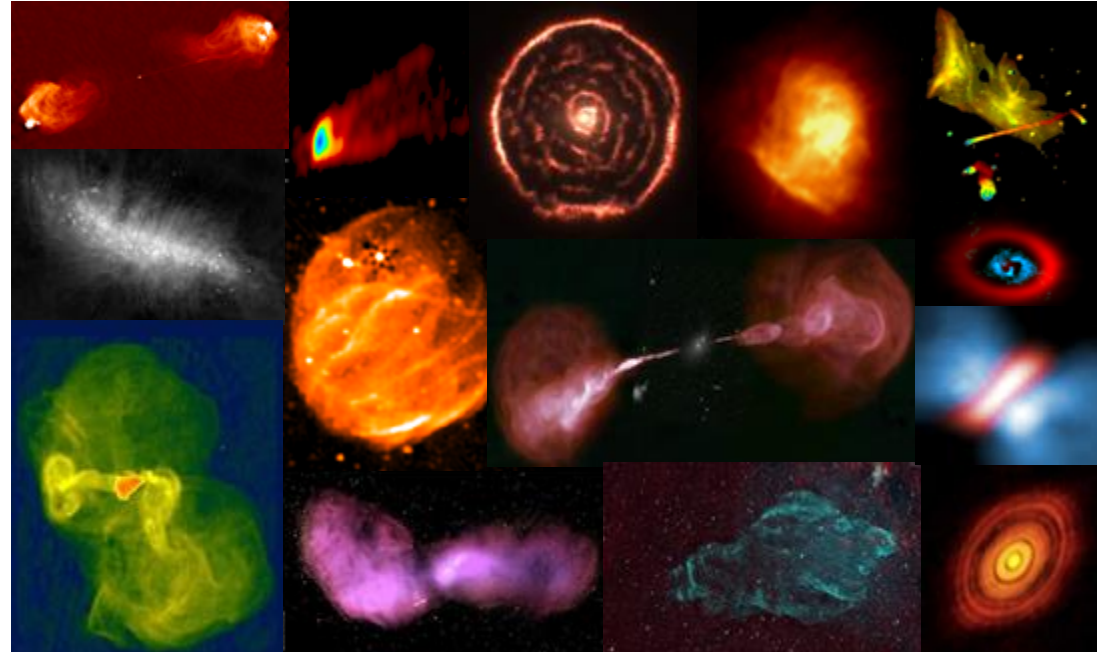


# Imaging in Radio Astronomy



**Urvashi Rau & Sanjay Bhatnagar**

National Radio Astronomy Observatory, USA



**National Radio  
Astronomy  
Observatory**

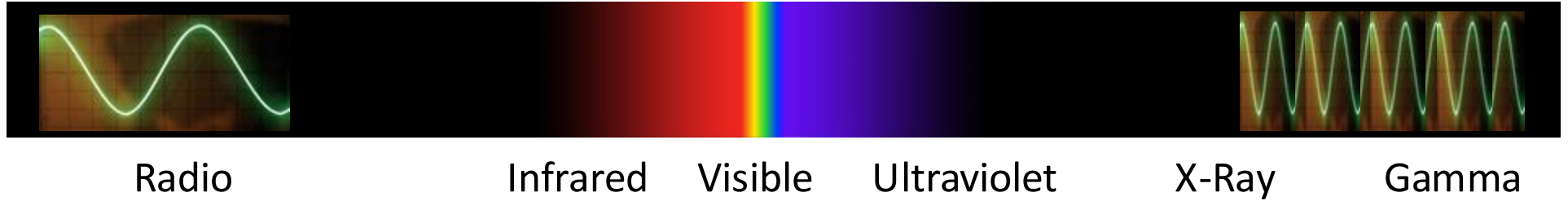
# Outline...

---

- Images in Radio Astronomy
- Signal measurement ( RF power, front-end systems, EMI )
- K-space sampling ( point spread functions )
- Image Reconstruction ( signal equations, algorithms )
- Operational workflows
- Future directions

( Extra slides – History of radio astronomy, and telescopes of the world )

# Imaging in Astronomy



## What we measure :

- Intensity of the received power
- EM Polarization
- Spectral structure
- Time-variability

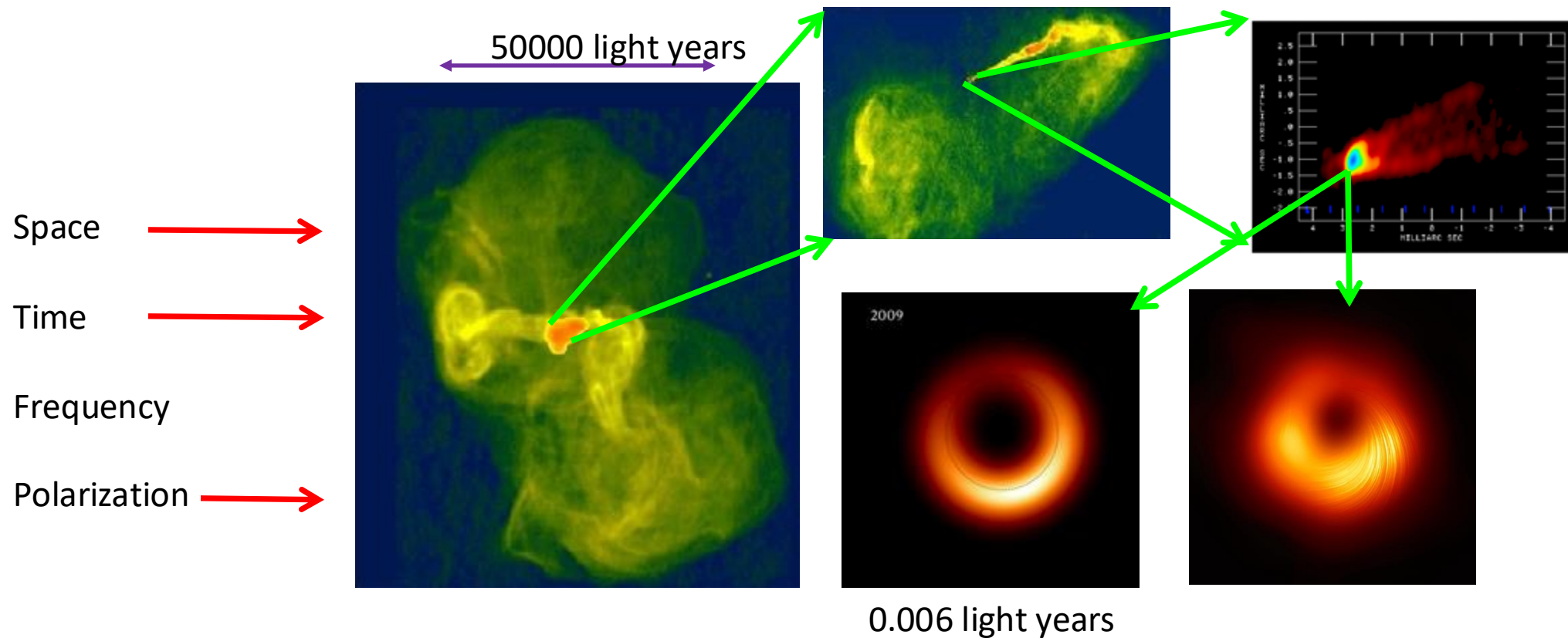
**Quantitative !**

## What we infer :

- Temperature, Energetics,
- Emission Physics, Chemical Composition
- Magnetic Fields
- Velocities , 3D structure
- Age of the source

**Why ? To study new Physics**

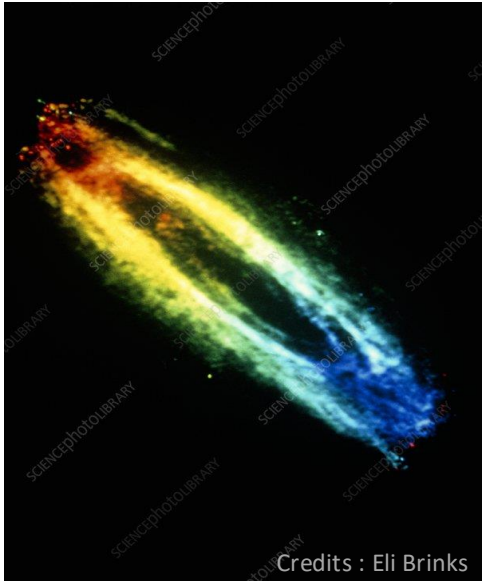
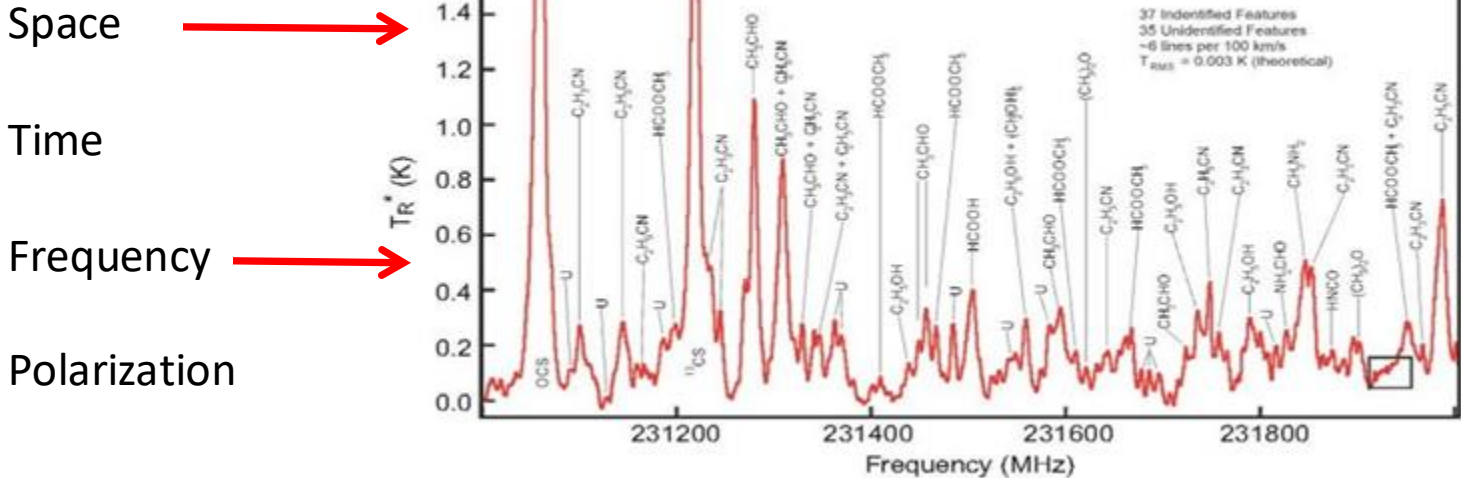
# Images in Radio Astronomy : 2D brightness distribution + B-fields



Credits :  
NRAO, ALMA,  
Event Horizon Telescope  
collaboration

\_\_\_\_\_

# Astro-chemistry tracers



Doppler shifts  
trace physical velocity

## M31 : Andromeda Galaxy Rotation

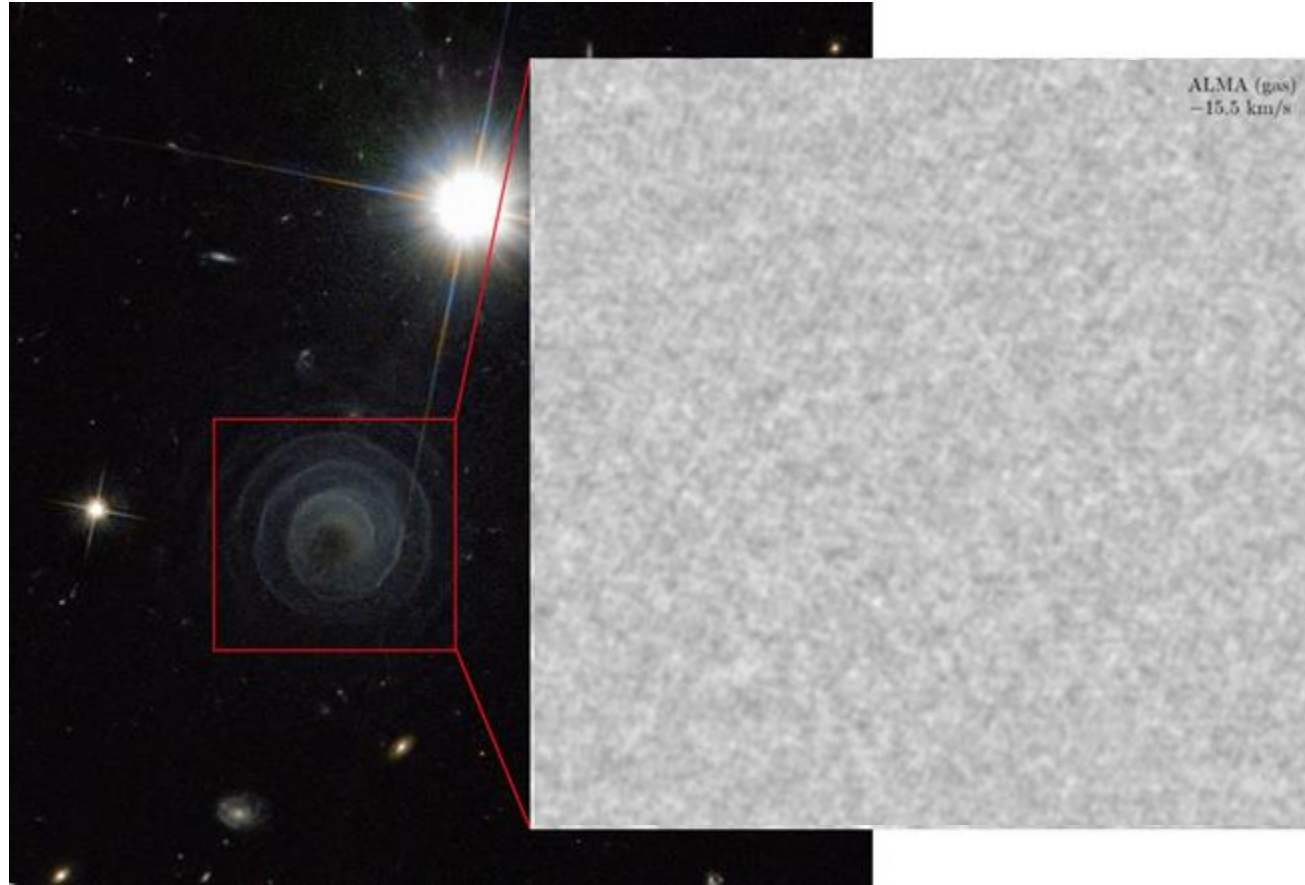
# Images in Radio Astronomy : 3D velocity structures (Doppler)

Space →

Time

Frequency →

Polarization



CO emission showing a spiral-shell structure around the AGB star LL Pegasi and its stellar companion

( Kim et al, Nature Astro 2017.)



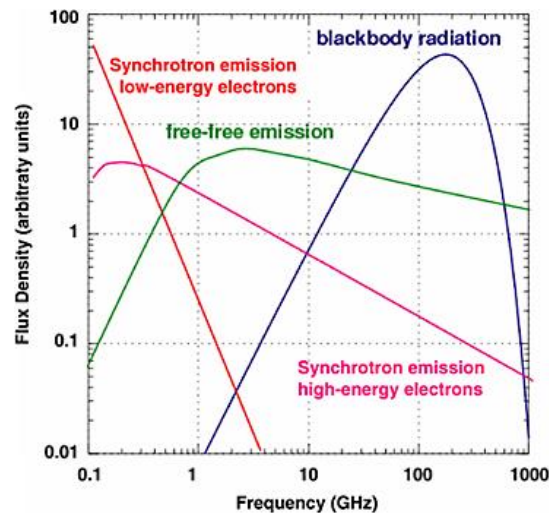
# Images in Radio Astronomy : Emission Physics

Space →

Time

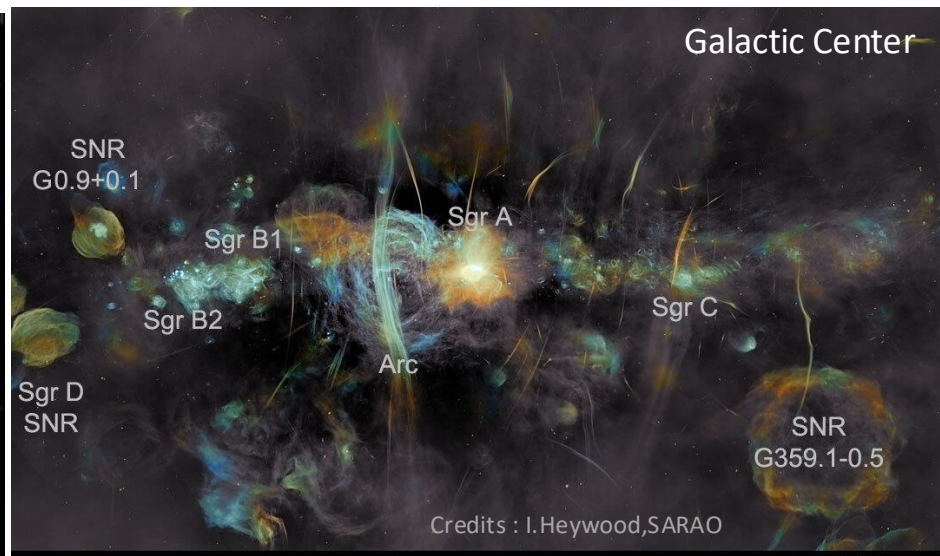
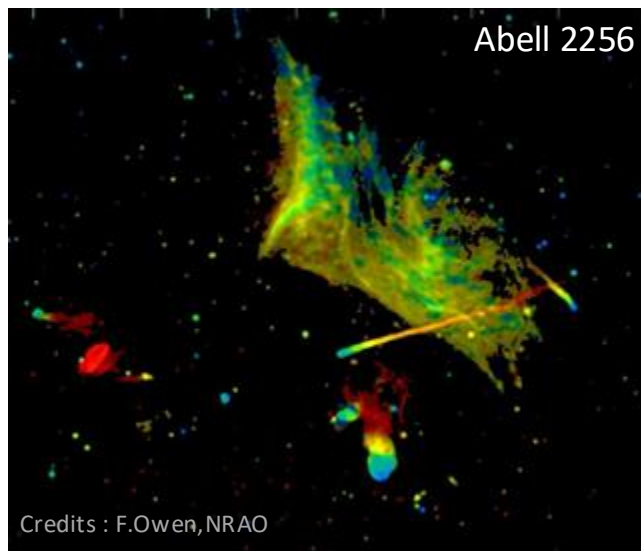
Frequency →

Polarization



Color : Spectral slope

- Identify radiation process
- Tracer of “age”



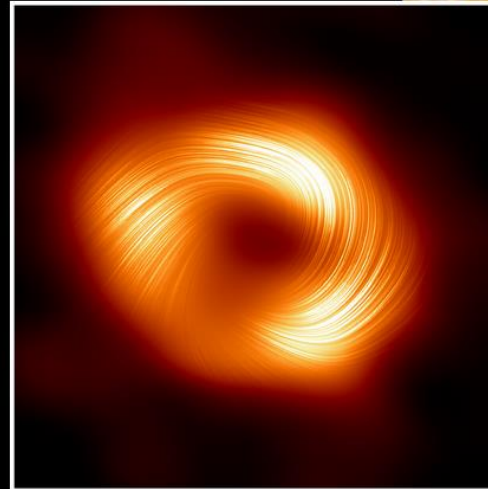
# Images in Radio Astronomy : Magnetic Field Direction

Space →

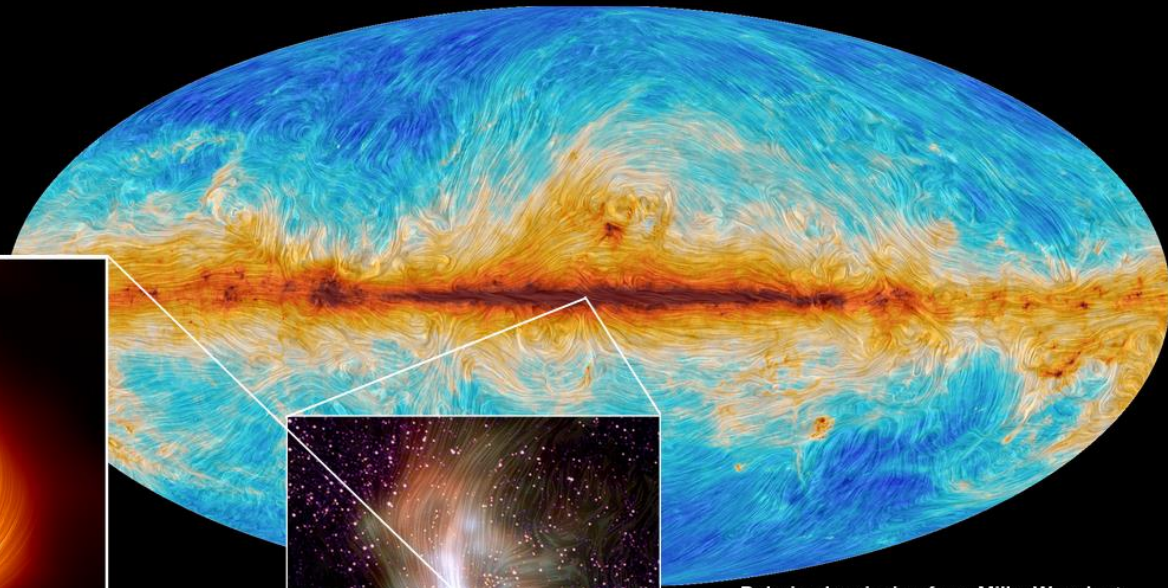
Time

Frequency

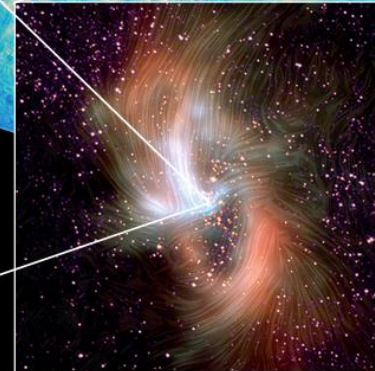
Polarization →



**Polarised emission from plasma around our Milky Way supermassive black hole Sagittarius A\***  
*Cr. Event Horizon Telescope Collaboration*



**Polarised emission from Milky Way dust**  
*Cr. ESA and the Planck Collaboration*



**Polarized emission from dust at the center of our Milky Way**  
*Cr. NASA/SOFIA, NASA/Hubble Space Telescope/NICMOS.*



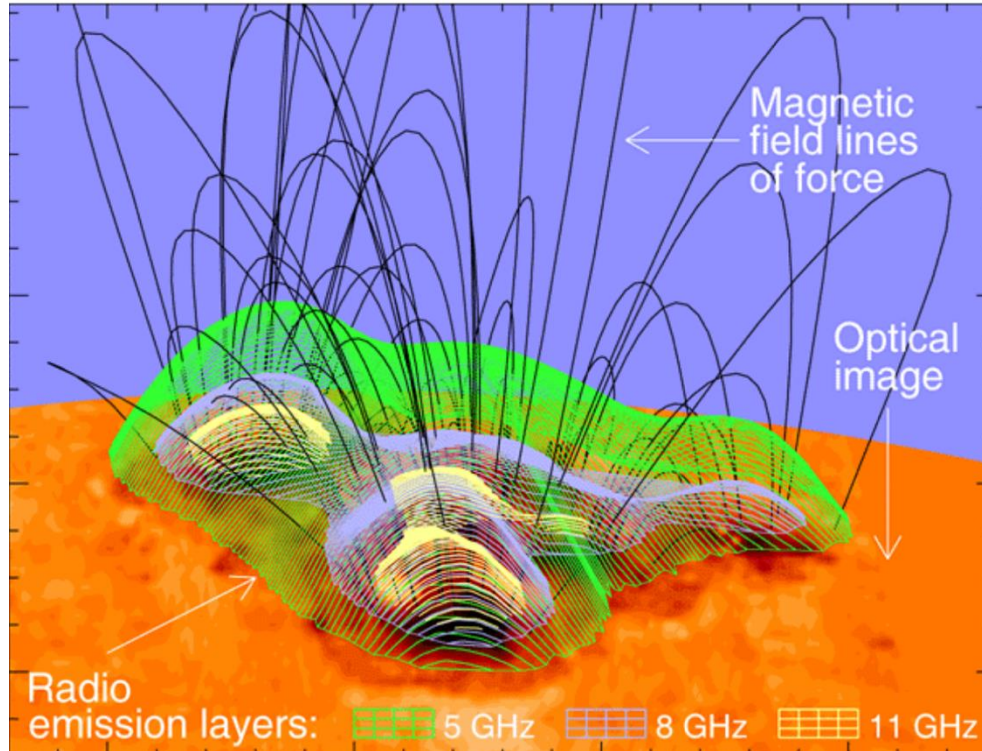
# Images in Radio Astronomy : Coronal Magnetography

Space →

Time

Frequency →

Polarization

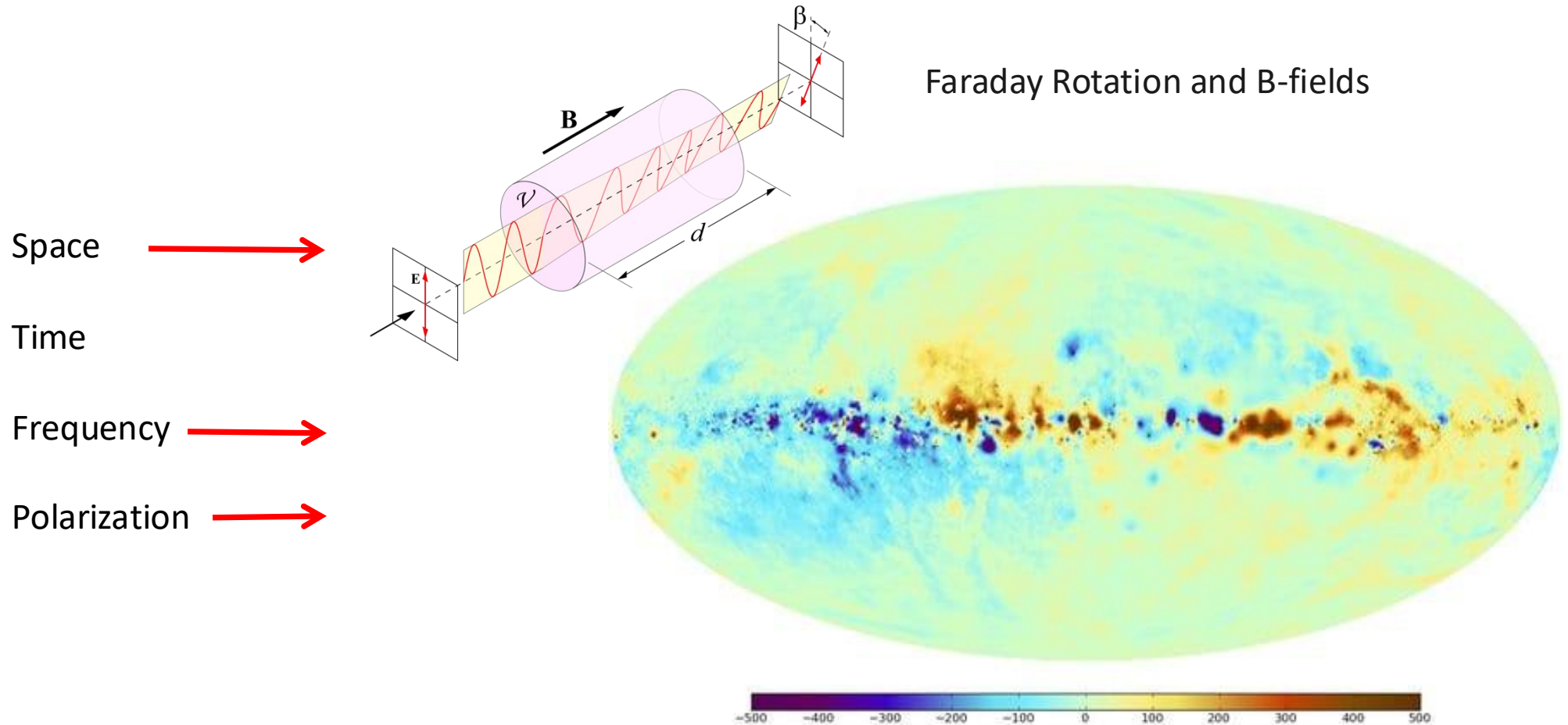


Gyro-synchrotron radio emission from the solar corona

Between 1 GHz and 20 GHz

**Traces B-field strengths** at different heights above the surface of the sun.

# Images in Radio Astronomy : 3D Magnetic Tomography



All sky map of Faraday Depth, as a tracer of magnetic fields of the Milky Way.

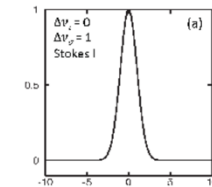
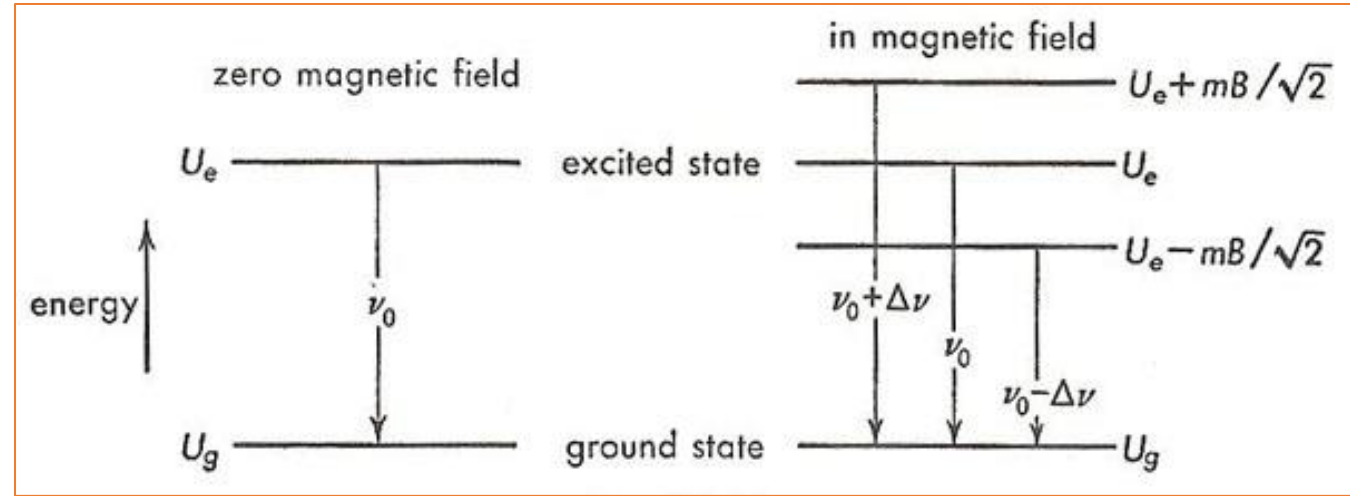
# Images in Radio Astronomy : Zeeman Splitting & B-field strength

Space →

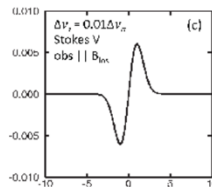
Time

Frequency →

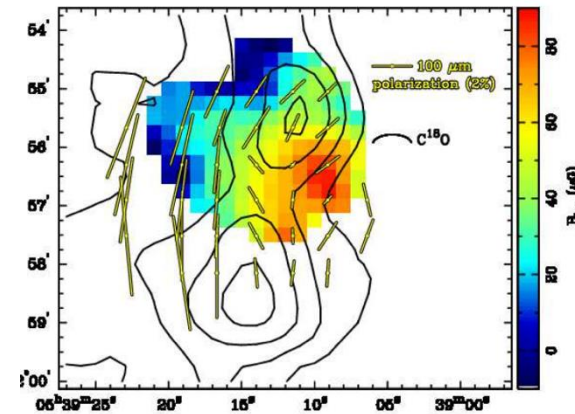
Polarization →



Stokes I



Stokes V

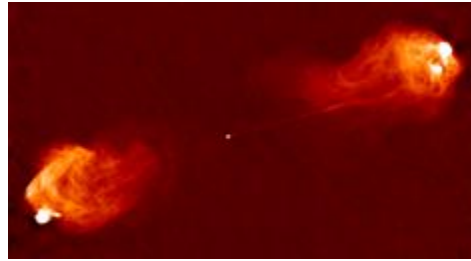


B-field from  
OH Zeeman lines

NGC2024  
star-forming  
region

# Measurements – Power Levels and Frequency Ranges

Electromagnetic radiation  
from objects in space

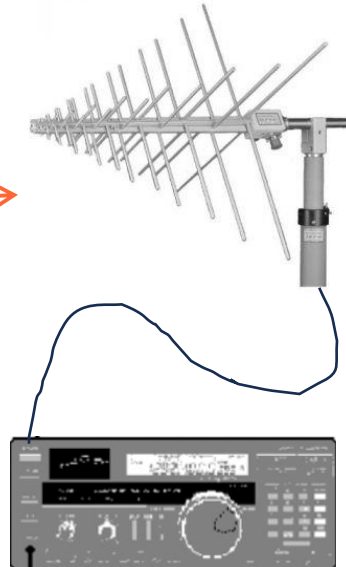
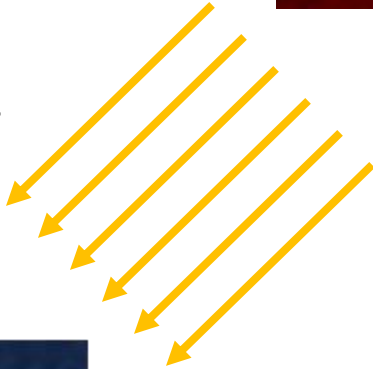


Spectral Power Flux Density

$$1 \text{ Jansky} = \frac{10^{-26} \text{ W}}{\text{m}^2 \text{ Hz}}$$

Measured range :  $10^4 \text{ Jy}$  to  $10^{-6} \text{ Jy}$

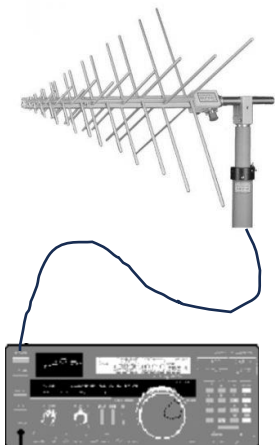
Measure Power  
Received at  
Radio  
Frequencies



Frequency Ranges

30 MHz -100 MHz	LOFAR
300 MHz – 1.5GHz	GMRT
900 MHz – 50 GHz	VLA
( 40 GHz – 120 GHz )	( ngVLA )
100 GHz – 900 GHz	ALMA

# Measurements – Front-end systems



Cooled Receivers

Low Noise Amplifiers

Sky : 2.7K

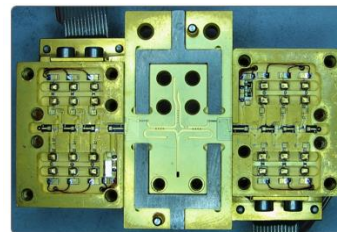
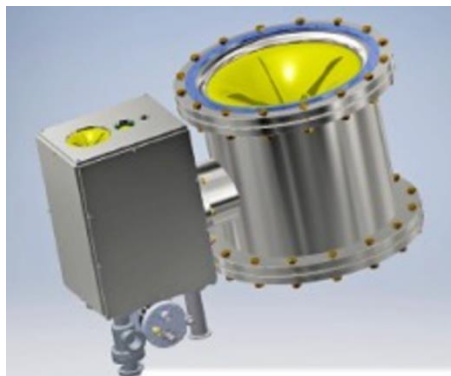
Ambient : 300K

Current instruments :  $\sim 20$  K

Spectral Power Flux Density

$$1 \text{ Jansky} = \frac{10^{-26} \text{ W}}{\text{m}^2 \text{ Hz}}$$

Measured range :  $10^4$  Jy to  $10^{-6}$  Jy



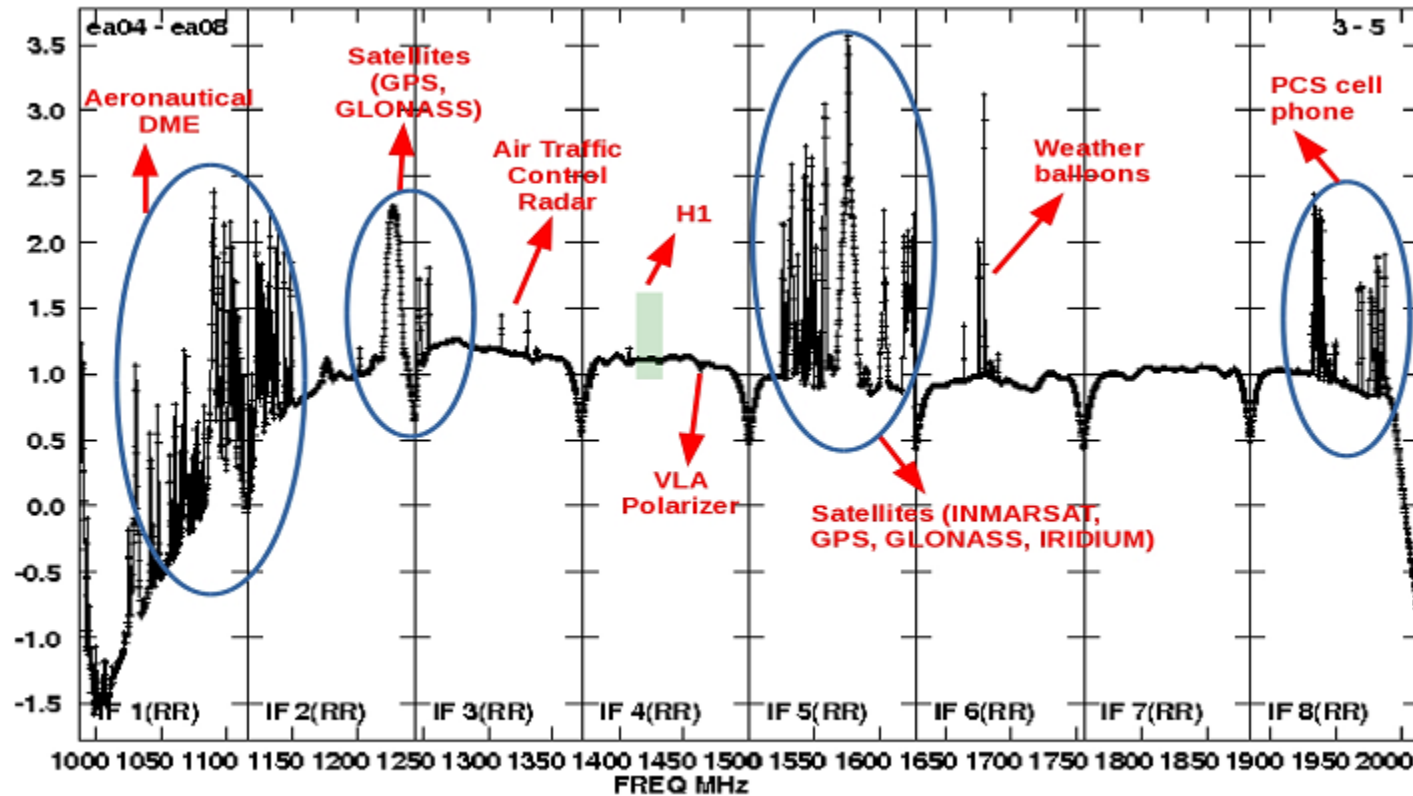
Central Dev Lab,  
NRAO



LNAs, Cryogenics,  
Mixers, waveguides,



# Radio Frequency Interference



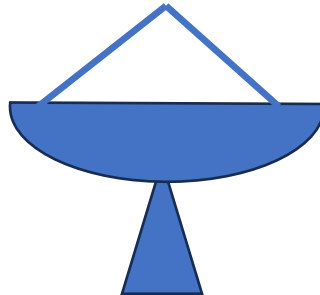
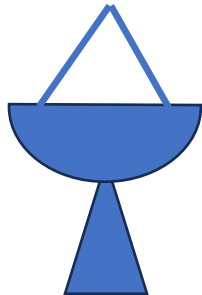
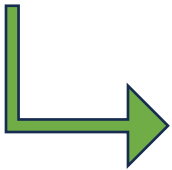
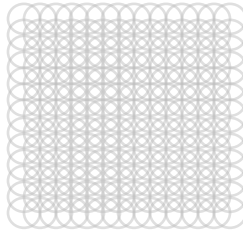
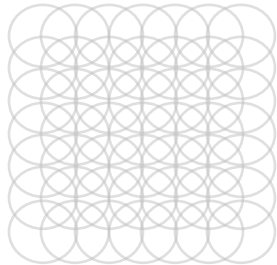
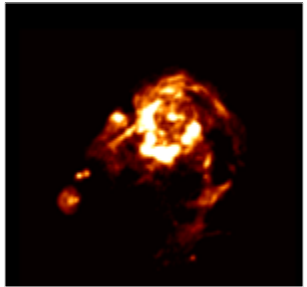
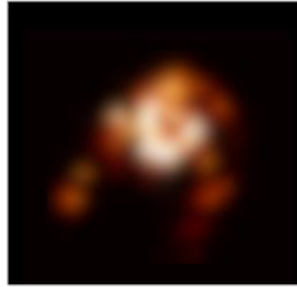
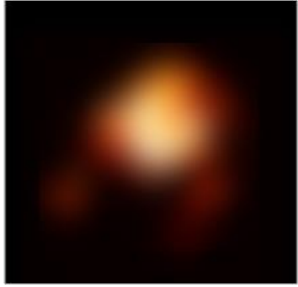
Man-made transmissions

- Satellite radio, internet, navigation comms, cell services, etc.
- Only very small protected bands

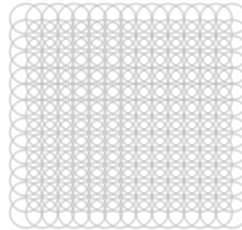
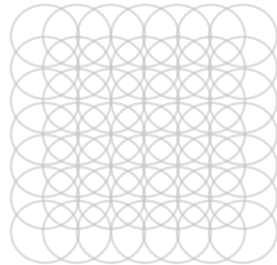
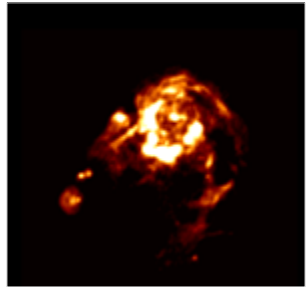
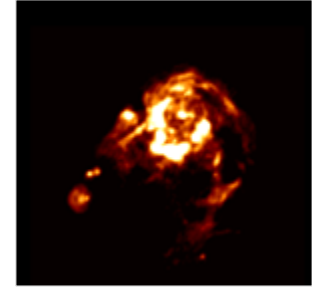
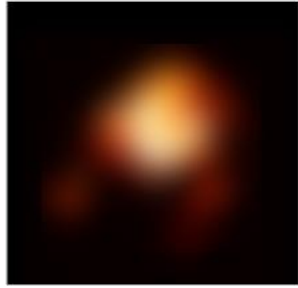
Algorithms : **Flagging** - Outlier detection + discarding corrupted samples.

Challenges : ~30-40% data loss in commercial bands → R&D on ways to recover data

# Images in Radio Astronomy



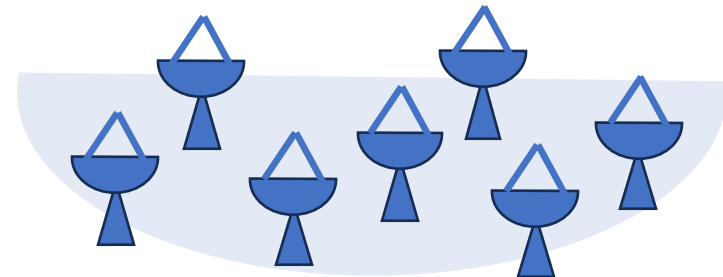
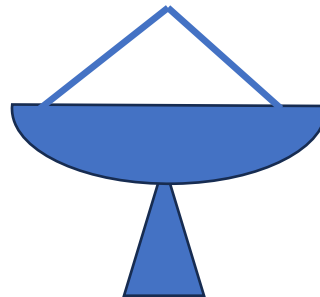
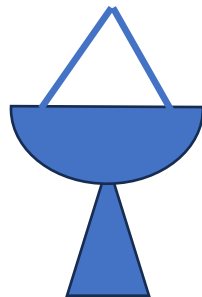
# Images in Radio Astronomy



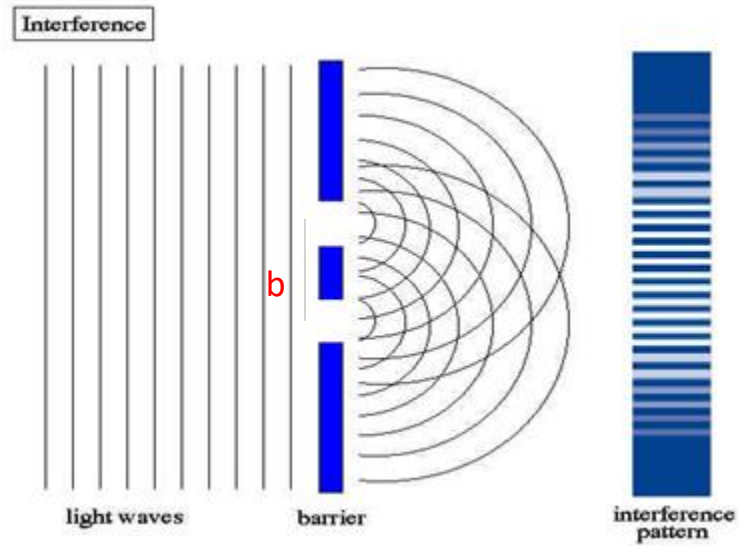
Aperture Synthesis

Measure spatial frequencies...

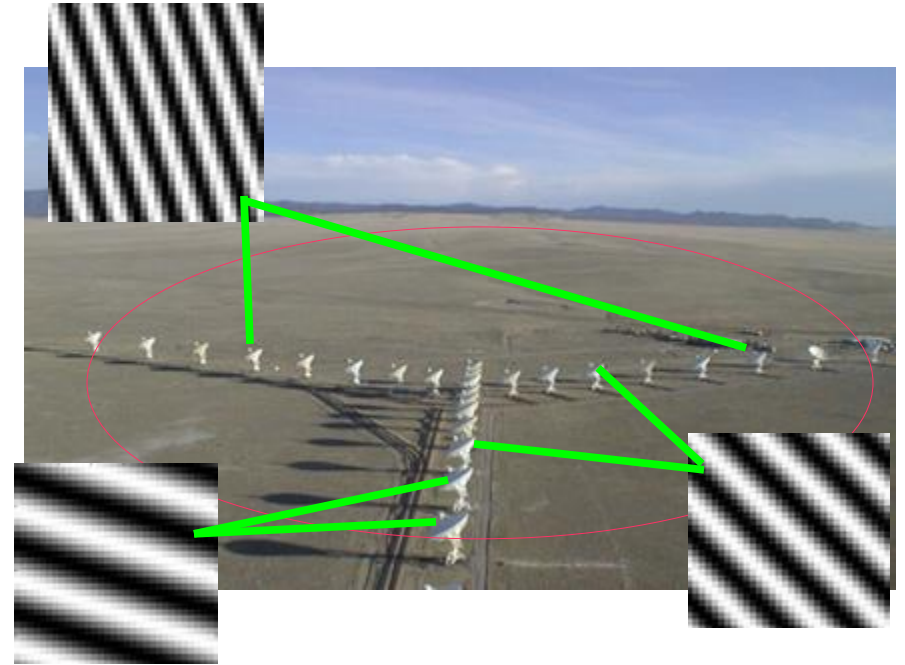
**“ K-space Sampling ”**



# An indirect imaging device

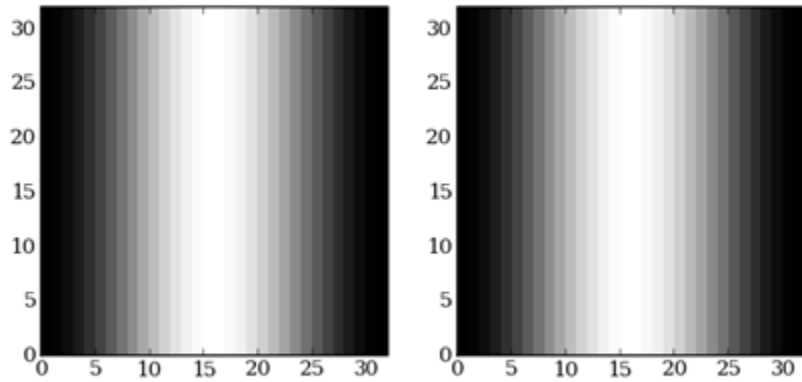


Young's double slit experiment

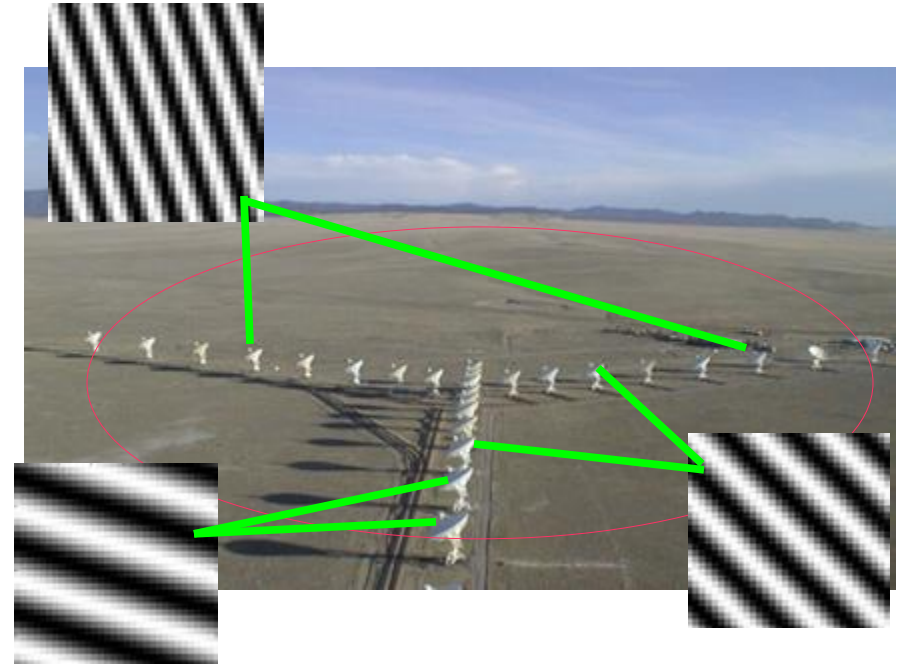


Imaging Interferometer : A detector array

# An indirect imaging device



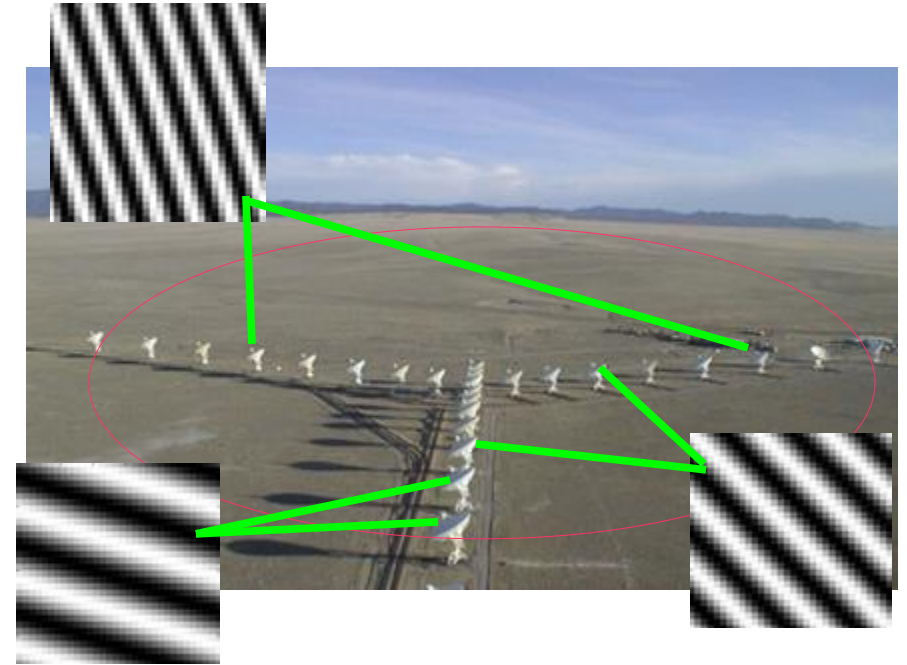
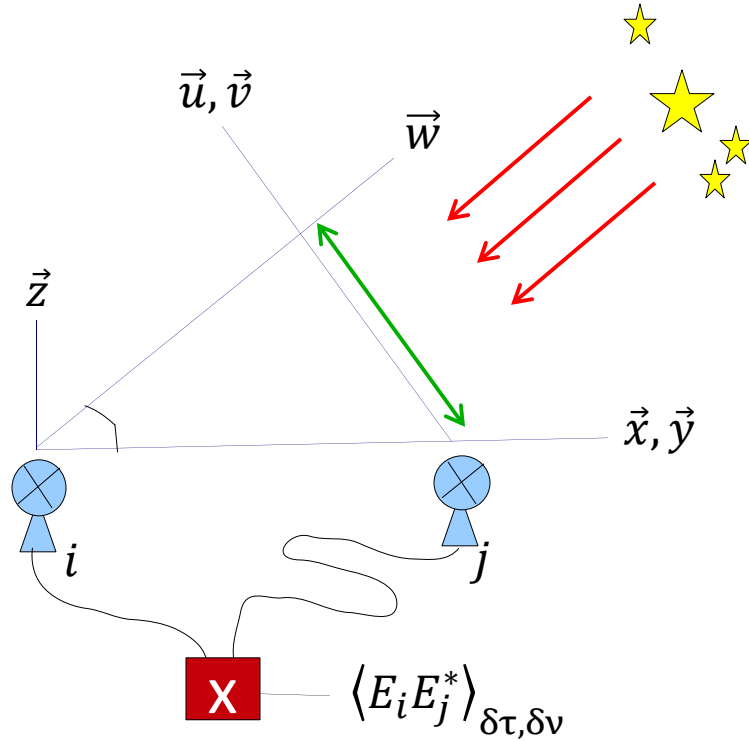
2D Fourier Series



Imaging Interferometer : A detector array



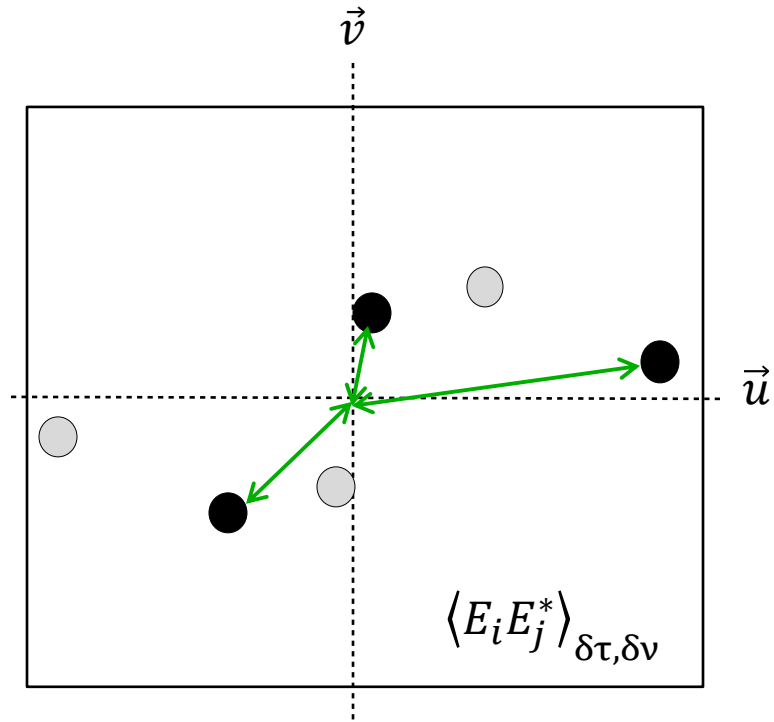
# Measuring spatial frequency components



Imaging Interferometer : A detector array

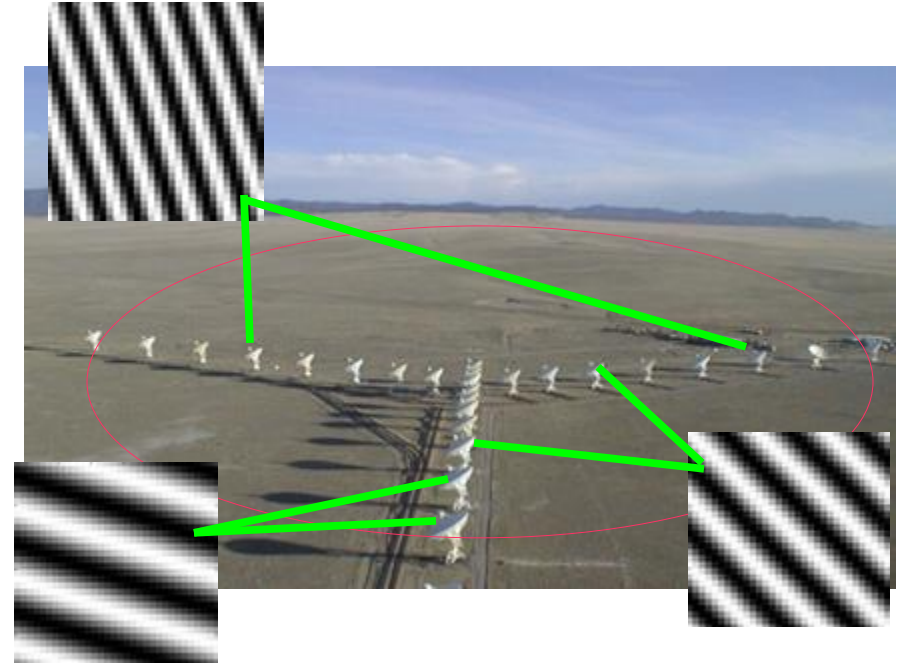
Measure spatial coherence of incident E-field

# Filling up the K-space



**Sampling the “ K-Space ”**

( Spatial Frequency Domain, UV-domain )



**Imaging Interferometer : A detector array**

# Spatial Frequency (uv) coverage + Observed Image

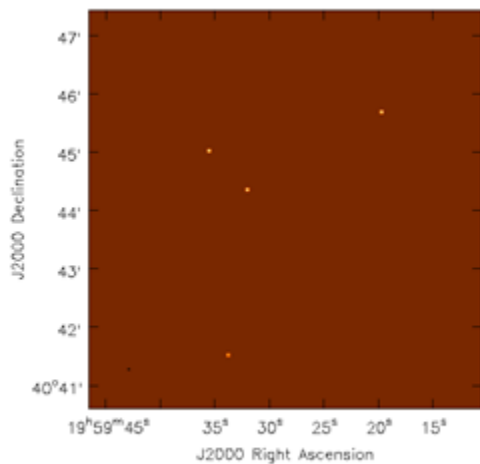
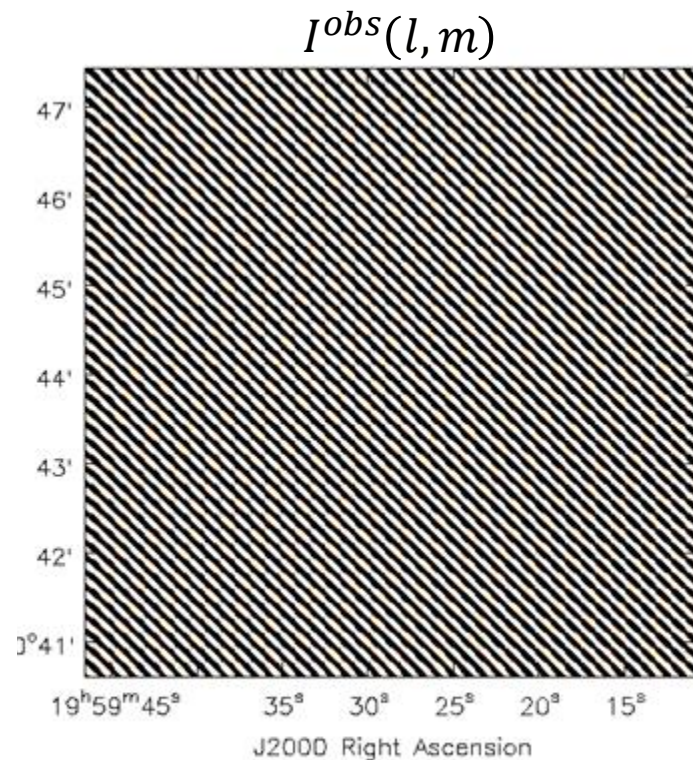
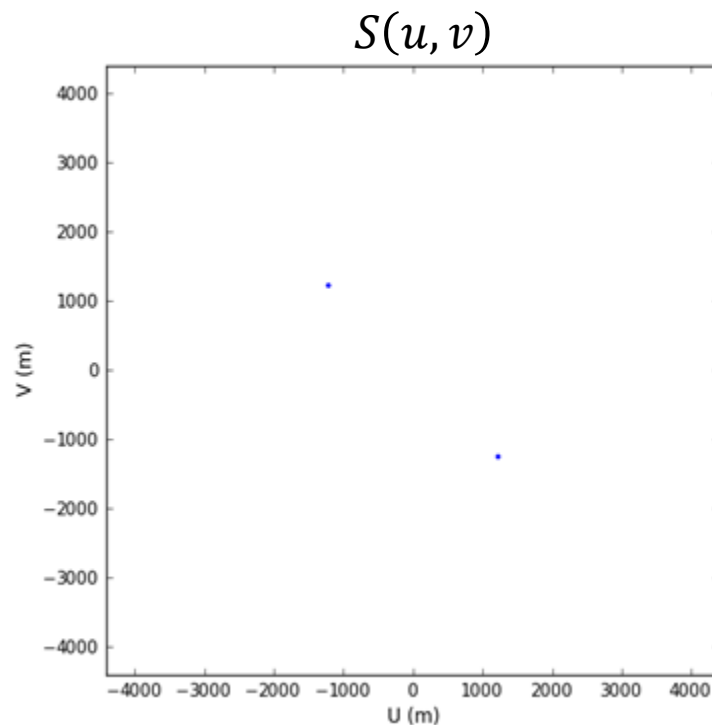


Image of the sky  
using 2 antennas



# Spatial Frequency (uv) coverage + Observed Image

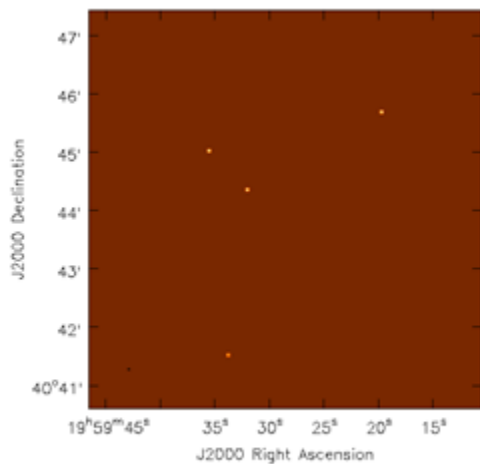
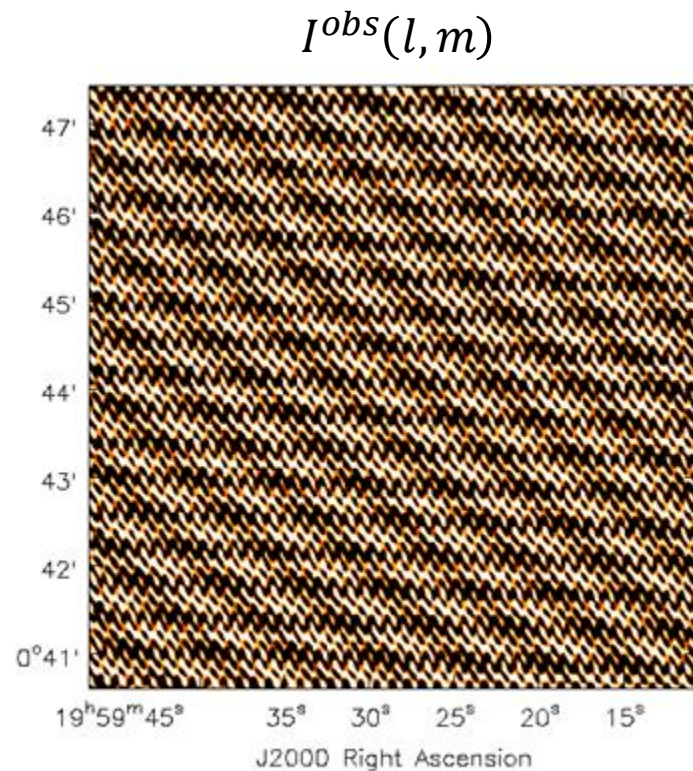
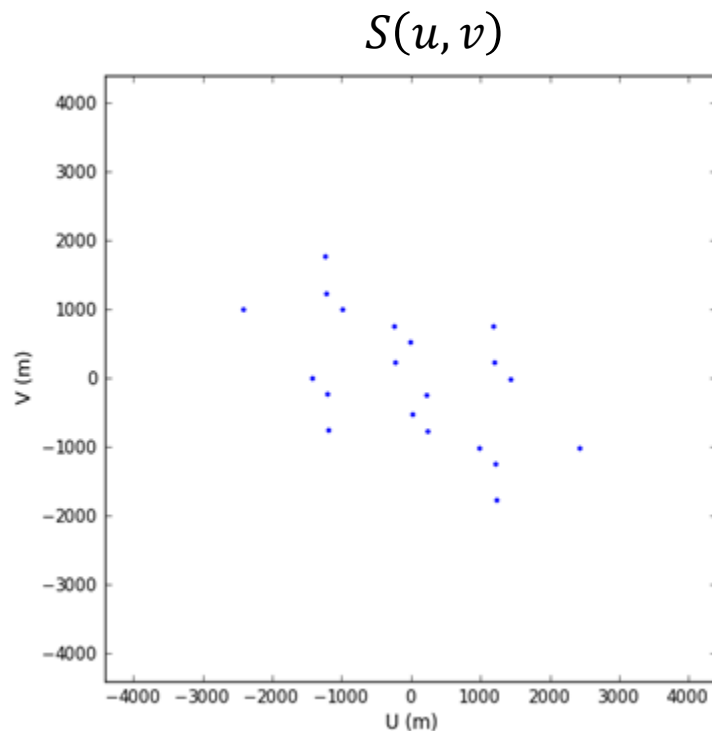


Image of the sky  
using 5 antennas

“Aperture Synthesis”





# Spatial Frequency (uv) coverage + Observed Image

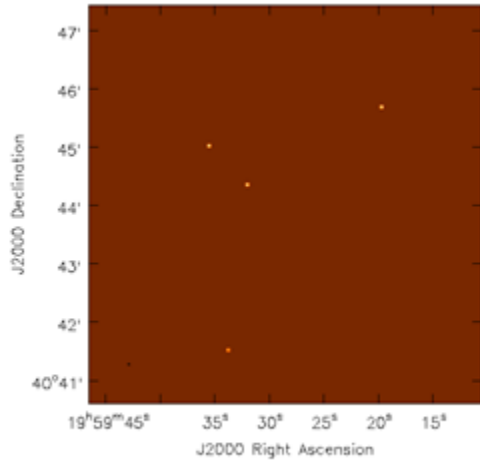
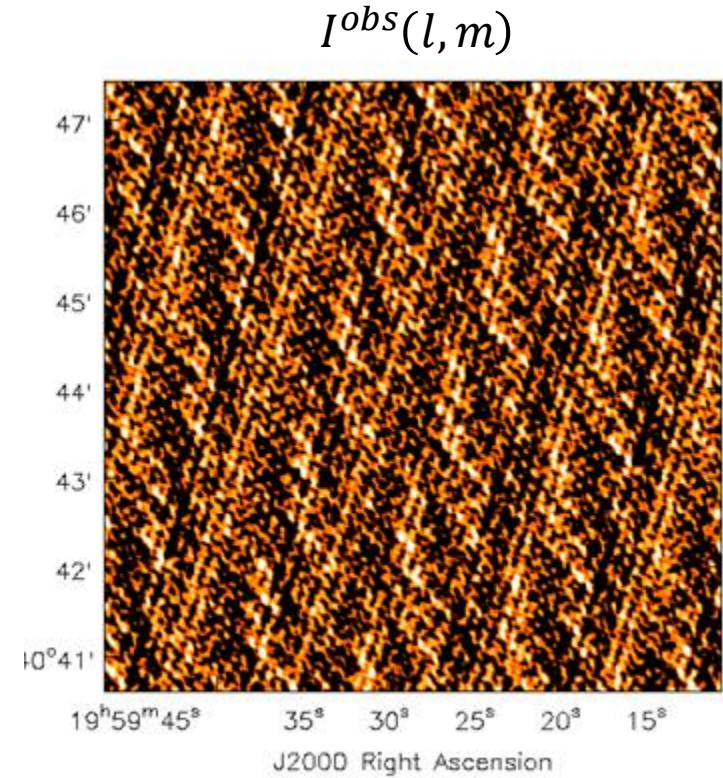
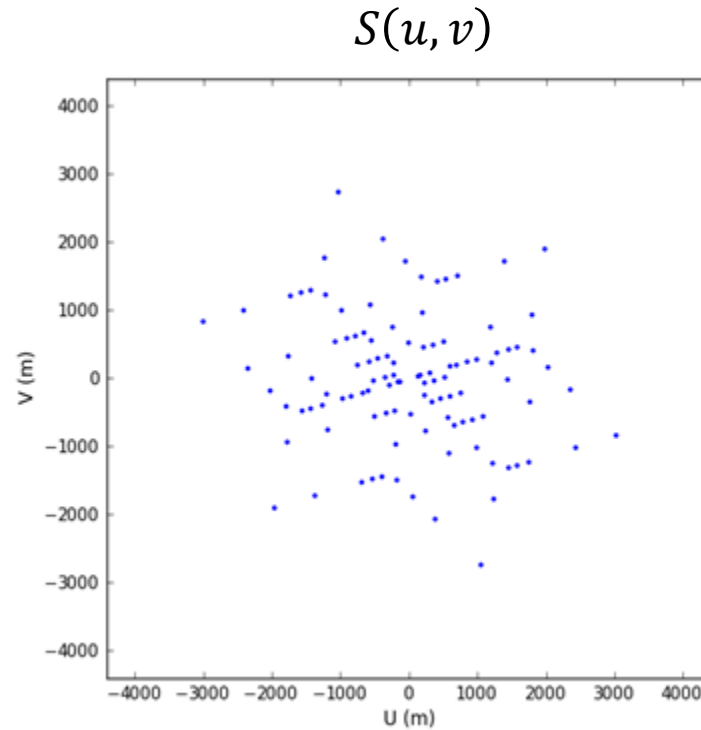


Image of the sky  
using **11** antennas

“Aperture Synthesis”





# Spatial Frequency (uv) coverage + Observed Image

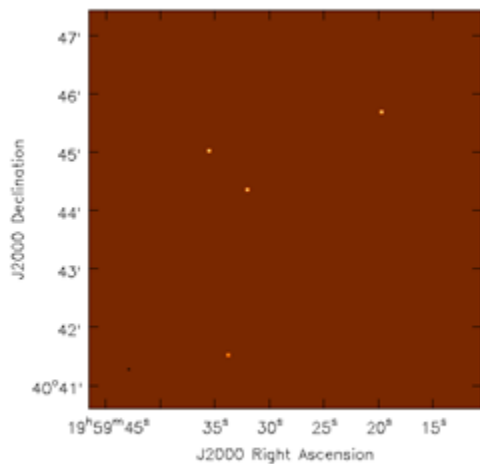
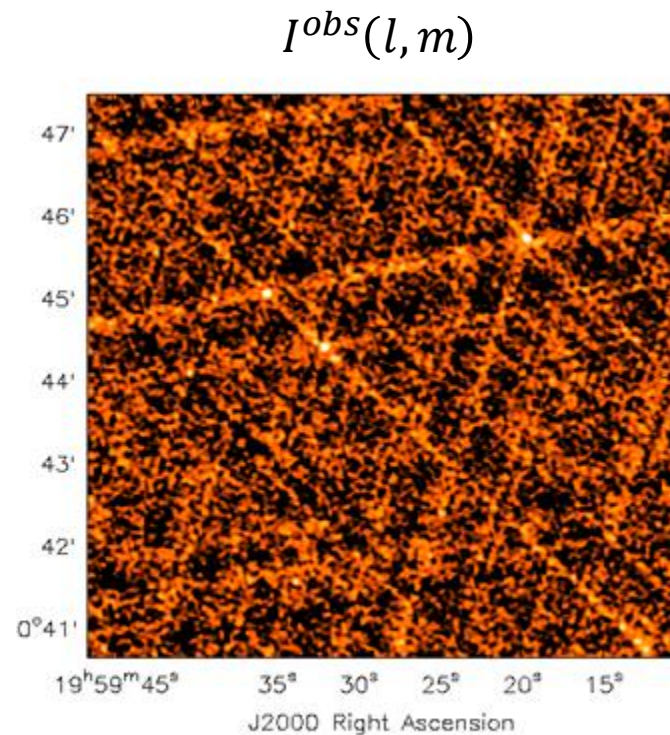
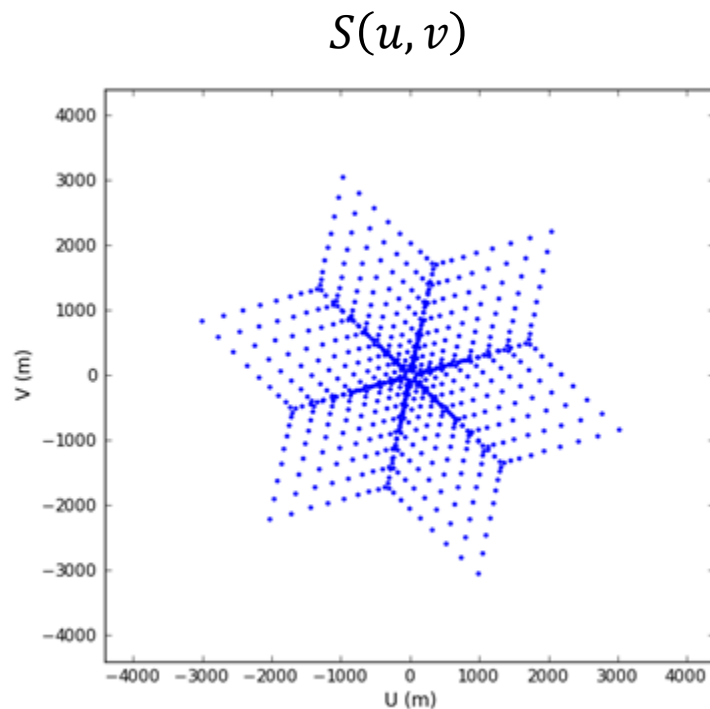


Image of the sky  
using **27** antennas

“Aperture Synthesis”



# Spatial Frequency (uv) coverage + Observed Image

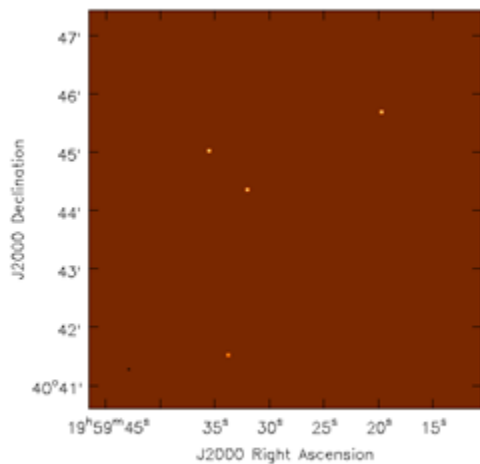
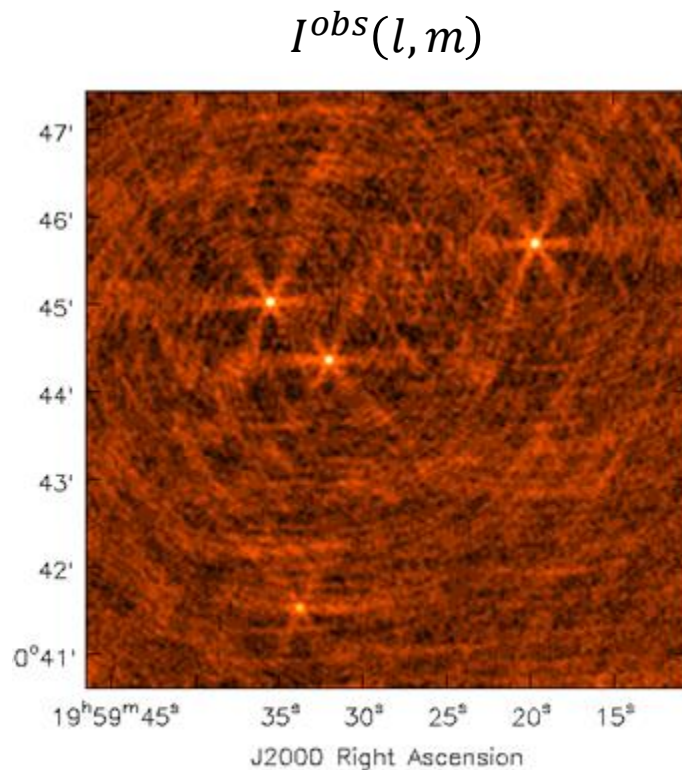
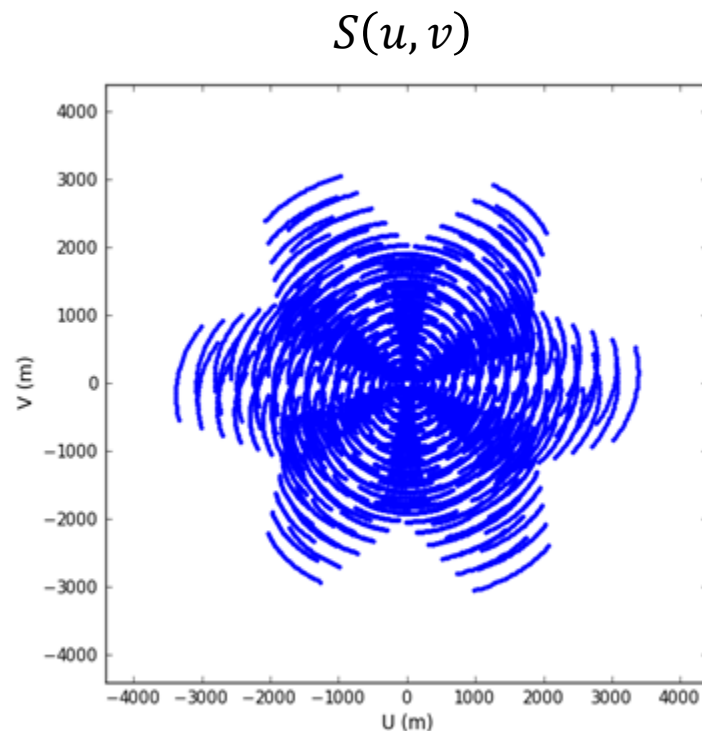


Image of the sky  
using 27 antennas

Observation : **2 hours**

“Earth Rotation Synthesis”



# Spatial Frequency (uv) coverage + Observed Image

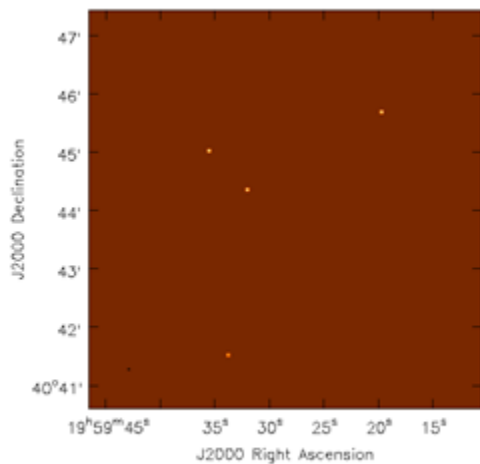
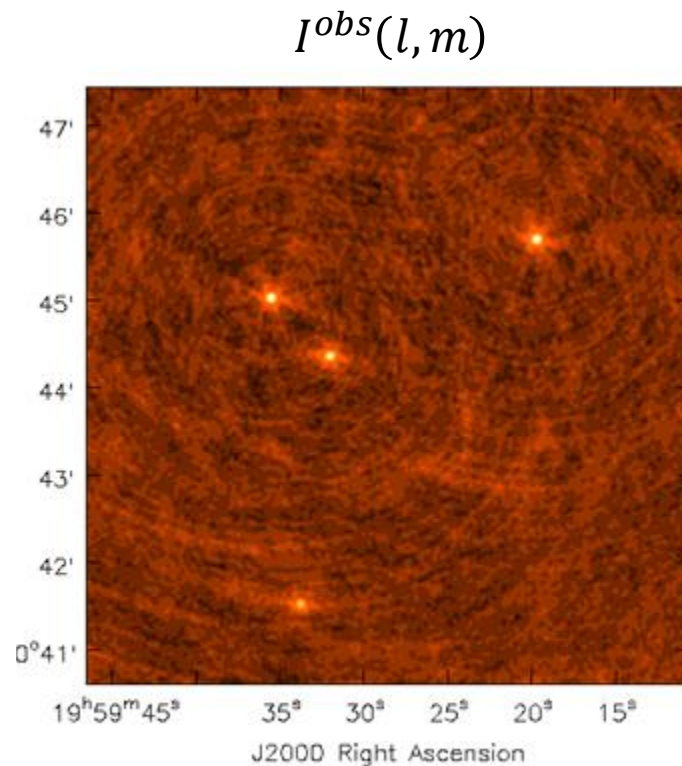
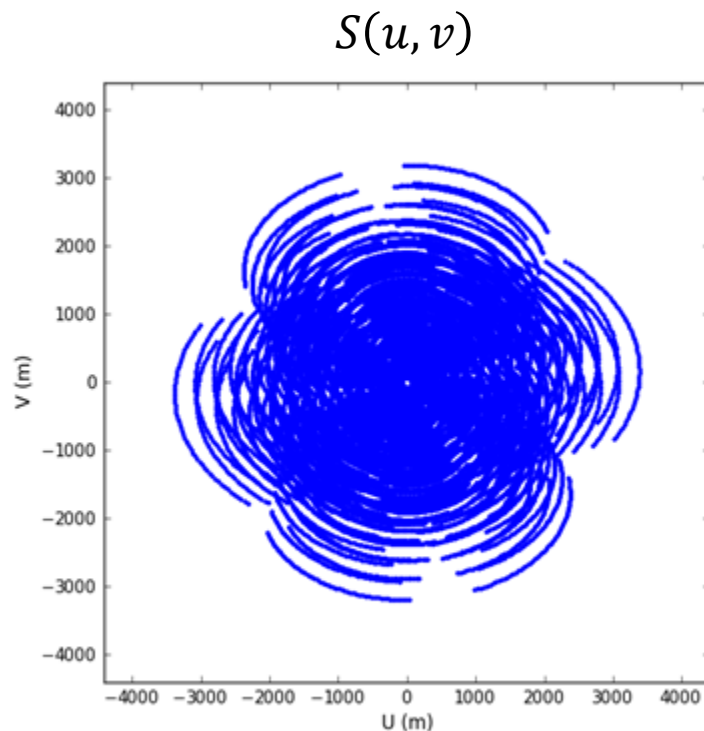


Image of the sky  
using 27 antennas

Observation : **4 hours**

“Earth Rotation Synthesis”



# Spatial Frequency (uv) coverage + Observed Image

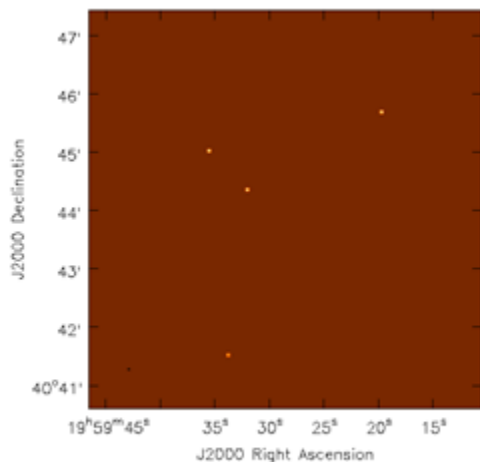
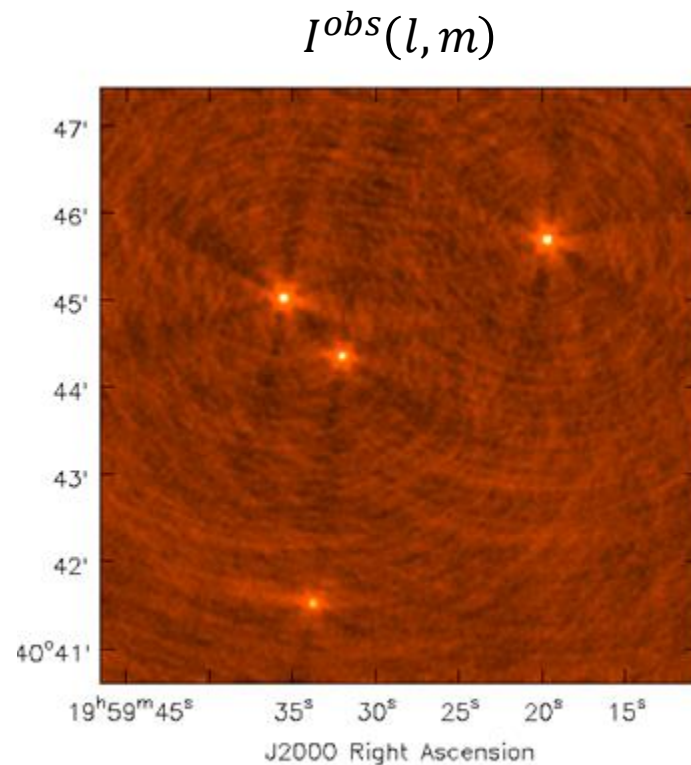
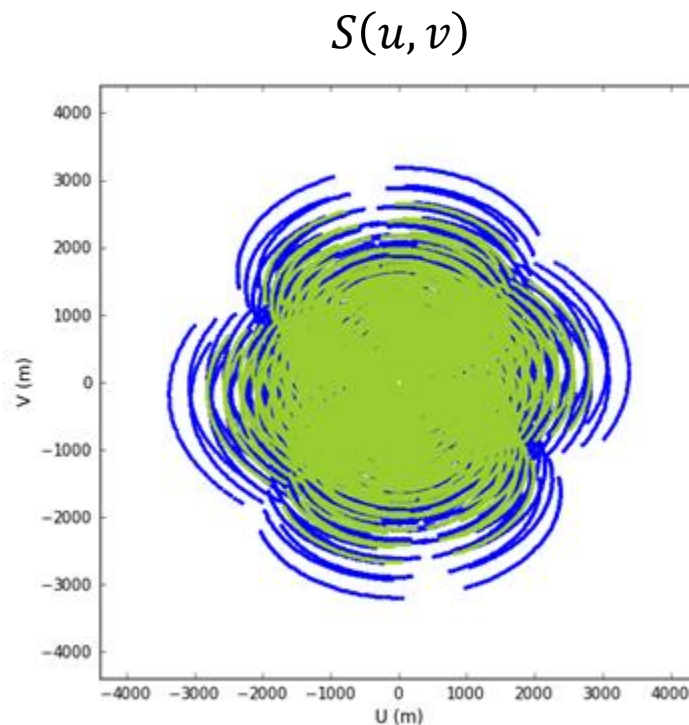


Image of the sky  
using 27 antennas

Observation : 4 hours, **2 frequency channels**

“Multi-Frequency Synthesis”





# Spatial Frequency (uv) coverage + Observed Image

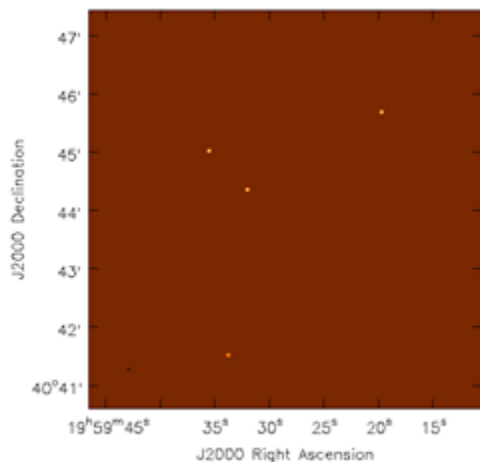
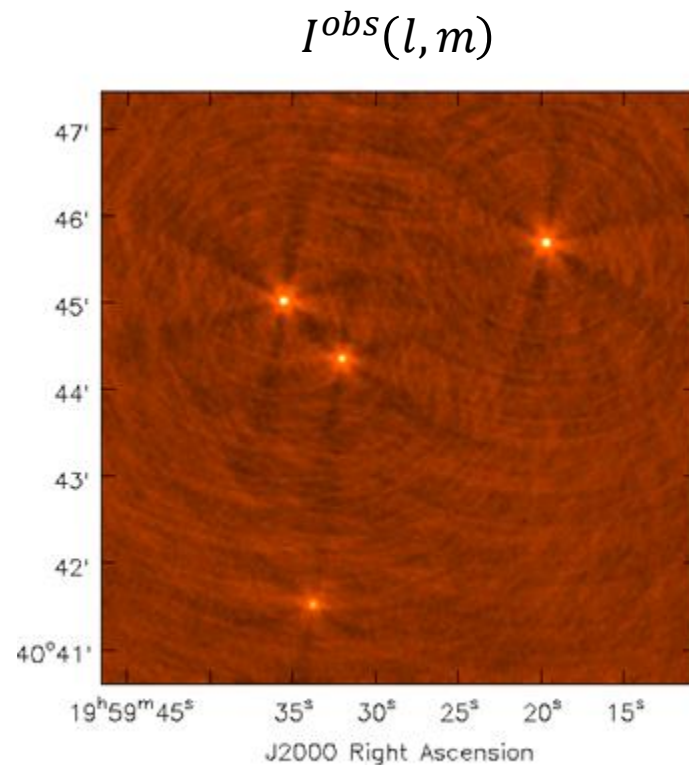
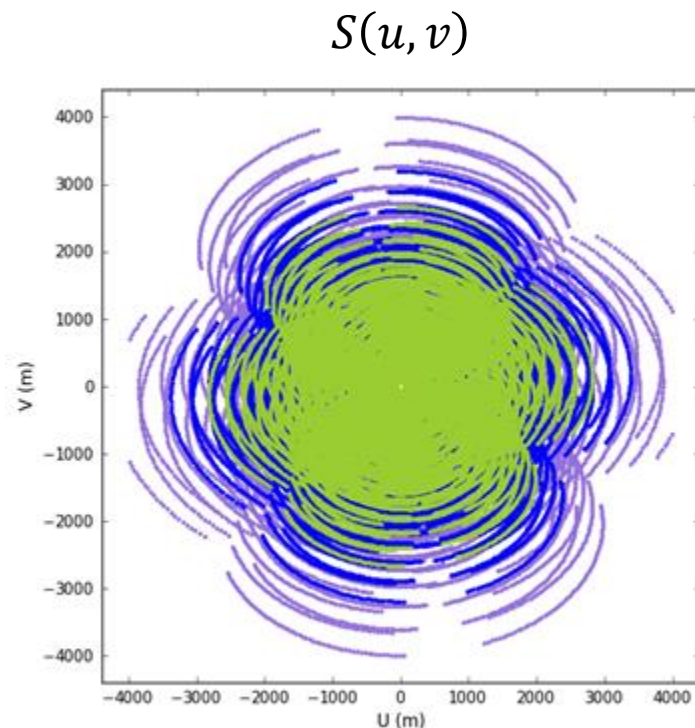


Image of the sky  
using 27 antennas

Observation : 4 hours, **3 frequency channels**

“Multi-Frequency Synthesis”





$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \iint I(l, m) e^{2\pi i(ul + vm)} dl dm$$

**The forward  
problem**

Observed  
visibilities  
(Data)

Direction  
Independent  
Gains

UV sampling  
pattern

Sky Brightness  
(Image )

Fourier transform  
kernel

$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \iint I(l, m) e^{2\pi i(ul+vm)} dl dm$$

The forward problem

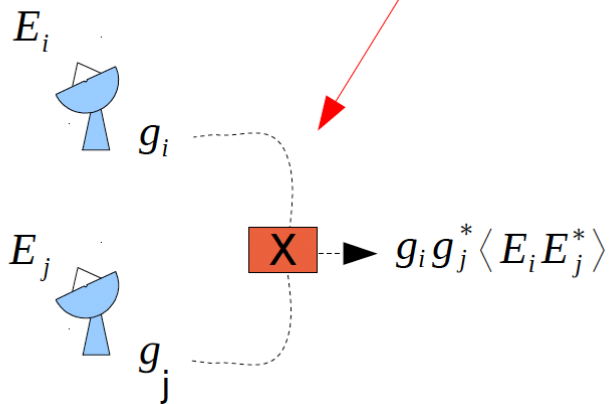
Observed visibilities (Data)

Direction Independent Gains

UV sampling pattern

Sky Brightness (Image)

Fourier transform kernel



Calibration

Solve for  $g_i$  and divide out  $g_i g_j^*$

N antennas  
N(N-1)/2 antenna-pairs (baselines)

$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \iint I(l, m) e^{2\pi i(ul + vm)} dl dm$$

**The forward problem**

Observed or  
Calibrated  
visibilities  
(Data)

Direction  
Independent  
Gains

UV sampling  
pattern

Sky Brightness  
(Image )

Fourier transform  
kernel

Gridding  
+  
IFFT  
+  
Normalization

$I^{obs}$

$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \iint I(l, m) e^{2\pi i(ul+vm)} dl dm$$

**The forward problem**

Observed or  
Calibrated  
visibilities  
(Data)

Direction  
Independent  
Gains

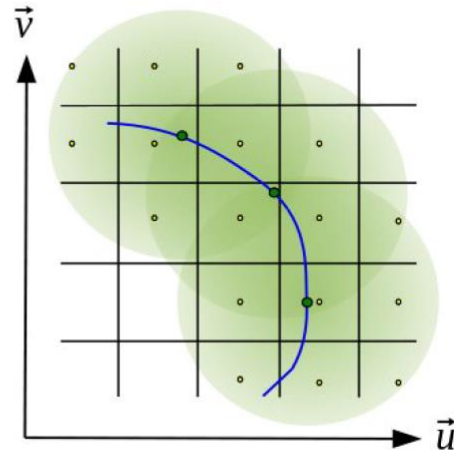
UV sampling  
pattern

Sky Brightness  
(Image )

Fourier transform  
kernel

Gridding  
+  
IFFT  
+  
Normalization

$I^{obs}$



Convolutional  
resampling

$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \iint I(l, m) e^{2\pi i(ul+vm)} dl dm$$

**The forward problem**

Observed or  
Calibrated  
visibilities  
(Data)

Direction  
Independent  
Gains

UV sampling  
pattern

Sky Brightness  
(Image)

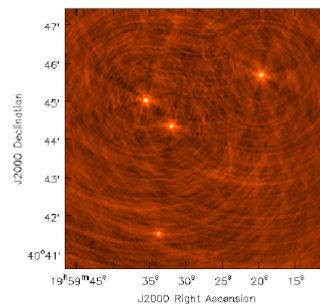
Fourier transform  
kernel

Gridding  
+  
IFFT  
+  
Normalization

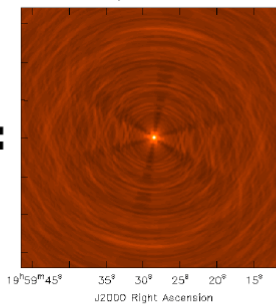
**The inverse problem**

$I^{obs}$

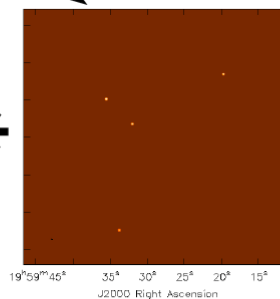
$$I^{obs}(l, m) = I^{PSF}(l, m) * I^{sky}(l, m)$$



=



\*



**Image Reconstruction** (deconvolution)



$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \iint I(l, m) e^{2\pi i(ul+vm)} dl dm$$

**The generalized  
forward problem**

$$V_{ij}^{obs}(\nu, t) = M_{ij}(\nu, t) S_{ij}(\nu, t) \iiint M_{ij}^s(l, m, \nu, t) I(l, m, \nu, t) e^{2\pi i(ul+vm+w(n-1))} dl dm dn$$

$$V_{ij}^{obs}(\mathbf{v}, t) \approx M_{ij}(\mathbf{v}, t) S_{ij}(\mathbf{v}, t) \iint I(l, m) e^{2\pi i(ul+vm)} dl dm$$

**The generalized forward problem**

$$V_{ij}^{obs}(\mathbf{v}, t) = M_{ij}(\mathbf{v}, t) S_{ij}(\mathbf{v}, t) \iiint M_{ij}^s(l, m, \mathbf{v}, t) I(l, m, \mathbf{v}, t) e^{2\pi i(ul+vm+w(n-1))} dl dm dn$$

General Form :  $V(\vec{b}_{ij}) = \iiint I^{sky}(\vec{s}) e^{2\pi i(\vec{b}_{ij} \cdot \vec{s})} d^3 \vec{s}$

$$V(\vec{b}_{ij}) = \iiint M(\vec{s}, t) I^{sky}(\vec{s}, t) e^{2\pi i(\vec{b}_{ij} \cdot \vec{s}) + \phi(\vec{s}, t)} d^3 \vec{s}$$

Interferometry & MRI share the same functional form (with different Physics)

=> Interesting overlap in solution techniques.

$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \iint I(l, m) e^{2\pi i(ul+vm)} dl dm$$

**The generalized forward problem**

$$V_{ij}^{obs}(\nu, t) = M_{ij}(\nu, t) S_{ij}(\nu, t) \iiint M_{ij}^s(l, m, \nu, t) I(l, m, \nu, t) e^{2\pi i(ul+vm+w(n-1))} dl dm dn$$

Direction  
Independent  
Gains

Direction  
Dependent  
Effects

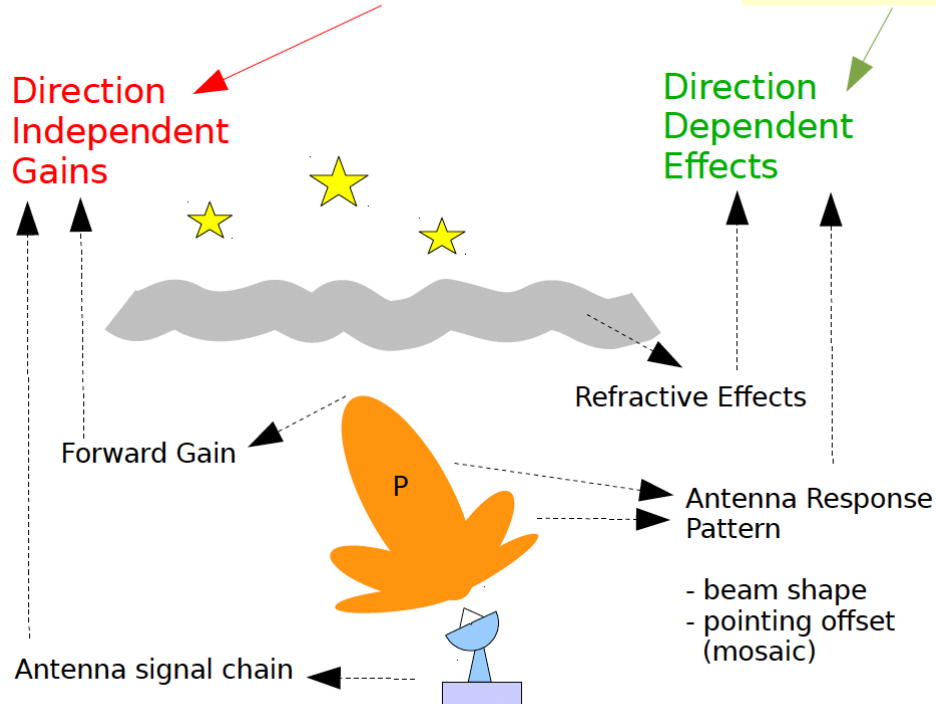
Sky-brightness varies  
with frequency (time)

W-Term

$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \iint I(l, m) e^{2\pi i(ul+vm)} dl dm$$

**The generalized forward problem**

$$V_{ij}^{obs}(\nu, t) = M_{ij}(\nu, t) S_{ij}(\nu, t) \iiint M_{ij}^s(l, m, \nu, t) I(l, m, \nu, t) e^{2\pi i(ul+vm+w(n-1))} dl dm dn$$



$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \iint I(l, m) e^{2\pi i(ul+vm)} dl dm$$

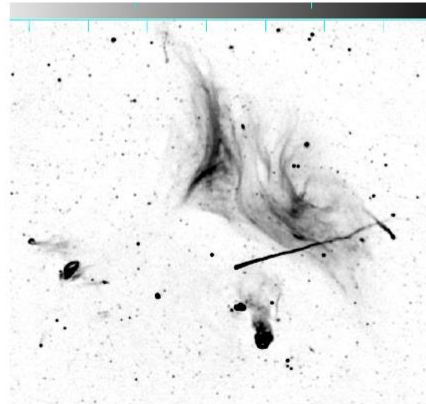
The generalized forward problem

$$V_{ij}^{obs}(\nu, t) = M_{ij}(\nu, t) S_{ij}(\nu, t) \iiint M_{ij}^s(l, m, \nu, t) I(l, m, \nu, t) e^{2\pi i(ul+vm+w(n-1))} dl dm dn$$

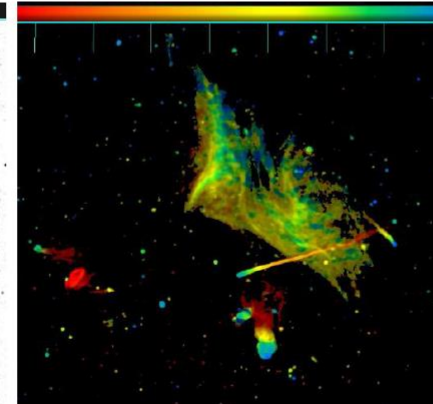


Sky-brightness varies with frequency

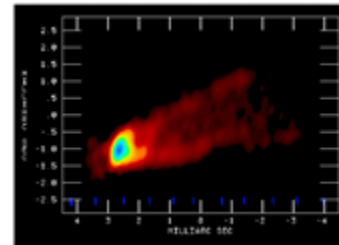
and time



Intensity



Intensity weighted Spectral Index



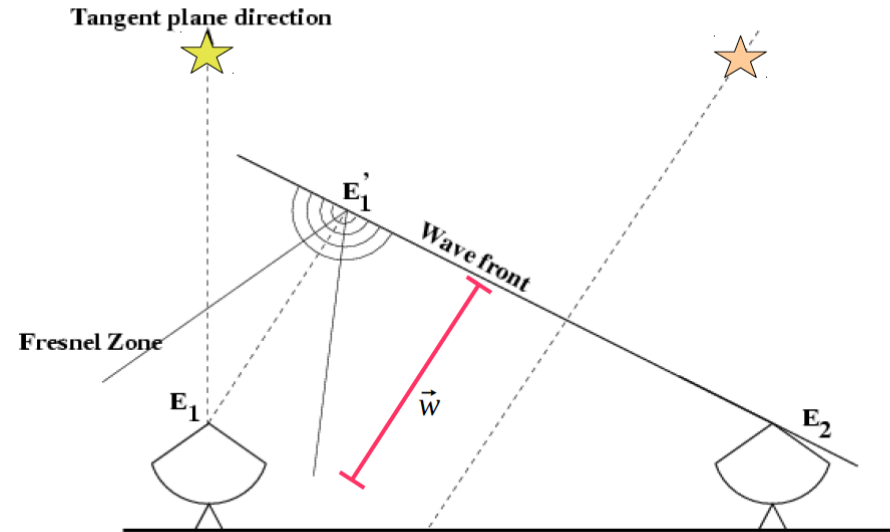
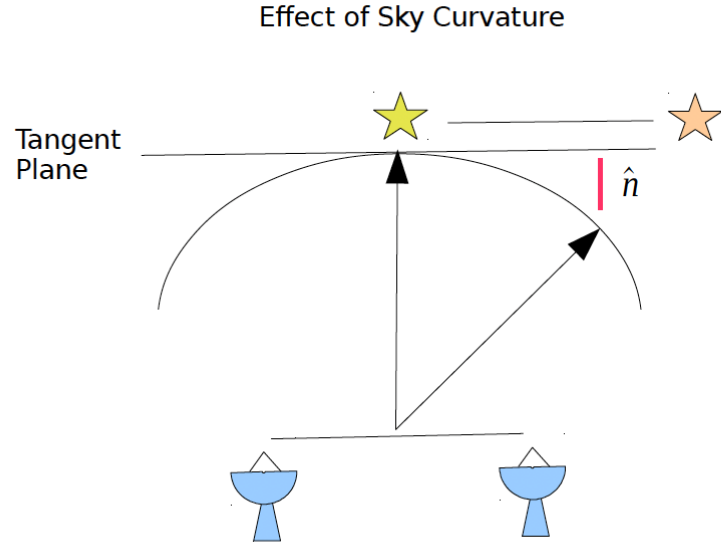


$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \iint I(l, m) e^{2\pi i(ul+vm)} dl dm$$

The generalized forward problem

$$V_{ij}^{obs}(\nu, t) = M_{ij}(\nu, t) S_{ij}(\nu, t) \iiint M_{ij}^s(l, m, \nu, t) I(l, m, \nu, t) e^{2\pi i(ul+vm+w(n-1))} dl dm dn$$

W-Term



$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \iint I(l, m) e^{2\pi i(ul+vm)} dl dm$$

**The generalized forward problem**

$$V_{ij}^{obs}(\nu, t) = M_{ij}(\nu, t) S_{ij}(\nu, t) \iiint M_{ij}^s(l, m, \nu, t) I(l, m, \nu, t) e^{2\pi i(ul+vm+w(n-1))} dl dm dn$$

Direction  
Independent  
Gains

UV sampling  
function

Direction  
Dependent  
Effects

Sky-brightness varies  
with frequency and time

W-Term

How do we solve these systems of equations ?

$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \iint I(l, m) e^{2\pi i(ul+vm)} dl dm$$

The generalized forward problem

$$V_{ij}^{obs}(\nu, t) = M_{ij}(\nu, t) S_{ij}(\nu, t) \iiint M_{ij}^s(l, m, \nu, t) I(l, m, \nu, t) e^{2\pi i(ul+vm+w(n-1))} dl dm dn$$

Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency and time

W-Term

Calibration

$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \iint I(l, m) e^{2\pi i(ul+vm)} dl dm$$

The generalized forward problem

$$V_{ij}^{obs}(\nu, t) = M_{ij}(\nu, t) S_{ij}(\nu, t) \iiint M_{ij}^s(l, m, \nu, t) I(l, m, \nu, t) e^{2\pi i(ul+vm+w(n-1))} dl dm dn$$

Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency and time

W-Term

Calibration

=> Multiplicative effect in the image domain  
 => Convolutions in the visibility domain  
 ( corrected during gridding + iFFT + normalization )

Inverse Imaging

$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \iint I(l, m) e^{2\pi i(ul+vm)} dl dm$$

**The generalized forward problem**

$$V_{ij}^{obs}(\nu, t) = M_{ij}(\nu, t) S_{ij}(\nu, t) \iiint M_{ij}^s(l, m, \nu, t) I(l, m, \nu, t) e^{2\pi i(ul+vm+w(n-1))} dl dm dn$$

Direction  
Independent  
Gains

UV sampling  
function

Direction  
Dependent  
Effects

Sky-brightness varies  
with frequency and time

W-Term

**Calibration**

**Image  
Reconstruction**

(De)convolution in the image domain

=> Multiplicative effect in the image domain  
=> Convolutions in the visibility domain  
( corrected during gridding + iFFT + normalization )

**Inverse  
Imaging**



# Forward and Inverse Problems : Iterative Optimization algorithms

$$V^{obs} = [A]I^m + n \quad \longrightarrow \quad I^m = [A]^{-1}V^{obs}$$

**Data regularization** : L2 ( chi-square )

**Sky model**

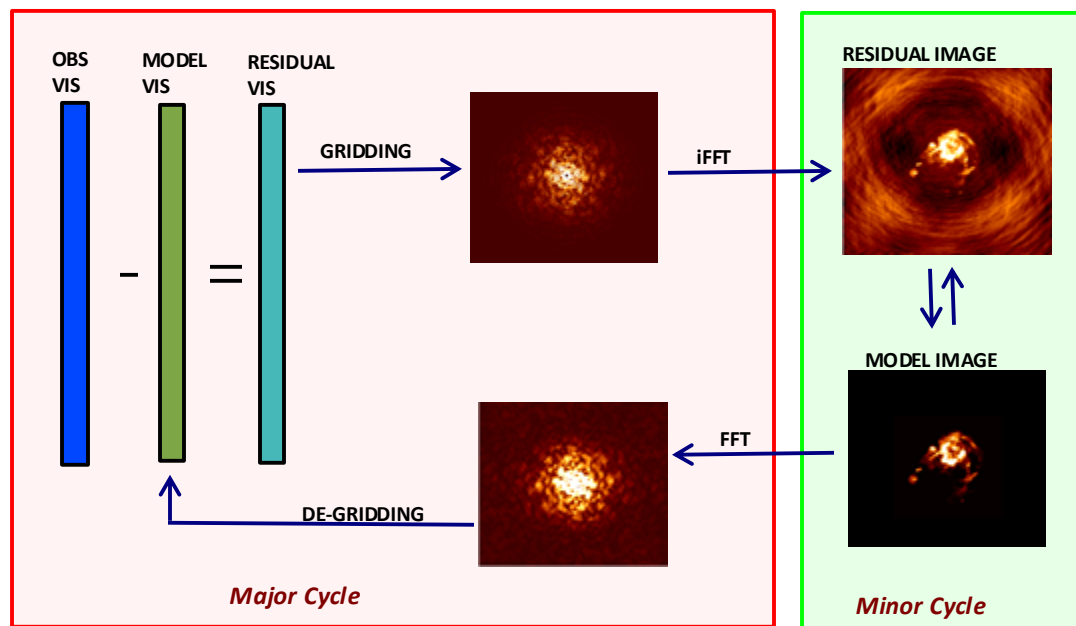
- Delta functions, Gaussians, Wavelets, etc, etc..
- Multi-frequency and time-variable models
- Astrophysics models ( non-imaging )

**Constrained Optimization**

- Log power spectrum, positivity, smoothness
- Manual constraints : spatial masks, iteration control
- Greedy algorithms vs Parameterized solvers
- L1 , TV norm, etc...

**Instrumental corrections**

- Wide-field and wide-band antenna response patterns,
- Ionospheric refraction corrections
- 3D to 2D effects, K-space 'hole' effects.



# Forward and Inverse Problems : Algorithm Variability

$$V^{obs} = [A]I^m + n \quad \longrightarrow \quad I^m = [A]^{-1}V^{obs}$$

**Data regularization** : L2 ( chi-square )

**Sky model**

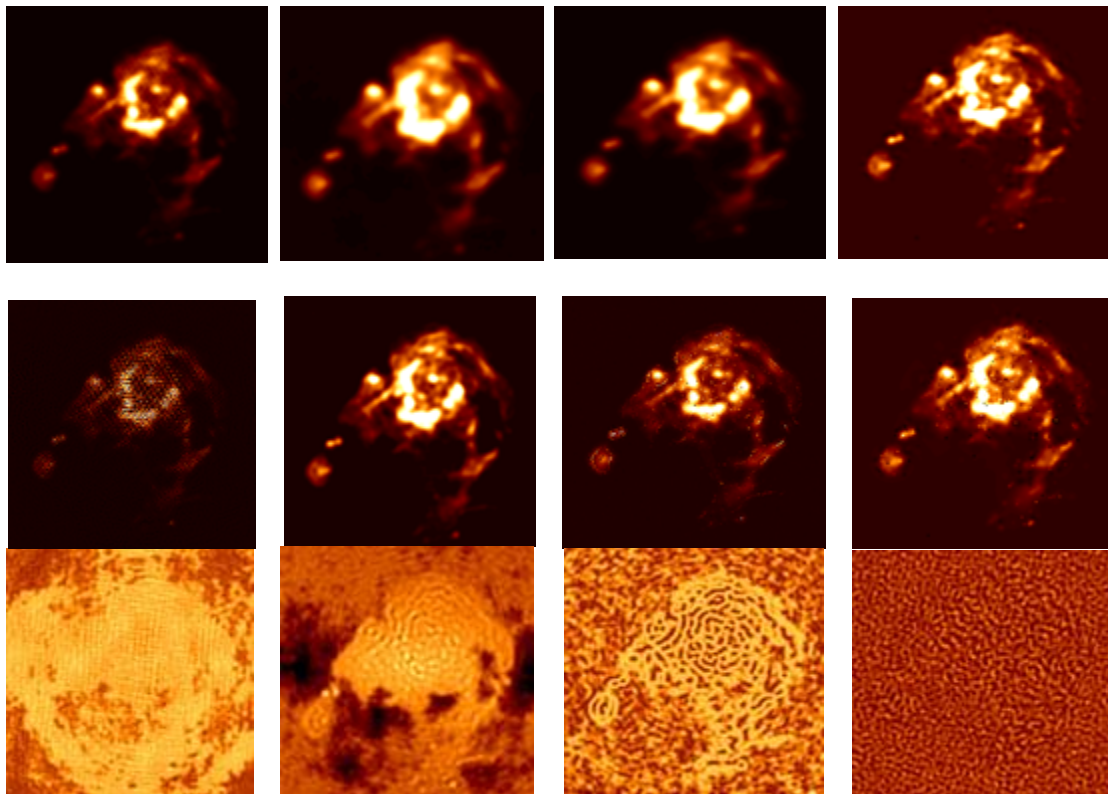
- Delta functions, Gaussians, Wavelets, etc, etc..
- Multi-frequency and time-variable models
- Astrophysics models ( non-imaging )

**Constrained Optimization**

- Log power spectrum, positivity, smoothness
- Manual constraints : spatial masks, iteration control
- Greedy algorithms vs Parameterized solvers
- L1 , TV norm, etc...

**Instrumental corrections**

- Wide-field and wide-band antenna response patterns,
- Ionospheric refraction corrections
- 3D to 2D effects, K-space 'hole' effects.



# Forward and Inverse Problems : Instrumental Corrections

$$V^{obs} = [A]I^m + n \quad \longrightarrow \quad I^m = [A]^{-1}V^{obs}$$

**Data regularization** : L2 ( chi-square )

**Sky model**

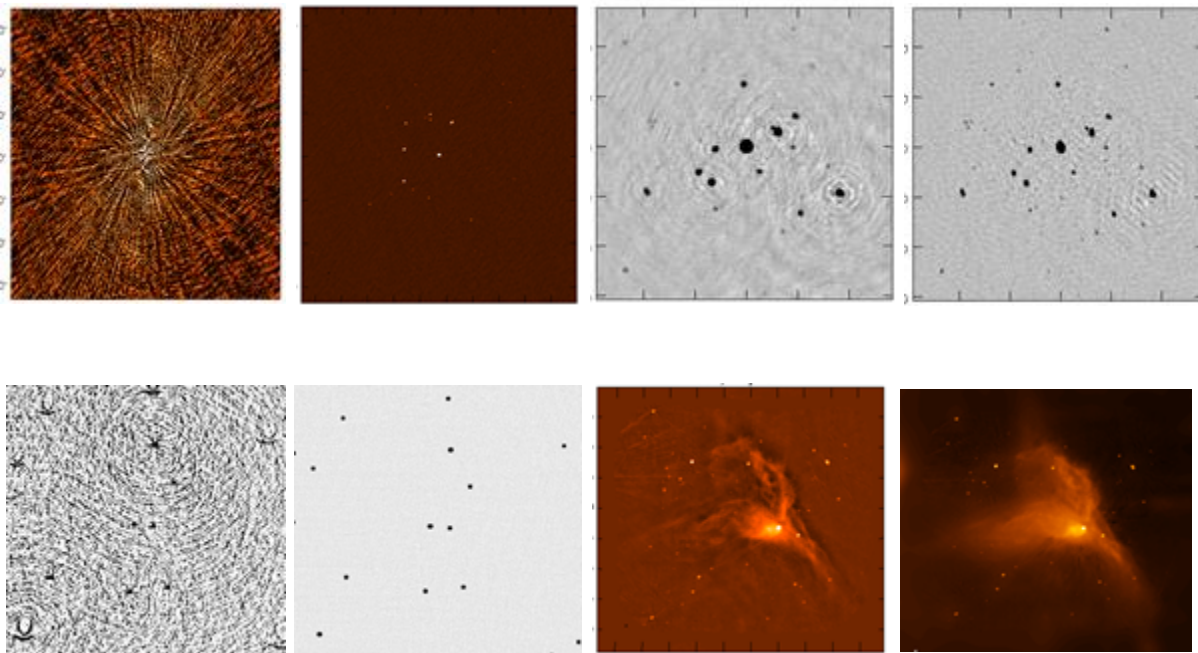
- Delta functions, Gaussians, Wavelets, etc, etc..
- Multi-frequency and time-variable models
- Astrophysics models ( non-imaging )

**Constrained Optimization**

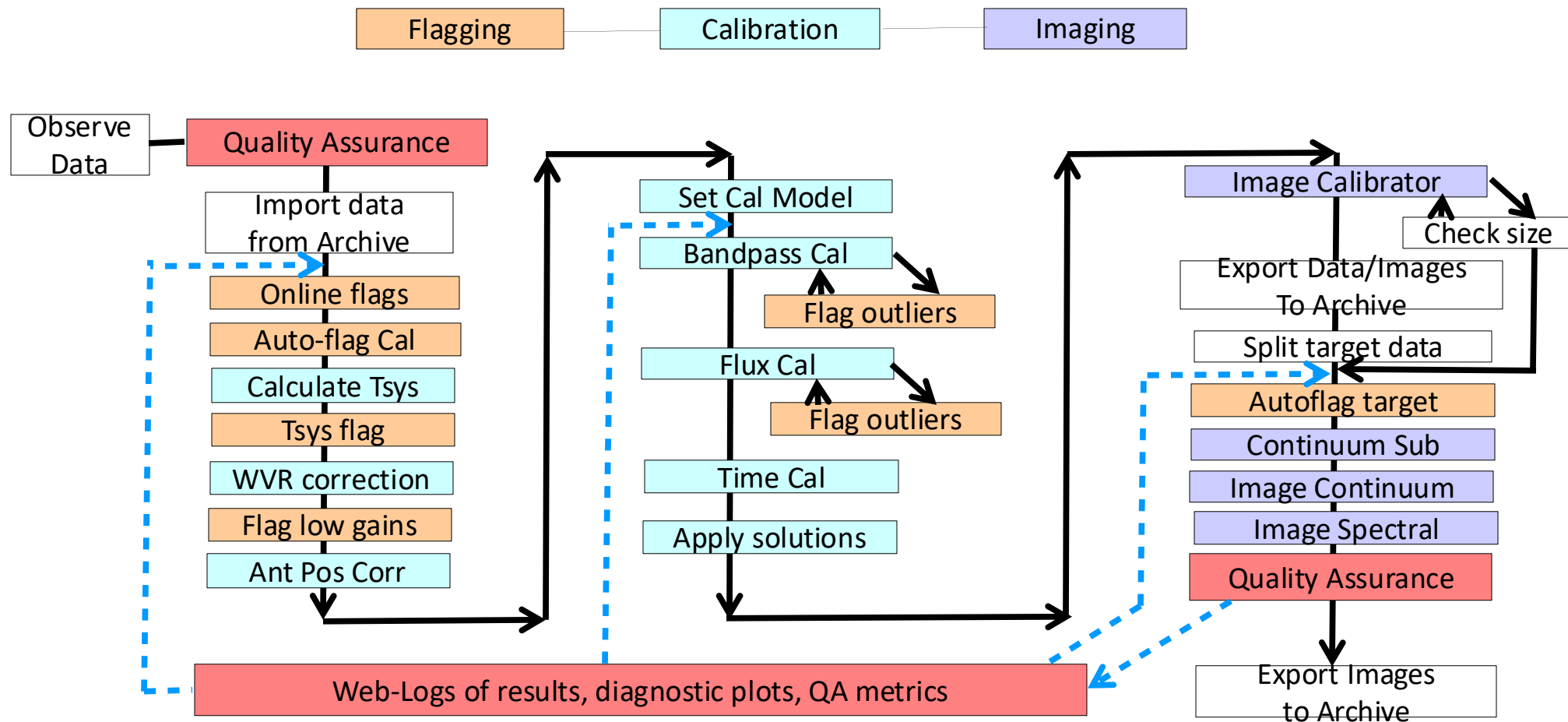
- Log power spectrum, positivity, smoothness
- Manual constraints : spatial masks, iteration control
- Greedy algorithms vs Parameterized solvers
- L1 , TV norm, etc...

**Instrumental corrections**

- Wide-field and wide-band antenna response patterns,
- Ionospheric refraction corrections
- 3D to 2D effects, K-space 'hole' effects.



# Operational Workflows



# The R&D frontier

**New Instruments** : More sensitive, Lower image noise, Detect Fainter Sources  
Larger Data Volume, Greater Algorithm Complexity

## Algorithms :

- A variety of sky models, instrument models, objective functions and regularizers, optimization strategies, the use of priors, etc..

=> Increased exploration of Machine Learning.

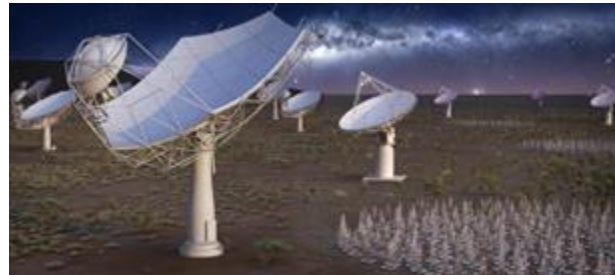
## Compute Load :

- Data volumes : 10s to 100s of GB → ngVLA/SKA : TeraBytes/PetaBytes/ExaBytes
- Image sizes : 10kx10k → 200k x 200k pixels ( with 10k channels and 4 pols )

=> High Performance and High Throughput Computing

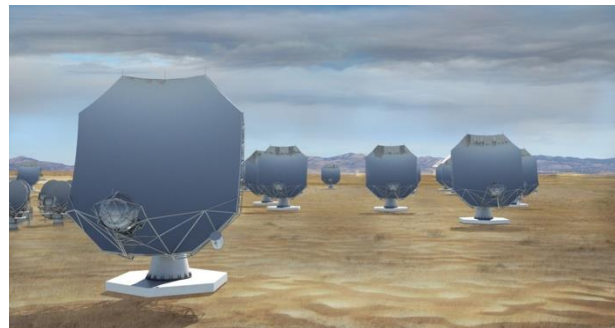
## Workflow Automation :

- Data analysis pipelines that tune parameters for each dataset



**Square Kilometer Array** ([skatelescope.org](http://skatelescope.org))

2K dishes, 1M antennas , 50 MHz – 30 GHz



**Next Generation VLA** ([ngvla.nrao.edu](http://ngvla.nrao.edu))

263 dishes (2 types) , 1-100 GHz

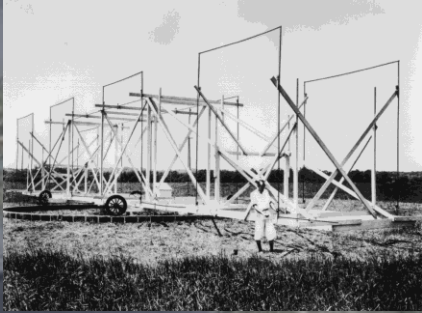


# Radio Telescopes around the World





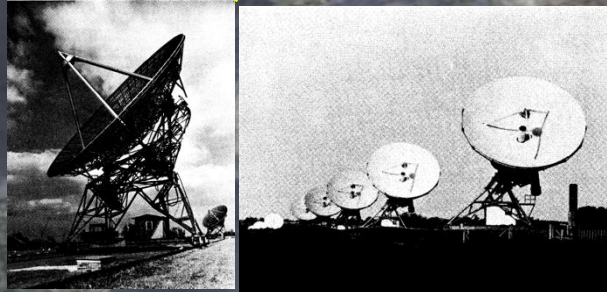
# Radio Telescopes around the World



Karl Jansky (1933)  
"Radio Waves from the Milky Way"



Grote Reber (1936)  
"First All-Sky Radio Map"



Martin Ryle (1960+)  
"First intentional sampling of spatial  
frequency (k-space) in radio astronomy"

Physics Nobel Prize : 1974

McCready, Pawsey, Payne-Scott. (1946)  
"Used wave interference to infer spatial scale"

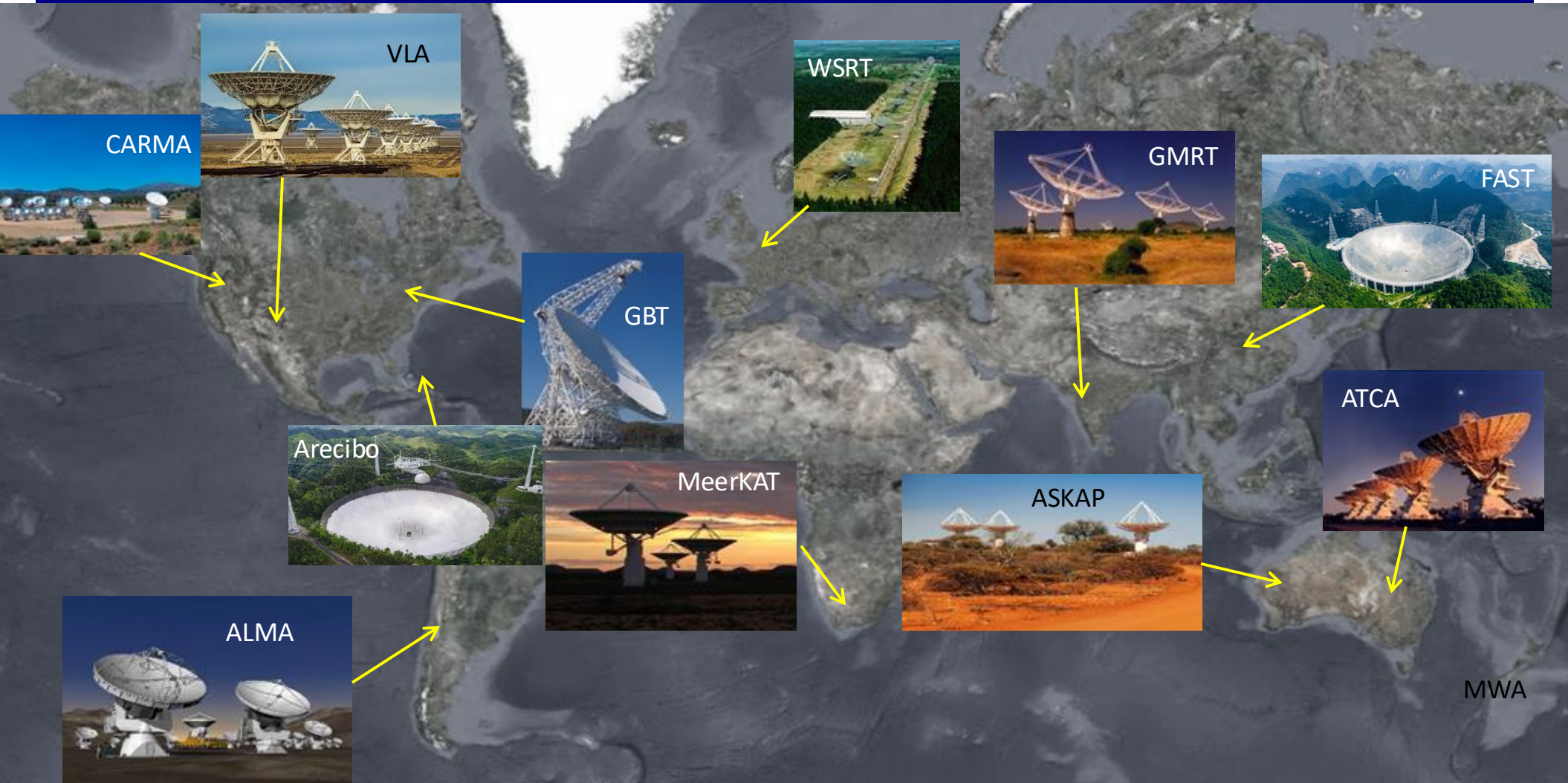


# Radio Telescopes around the World



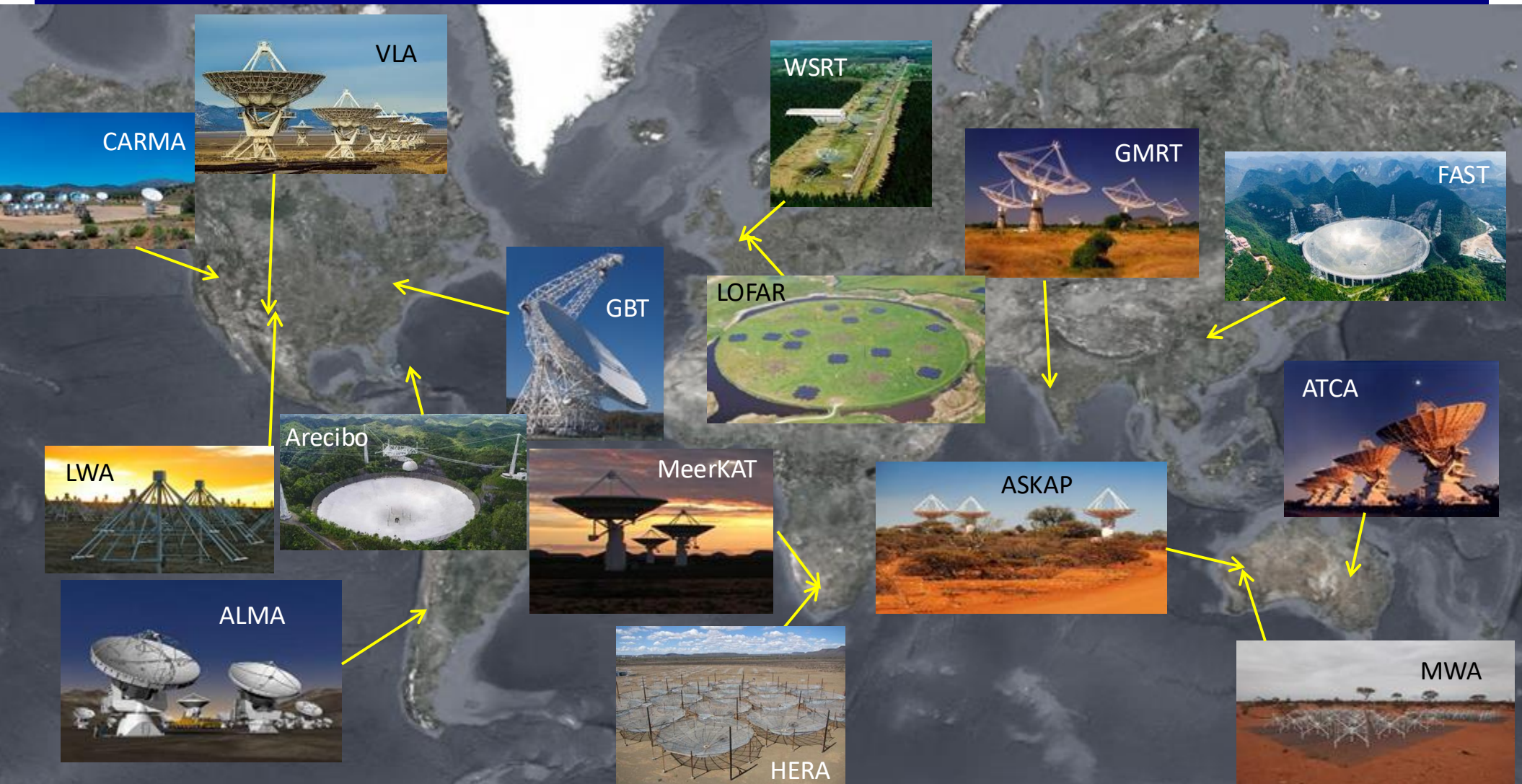


# Radio Telescopes around the World



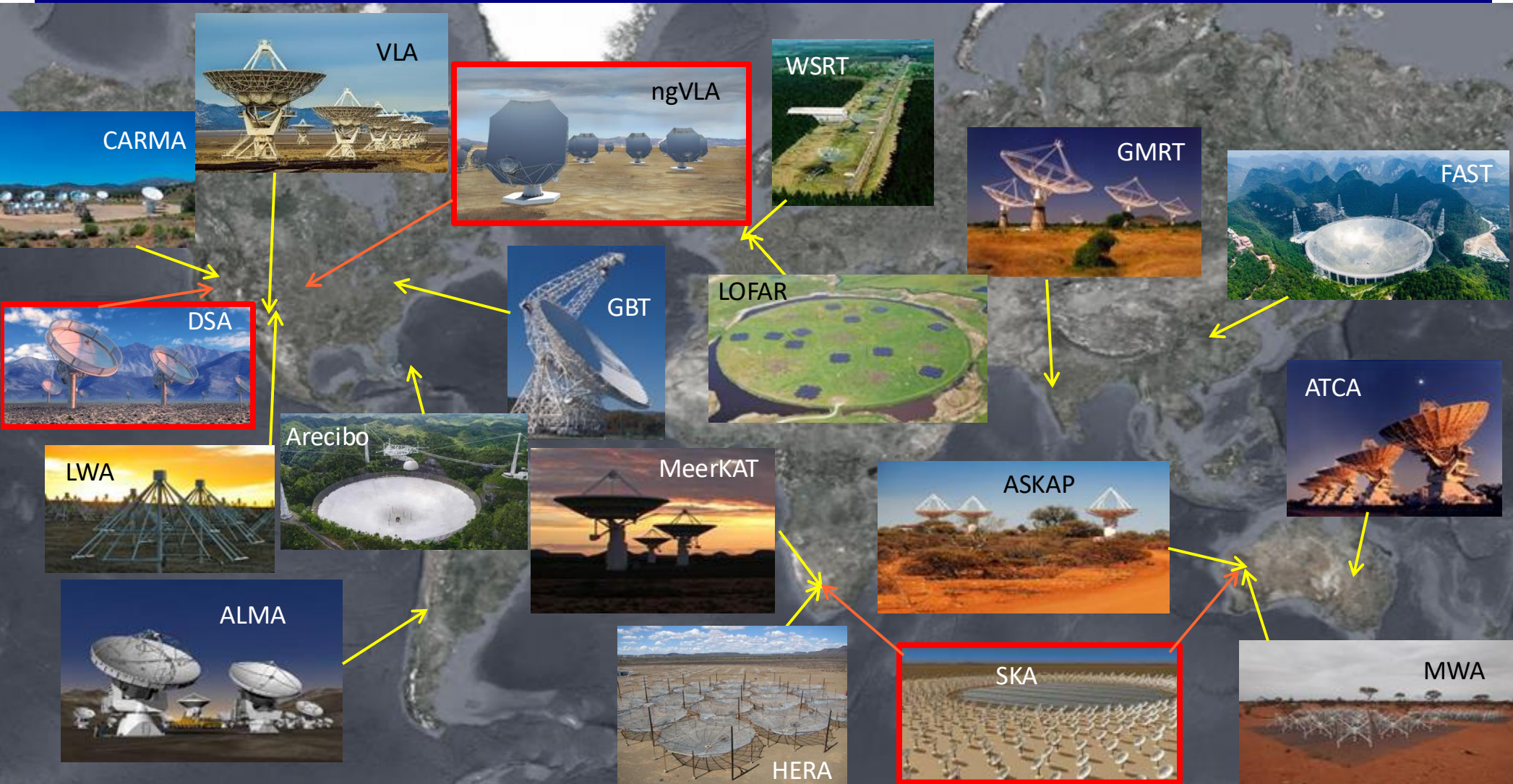


# Radio Telescopes around the World



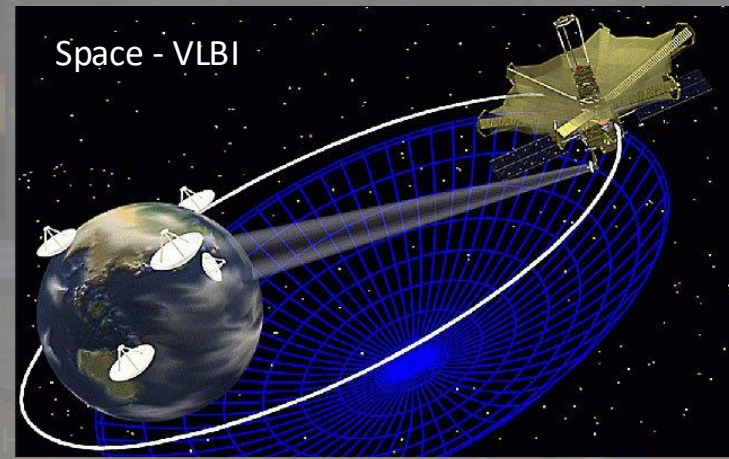
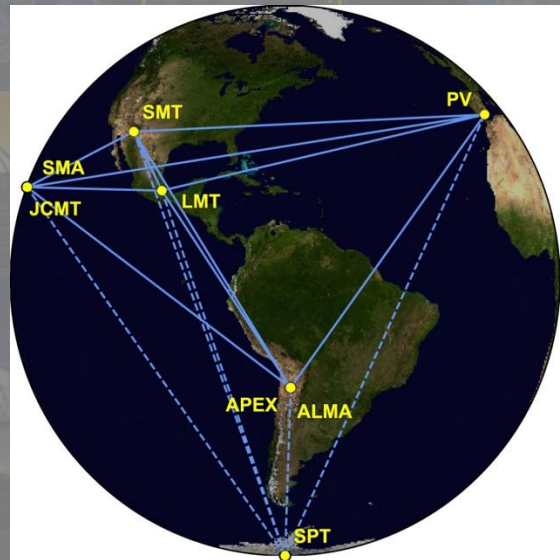


# Radio Telescopes around the World



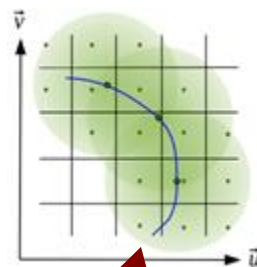


# Radio Telescopes around the World





# Compute Cost



Gridding : Convolutional resampling

$O(N_{\text{data}}) \times (n \times n)$  complex multiply/add ( $n=5 - 100$ )

Example : Major cycle : 1hr  $\rightarrow$  10 days (Diff Algorithms)

Data volume

$N_{\text{data}} =$   
 $N_{\text{ant}}^2 \times N_{\text{time}}$   
 $\times N_{\text{chan}} \times N_{\text{pol}}$

Complex numbers

Lustre I/O

Example :  
8hr data 300 GB

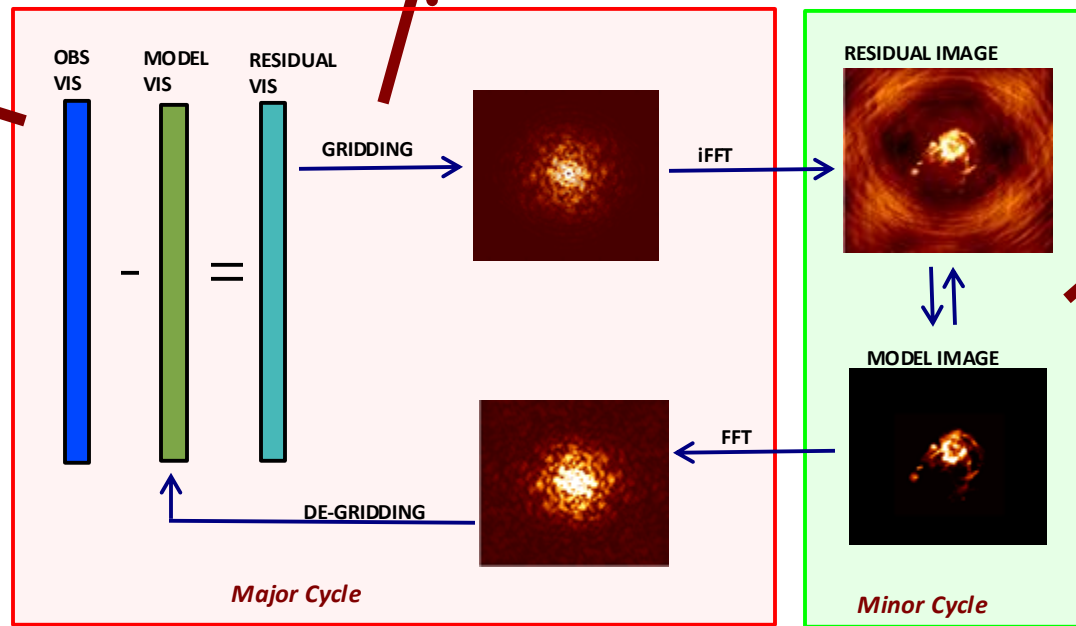


Image sizes

$N_{\text{pix}} = N_x \times N_y \times$   
 $N_{\text{chan}} \times N_{\text{pol}}$

FFTs :  $O(N \log N)$   
Pixel math:  $O(N^2)$   
Mem :  $\sim 8$  copies

$N_x : 1k \rightarrow 40k$   
 $N_{\text{chan}} : 200 - 16K$

Example :  
 $1K \times 1K \times 256$   
 $\sim 1$  GB per image

Number of iterations : 5 – 10 outer loops ,  $10^2$  to  $10^4$  for regularization  
Runtime varies by 1-2 orders of magnitude. Depends on data.