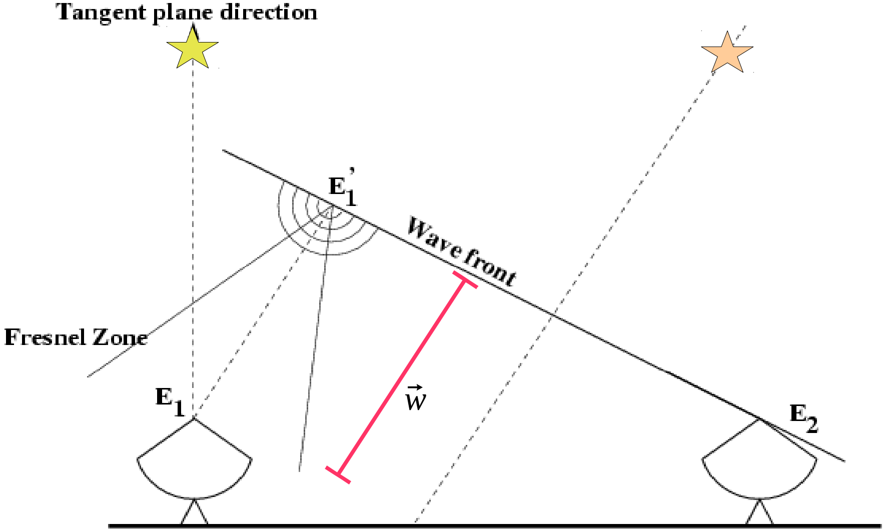
W-Term

Effect of Sky Curvature





Radio Interferometry – Measurement Equations

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- $M_{ij}(\nu,t)$: Direction Independent Gains
- $S_{ij}(\nu,t)$: UV sampling function
- $M_{ij}^{s}(l,m,\nu,t)$: Direction Dependent Effects
- $I(l,m,\nu,t)$: Sky-brightness varies with frequency and time
- $w(n-1)$: W-Term

How do we solve these systems of equations ?

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} dl\, dm$$

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Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency and time

W-Term

Calibration

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} \, dl \, dm$$

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Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency and time

W-Term

Calibration

=> Multiplicative effect in the image domain
=> Convolutions in the visibility domain
(corrected during gridding + iFFT + normalization)

Inverse Imaging

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} dl\, dm$$

The generalized forward problem

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Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency and time

W-Term

Calibration

Image Reconstruction

(De)convolution in the image domain

=> Multiplicative effect in the image domain
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Inverse Imaging

Radio Interferometry – Measurement Equations

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Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency and time

W-Term

Calibration

Image Reconstruction

(De)convolution in the image domain

Self-Calibration

Direction-dependent Self-Cal

Wideband Self-Cal

=> Multiplicative effect in the image domain
=> Convolutions in the visibility domain
(corrected during gridding + iFFT + normalization)

Inverse Imaging

Outline

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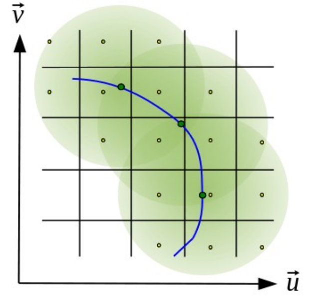
- Current limits and future trends

Inverse Imaging = Gridding + iFFT

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

Direction Dependent Effects

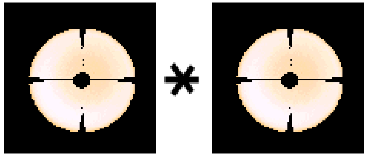
W-Term



Correct multiple direction dependent effects together, during gridding, before the iFFT

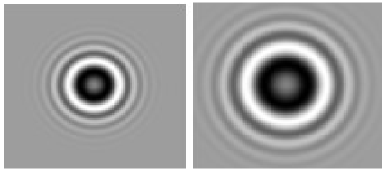
The gridding convolution kernel (per visibility) is a combination of the following......

Aperture illumination functions (ij, time, freq, pol)



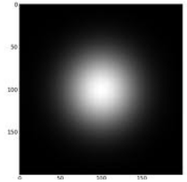
Antenna Primary Beams

FT of Fresnel kernels



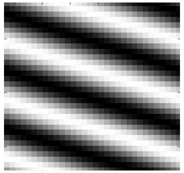
W-term effects

Prolate Spheroidal



Anti-Aliasing Filter

Phase gradient



Mosaic

Iterative Image Reconstruction

The generalized forward problem $V^{obs} = [A] I^m + n$

The generalized inverse problem $I^m = [A]^{-1} V^{obs}$

L2 data regularization
(maximum likelihood , chi-square minimization)

+ Sky model (multiscale, wideband, timevar)
+ Solver/Optimizer with constraints/biases

Iterative Image Reconstruction

The generalized forward problem $V^{obs} = [A] I^m + n$

The generalized inverse problem $I^m = [A]^{-1} V^{obs}$

L2 data regularization
(maximum likelihood , chi-square minimization)

+ Sky model (multiscale, wideband, timevar)
+ Solver/Optimizer with constraints/biases

Forward and Reverse transforms

Calc $\frac{\delta \chi^2}{\delta I^m}$

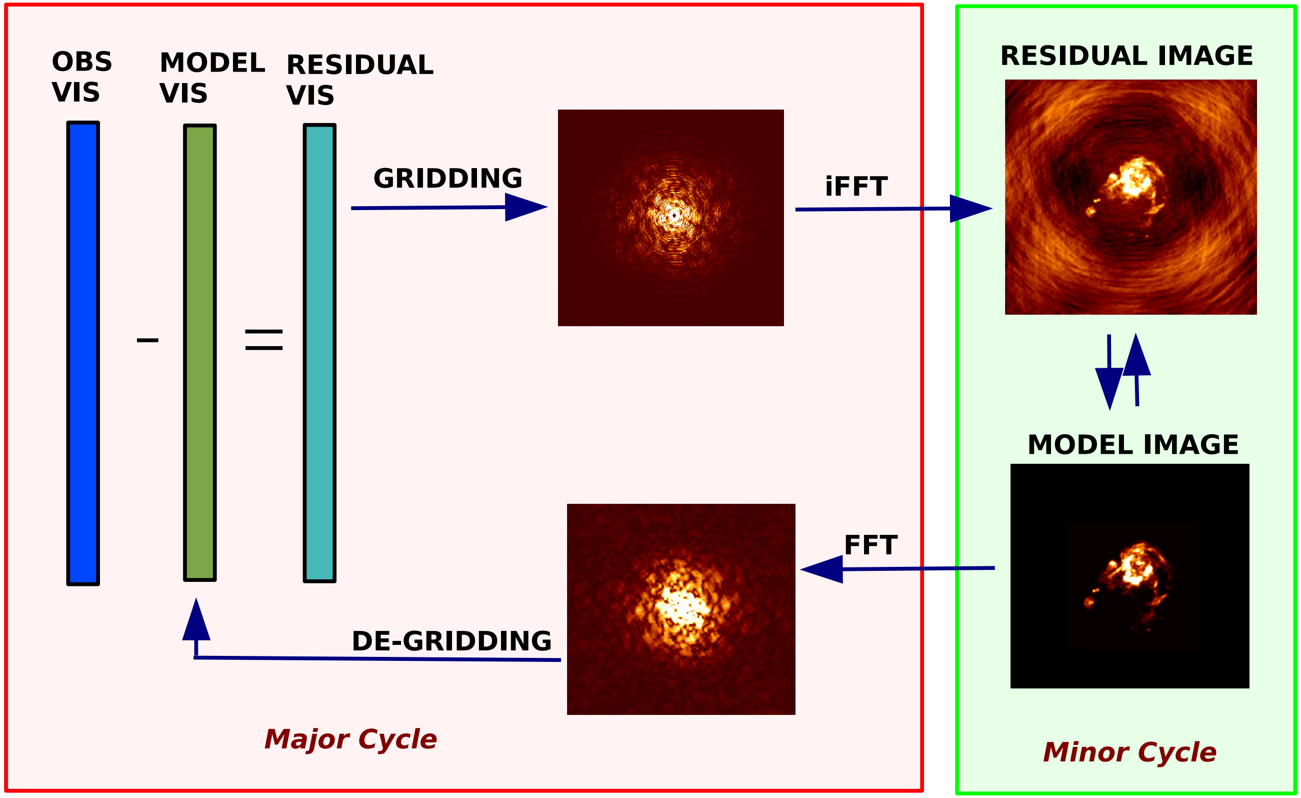


Image Reconstruction

Sky models
- Delta function
- Gaussians
- .....

Algorithms
- Clean (greedy)
- Many other CS Ideas

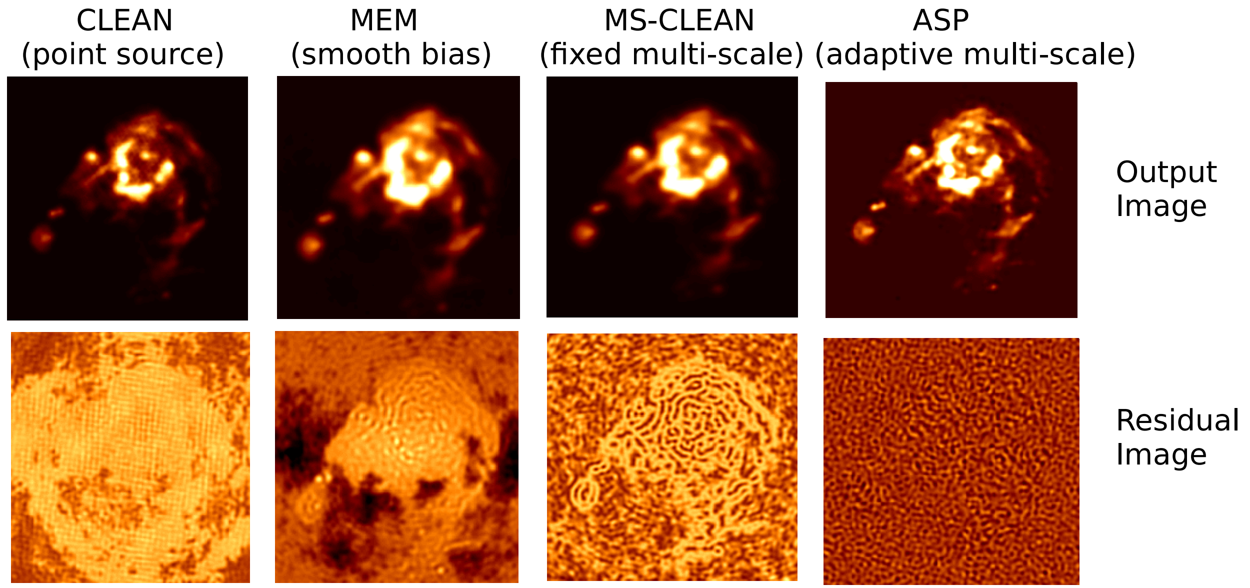
Reconstruction Algorithms

(1) CLEAN (and its variants for multi-scale/adaptive-scale, broad-band and time-variable structures)

- Matching pursuit / greedy algorithms for the reconstruction step.

(2) RESOLVE, Fast-RESOLVE, MEM, SARA / PURIFY, MORESANE, and several more....

- Direct modeling approaches.



Reconstruction Algorithms

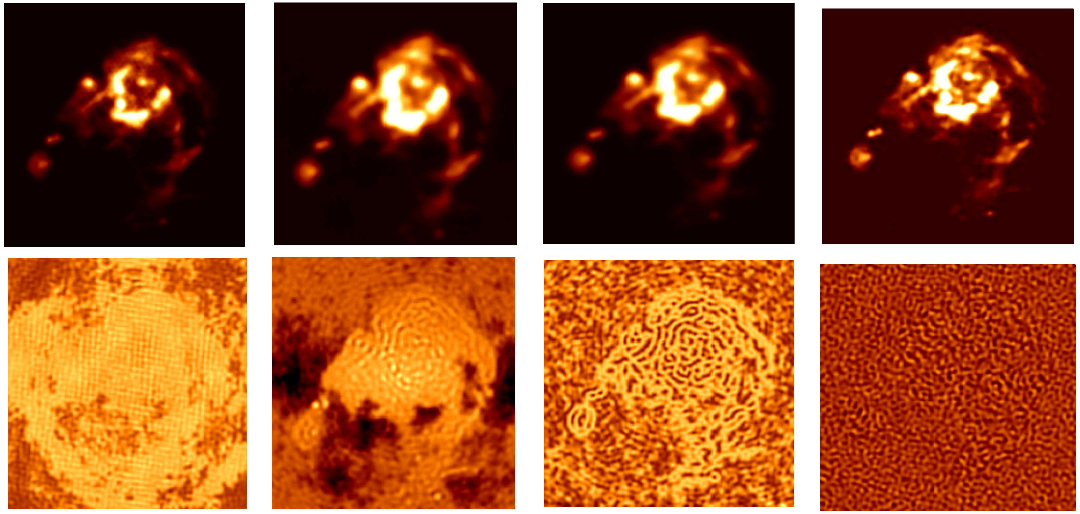
(1) CLEAN (and its variants for multi-scale/adaptive-scale, broad-band and time-variable structures)

- Matching pursuit / greedy algorithms for the reconstruction step.

(2) RESOLVE, Fast-RESOLVE, MEM, SARA / PURIFY, MORESANE, and several more....

- Direct modeling approaches.

CLEAN (point source) | MEM (smooth bias) | MS-CLEAN (fixed multi-scale) | ASP (adaptive multi-scale)



Output Image

Residual Image

Current ALMA Pipelines use Point-Source CLEAN with region constraints

Reconstruction Algorithms

(1) CLEAN (and its variants for multi-scale/adaptive-scale, broad-band and time-variable structures)

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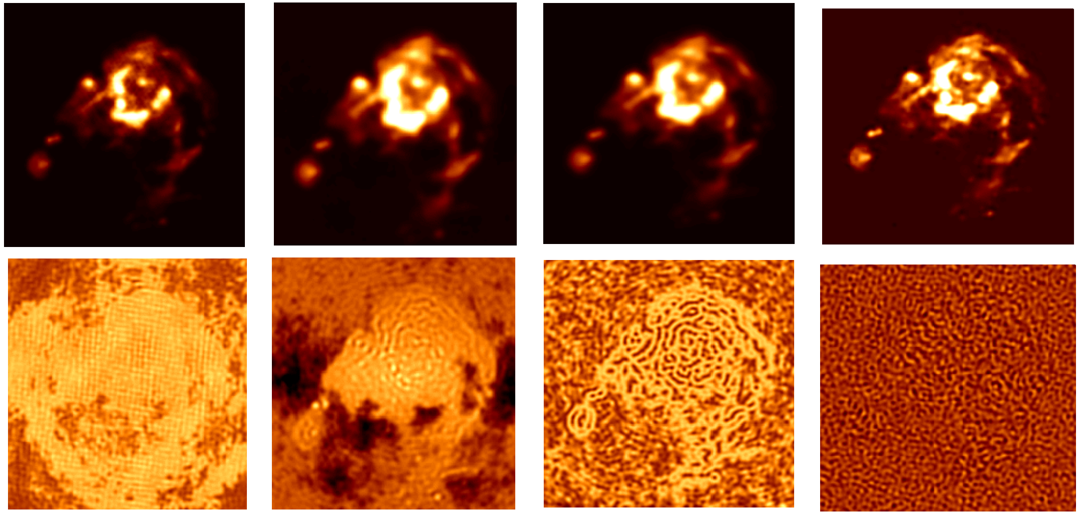


Current ALMA Pipelines use Point-Source CLEAN with region constraints

Note :

Algorithms developed for the Event Horizon Telescope (also VLBI) may be closest to Optical Interferometry

→ very sparse uv-coverage

→ use closure-phase as an extra constraint

Software Tools

Major Software Packages

- AIPS, Miriad, Gildas, CASA, ASKAPsoft, Difmap, **etc…** : Interactive end-to-end data analysis platforms
- Resolve, Quartical/Cubical, WSClean, **etc…** : Modular packages for separate analysis steps

Data and Image Formats

- FITS, UVFITS/FITSIDI, Measurement Set, CASAcore images/tables, etc…
- *Allows inter-operability across observatories and software packages*

Scientific Usage

- **Manual data reduction** : Individual PIs start with raw data from telescope archives, and run the above software/algorithms one step at a time, inspecting the data in-between.

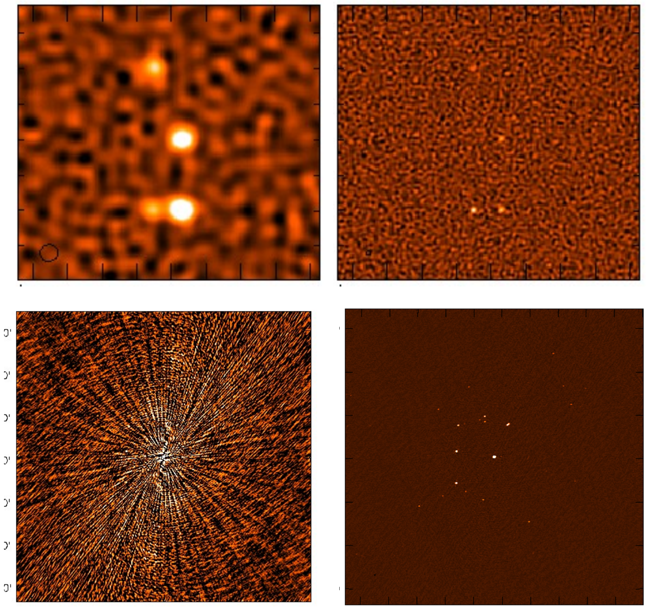
- **Automated Pipelines** : Observatories or Project teams construct end-to-end pipelines with heuristics

  – Project based : Tuned to specific large-project science use cases (e.g. ASKAP, MeerKAT….)
  – General purpose : Must be robust to a variety of individual science use cases (e.g. VLA, ALMA,...)

Outline

- Imaging with a radio interferometer
    - Image formation
    - The forward and inverse problems
    - Algorithms and software
- Current limits and future trends

Limits of Interferometric Imaging



Angular resolution & sensitivity

Dynamic range

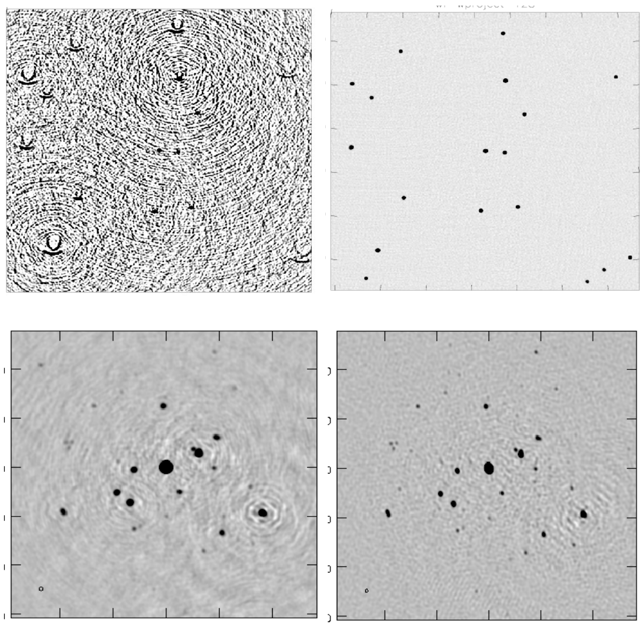
Un-measured large spatial scales

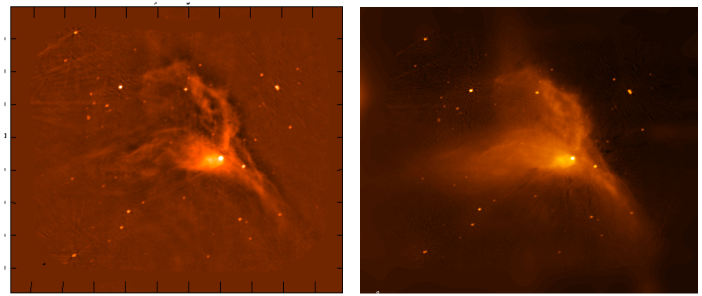
Accuracy of instrument models

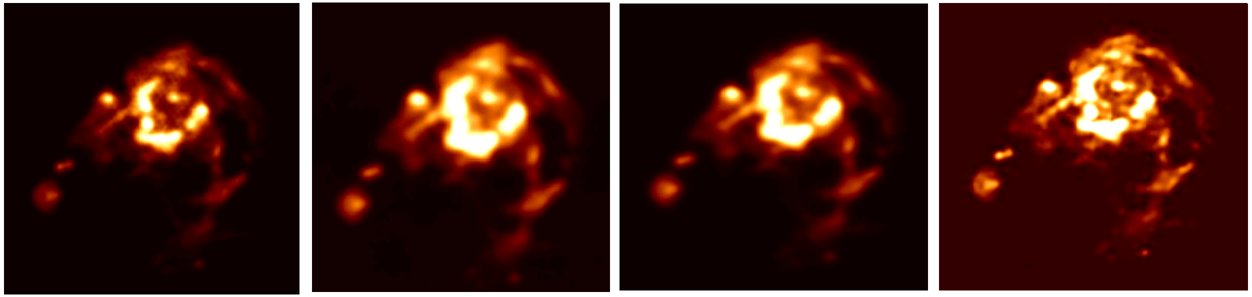
Algorithmic variability

Data volume and compute cost

Manual tuning of heuristics





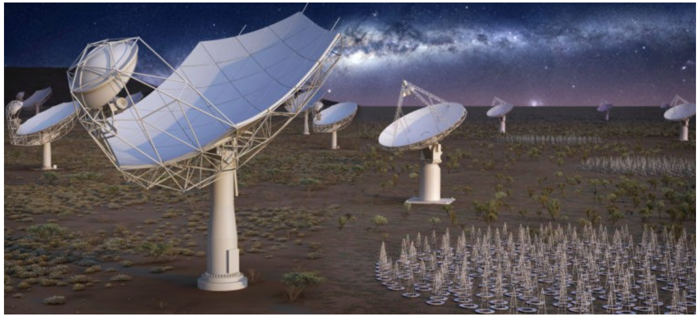


Going forward……

Data volumes will only increase (e.g. ALMA-WSU, ngVLA, SKA….)

=> image noise reduces => instrumental effects easily seen => need complex algorithms

=> compute cost increases => manual intervention is harder => need HPC and automation



Square Kilometer Array
(skatelescope.org)

2K dishes, 1M antennas
50 MHz – 30 GHz

Next Generation VLA
(ngvla.nrao.edu)

263 dishes (2 types)
1-100 GHz

ALMA upgrades
(alma.nrao.edu)

WSU : Increased bandwidth and data volumes

The R&D frontier

Algorithms :

- A variety of sky models, instrument models, objective functions and regularizers, optimization strategies, the use of prior knowledge versus unknowns to solve for, etc

- Machine learning to move beyond using only pre-determined models

Compute Load :

- Native parallelization of data and algorithms, GPUs for the gridding compute hotspot, etc..

Automation :

- Data analysis pipelines that know how to tune the details of each step (data editing, calibration, imaging) to each specific dataset

Imaging Instruments in Radio Astronomy