Introduction to Radio Astronomy (for Medical Imaging Professionals)



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From Cells to Galaxies (13 October 2021)

Outline

- Images in radio astronomy

- Imaging with a radio interferometer

- Image formation

- The forward and inverse problems

- Current limits and future trends



Flux Density :
$$\frac{10^{-26}W}{m^2 Hz Beam} = \frac{Jy}{Beam}$$

- Spatial Structure



The M87 Radio Galaxy

Flux Density : $\frac{10^{-26}W}{m^2 Hz Beam} = \frac{Jy}{Beam}$

- Spatial Structure
- Time Variability



The M87 Radio Galaxy



Continuum Spectrum :

- Energetics, Temperature
- Emission mechanism

Flux Density : $\frac{10^{-26}W}{m^2 Hz Beam} = \frac{Jy}{Beam}$

- Spatial Structure
- Time Variability
- Spectral Structure
 - Continuum Emission



VLA Image of the synchrotron spectral slope in Abell 2256, a galaxy cluster radio relic

(credits : F.Owen, NRAO)



 $10^{-26}W$

 m^2 Hz Beam

Beam

- Time Variability

Flux Density :

- Spectral Structure
 - Continuum Emission
 - Spectral Lines



Spectral Lines :

- Astro-Chemistry
- Doppler shifts \rightarrow 3D structure

Flux Density : $\frac{10^{-26}W}{m^2 Hz Beam} = \frac{Jy}{Beam}$

- Spatial Structure
- Time Variability
- Spectral Structure
 - Continuum Emission
 - Spectral Lines



ALMA image of CO emission from the molecular shell around the AGB star LL Pegasi and its stellar companion

(Kim et al, Nature Astro 2017.)

Flux Density : $\frac{10^{-26}W}{m^2 Hz Beam} = \frac{Jy}{Beam}$

- Spatial Structure
- Time Variability
- Spectral Structure
 - Continuum Emission
 - Spectral Lines
- Polarization (Stokes)



Magnetic Field direction in the Whirlpool Spiral Galaxy.

Single Dish



Image : Raster Scan on the Sky



Single Dish



Interferometer



Image : iFFT of partially sampled spatial frequency domain

Image : Raster Scan on the Sky





Single Dish



Interferometer





Image : iFFT of partially sampled spatial frequency domain

Image : Raster Scan on the Sky





Single Dish + Interferometer

Mosaics :

- Single pixel/beam feeds
- Phased-Array Feeds

Tomography :

 e.g. Make images of a planet's emission region as it rotates

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Young's double slit experiment



Young's double slit experiment

Interference



Instrument : An array of detectors



Young's double slit experiment

Interference



Each antenna-pair measures the parameters of one 'fringe'.



Measured Fringe Parameters :

Amplitude, Phase Orientation, Wavelength

Young's double slit experiment

Interference



2D Fourier transform :



Image = sum of cosine 'fringes'.

Each antenna-pair measures the parameters of one 'fringe'.



Measured Fringe Parameters :

Amplitude, Phase Orientation, Wavelength

Measuring the visibility function

Measure the spatial correlation of the E-field incident at each pair of antennas



N antennas N(N-1)/2 antenna-pairs (baselines)

Measuring the visibility function

Measure the spatial correlation of the E-field incident at each pair of antennas



Parameters of a Fringe : Amplitude, Phase : $\langle E_i E_j^* \rangle$ is a complex number. Orientation, Wavelength : $\vec{u}, \vec{v}, \vec{b}$ (geometry)

N antennas N(N-1)/2 antenna-pairs (baselines)

Measuring the visibility function

Measure the spatial correlation of the E-field incident at each pair of antennas



N antennas N(N-1)/2 antenna-pairs (baselines) Parameters of a Fringe : Amplitude, Phase : $\langle E_i E_j^* \rangle$ is a complex number. Orientation, Wavelength : $\vec{u}, \vec{v}, \vec{b}$ (geometry)

Van Cittert Zernicke theorem (far-field)

$$\langle E_i E_j^* \rangle \propto V_{ij}(u,v) = \iint I^{sky}(l,m) e^{2\pi i(ul+vm)} dldm$$

General Form :
$$V(\vec{b}_{ij}) = \iiint I^{sky}(\vec{s})e^{2\pi i(\vec{b}_{ij}\cdot\vec{s})}d^{3}\vec{s}$$

Visibilities on the UV plane

Measure the spatial correlation of the E-field incident at each pair of antennas





N antennas N(N-1)/2 antenna-pairs (baselines)

Visibilities on the UV plane

Measure the spatial correlation of the E-field incident at each pair of antennas





N antennas N(N-1)/2 antenna-pairs (baselines)





"Aperture Synthesis"



"Aperture Synthesis"



"Aperture Synthesis"



Observation : 2 hours

"Earth Rotation Synthesis"



Observation : 4 hours

"Earth Rotation Synthesis"



Observation : 4 hours, 2 frequency channels

"Multi Frequency Synthesis"



Observation : 4 hours, 3 frequency channels

"Multi Frequency Synthesis"



J2000 Right Ascension

Point Spread Function

=> Imaging Properties

Observation : 4 hours, 3 frequency channels

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N antennas N(N-1)/2 antenna-pairs (baselines)









Image Reconstruction (deconvolution)

$$V_{ij}^{obs}(\mathbf{v},t) \approx \frac{M_{ij}(\mathbf{v},t)}{S_{ij}(\mathbf{v},t)} \int \int I(l,m) e^{2\pi i (ul+vm)} dl dm$$

The forward problem

$$V_{ij}^{obs}(\mathbf{v},t) \approx \frac{M_{ij}(\mathbf{v},t)}{S_{ij}(\mathbf{v},t)} \int \int I(l,m) e^{2\pi i(ul+vm)} dl dm$$

$$V_{ij}^{obs}(\mathbf{v},t) = \frac{M_{ij}(\mathbf{v},t)}{S_{ij}(\mathbf{v},t)} S_{ij}(\mathbf{v},t) \iiint \frac{M_{ij}^{s}(l,m,v,t)}{M_{ij}^{s}(l,m,v,t)} I(l,m,v,t) e^{2\pi i (ul+vm+w(n-1))} dl dm dn$$

$$V_{ij}^{obs}(\mathbf{v},t) \approx M_{ij}(\mathbf{v},t) S_{ij}(\mathbf{v},t) \iint I(l,m) e^{2\pi i (ul+vm)} dl dm$$

The generalized forward problem

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General Form :
$$V(\vec{b}_{ij}) = \iiint I^{sky}(\vec{s})e^{2\pi i (\vec{b}_{ij}\cdot\vec{s})}d^{3}\vec{s}$$

$$V(\vec{b}_{ij}) = \iiint M(\vec{s},t)I^{sky}(\vec{s},t)e^{2\pi i (\vec{b}_{ij}\cdot\vec{s})+\phi(\vec{s},t)}d^{3}\vec{s}$$

Interferometry & MRI share the same functional form (with different Physics)

=> Interesting overlap in solution techniques.

$$V_{ij}^{obs}(\mathbf{v},t) \approx \frac{M_{ij}(\mathbf{v},t)}{S_{ij}(\mathbf{v},t)} \int I(l,m) e^{2\pi i(ul+vm)} dl dm$$

$$V_{ij}^{obs}(v,t) = M_{ij}(v,t) S_{ij}(v,t) \iiint M_{ij}^{s}(l,m,v,t) I(l,m,v,t) e^{2\pi i (ul+vm+w(n-1))} dl dm dn$$
Direction
Direction
Dependent
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Effects
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$$V_{ij}^{obs}(\mathbf{v},t) \approx \frac{M_{ij}(\mathbf{v},t)}{S_{ij}(\mathbf{v},t)} \int \int I(l,m) e^{2\pi i (ul+vm)} dl dm$$

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Direction
Independent
Gains

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Direction
Independent
Gains
Forward Gain
P
Antenna signal chain

$$V_{ij}^{obs}(\mathbf{v},t) \approx \frac{M_{ij}(\mathbf{v},t)}{S_{ij}(\mathbf{v},t)} \int \int I(l,m) e^{2\pi i (ul+vm)} dl dm$$

$$V_{ij}^{obs}(\mathbf{v},t) = M_{ij}(\mathbf{v},t) S_{ij}(\mathbf{v},t) \iiint M_{ij}^{s}(l,m,\mathbf{v},t) I(l,m,\mathbf{v},t) e^{2\pi i (ul+vm+w(n-1))} dl dm dn$$
Direction
Independent
Gains
Refractive Effects
Refractive Effects
Antenna Response
Pattern
- beam shape
- pointing offset
(mosaic)

$$V_{ij}^{obs}(\mathbf{v},t) \approx \frac{M_{ij}(\mathbf{v},t)}{S_{ij}(\mathbf{v},t)} \int I(l,m) e^{2\pi i(ul+vm)} dl dm$$

The generalized forward problem

$$V_{ij}^{obs}(v,t) = M_{ij}(v,t) S_{ij}(v,t) \iiint M_{ij}^{s}(l,m,v,t) I(l,m,v,t) e^{2\pi i (ul+vm+w(n-1))} dl dm dn$$
Direction
Direction
Dependent
Dependent
Effects
Direction
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Effects
Direction
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Effects
Direction
Dependent

Gains

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$$\downarrow$$
Sky-brightness varies

with frequency

and time







Intensity

Intensity weighted Spectral Index

$$V_{ij}^{obs}(\mathbf{v},t) \approx \frac{M_{ij}(\mathbf{v},t)}{S_{ij}(\mathbf{v},t)} \int I(l,m) e^{2\pi i(ul+vm)} dl dm$$

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Direction
Direction
Dependent
Dependent
Effects
Direction
Dependent
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W-Term
Effect of Sky Curvature
Tangent
Plane
$$\widehat{h}$$

$$\widehat{h}$$
Fresnel Zone
$$E_{1}$$

$$\widehat{w}$$

$$E_{2}$$

$$V_{ij}^{obs}(\mathbf{v},t) \approx \frac{M_{ij}(\mathbf{v},t)}{S_{ij}(\mathbf{v},t)} \int \int I(l,m) e^{2\pi i (ul+vm)} dl dm$$

The generalized forward problem



How do we solve these systems of equations ?

$$V_{ij}^{obs}(\mathbf{v},t) \approx \frac{M_{ij}(\mathbf{v},t)}{S_{ij}(\mathbf{v},t)} \int \int I(l,m) e^{2\pi i (ul+vm)} dl dm$$

The generalized forward problem



Calibration



$$V_{ij}^{obs}(\mathbf{v},t) \approx M_{ij}(\mathbf{v},t) S_{ij}(\mathbf{v},t) \iint I(l,m)e^{2\pi i(ul+vm)} dl dm$$
The generalized forward problem
$$V_{ij}^{obs}(\mathbf{v},t) = M_{ij}(\mathbf{v},t) S_{ij}(\mathbf{v},t) \iiint M_{ij}^{s}(l,m,\mathbf{v},t) I(l,m,\mathbf{v},t)e^{2\pi i(ul+vm+w(n-1))} dl dm dn$$
Direction
Dir

Iterative Image Reconstruction

The generalized forward problem $V^{obs} = [A]I^m + n$

The generalized inverse problem $I^m = [A]^{-1} V^{obs}$

L2 data regularization

+ Sky model (multiscale, wideband, timevar)+ Solver/Optimizer with constraints/biases

Iterative Image Reconstruction

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Image Reconstruction

Sky models - Delta function - Gaussians

-

Algorithms - Clean (greedy) - Many other CS ideas

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Limits of Interferometric Imaging



Signal-to-noise ratio (SNR) Angular resolution & sensitivity Dynamic range (CNR) Un-measured large spatial scales Accuracy of instrument models Algorithmic variability Data volume and compute cost















Going forward.....

Data volumes will only increase (e.g. ngVLA, SKA....)

- => image noise reduces => instrumental effects easily seen => need complex algorithms
- => compute cost increases => manual intervention is harder => need HPC and automation





Square Kilometer Array (skatelescope.org) 2K dishes, 1M antennas , 50 MHz – 30 GHz Next Generation VLA(ngvla.nrao.edu)263 dishes (2 types) ,1-100 GHz

The R&D frontier

Algorithms :

- A variety of sky models, instrument models, objective functions and regularizers, optimization strategies, the use of prior knowledge versus unknowns to solve for, etc..

- Talks by Maxim Voronkov and Kazunori Akiyama

Compute Load :

- Parallelization of data and algorithms, GPUs for the gridding compute hotspot, etc..
- Talk by Kumar Golap

Automation :

- Data analysis pipelines that know how to tune the details of each step (data editing, calibration, imaging) to each specific dataset

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