Understanding imaging limits due to approximations in ALMA primary beam models

Urvashi Rau  
NRAO, Socorro

Kara Kundert  
Intern from  
U.Michigan / U.C.Berkeley

Sanjay Bhatnagar  
NRAO, Socorrollo

ALMA Future Science Development Program Workshop  
24-25 August 2016, Charlottesville, VA
Outline

Problem: ALMA antenna aperture illuminations vary a lot within an observation
- DA,DV,PM, illumination offsets, Pointing, Parallactic angle rotation

Imaging algorithms can account for this via A-Projection but at a very high computing cost.

=> Need to understand when approximations can be used.

Simulations: Use measured aperture illumination functions to simulate data and perform only standard Stokes I imaging.
[ Similar to a study for CARMA by S.Corder 2009]

Results: DR < 1000: Only dish sizes matter (7m/12m).

DR > 1000: Pointing offsets (uncorrected, 2-4arcsec)

DR > 5000: Illumination offsets, variations between antennas, corrected pointing offsets (<0.5arcsec)

DR > 10000: Parallactic angle rotation, DA/DV combination
Wide-Field Imaging – Primary Beams

The Sky is multiplied by a PB, before being sampled by each baseline

\[ I^{obs}(l, m) = \sum_{ij,t} I_{ij}^{PSF}(l, m, t) \ast \left[ P_{ij}(l, m, t) \cdot I^{sky}(l, m) \right] \]

Primary Beam for baseline \( ij \)

\[ P_{ij} = V_i \cdot V_j^* = FT[A_i \ast A_j^*] = FT[A_{ij}] \]

Aperture Illumination for antennas \( i \) and \( j \) : \( A_i, A_j \)

The antenna field of view : \( D = \) antenna diameter

\( \lambda / D \)

Baseline aperture Illumination

\( A_{ij} = \)
Primary beam variations

- Different antenna structures – 3 types for 12m and 1 for 7m
- Illumination offsets – all antennas
- Pointing errors and parallactic angle rotation – all antennas/times
Primary Beam – Effect on images (VLA simulated example)

(1) Multiplicative gain pattern

PBCOR : Divide out an average PB

(2) Artifacts around bright sources

\[ \delta I^{\text{obs}} = \sum_t I^{\text{PSF}}(t) \ast [\delta P(t) \cdot I^{\text{sky}}] \]

A-PROJECTION : Partial UV-domain correction before combining visibilities

CASA `grdder=’mosaic’` : Accounts for different antenna sizes (7m, 12m) by default and allows specification of separate models for each antenna. [No parallactic angle rotation or squint corrections or full-pol.]

CASA `grdder=’awproject’` : Rotationally asymmetric beams with parallactic angle rotation and squint correction (i.e. uses complex conjugates to undo systematic phase structures). Handles full-pol. [Uses ray-traced models for EVLA and assumes identical antennas. Not ready for ALMA yet.]

( Mosaics : Additional phase gradient on the baseline aperture functions )
Primary Beam Correction: A-Projection

Bhatnagar et al, 2008

Apply PB correction in the UV-domain before visibilities are combined.

\[ I_{ij}^{\text{obs}} = I_{ij}^{\text{psf}} \star \left[ P_{ij} \cdot I_{i}^{\text{sky}} \right] \quad \Rightarrow \quad V_{ij}^{\text{obs}} = S_{ij} \cdot \left[ A_{ij} \star V_{i}^{\text{sky}} \right] \]

For each visibility, apply \( A_{ij}^{-1} \approx \frac{A_{ij}^{T}}{A_{ij}^{T} \star A_{ij}} \)

1. Use \( A_{ij}^{T} \) as the convolution function during gridding

2. Divide out \( \text{FT} \left[ \sum_{ij} A_{ij}^{T} \star A_{ij} \right] \) from the image (in stages).

- Conjugate transpose during imaging corrects for phase structures in the baseline aperture functions.

  e.g. : pointing offsets such as beam squint.
Computational Cost of full A-Projection

- Number of convolution kernels to be computed: \( \frac{Na(Na-1)}{2} \times Nt \times Nf \) (for \( Na \) antennas, \( Nt \) steps in PA, \( Nf \) channels)

- Each kernel has \([ N_{\text{support}} \times \text{oversampling} ]\) pixels on a side.

  \[ N_{\text{support}} : \text{approximately } 7 - 20 \text{ (for a f-o-v that avoids aliasing)} \]
  \[ \text{Oversampling} : 20 - 100 \text{ (to account for sub-uv-pixel shifts)} \]

- Combining with W-Projection: Multiply \( N \) by \( N_{\text{wplanes}} \)
  \[ N_{\text{support}} \text{ can be } >100 \text{ pixels} \]

- Full polarization: multiply \( N \) by 16 to get the full Mueller matrix

- Combine A-proj, W-proj, anti-aliasing func \( \Rightarrow 3 \) convolutions per kernel.

  \[ \Rightarrow \text{Need viable approximations!} \]


But, for high dynamic range and full-pol imaging, both need components from each other and computing costs escalate quickly.
Simulations to test what features we really need

**Data**: Each antenna has a:

- (complex) aperture illumination function
- pointing offset as a phase gradient
- parallactic angle rotation (numerical)

For each timestep and antenna pair,
- \( PB = \text{product of complex antenna voltage patterns} \)
- Predict visibilities for real\((PB) \times \text{sky}\)

**Imaging**:

Standard imaging and deconvolution with post-deconvolution (average) PB-correction

**Variants**: Stage 1: toy beam models
Stage 2: measured beams

( Simulations done at 100 GHz )

Kundert, Rau, Bhatnagar, Bergin (in prep), 2016
Stage 1 – simple aperture models

Tests:

Round disk with feed leg shadows
+ dish sizes (7m, 12m)
+ pointing offsets (<0.5asec)
+ ‘noise’ on the aperture
+ ellipticity (few %)
+ rotation

For practical reasons, we used only 10 antennas and 20 timesteps spanning a parallactic angle range of upto 90deg.

Results:

- Verified that the simulation code is working.
- Artifacts due to PA rotation peak at 45deg.
- This is similar to just combining DA and DV antennas
- It is a smaller effect than corrected pointing offsets.
- Rotation at native resolution is error prone and doing it for every timestep is very expensive.

=> Ignore parallactic angle rotation for Stage 2
Available aperture illumination models

- DA Measured: Complex
- DV Measured: Complex
- TICRA: Complex
- DA Measured, (Imaginary part)
- 7m Measured, Complex
- CASA Ray-Traced: Real

Measured beams from S.Corder & D.Gunawan
Stage 2 – Measured aperture illumination functions

Tests:

(1) Dish sizes (7m+12m)

(2) Pointing offsets
   (corrected : <0.5 arcsec vs Uncorrected : 2-4 arcsec )

Apply random pointing offsets to a single beam model.

(3) Illumination offsets
   Pick N different beams of one type (DA)

(4) Combine all effects

- Parallactic angle rotation and DA/DV combination were left out
  - A small effect in comparison to antenna-to-antenna variability
  - Computational cost.

- Only real part of the complex baseline PB was used
  - A software restriction at the time
  - Leftover (gain) phase variability would be <2deg

( Still, these should be included in the next version )
Results: Example images

No perturbations

Corrected Pointing

Antenna size diff

Illumination offsets

Uncorrected Pointing

All effects
### Results: Effects and their dynamic range limit (in order)

<table>
<thead>
<tr>
<th></th>
<th>No Perturbation</th>
<th>Corrected Pointing</th>
<th>Illumination Offset</th>
<th>Uncorrected Pointing</th>
<th>Size Difference</th>
<th>All Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Point Source</strong></td>
<td>$5.96 \times 10^{-8}$</td>
<td>$2.06 \times 10^{-4}$</td>
<td>$2.76 \times 10^{-4}$</td>
<td>$1.02 \times 10^{-3}$</td>
<td>$3.28 \times 10^{-3}$</td>
<td>$3.46 \times 10^{-3}$</td>
</tr>
<tr>
<td><strong>Small Extended</strong></td>
<td>$7.64 \times 10^{-5}$</td>
<td>$2.62 \times 10^{-4}$</td>
<td>$4.60 \times 10^{-4}$</td>
<td>$9.60 \times 10^{-4}$</td>
<td>$5.74 \times 10^{-3}$</td>
<td>$6.06 \times 10^{-3}$</td>
</tr>
<tr>
<td><strong>M51-type Galaxy</strong></td>
<td>0.0128</td>
<td>0.0129</td>
<td>0.0128</td>
<td>0.0127</td>
<td>0.0139</td>
<td>0.0140</td>
</tr>
</tbody>
</table>

RMS near the source, relative to a peak of 1.0 Jy.

DR < 1000: only dish sizes matter.
DR > 1000: pointing offsets (uncorrected, 2-4arcsec)
DR > 5000: Illumination offsets, variations between antennas, corrected pointing offsets (<0.5arcsec)
DR > 10000: Parallactic angle rotation, DA/DV combination
Conclusions

(1) DR $<~ 5000$: current approximations suffice (rotational symmetry with Airy disk models of the appropriate size, no phase corrections)

(2) DR $> 5000$, need antenna-to-antenna variations in illumination offsets
   $\Rightarrow$ TICRA models will not help $\Rightarrow$ Need measured models.
   $\Rightarrow$ Need PA rotation during imaging $\Rightarrow$ huge A-Projection compute load.
   $\Rightarrow$ Is it feasible to correct/fix the illumination offsets on each antenna so that we can use identical PB models for all antennas of a given type during imaging?

   It may be possible to define tolerances on the spatial scale at which variations between antennas can be ignored.

(3) Corrected Pointing offsets at 100GHz will have the same effect as uncorrected pointing offsets at (say) 800GHz to limit DR to $\sim 1000$.
   (Need pointing self-calibration?)

Stage 3 tests:
- Use unmodified complex baseline PBs during visibility simulation
- Full Stokes imaging (w/squint): Does it limit you at a lower DR than Stokes I?
- Include PA rotation and DA/DV combination in simulations and imaging
- Make mosaics since every point is away from some PB center
Primary beams vary within an observation
Primary beams vary within an observation