Deconvolution and Wide-Band Imaging

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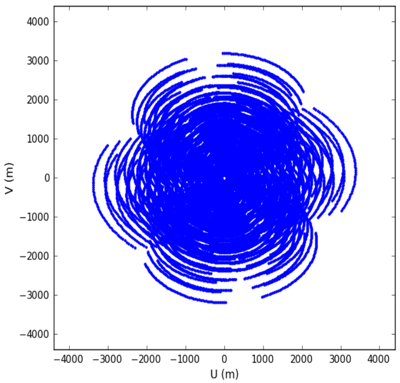
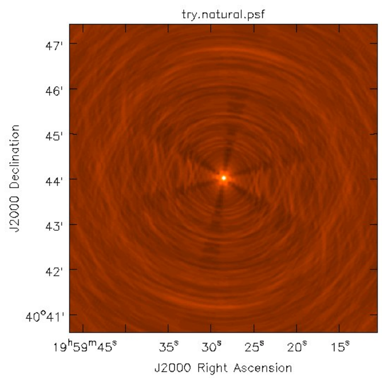
Image from an interferometer : Convolution eqn : $I^{obs} = I^{PSF} * I^{sky}$

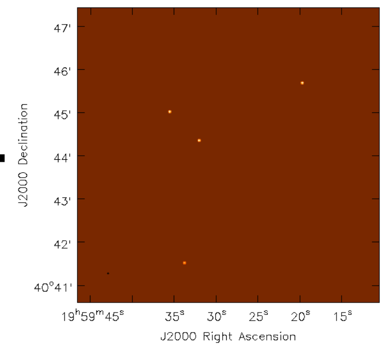


$F[I^{PSF}]$



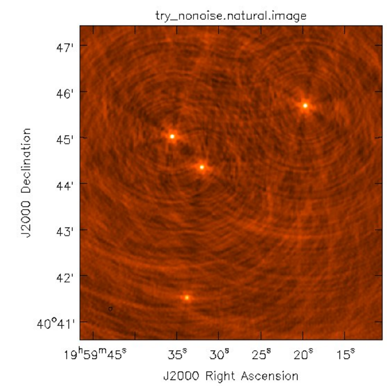
I^{PSF}

$*$



I^{sky}

$=$



I^{obs}

Image Reconstruction or Deconvolution : Extract I^{sky} from I^{obs} .

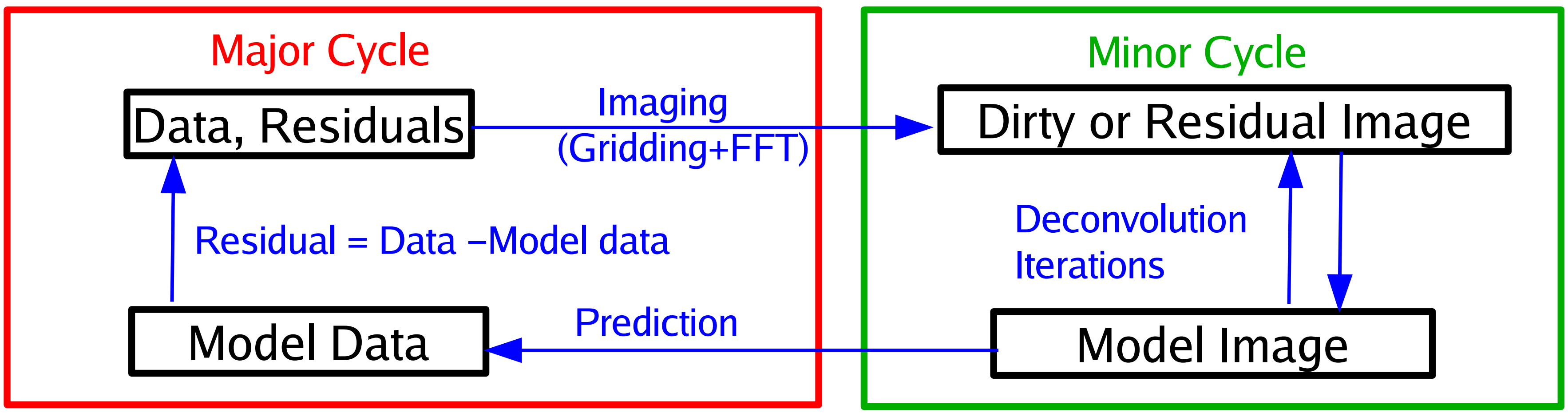
- Estimate the visibility function in unsampled regions of UV-space, such that it fits the data.
- There is no unique solution. In fact, there are infinite solutions.
- Constrain the solution by forcing astrophysical plausibility
  (point-like compact structure, positive intensity,smooth extended emission, etc...)

- The reconstruction process is always 'non-linear'.
  => Use methods of successive approximation (iterative model-fitting).

- There are limits to the largest and smallest features that can be trusted (set by uv-coverage)

Deconvolution – Iterative Model Fitting (χ^2 minimization)

Solve $[A]I^m = V^{obs}$ to fit a sky-model to the observed visibilities



Normal Equations : $[A^T W A]I^m = [A^T W]V^{obs}$

– This describes an image-domain convolution $I^{psf} * I^m = I^{dirty}$

Iterative Solution : $I^m_{i+1} = I^m_i + g[A^T W A]^+ \left(A^T W \left(V^{obs} - A I^m_i\right)\right)$

Deconvolution

Imaging
(Gridding + iFT)

Prediction
(FT + de-Gridding)

Deconvolution Algorithms + Image Restoration

(Minor cycle) Deconvolution algorithms differ in choice of sky-model, optimization scheme, and how they handle parameters that depend on each other.

Classic CLEAN : Point-source sky model, Steepest-descent optimization

Maximum Entropy Method : Point-source sky model with a smoothness constraint.
Steepest-descent optimization with backtracking

Multi-Scale CLEAN : Sky is a linear combination of components of different known shapes/sizes. Steepest-descent optimization

Adaptive-Scale-Pixel CLEAN : Sky is a linear combination of best-fit Gaussians.
BFGS optimization with subspace filtering.

[Several other adaptations of compressed-sensing reconstruction techniques (R&D)]

Output of deconvolution (minor cycle) : A model image (units : Jy/pixel)
A residual image (units : Jy/beam)

Restoration : Convolve model with a 'clean beam' (Gaussian fit to PSF main lobe)
Add in residual image. (units : Jy/beam)

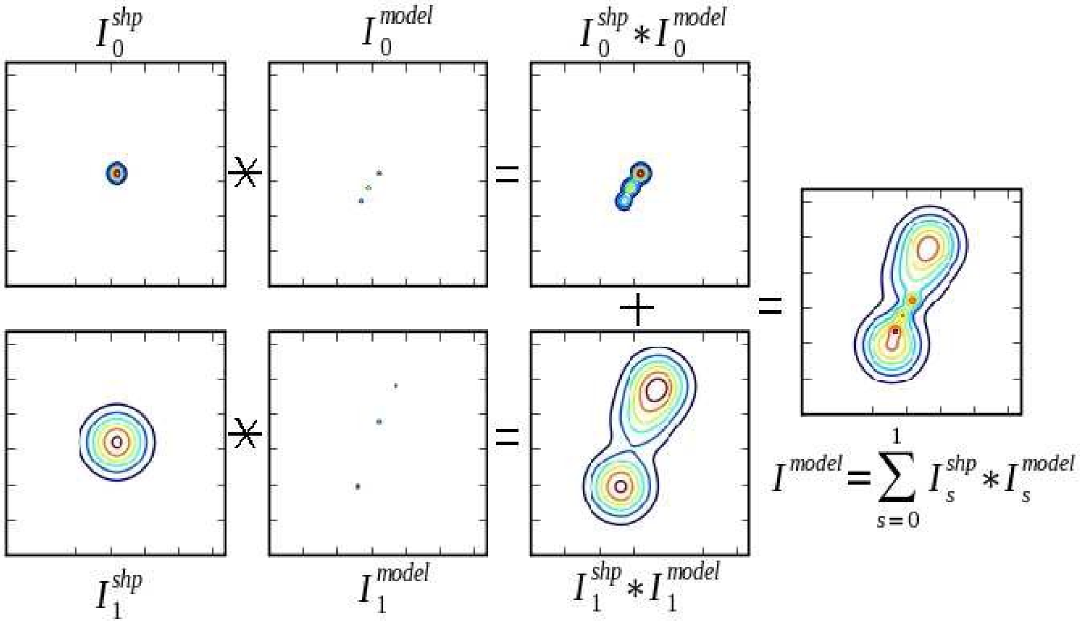
Deconvolution – MS-CLEAN

Multi-Scale Sky Model : Linear combination of 'blobs' of different scale sizes

$$I^{sky} = \sum_s [I_s^{shp} * I_s^m]$$

where I_s^{shp} is a blob of size 's'

and $I_s^m = \sum_i a_{s,i} \delta(l - l_{s,i})$



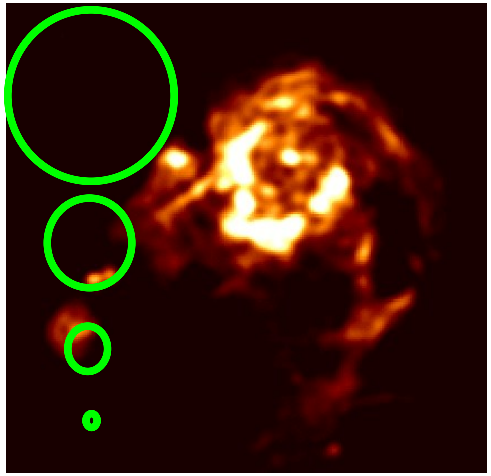
A scale-sensitive algorithm

(1) Choose a set of scale sizes (basis set)

(2) Calculate residual images smoothed to several scales
 – Normalize by the instrument's relative sensitivity to each scale

(3) Find the peak across all scales, update a multi-scale model
 and all residual images (accounting for coupling between scales)

Iterate, similar to Classic CLEAN with Major and Minor cycles



Deconvolution – Adaptive Scale Pixel (ASP) CLEAN

Sky Model : List of Gaussians $I^{sky} = \sum_c a_c e^{-\frac{(x-x_c)^2}{\sigma^2}}$

(1) Calculate the dirty image, smooth to a few scales.

(2) Find the peak across scales to identify a good initial guess of a_c, x_c, σ_c for a new component.

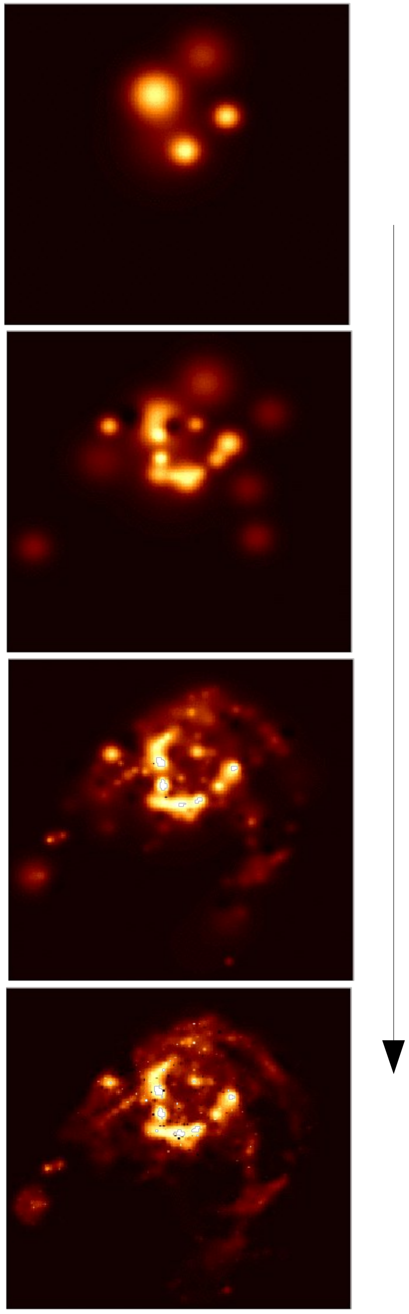
(3) Find best-fit parameters, and add this component to a list.

(4) Choose a subset of components most likely to have a significant impact on convergence. Re-fit Gaussian parameters for new and old components together.

(5) Subtract the contribution of all updated components from the dirty image.

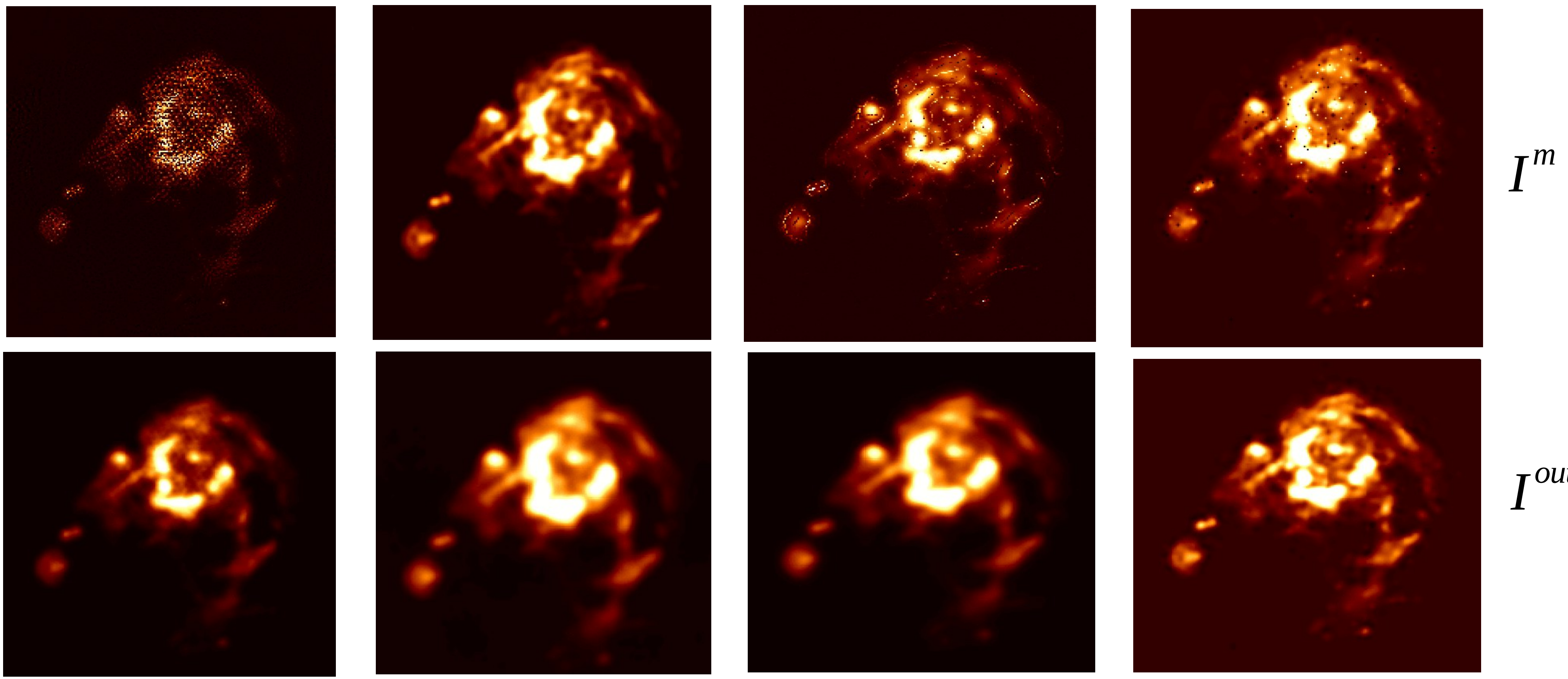
Repeat steps (2)-(5) until a stopping criterion is reached.

Adaptive Scale sizes leads to better reconstruction than MS-Clean, and more noise-like residuals.



Deconvolution – Comparison of Algorithms

CLEAN	MEM	MS-CLEAN	ASP
Point-source model	Point-source model with a smoothness constraint	Fit using a set of multi-scale basis functions.	Fit for parameters of compact and extended components

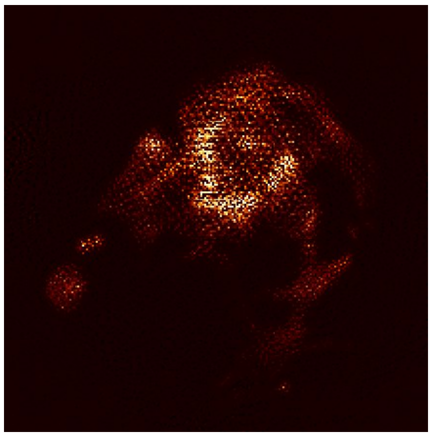


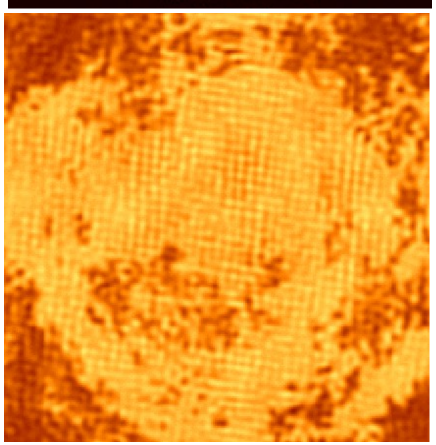
(Hogbom 1974, Clark 1980, Schwab & Cotton 1983)	(Cornwell & Evans, 1985)	(Cornwell, 2008)	(Bhatnagar & Cornwell 2004)

Deconvolution – Comparison of Algorithms

CLEAN

Point-source model

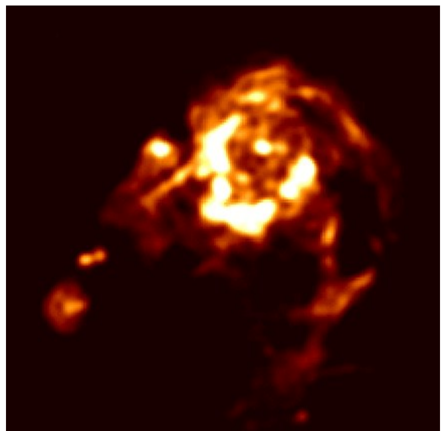


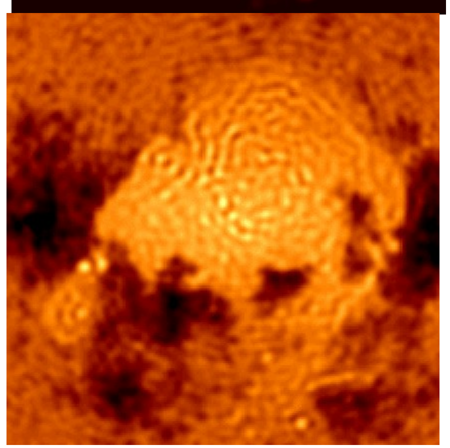


(Hogbom 1974, Clark 1980, Schwab & Cotton 1983)

MEM

Point-source model with a smoothness constraint

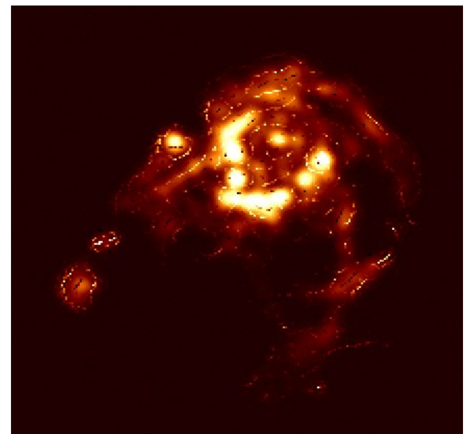


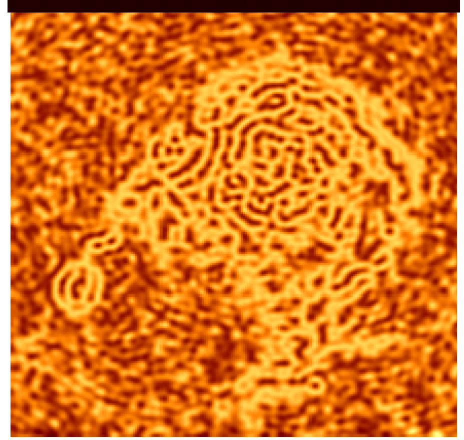


(Cornwell & Evans, 1985)

MS-CLEAN

Fit using a set of multi-scale basis functions.

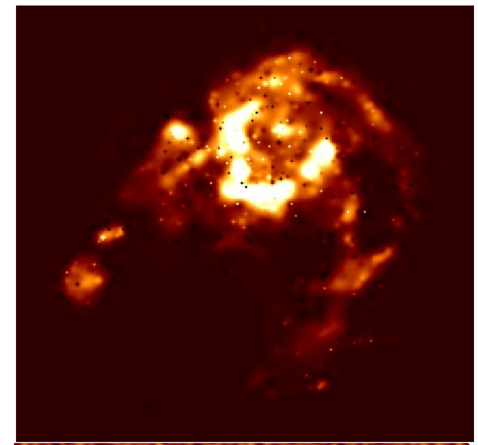


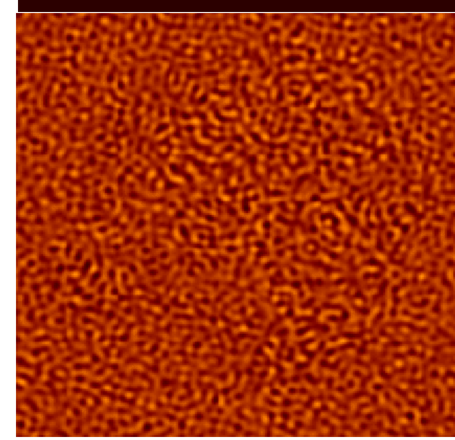


(Cornwell, 2008)

ASP

Fit for parameters of compact and extended components





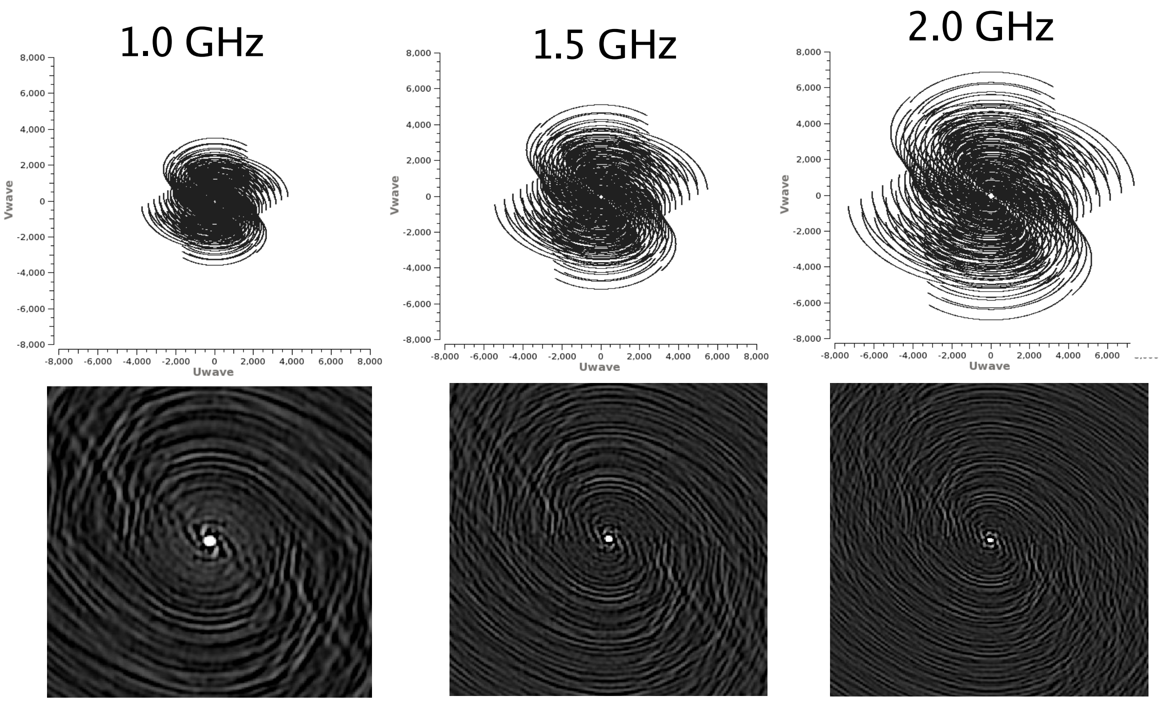
(Bhatnagar & Cornwell 2004)

I^m

I^{res}

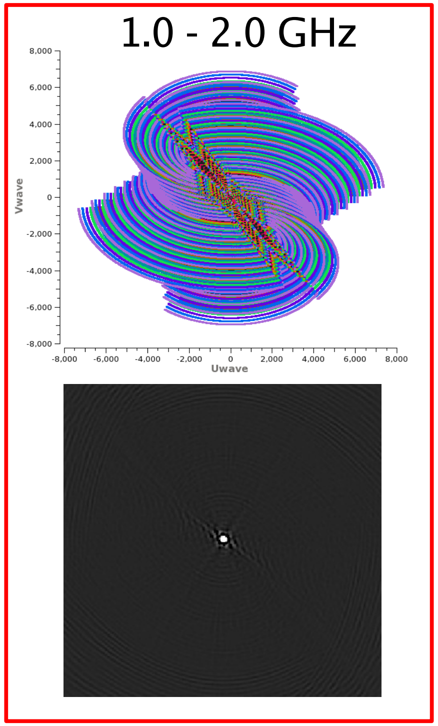
Wide Band Imaging + Multi-Frequency Synthesis (MFS)

Broad-band hardware => UV-coverage / imaging properties change with frequency



$$S(u,v)_\nu = \frac{\vec{b}}{\lambda} = \frac{\vec{b}\,\nu}{c}$$

=> combine multi-frequency measurements during imaging



But, the sky brightness distribution also changes with frequency
(astrophysical source spectrum and the antenna primary beam)

=> If you want to use the combined UV-coverage during image reconstruction, you need to model and reconstruct sky intensity and spectrum simultaneously.

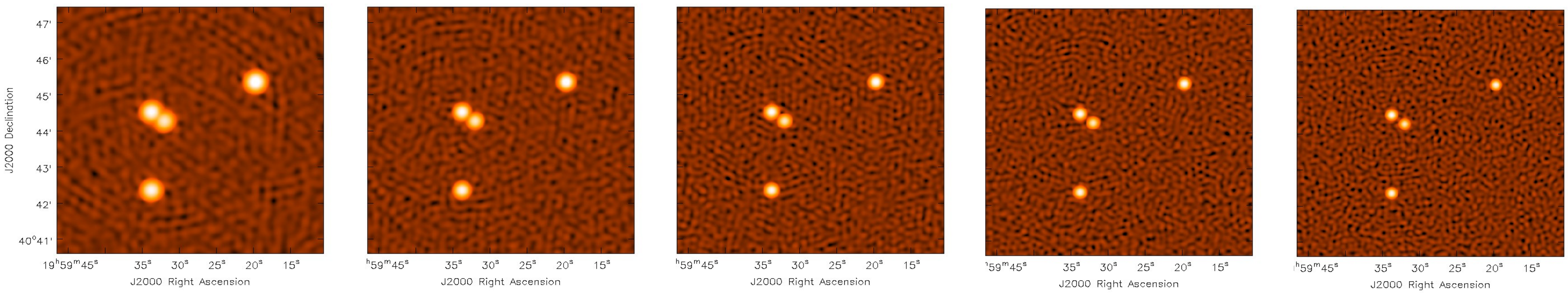
=> Or..... treat each frequency separately (limited uv-cov and sensitivity) + combine later.

Spectral Cube (vs) MFS imaging

Simulation : 3 flat-spectrum sources + 1 steep-spectrum source (1-2 GHz VLA observation)

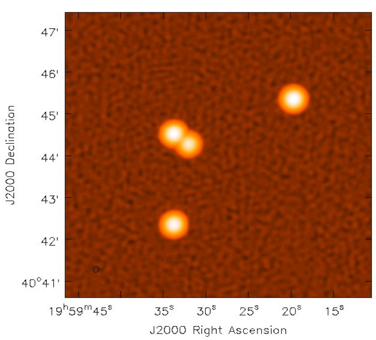
Images made at different frequencies between 1 and 2 GHz (limited to narrow-band sensitivity)

1 GHz

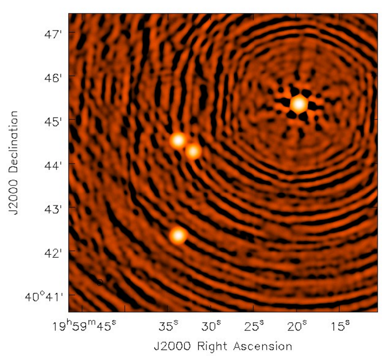


2 GHz

Add all single-frequency images (after smoothing to a low resolution)

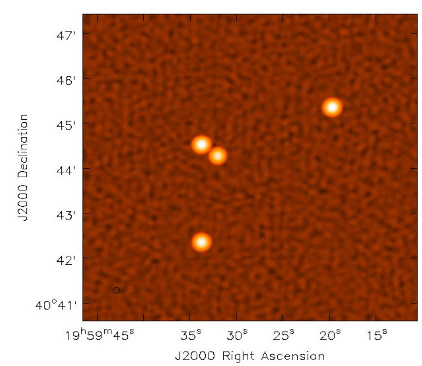


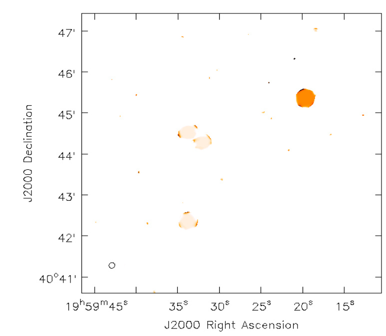
Use wideband UV-coverage, but ignore spectrum (MFS)



Use wideband UV-coverage
+ Model and fit for spectra too (MT-MFS)

Output : Intensity and Spectral-Index





Multi-term Multi-frequency-synthesis – fit a polynomial to the spectrum

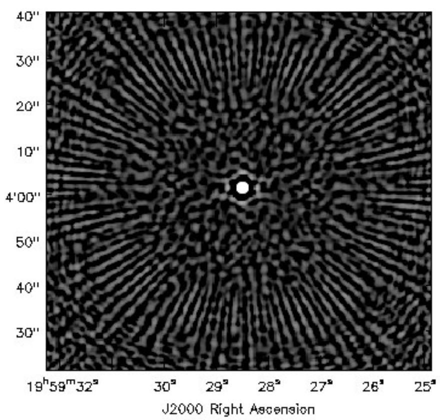
Sky Model : Collection of (multi-scale) flux components whose amplitudes follow a polynomial in frequency

$$I_\nu^{sky} = \sum_t I_t \left(\frac{\nu - \nu_0}{\nu_0} \right)^t \quad \text{where} \quad I_t = \sum_s [I_s^{shp} * I_{s,t}]$$

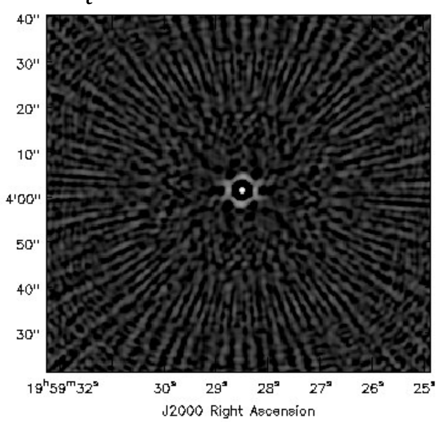
(1) Define "spectral PSFs": the instrument's response to each term of a Taylor polynomial in frequency :

$$I_t^{psf} = \sum_\nu \left(\frac{\nu - \nu_0}{\nu_0} \right)^t I_\nu^{psf}$$

I_0^{psf} Flat Spectrum



I_t^{psf} Linear Spectrum



The observed image is a sum of convolutions....

$$I^{obs} = \sum_t I_t^{psf} * I_t^{obs}$$

(this follows basic polynomial-fitting rules)

(2) Do a joint deconvolution of ALL Taylor-PSFs (spectral PSFs) from a series of dirty-images formed as Taylor-weighted averages of individual-frequency images.

(3) Interpret the output Taylor Coefficient maps in terms of a power law

$$I_\nu = I_{\nu_0} \left(\frac{\nu}{\nu_0} \right)^{\alpha + \beta \log(\nu / \nu_0)}$$

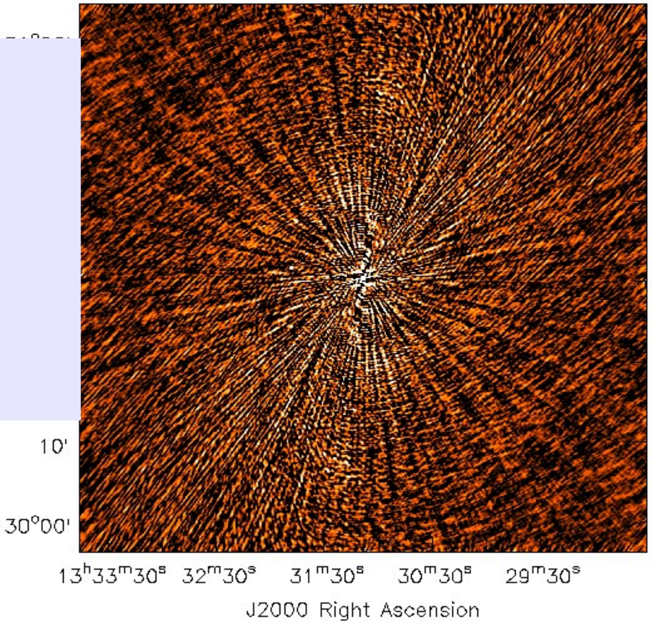
Intensity
Spectral Index

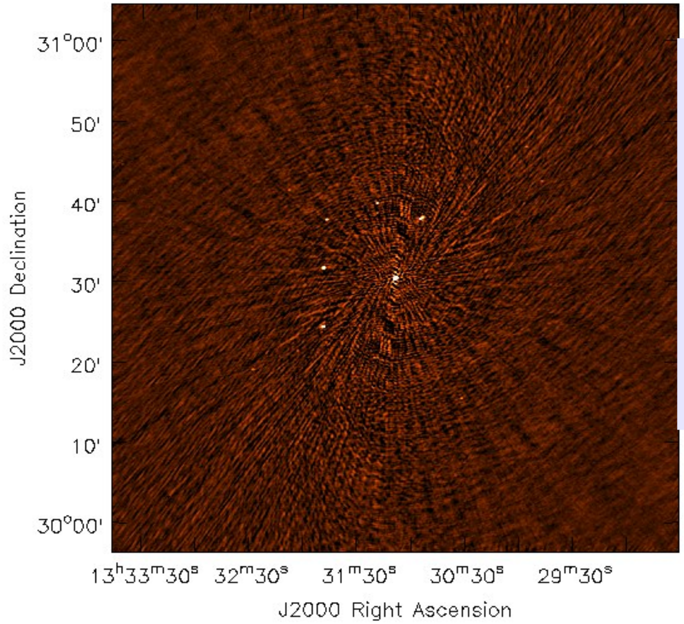
Dynamic-range with MS-MFS : 3C286 example : Nt=1,2,3,4

NTERMS = 1

Rms :
9 mJy -- 1 mJy

DR :
1600 -- 13000





NTERMS = 2

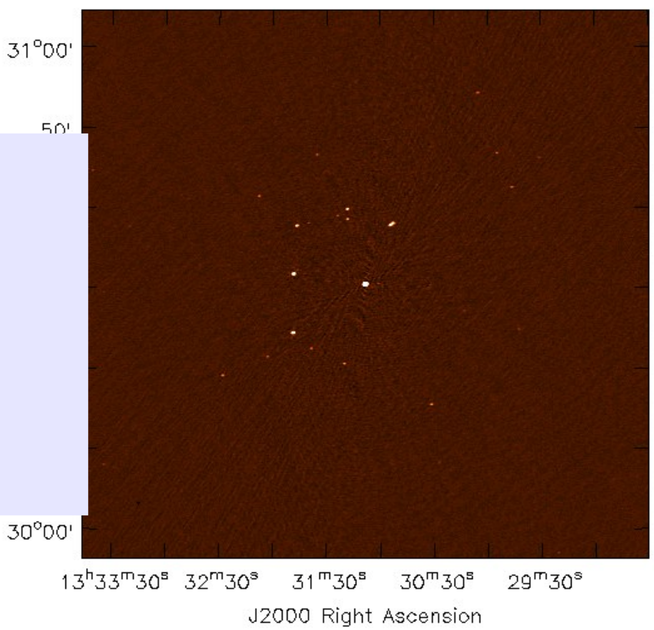
Rms :
1 mJy -- 0.2 mJy

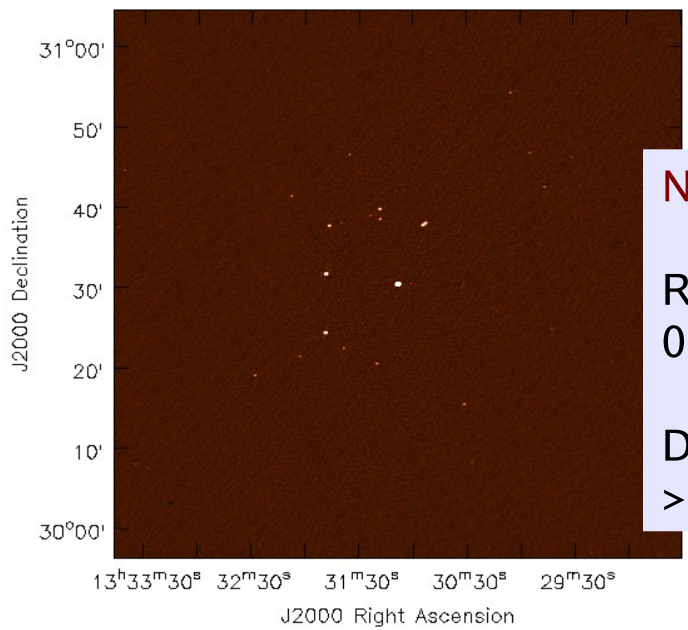
DR :
10,000 -- 17,000

NTERMS = 3

Rms :
0.2 mJy -- 85 uJy

DR :
65,000 -- 170,000





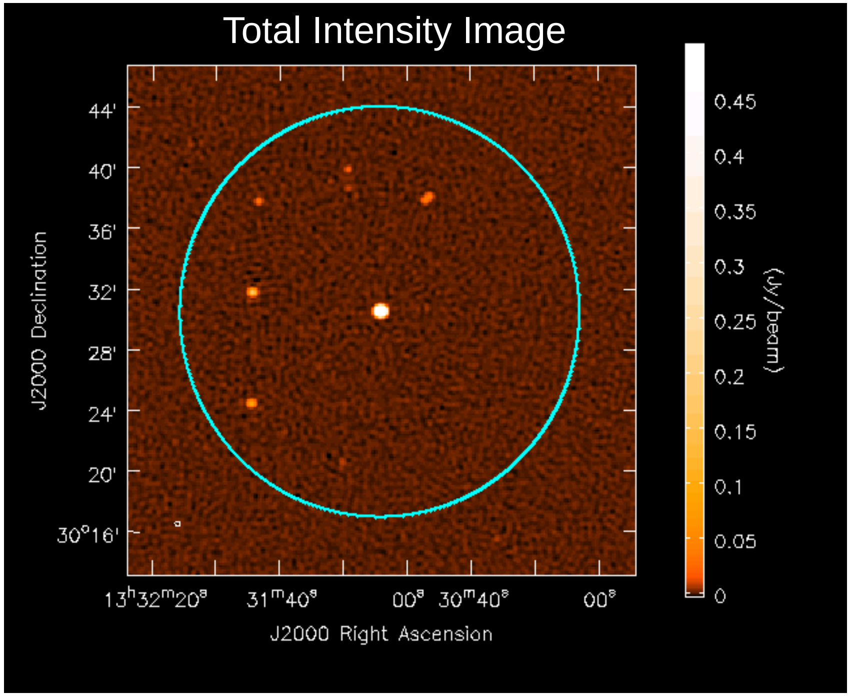
NTERMS = 4

Rms
0.14 mJy -- 80 uJy

DR :
>110,000 -- 180,000

Example of Imaging with wide-band PB (artificial spectrum)

3C286 field , C-config , L-band (30min)



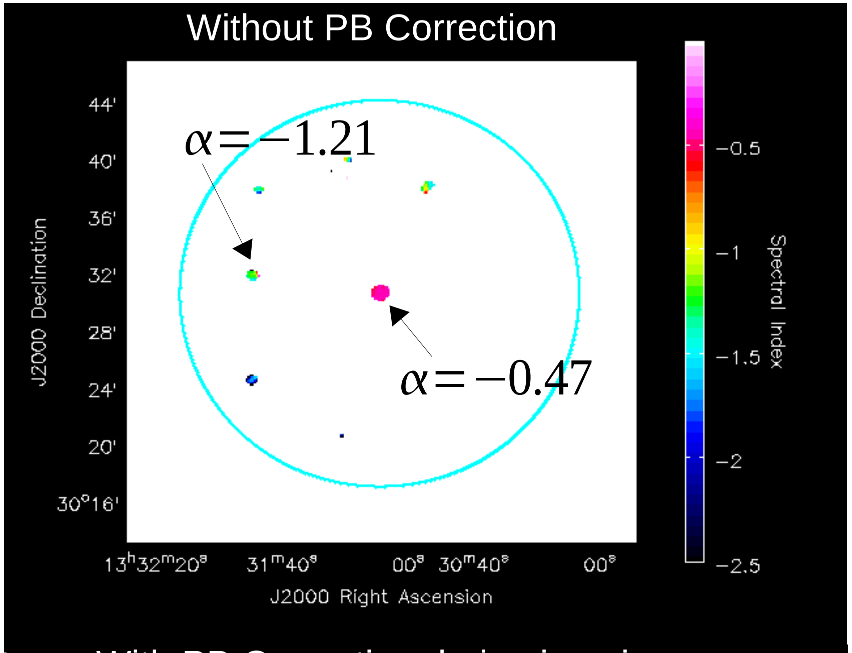
Sources away from the pointing center pick up an artificial spectrum due to the Primary Beam.

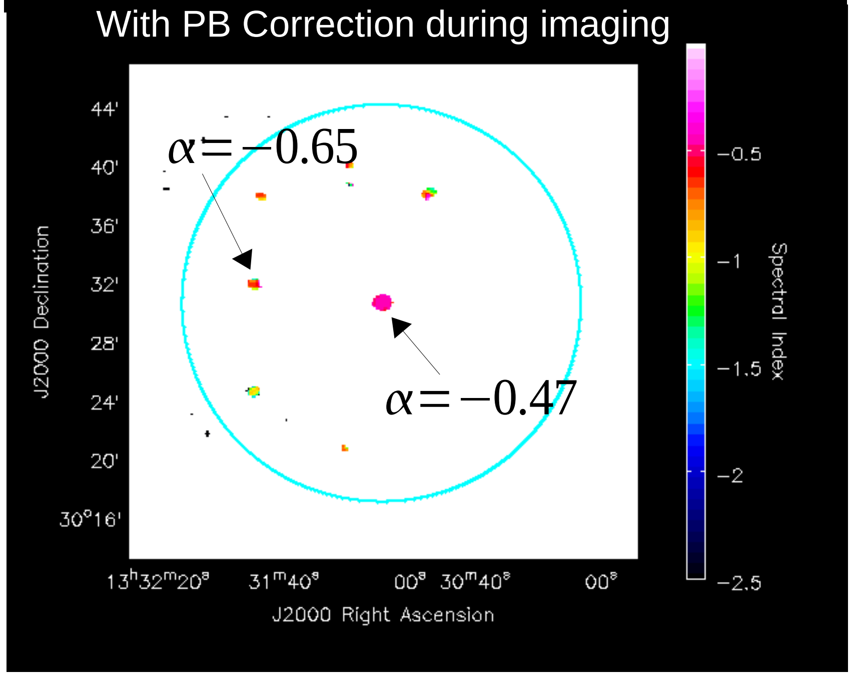
Do a post-deconvolution polynomial-division of the model spectrum by the PB-spectrum

Accuracy depends on how good the PB model is.

Also verified via holography observations at two frequencies

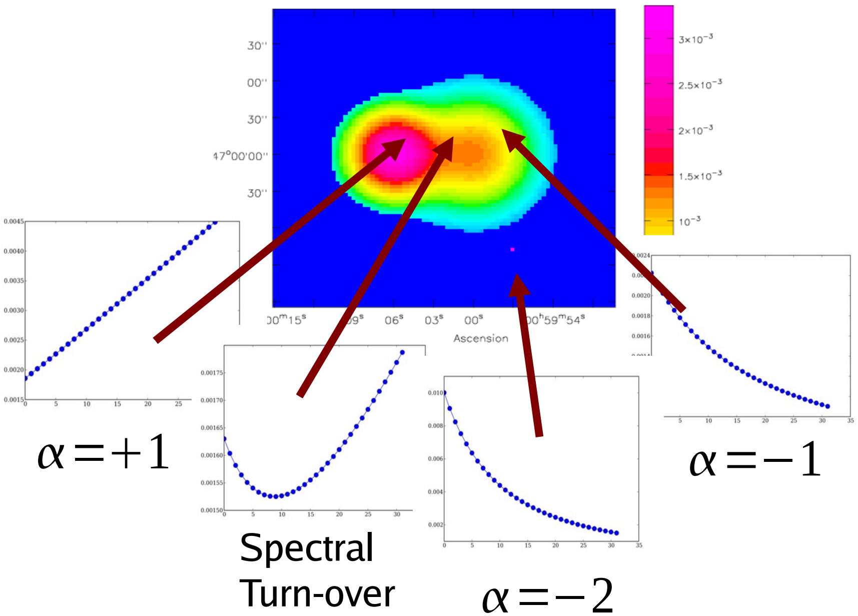
Obtained $\delta\alpha$ = 0.05 to 0.1 for SNR of 1000 to 20





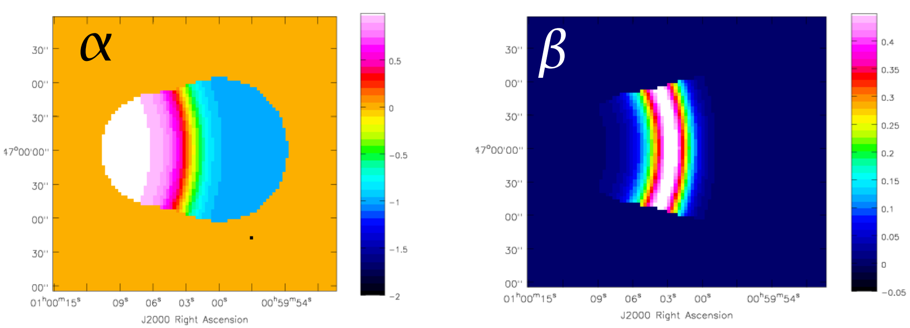
Example of wideband-imaging on extended-emission

Intensity Image



Average Spectral Index

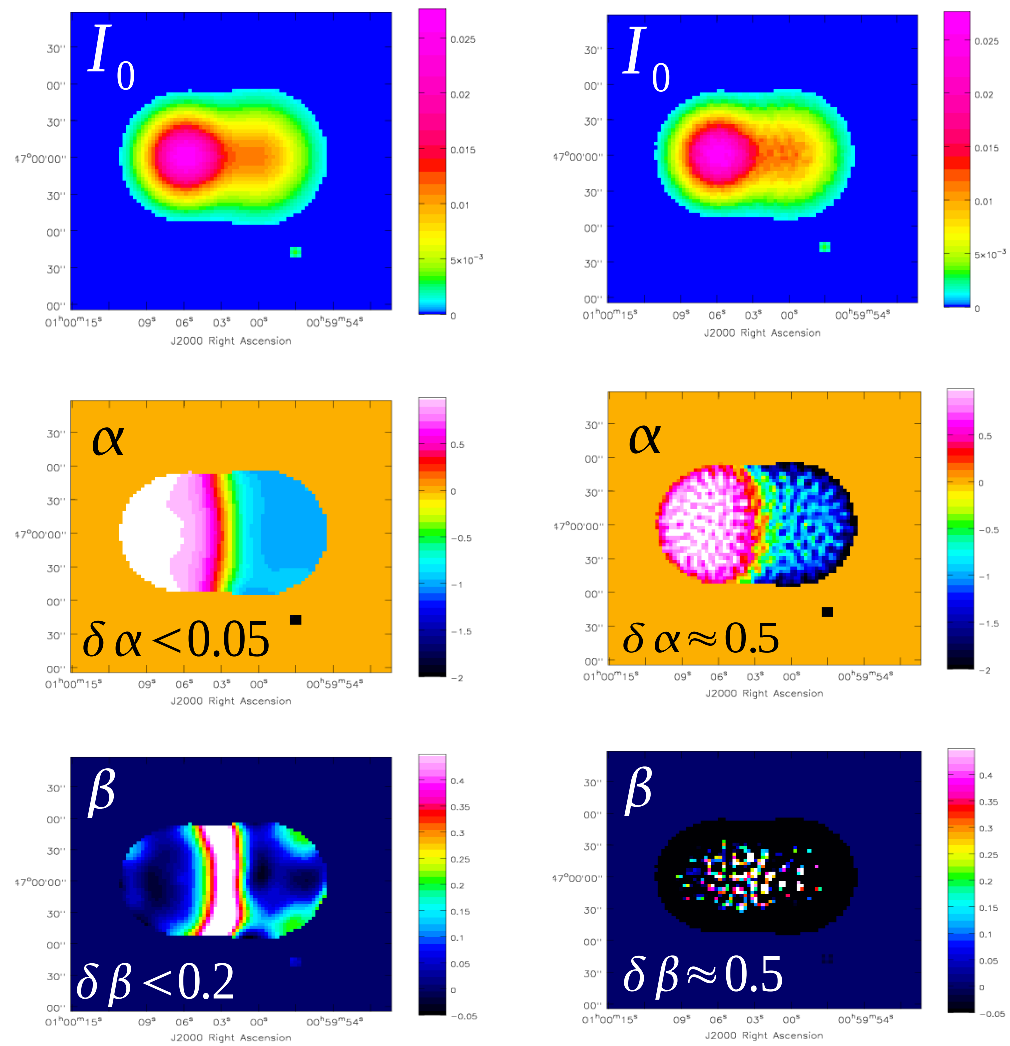
Gradient in Spectral Index



MFS

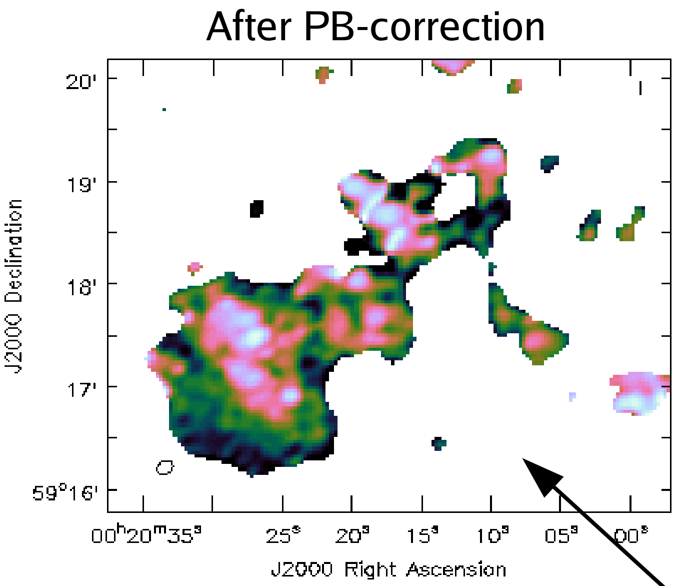
multi-scale

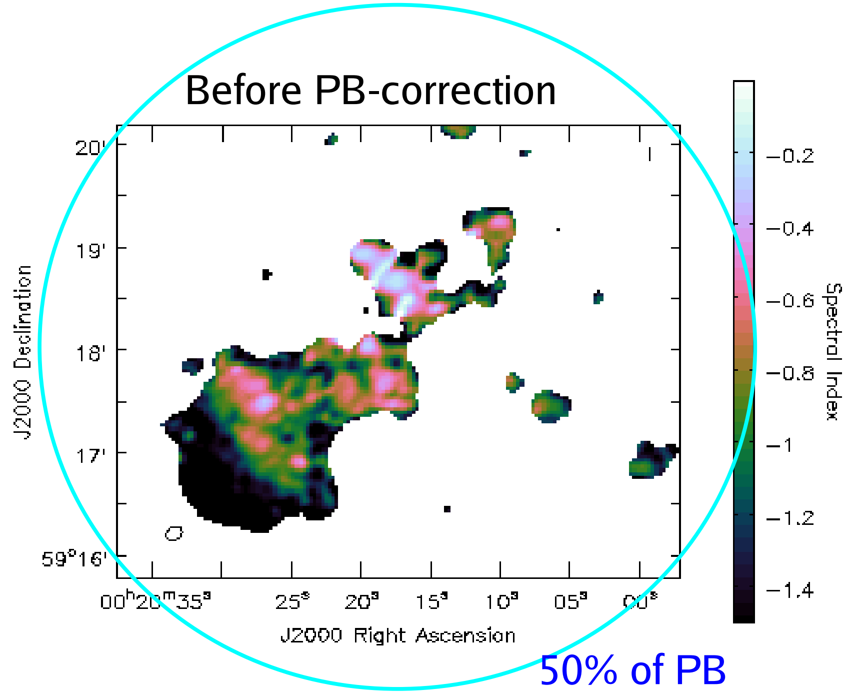
point-source



=> For extended emission - spectral-index error is dominated by 'division between noisy images'
– a multi-scale model gives better spectral index and curvature maps

Continuum (MS-MFS) vs Cube Imaging (with PB-correction)

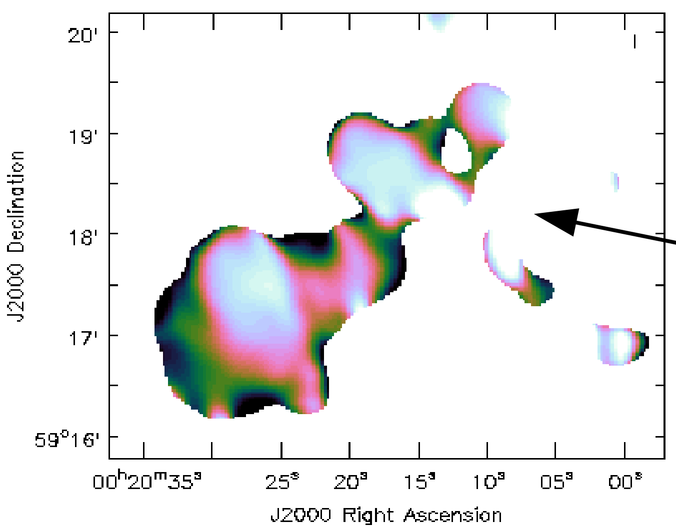




IC10 Dwarf Galaxy :

Spectral Index across C-Band.

Dynamic-range ~ 2000 (~ noise-limited image obtained)



MS-MFS :

Result of wide-band PB-correction after MT-MS-MFS.

Cube :

Spectral-index map made by PB-correcting single-spw images smoothed to the lowest resolution.

This is an example of MFS with a spectral model extracting more information compared to the traditional method.

Summary

Several image reconstruction (deconvolution) algorithms exist.

- Point source flux models (CLEAN)
- Point source model with smoothness constraints (MEM)
- Multi-scale flux models (MS-CLEAN, ASP)
- Wide-band flux models (MS/MT-MFS)

All are iterative, constrained, non-linear optimizations : fit a model to the data.

- Traditional : chi-square minimization
- New : compressed sensing methods

Choose/constrain your deconvolution algorithm based on

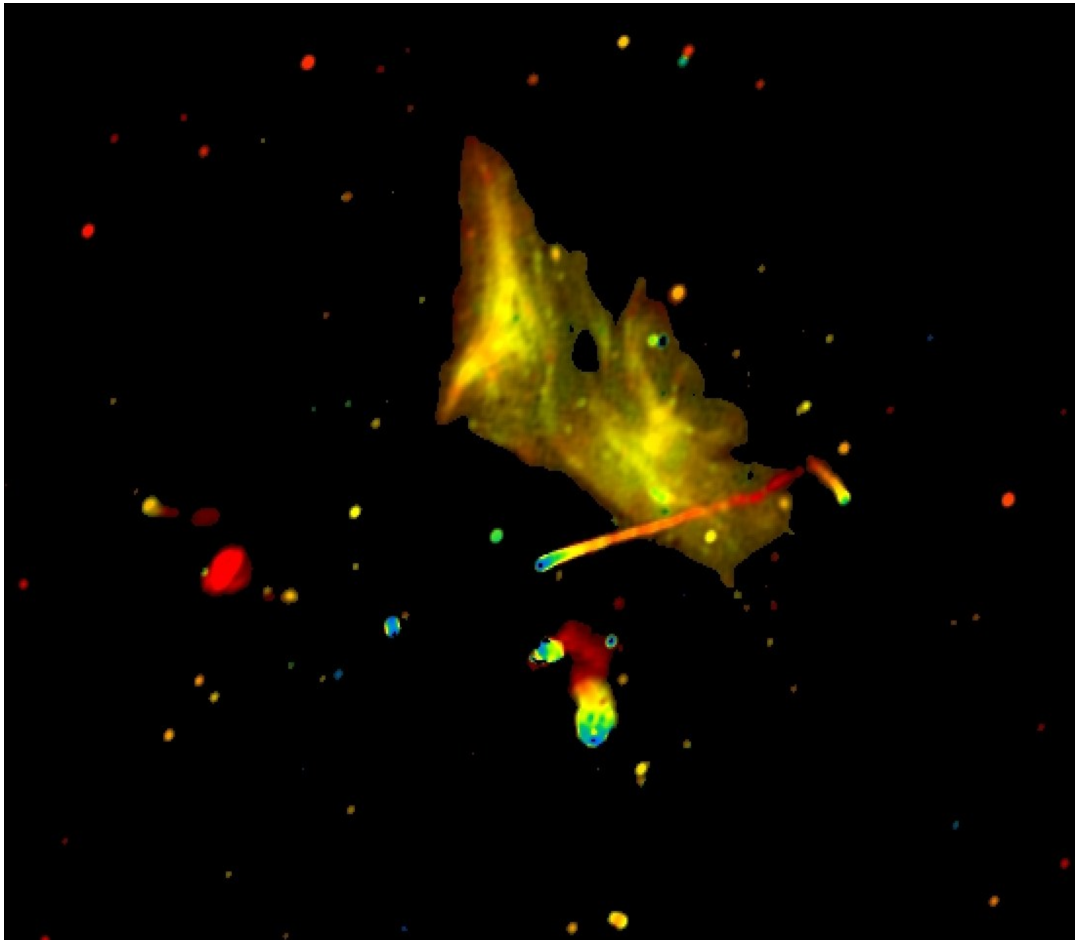


Image from F.Owen : Intensity-weighted Spectral Index of Abell 2256

- Source structure : point sources only, extended emission, flat/steep spectrum, wide-field...
- UV-coverage : choose weighting schemes to match the sky structure, use masks if the model is ill-constrained, choose a model that is well-constrained by the data, etc...