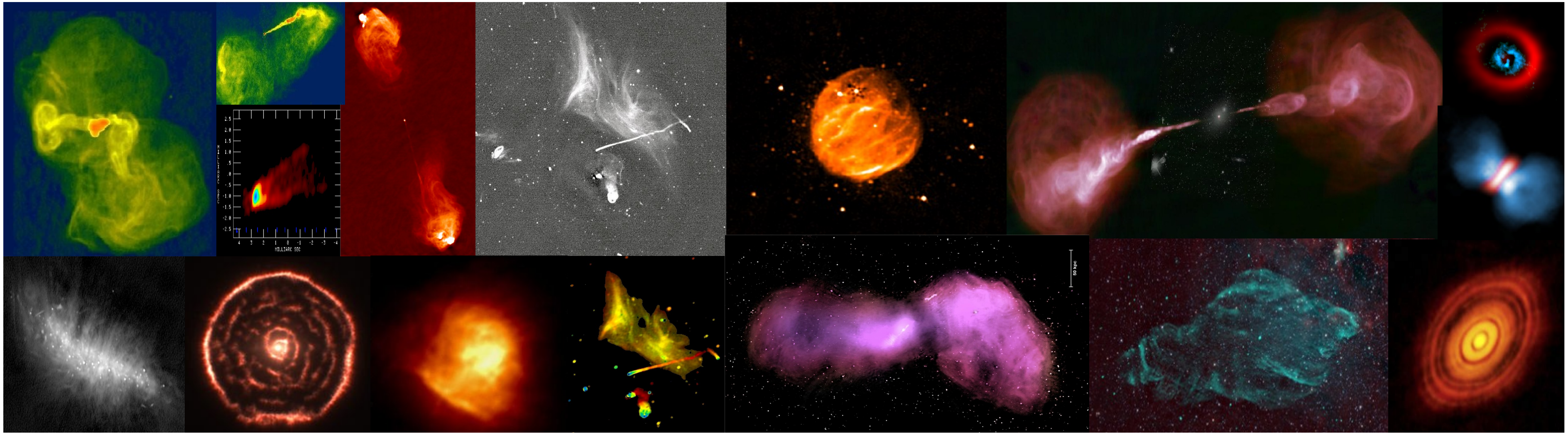
Radio Interferometry

Image Reconstruction, Calibration and Data Analysis



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From Cells to Galaxies 2019
17 April 2019, Charlottesville, Virginia.

Outline

- Measurement Equation
- Image Reconstruction
  - Sky models and solvers
  - Calibrating for instrumental effects
  - Modeling instrumental effects
- Joint reconstructions (multi-spectral, filled-aperture + interferometer, etc)
- Typical compute costs
- Data reduction pipelines and algorithm automation

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} dl\, dm$$

Observed visibilities (Data)

Direction Independent Gains

UV sampling pattern

Sky Brightness (Image)

Fourier transform kernel

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$g_i g_j^*$

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Calibration

(1) External models
(2) Self-Calibration

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Fourier transform kernel

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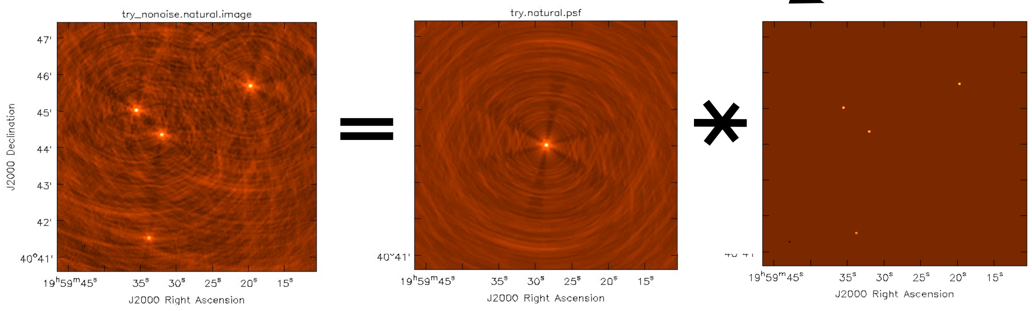
Imaging + Deconvolution

$$I^{obs}(l,m) = I^{PSF}(l,m) * I^{sky}(l,m)$$



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Direction
Independent
Gains

- feed gains,
delays,bandpass

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Direction Independent Gains

- feed gains, delays,bandpass

Antenna primary beam

- Power pattern varies with time, frequency and baseline

- Ionospheric refraction

W-Term

-Non-coplanar baselines

-Sky curvature

Direction Dependent Instrumental Effects

=> Multiplicative effect in the image domain

=> Convolutions in the visibility domain

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Sky-brightness varies with frequency (time)

- Include source spectra and time variability into the sky model and/or the regularization process.

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Image Reconstruction

Calibration (DI & DD)

(1) External models
(2) Self-Calibration

Direction Dependent Instrumental Effects

=> Multiplicative effect in the image domain
=> Convolutions in the visibility domain

Iterative Image Reconstruction

L2 data regularization
+ Non-linear sky model
and solver/constraints.

$$[A] I^m = V^{obs} \qquad \frac{\delta \chi^2}{\delta I^m} = 0$$

Normal Equations
(convolution eqn)

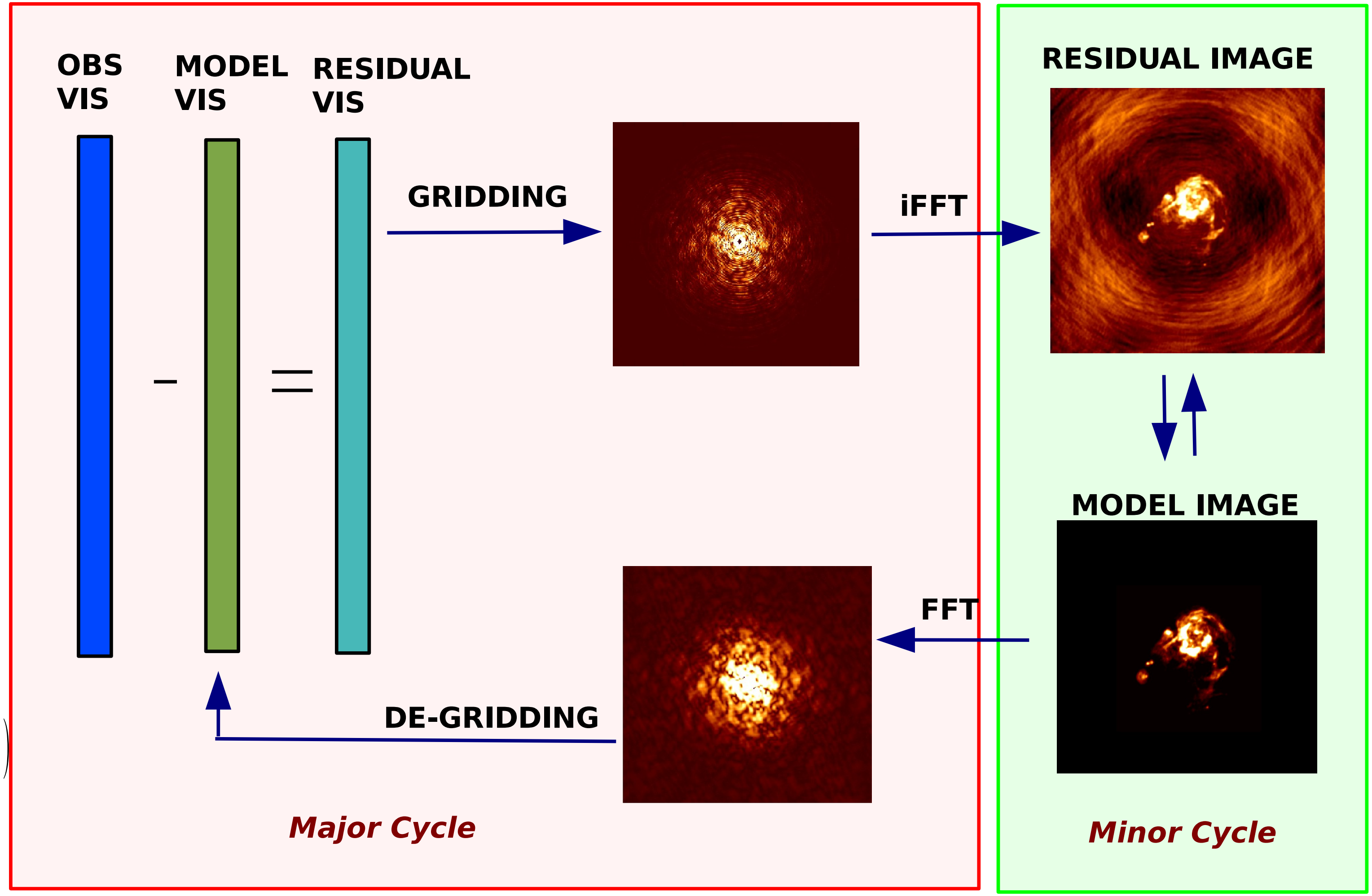
$$[A^T W A] I^m = [A^T W] V^{obs}$$

Sky model solver

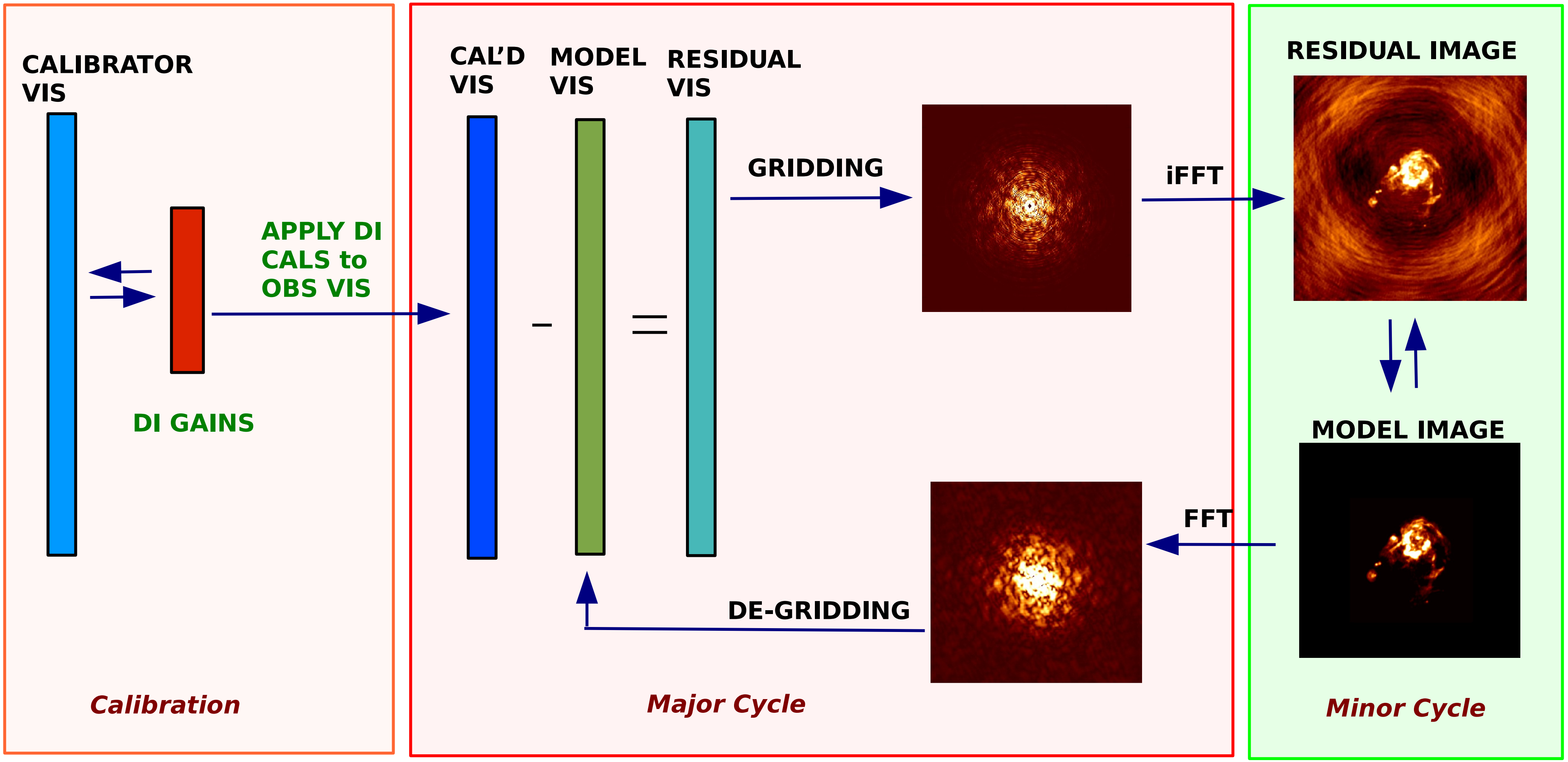
$$I^m_{i+1} = I^m_i + g[A^T W A]^+ \boxed{R}$$

$$\boxed{R} = \left(A^T W \left(V^{obs} - A I^m_i\right)\right)$$

Calculate Forward Model
& Residual Image



Iterative Image Reconstruction + External calibration



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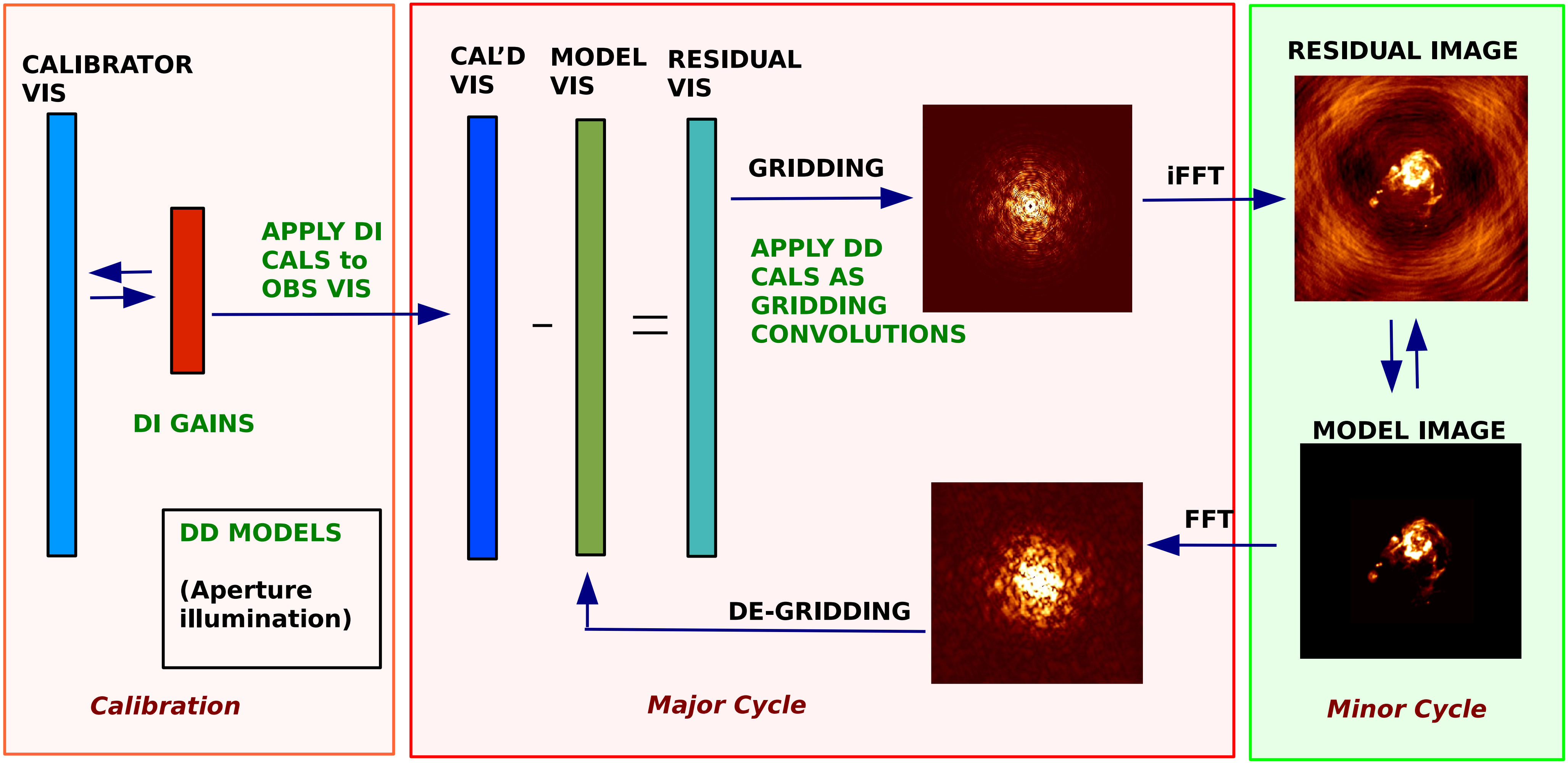
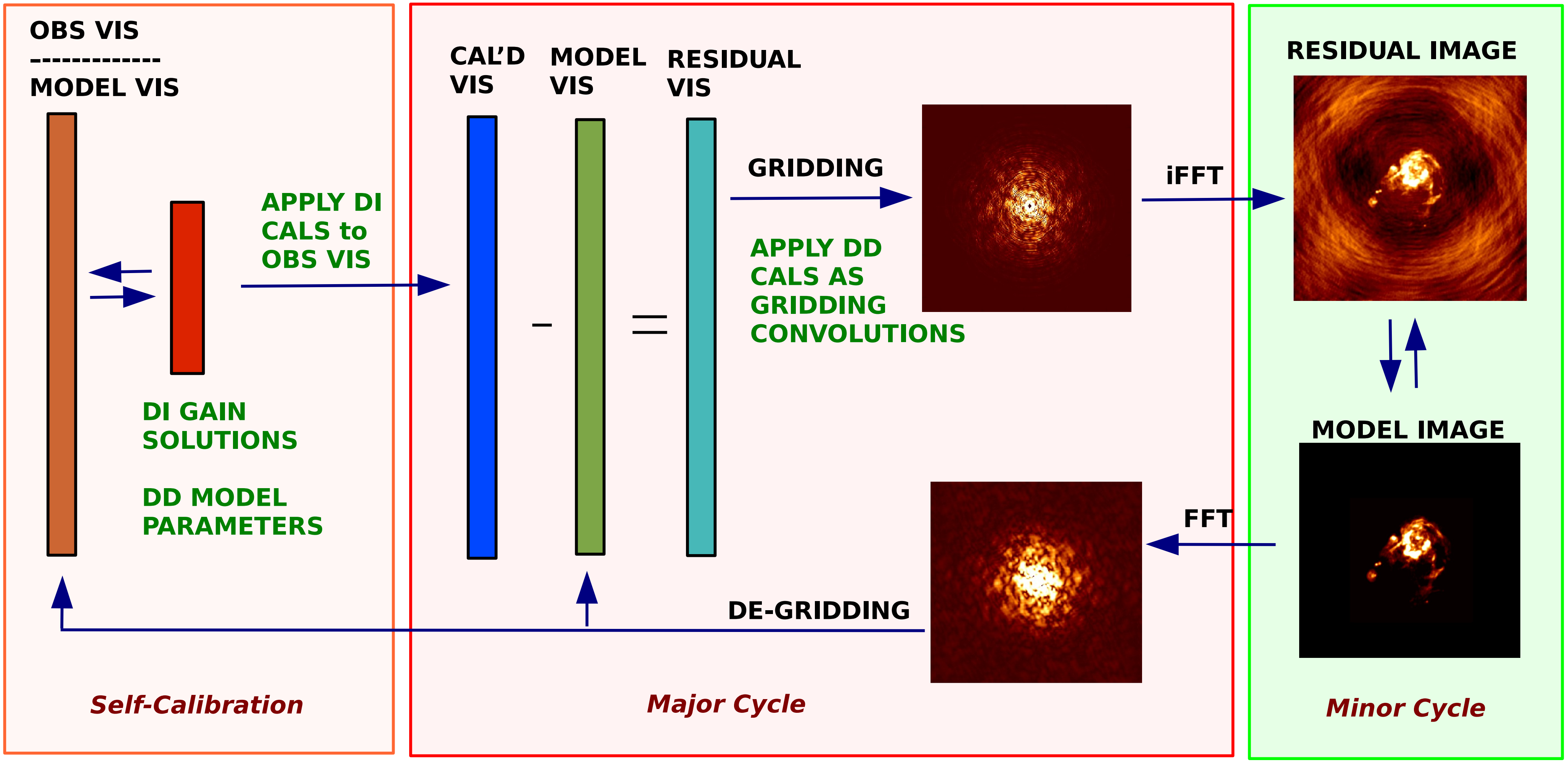
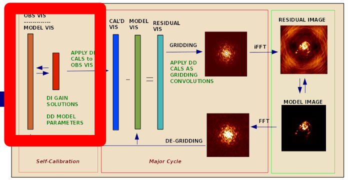
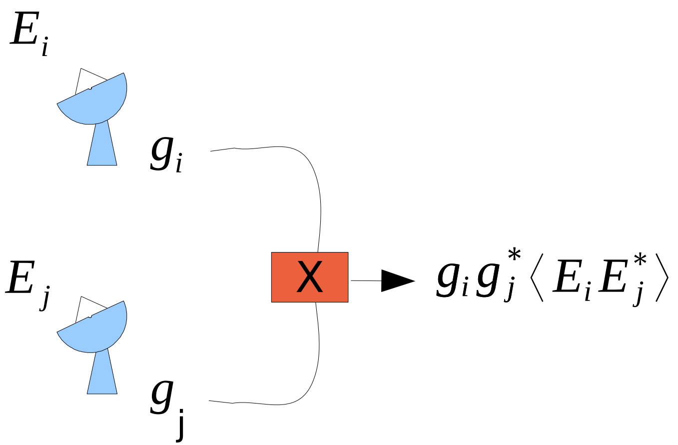


Iterative Image Reconstruction + Self-Calibration



Calibration – Direction Independent effects





(1) Observe a known source

$$\langle E_i E_j^* \rangle \text{ is known}$$

(2) Use data from all correlation pairs ij
Solve for complex gains g_i

(3) Apply corrections to target data :

$$\frac{g_i g_j^* \langle E_i E_j^* \rangle}{g_i g_j^*}$$

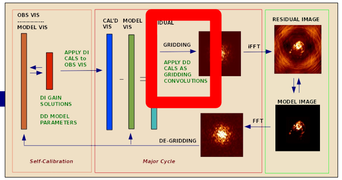
Typically, solutions are done in a sequence, with averaging to increase SNR for the solutions.

E.g. Average in time and solve for average stable bandpass
Apply bandpass solutions, average in frequency, solve for time-variable gains.

Baseline based calibration : Possible, but to be used with caution.

(Equivalence between solving for antenna-based terms and satisfying closure relations)

Direction Dependent Corrections – Using known models



Models of DD instrumental effects can be used to correct the data

Antenna Beams, Ionospheric Phase screen, Non-coplanar Baselines, Sky curvature

$$I^{obs} = \sum_{ij} I_{ij}^{psf} * \left[P_{ij} . I^{sky} \right] \longleftrightarrow V_{ij}^{obs} = S_{ij} . \left[A_{ij} * V^{sky} \right]$$

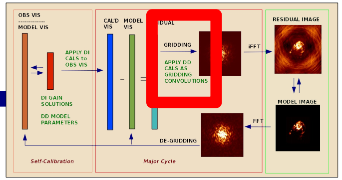
For each visibility, apply $A_{ij}^{-1} \approx \dfrac{A_{ij}^T}{A_{ij}^T * A_{ij}}$

(1) Use A_{ij}^T as the convolution function during gridding

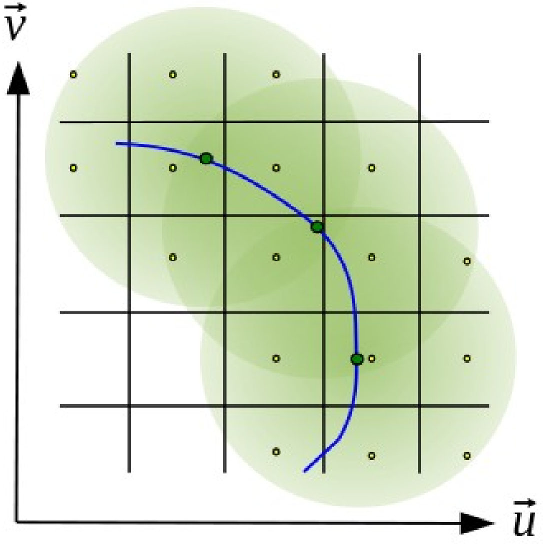
(2) Divide out $FT\left[\sum_{ij} A_{ij}^T * A_{ij} \right]$ from the image (in stages).

- If P_{ij} is invariant, an image-domain correction can be done instead (flat-fielding)
- Phase gradients across A_{ij}^T can fix known pointing offsets (tip-tilt) and make mosaics
- A_{ij}^{-1} is often only approximate => Convergence depends on an accurate forward model

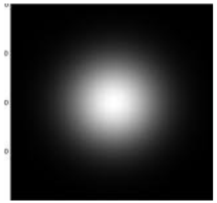
Applying DD corrections (A_{ij}^T) during gridding



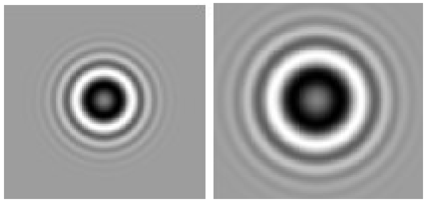
Gridding = Convolutional Resampling of visibilities to a regular grid



Standard Imaging :
Prolate Spheroidal



W-Projection :
FT of a Fresnel kernel

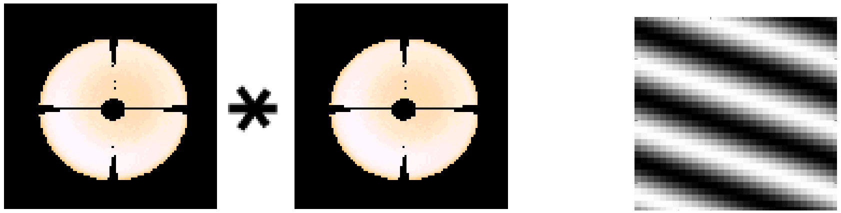


A-Projection : Baseline aperture illumination functions
+ phase gradients for pointing offsets
+ ionospheric refraction models

Convolution in UV-domain (per vis)

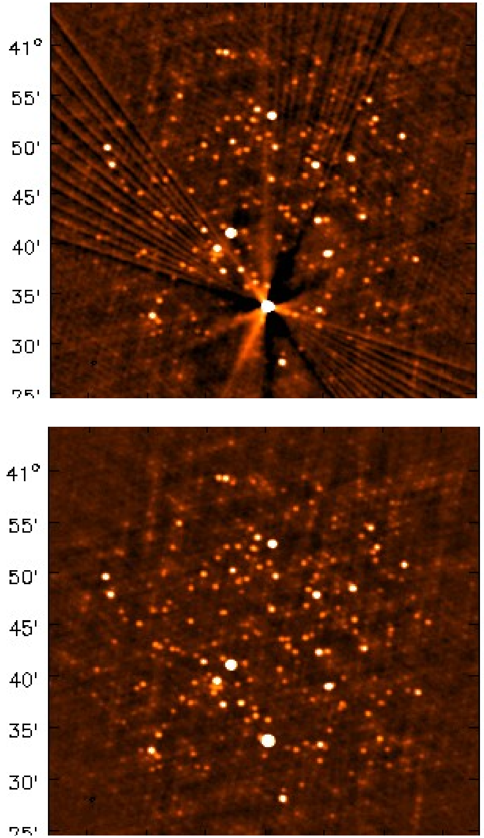
=> Handle wide-field imaging effects
before averaging in time/baseline

(Vis Prediction : Calculate Forward Model)

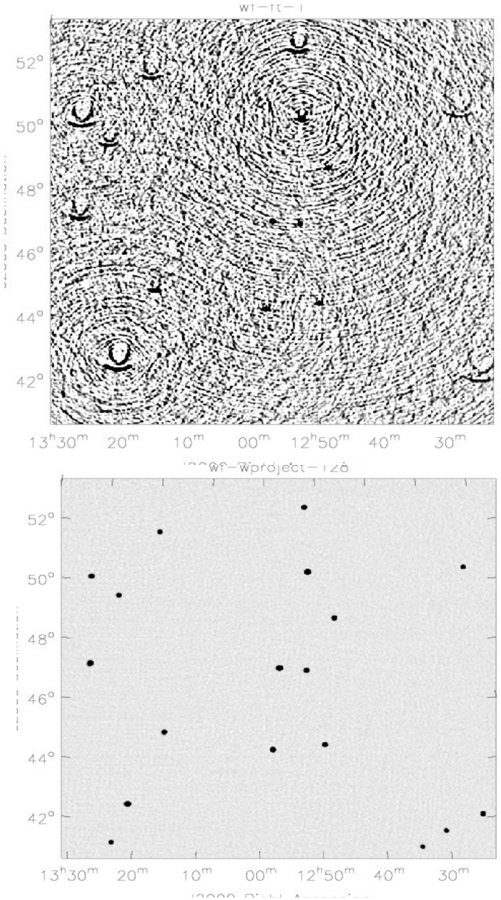


Examples of wide-field instrumental corrections

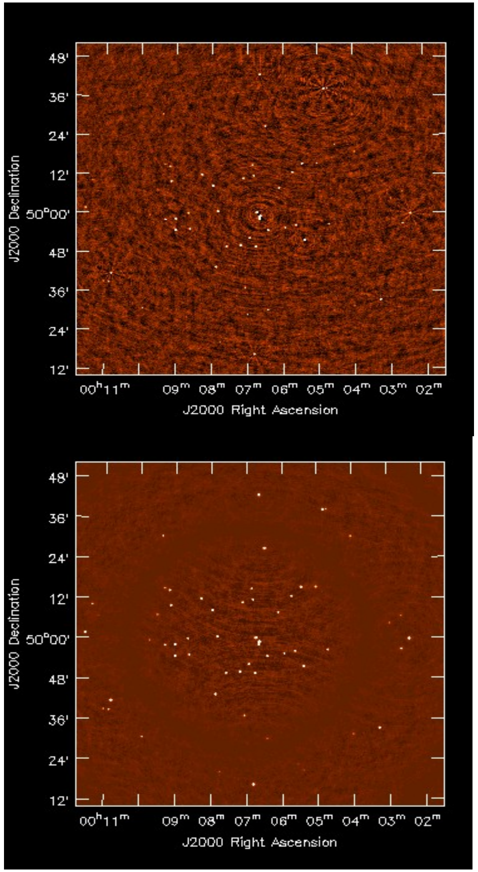
Antenna Beam rotation with parallactic angle



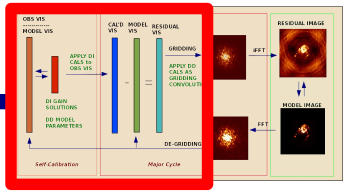
Sky curvature, non-coplanarity (W-term)



Antenna-based, time-variable pointing offsets



Unknown instrumental models : DD Self-Calibration



(1) Solve for parameters of instrument-specific models in a self-calibration loop

E.g. Pointing Self-Cal : Solve for antenna-based phase gradients across each aperture illumination function (Adaptive Optics !)

=> Correction : Apply opposite phase gradient during gridding

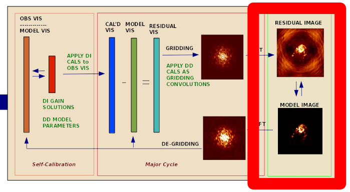
(2) Perform direction-independent (DI) calibrations for multiple directions on the sky

E.g. DD-Facets : Define regions around all bright sources.

Solve V = $g_i g_j^* \langle E_i E_j^* \rangle$ to get N complex gains for each selected direction

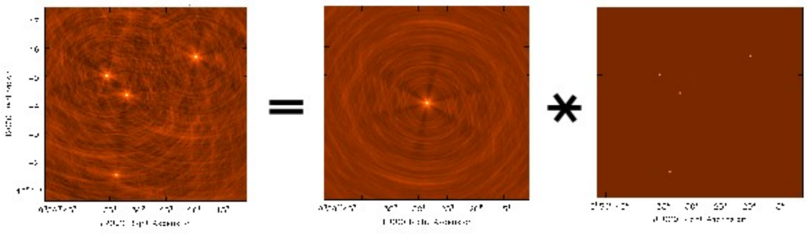
=> Correction : Image each region using different DI gain solutions.

Sky models and solvers (image reconstruction)



Most commonly used algorithms

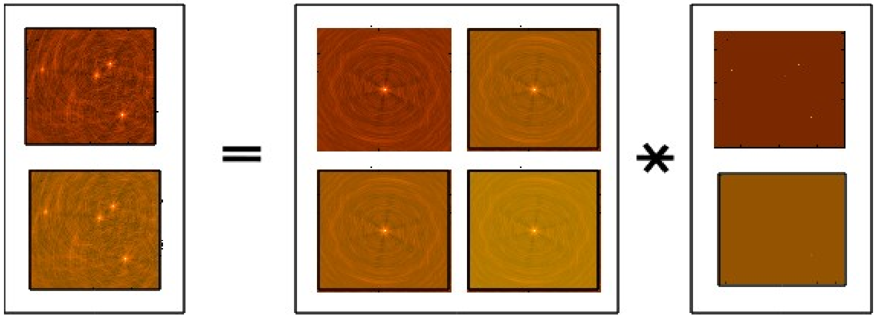
Convolution Equation ==> Deconvolution



For Point Sources :

– Hogbom CLEAN
– Clark CLEAN

Multi-Term Convolution Equation
==> Joint Deconvolution



For Point/Extended Sources :

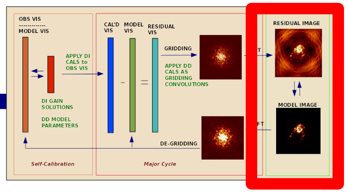
– Multi-Scale-CLEAN

For Wide-band Sky models

– Multi-Term Multi-Frequency Synthesis
(with or without Multi-Scale / Time variability)

Others : Any non-linear image-domain solver (many compressed sensing formulations)

Sky models and solvers (image reconstruction)



Algorithms : Parameterized models + Iterative model fitting.
Feature extraction + classification, Mixed models

– Basis functions : Delta functions, Gaussians, Wavelets, Shapelets, Polynomials to represent spectral structure or time-variability, 2D,3D,4D models

– Metrics being optimized : L2 or L1 or TV norms, weighted combination of different norms and a-priori bias terms, Bayesian formulations, etc..

– Optimization schemes : Greedy algorithms + gradient descent, primal-dual methods, etc

Examples from Radio Interferometry : Gaussians (ASP), Wavelets and other Atoms with Sparsity (SARA / PURIFY / MORESANE), Bayesian forms (MEM, RESOLVE), wide-band non-parametric models, CHIRP/EHT (direct solvers for VLBI).

Examples from Optical Interferometry : BSMEM, IRBis, MACIM, MiRA, SQUEEZE, SPARCO, PAINTER, MiRA-3D,...

Algorithm Comparison

Algorithm choice depends on sky structure, data quality, target science.

Each algorithm needs (different) tuning for best results.

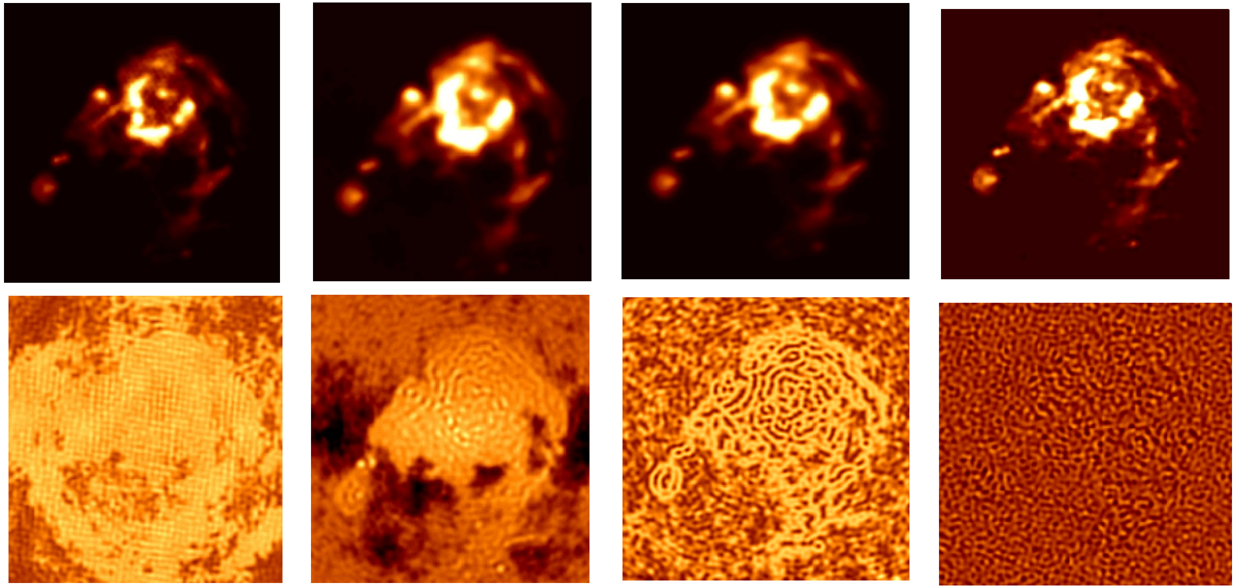
Image Quality Metrics :

Noise RMS,

Peak residual,

Dynamic Range

CLEAN MEM MS-CLEAN ASP



Output Image

Residual Image

Joint Reconstructions : Wide-Band (multi-spectral) Solvers

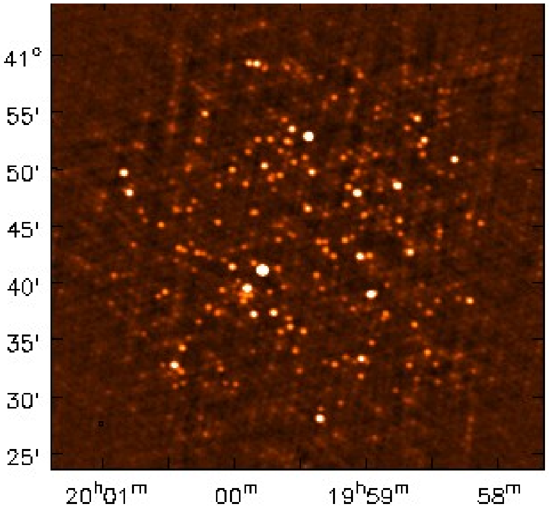
Model the spectrum per ‘atom’ as a smooth polynomial

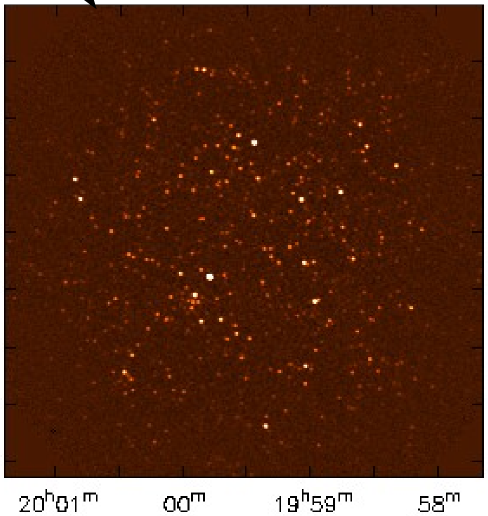
$$I_\nu^{sky} = \sum_t I_t^m \left(\frac{\nu - \nu_0}{\nu_0}\right)^t$$

MT-MFS : Multi-term linear least squares + CLEAN-based greedy algorithm in a transformed (sparse) space.

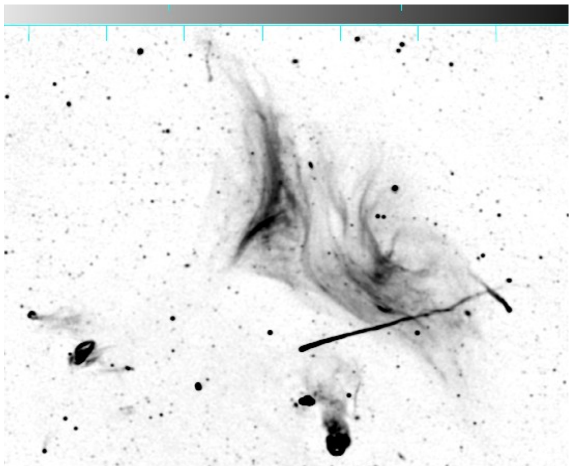
Improve

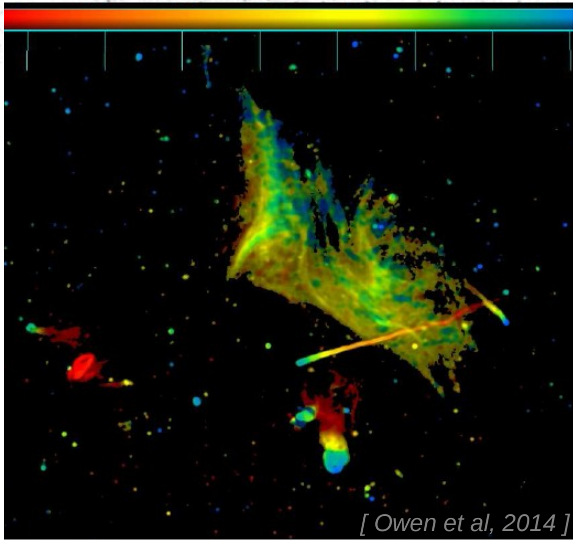
Angular Resolution & Imaging Fidelity & Dynamic Range



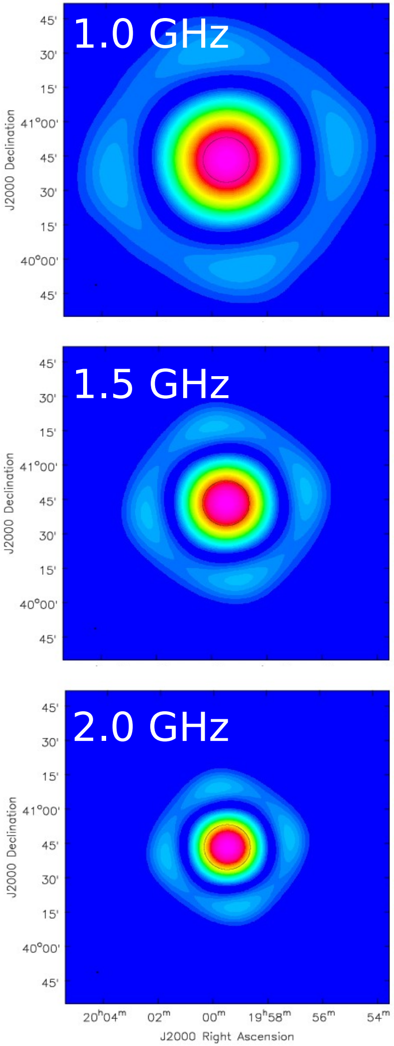


Intensity and Spectral Index





Joint Reconstructions : Wide-Band+ Wide-Field

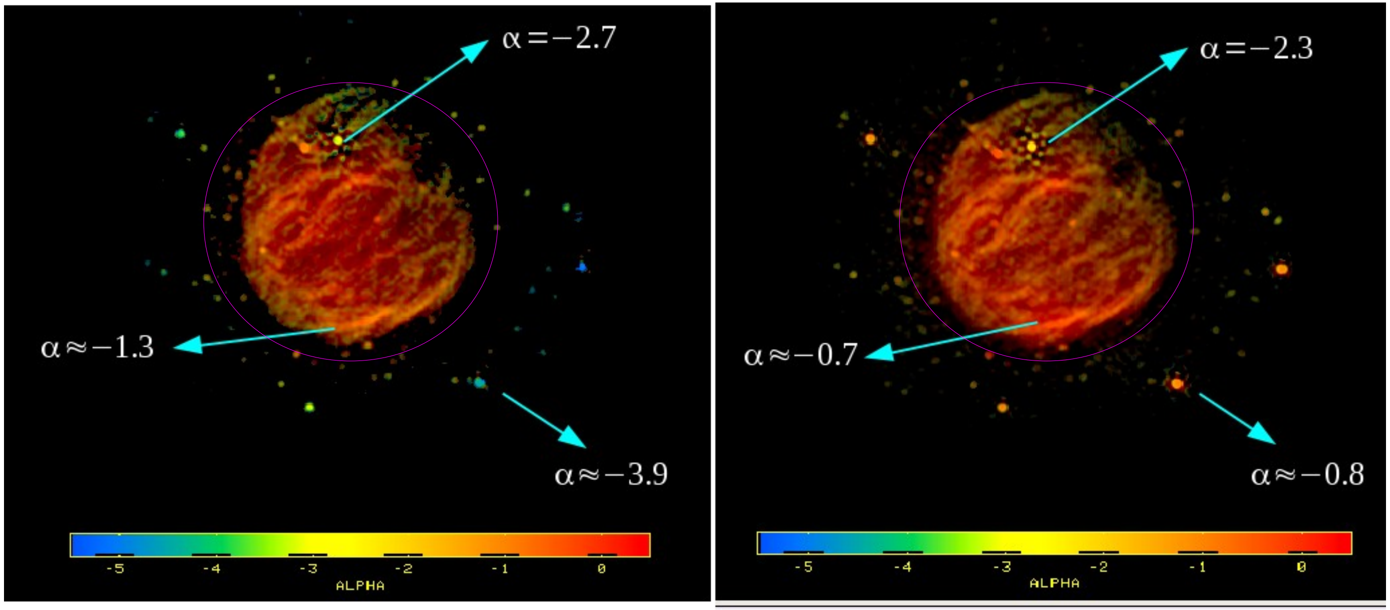


Antenna Power Patterns scale with observing frequency

=> Artificial spectral structure for all sources away from center

(1) Include PB spectrum in Sky Model. Remove post-reconstruction.

(2) Eliminate as an instrumental correction before modeling sky



Joint Reconstructions : Adding single-dish (low resolution) data

Missing Short Spacings

=> Negative Bowls
=> Unconstrained Spectra

Use Low-Resolution information from single dish maps.

Methods :

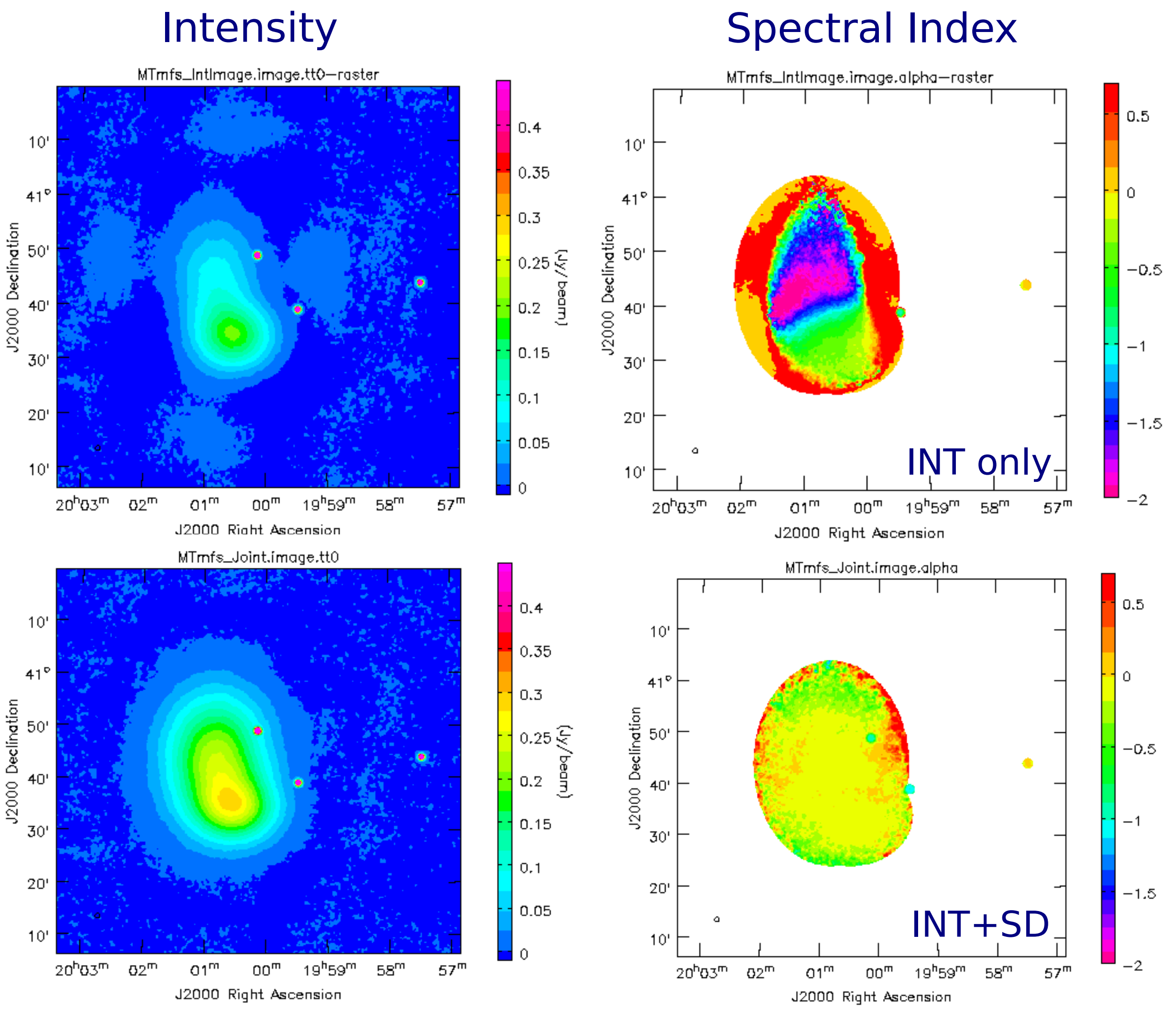
(1) Joint Reconstructions (better!)

Add another data regularization term to any existing solver

E.g. Joint PSFs and Residuals

(2) Feathering

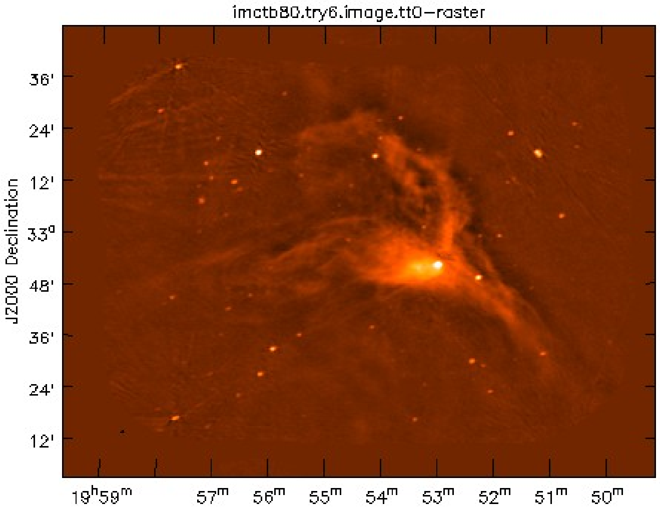
Weighted UV-domain average of INT-only and SD-only images

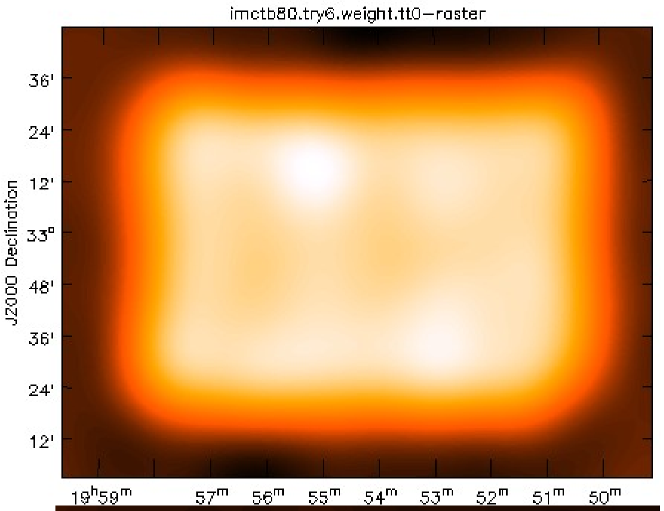


Joint Reconstructions : Mosaics + Wide-Band + Wide-Field + Single Dish

Interferometer joint mosaic intensity

(used phase gradients across aperture functions during gridding)

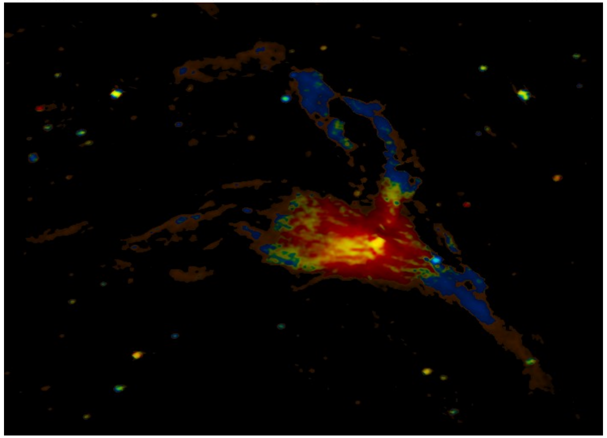


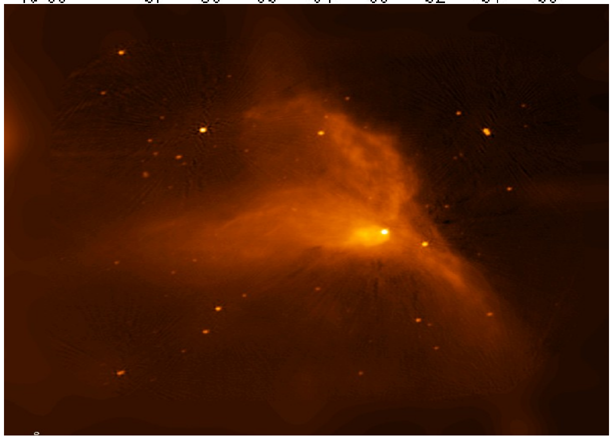


Joint mosaic primary beam from 106 VLA pointings

Wide-Field Spectral Index

(with corrections for PB spectrum)





Interferometer + Single dish

(intensity only)

An example of the current state-of-the-art ...

Compute Costs

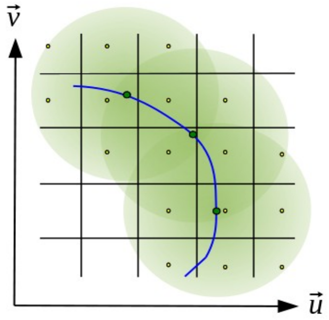
Data volume

N_data =
N_ant^2 x
N_chan x
N_pol x
N_time

Complex numbers

Lustre I/O

Example :
8hr data
300 GB



Gridding : Convolutional resampling

O(N_data) x (nxn) complex multiply/add (n=5 - 100)
=> Compute load : O(N_data) * 10^{2-5} flops

Data parallelization, Multi-threading, GPUs, etc…

Example : Major cycle : 1hr → 10 days (Diff Algorithms)

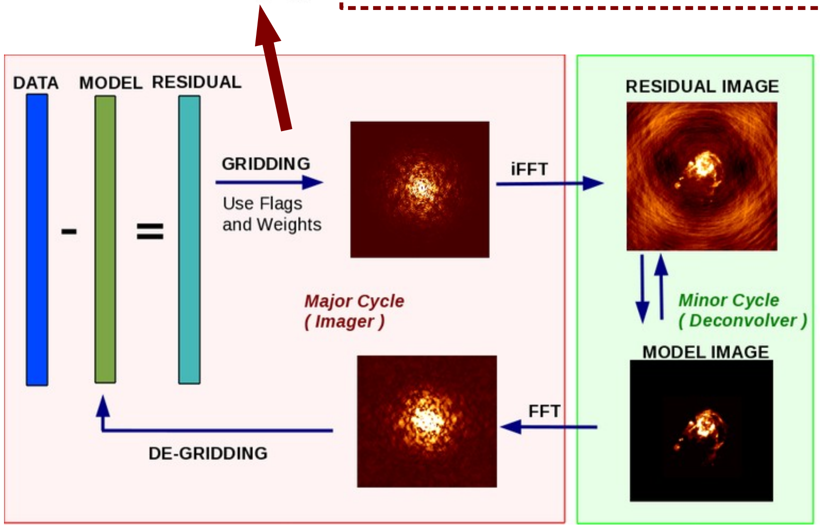


Image sizes : N_pix = Nx x Ny x N_chan x N_pol

Real / Complex
FFTs : O(NlogN) Pixel math: O(N^2)
Mem : ~8 copies
Multi-threading + Chan parallelization

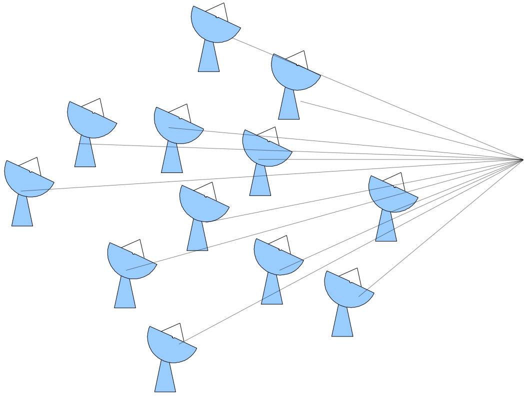
Nx : 1k → 40k N_chan : 200 - 16K

Example range :
1K x 1K x 256 ==> 20K x 20K x 8K

Number of iterations : 5 – 10 major cycle loops, 10^2 to 10^4 minor cycle steps

Runtime varies by 1-2 orders of magnitude. Depends on data.

Data Flow



Correlation (Real time system. FPGA/ASIC + backend cluster)

Time Series → Correlation → Spectral Channels → Integrate

Example Data rate : N(N-1)/2 * 1000 complex values per second

Data Archive (2.4 PB RAID storage)

Each observation is stored as a relational database

Example : VLA archive is 1.8 PB in size (+ 1 TB per day)

Post Processing – (1.6 PB Lustre FS, workstations, 90 node cluster, AWS)

Flagging → Calibration → Imaging

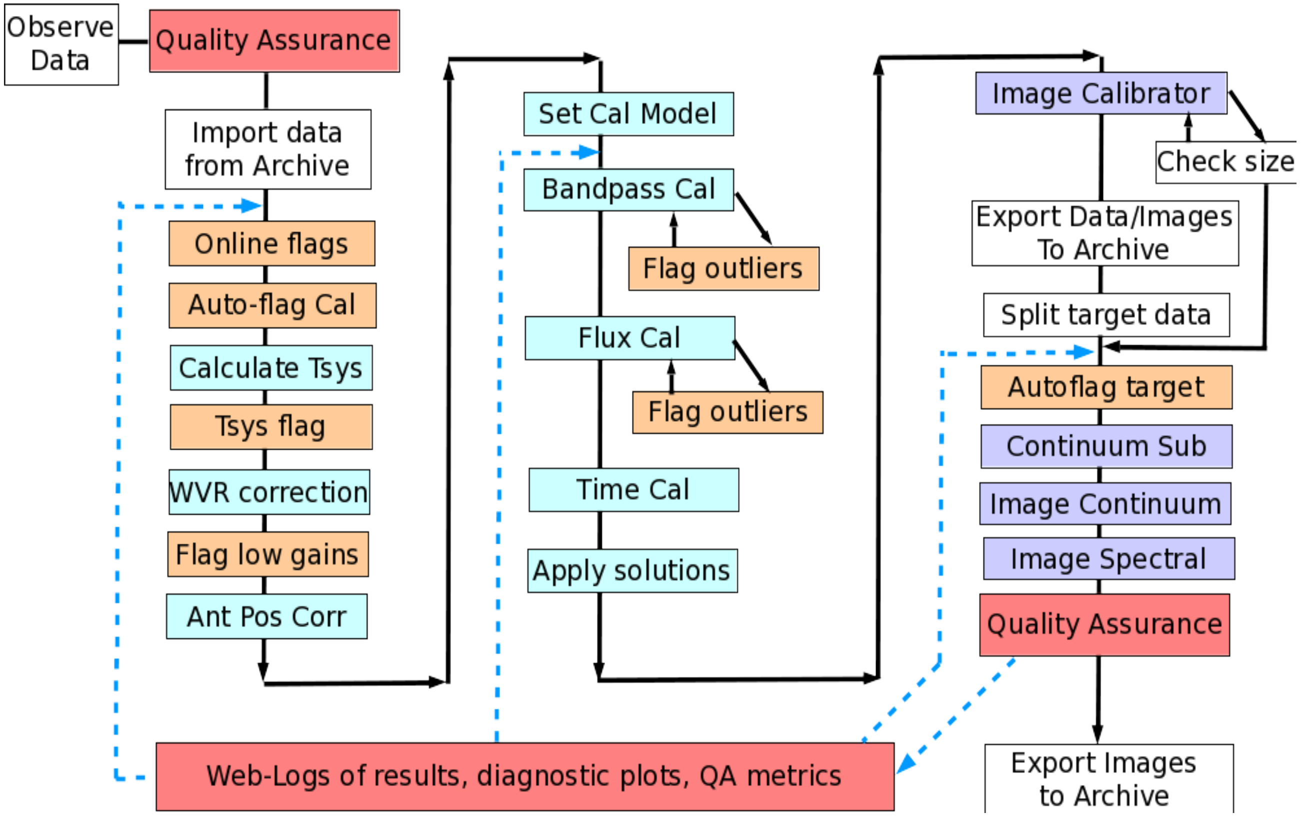
Flagging: Identify and mask corrupted data (RFI, Instrument errors, etc)

Calibration: Derive and apply corrections to undo the effects of complex valued antenna gains

Imaging: Reconstruct images by iterative model fitting while correcting for other instrumental effects

Automated Data Analysis : Science Ready Data Products

Our current end-to-end pipelines are the result of hand-optimized manual tuning by a team of scientists, validated on ~100 datasets, for a few standard imaging modes.



Over the last few years :

– Reduced need for manual intervention
– Increase in supported observing modes.

Ongoing R&D :

- Automate the quality checks

- Automate the decision tree

- Automate algorithm and parameter choices
(+ more robust imaging algorithms)

Machine Learning : potential areas of application

(1) Automating the data analysis decision tree :

It is possible to choose a sequence of steps and detailed parameter tunings that provides the best flagging, calibration and imaging outcome for any given dataset.   This may differ between types of datasets and science goals.

(2) Error recognition :

Humans are adept at identifying RFI patterns in plots of recorded data, non-standard antenna behaviour from calibration solution plots, and artifacts and other tell-tale shapes in images.

(3) Telescope monitoring and control :

Using telemetry and monitoring data to classify problems and their symptoms andpredict failures.
Use information about RFI sources, weather, to optimize the observation schedule and setup.

(4) Image and Spectrum analysis :

Feature detection/description and classification for surveys and catalogues
Spectral profile matching (mixed models)
Quality assessment : Have we gotten the best we can out of the data ?

Summary

- Measurement Equation
- Image Reconstruction
    - Sky models and solvers
    - Calibrating for instrumental effects
    - Modeling instrumental effects
- Joint reconstructions (multi-spectral, filled-aperture + interferometer, etc)
- Typical compute costs
- Data reduction pipelines and algorithm automation

Synergy between Biomedical and Radio Interferometric Imaging and Analysis ?