Synthesis Imaging in Radio Astronomy - reconstructing spatial and spectral structure of an astronomical source

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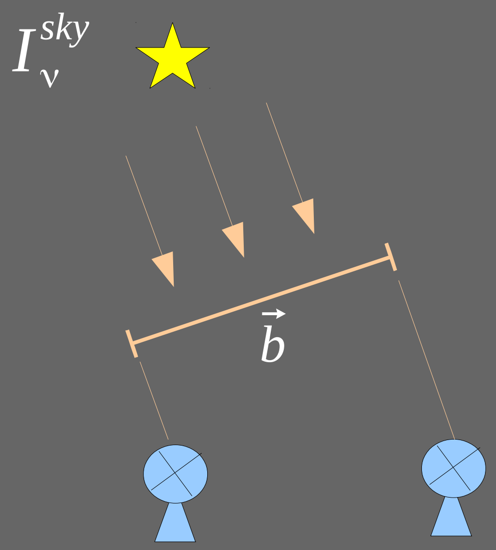
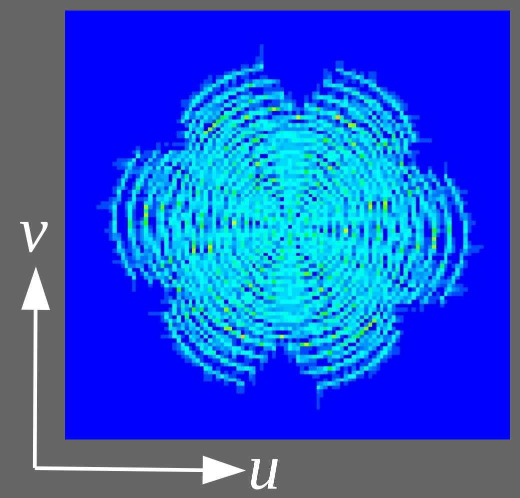
Imaging with an interferometer (narrow-band)

An interferometer samples the spatial Fourier transform of the “sky brightness”



spatial frequency coverage



$$[S_\nu] = \frac{\vec{b}}{\lambda} = \frac{\vec{b}\nu}{c}$$

Earth-Rotation Synthesis

Measurement Equation :

$$V_\nu = [S_\nu][F] I_\nu^{sky}$$

Data | Spatial frequency coverage | Fourier transform | Sky Brightness

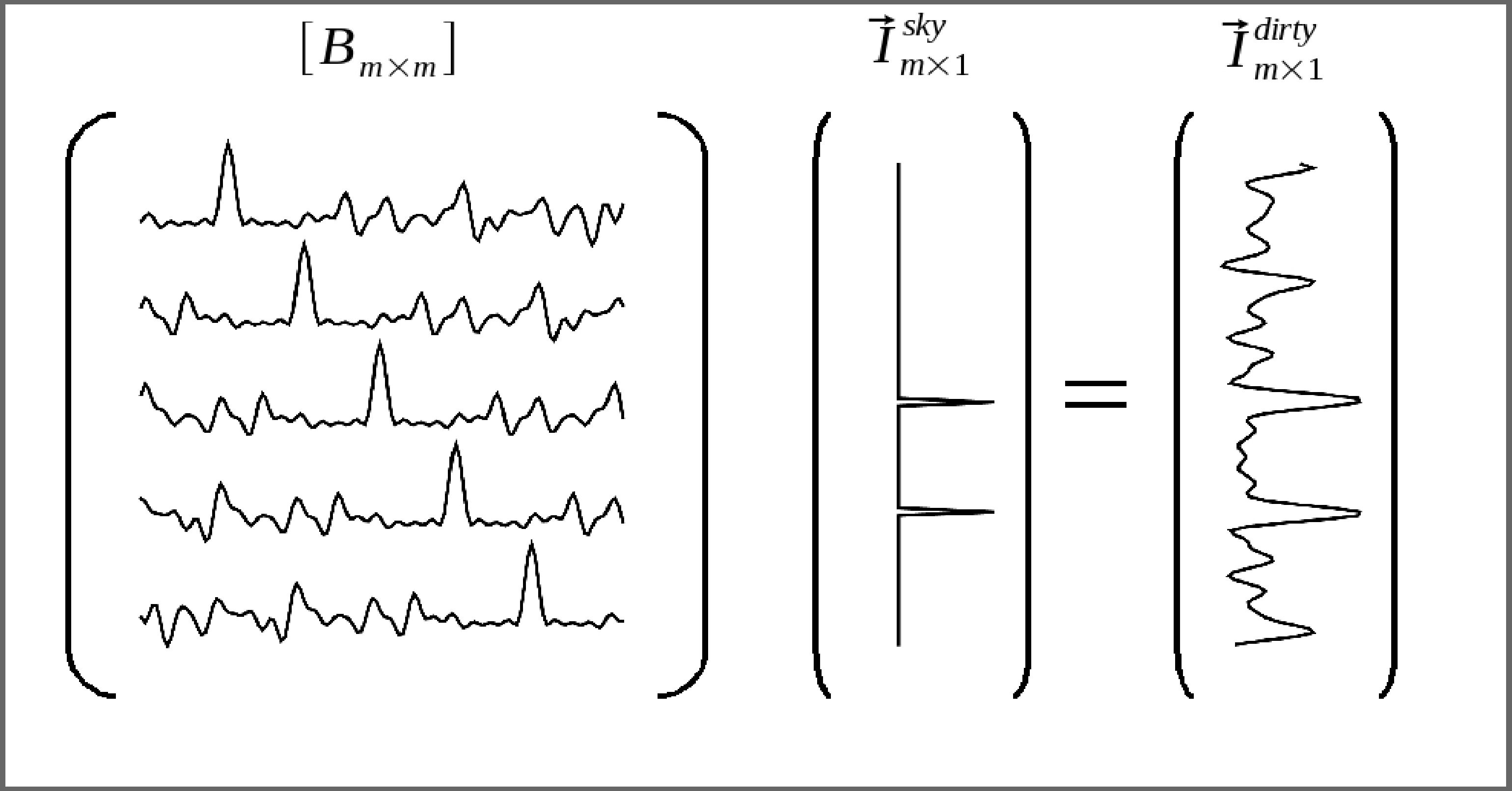
Solve this equation to reconstruct I_ν^{sky} from V_ν

$$min \ \|\cdot\|_2 \ \ s.t. \ \ \|\cdot\|_\infty < \epsilon$$

'CLEAN' – $min \; \|\cdot\|_2 \;\; s.t. \;\; \|\cdot\|_\infty < \epsilon$ (+ sparsity in the pixel-basis)

Measurement Eqns : $V^{obs}_\nu = [S_\nu][F] I^{sky}_\nu$ where $I^{sky}_\nu = \sum_i \delta(x - x_i)$

Normal Eqns : $[F^T S^T_\nu W S F] I^{sky}_\nu = [F^T S^T_\nu W] V^{res}_\nu = I^{dirty}_\nu$

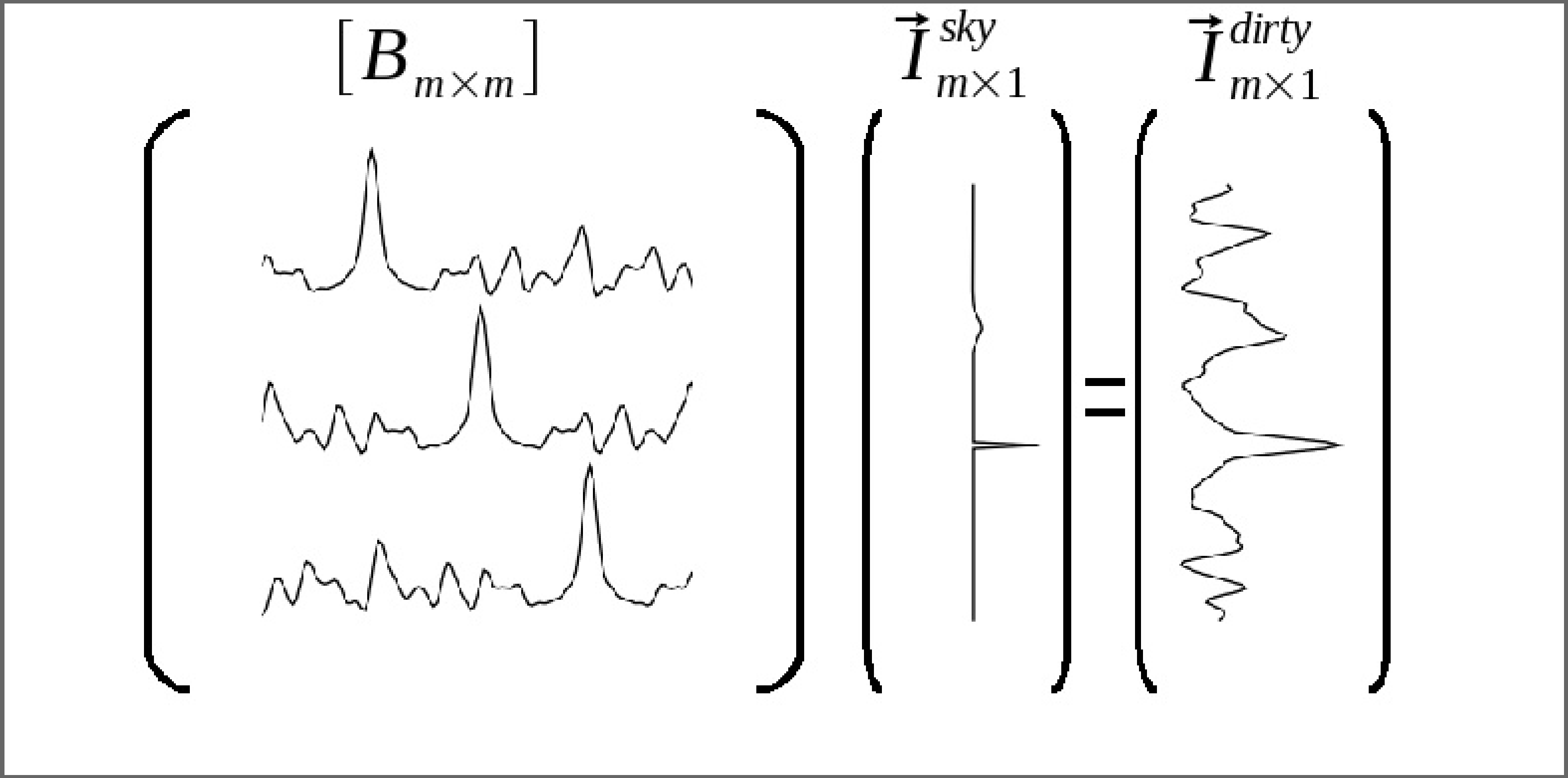


Choose update direction by applying an approximate Hessian inverse (diag.approx.) to the peaks of the residual image. Subtract response of new component. Iterate. Stop iterating when peak residual < user-specified threshold

'CLEAN' – $min \; \|\cdot\|_2 \;\; s.t. \;\; \|\cdot\|_\infty < \epsilon$

Measurement Eqns : $V_\nu^{obs} = [S_\nu][F] I_\nu^{sky}$ where $I_\nu^{sky} = \sum_i \delta(x - x_i)$

Multi-Scale Structure : The signal is no longer sparse in the pixel-basis



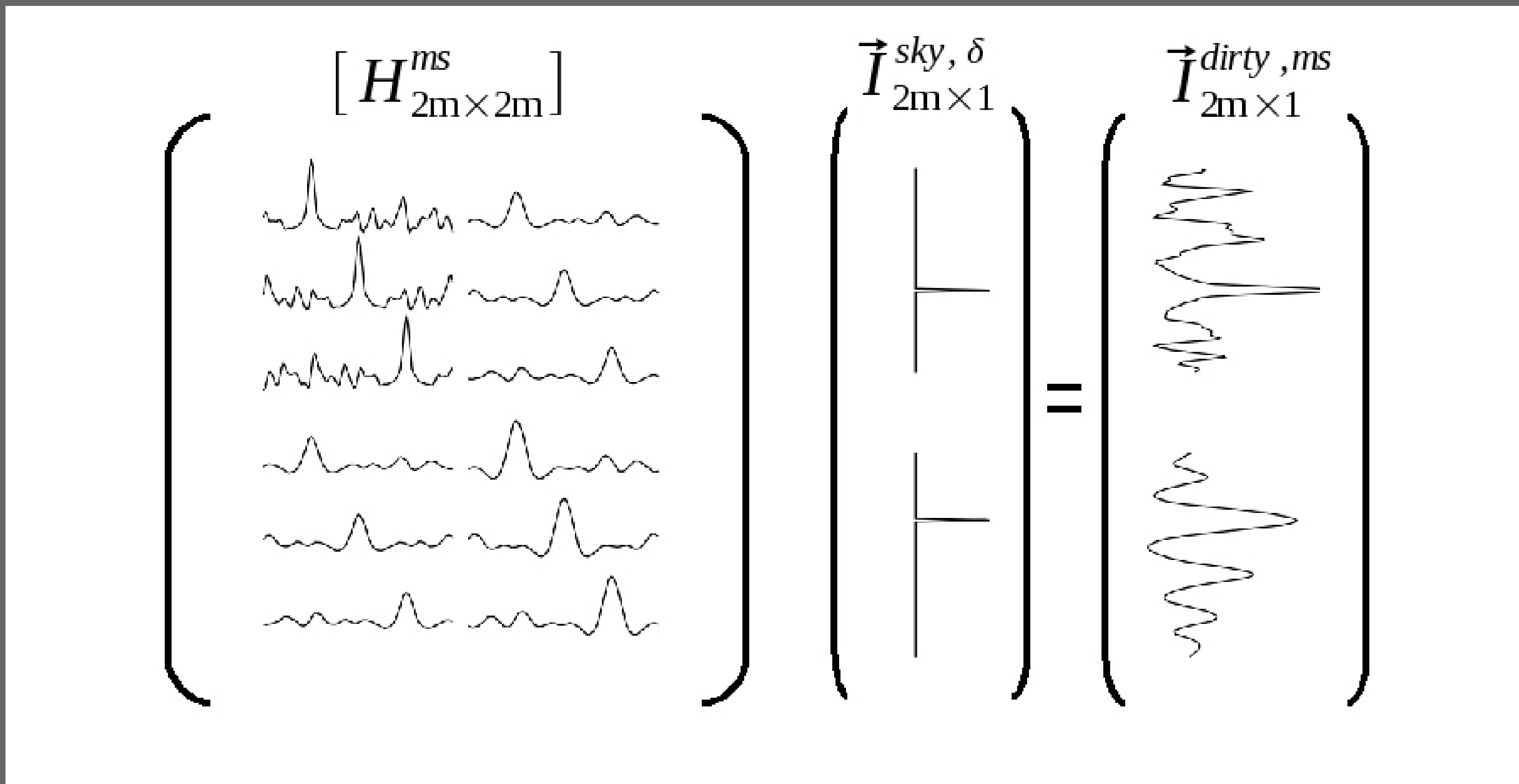
Standard CLEAN (using point-source basis functions) will not give an optimal reconstruction

=> Choose a multi-scale basis set => Multi-Scale CLEAN

'MS-CLEAN' – $min \; \|\cdot\|_2 \; s.t. \; \|\cdot\|_\infty < \epsilon$

Measurement Eqns : $V^{obs}_{\nu} = [S_{\nu}][F] I^{sky}_{\nu}$ where $I^{sky}_{\nu} = \sum_s [I^{shp}_s * I_s]$

Use Multi-Scale basis functions : The signal is again sparse.



Multi-Scale CLEAN : Inversion of a block-diagonal approximation of the Hessian, applied to the peaks of the RHS vectors, is a good estimator of the parameter values.

CLEAN

$$I_\nu^{sky} = \sum_i \delta(x - x_i)$$

Minimize L2
(assume sparsity in the image)

MEM

$$I_\nu^{sky} = \sum_i \delta(x - x_i)$$

Minimize L2 subject to an entropy-based prior (e.g. smoothness)

MS-CLEAN

$$I_\nu^{sky} = \sum_s [I_s^{shp} * I_s]$$

Minimize L2
(assume a set of spatial scales)

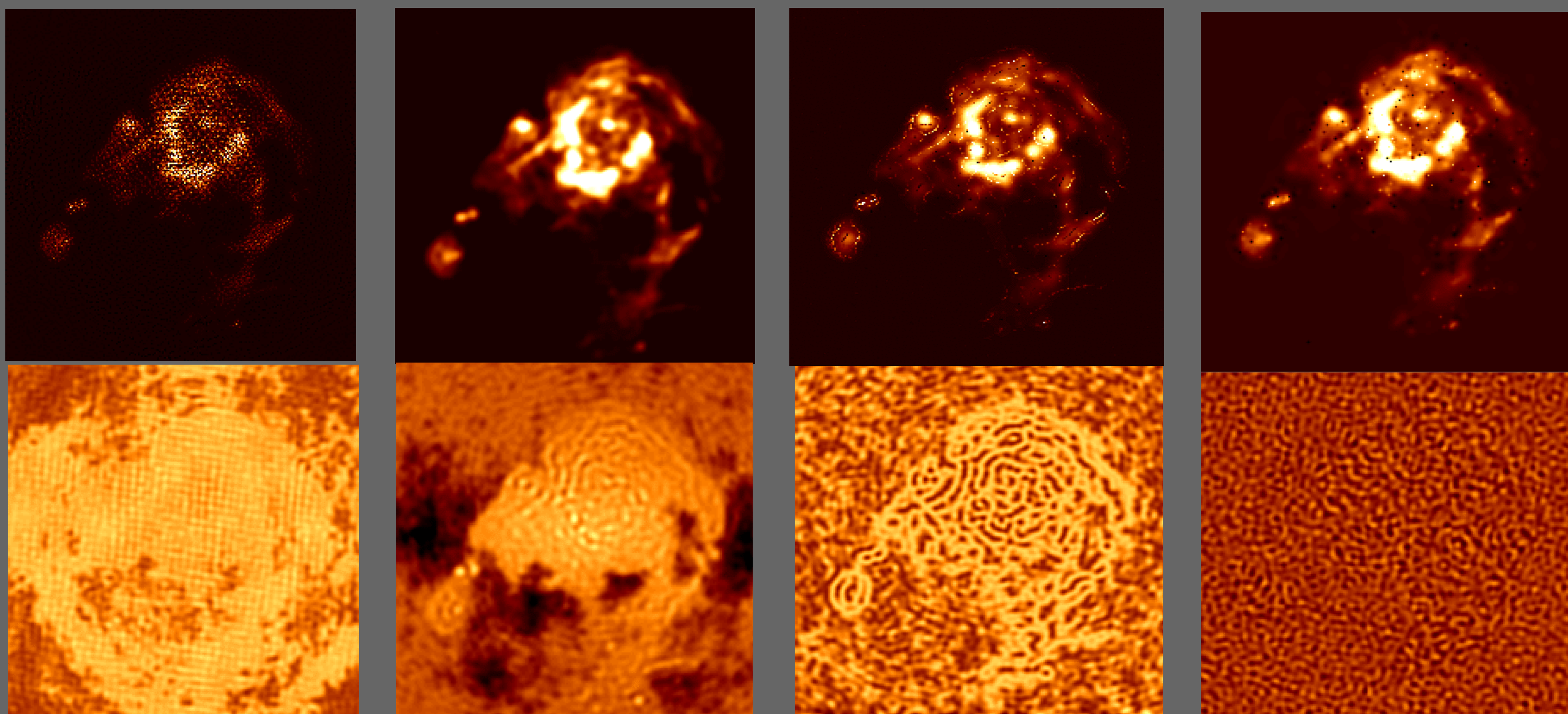
ASP

$$I_\nu^{sky} = \sum_i G(a_i, x_i, y_i, s_i)$$

Minimize L2 with TV-based subspace searches

I^M

I^R



(Hogbom 1974, Clark 1980, Schwab & Cotton 1983)

(Cornwell & Evans, 1985)

(Cornwell, 2008)

(Bhatnagar & Cornwell 2004)

Imaging with a broad-band interferometer

Goal : Use wide-band receivers to increase signal-to-noise ratio.

But, spatial-frequency coverage changes with frequency

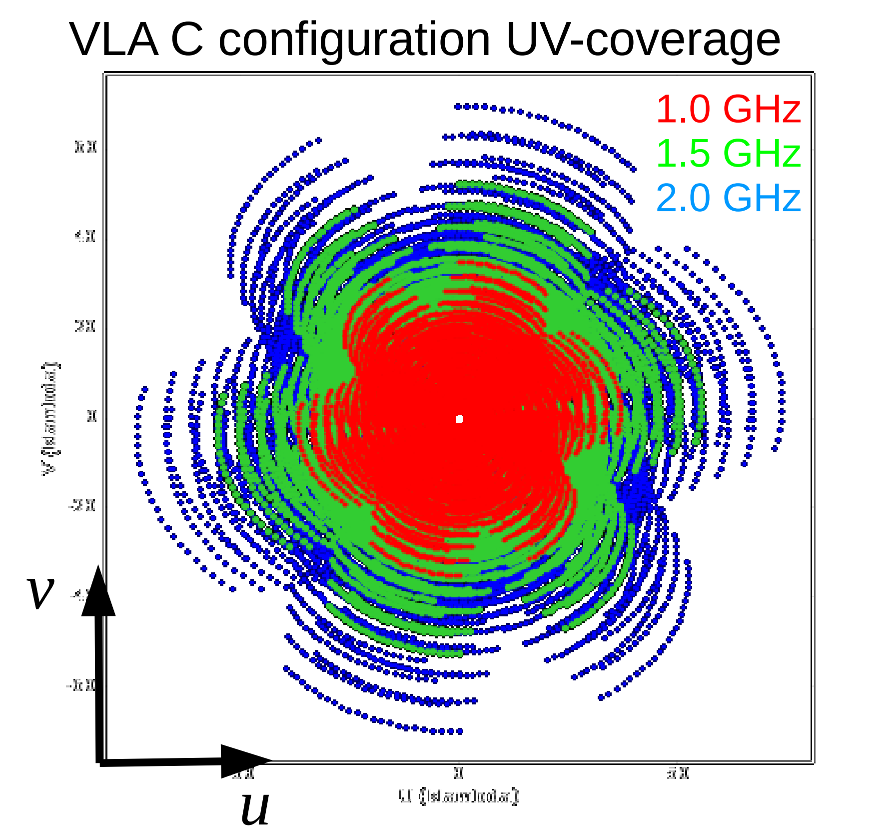
$$[S_\nu] = \frac{\vec{b}}{\lambda} = \frac{\vec{b}\,\nu}{c}$$

=> Must avoid chromatic aberration

Method : Measure data in multiple channels, and combine during imaging

Multi-Frequency Synthesis (MFS)

- Better imaging fidelity
- Higher angular resolution



But..... the Sky Brightness also changes with frequency.

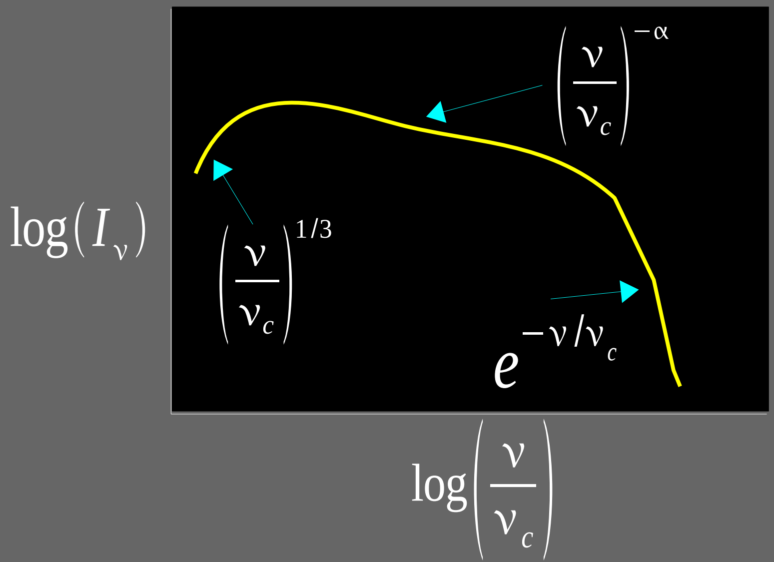
=> Can/Must reconstruct sky spectra too !

Frequency Dependent Sky Brightness

Each point on the source has a spectrum :

- the radio synchrotron spectrum is often a power law with varying index

$$I_\nu = I_{\nu_0}\left(\frac{\nu}{\nu_0}\right)^{\alpha+\beta\log(\nu/\nu_0)}$$



Source structure changes with frequency :

- spectrum traces velocity structure (doppler-shifted line emission)
- frequency probes depth in a 3D volume (solar flares/loops)

Measurement Equation

$$V_\nu = [S_\nu][F] I_\nu^{sky}$$

Image Model

$$I_\nu^{sky} = \sum_t I_t \left(\frac{\nu-\nu_0}{\nu_0}\right)^t$$

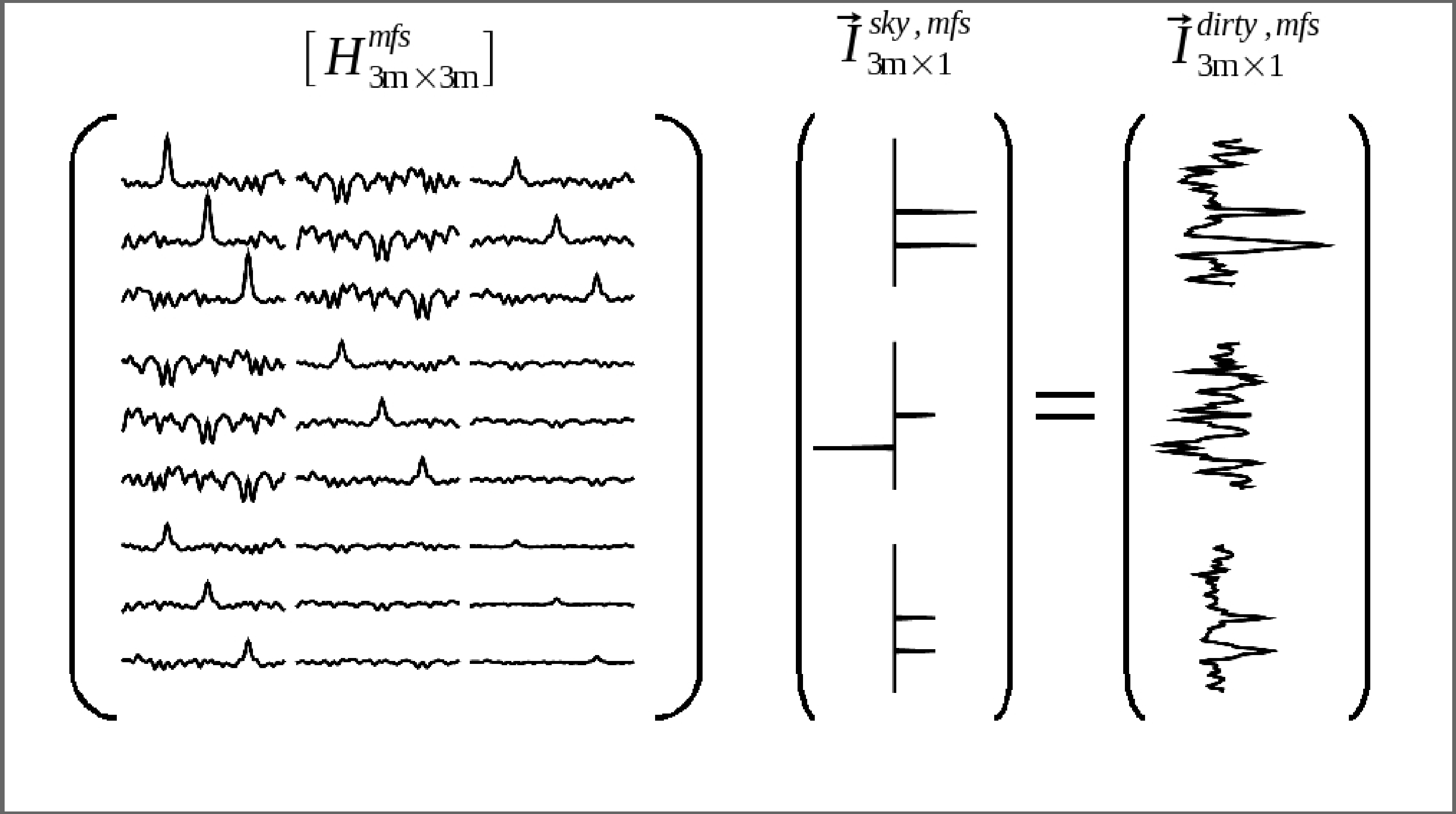
Algorithm : $min \; \|\cdot\|_2 \;\; s.t. \; \|\cdot\|_0 < \epsilon$

Combine with MS-Clean or ASP approaches to include multi-scale. Both work.

'MF-CLEAN' – $min \ \|\cdot\|_2 \ \ s.t. \ \ \|\cdot\|_\infty < \epsilon$ (+ sparsity)

Measurement Eqns : $V_\nu^{obs} = [S_\nu][F] I_\nu^{sky}$ where $I_\nu^{sky} = \sum_t I_t^{sky} \left(\frac{\nu - \nu_0}{\nu_0} \right)^t$

Multi-Frequency Data : A smooth spectrum is sparse in a Polynomial basis



(Sault & Wieringa, 1994, Rau & Cornwell, 2011)

MS-MFS CLEAN – Combine MS and MF ideas

Measurement Eqns : $V^{obs}_{\nu} = [S_{\nu}][F] I^{sky}_{\nu}$ where $I^{sky}_{\nu} = \sum_s \sum_t \left(\frac{\nu - \nu_0}{\nu_0} \right)^t [I^{shp}_s * I^m_{s,t}]$

Sky parameterization : multi-scale multi-frequency sparse basis

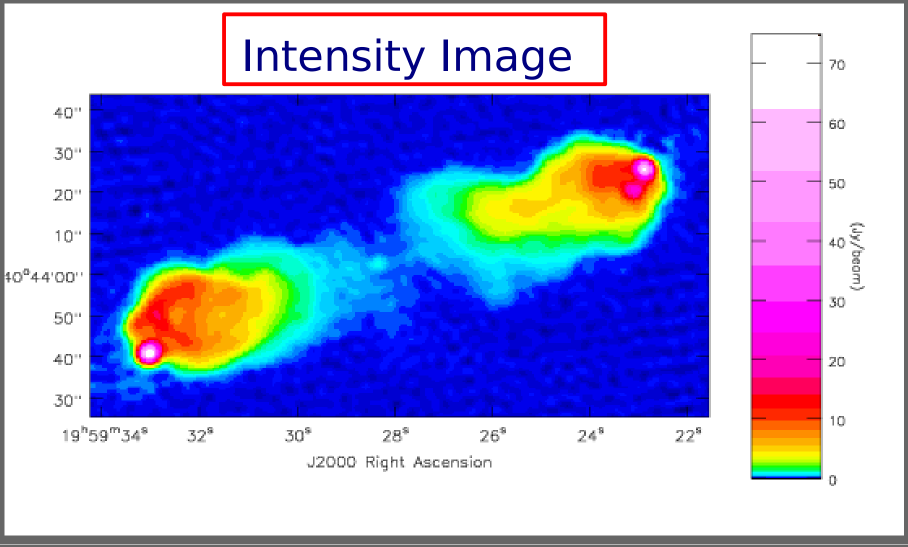
$$
\begin{bmatrix}
[H^{s=0,p=0}_{t=0,q=0}] & [H^{s=0,p=0}_{t=0,q=1}] & [H^{s=0,p=0}_{t=0,q=2}] & [H^{s=0,p=1}_{t=0,q=0}] & [H^{s=0,p=1}_{t=0,q=1}] & [H^{s=0,p=1}_{t=0,q=2}] \\
[H^{s=0,p=0}_{t=1,q=0}] & [H^{s=0,p=0}_{t=1,q=1}] & [H^{s=0,p=0}_{t=1,q=2}] & [H^{s=0,p=1}_{t=1,q=0}] & [H^{s=0,p=1}_{t=1,q=1}] & [H^{s=0,p=1}_{t=1,q=2}] \\
[H^{s=0,p=0}_{t=2,q=0}] & [H^{s=0,p=0}_{t=2,q=1}] & [H^{s=0,p=0}_{t=2,q=2}] & [H^{s=0,p=1}_{t=2,q=0}] & [H^{s=0,p=1}_{t=2,q=1}] & [H^{s=0,p=1}_{t=2,q=2}] \\
[H^{s=1,p=0}_{t=0,q=0}] & [H^{s=1,p=0}_{t=0,q=1}] & [H^{s=1,p=0}_{t=0,q=2}] & [H^{s=1,p=1}_{t=0,q=0}] & [H^{s=1,p=1}_{t=0,q=1}] & [H^{s=1,p=1}_{t=0,q=2}] \\
[H^{s=1,p=0}_{t=1,q=0}] & [H^{s=1,p=0}_{t=1,q=1}] & [H^{s=1,p=0}_{t=1,q=2}] & [H^{s=1,p=1}_{t=1,q=0}] & [H^{s=1,p=1}_{t=1,q=1}] & [H^{s=1,p=1}_{t=1,q=2}] \\
[H^{s=1,p=0}_{t=2,q=0}] & [H^{s=1,p=0}_{t=2,q=1}] & [H^{s=1,p=0}_{t=2,q=2}] & [H^{s=1,p=1}_{t=2,q=0}] & [H^{s=1,p=1}_{t=2,q=1}] & [H^{s=1,p=1}_{t=2,q=2}]
\end{bmatrix}
\begin{bmatrix}
\vec{I}^{sky}_{p=0,q=0} \\ \vec{I}^{sky}_{p=0,q=1} \\ \vec{I}^{sky}_{p=0,q=2} \\ \vec{I}^{sky}_{p=1,q=0} \\ \vec{I}^{sky}_{p=1,q=1} \\ \vec{I}^{sky}_{p=1,q=2}
\end{bmatrix}
=
\begin{bmatrix}
\vec{I}^{dirty}_{s=0,t=0} \\ \vec{I}^{dirty}_{s=0,t=1} \\ \vec{I}^{dirty}_{s=0,t=2} \\ \vec{I}^{dirty}_{s=1,t=0} \\ \vec{I}^{dirty}_{s=1,t=1} \\ \vec{I}^{dirty}_{s=1,t=2}
\end{bmatrix}
$$

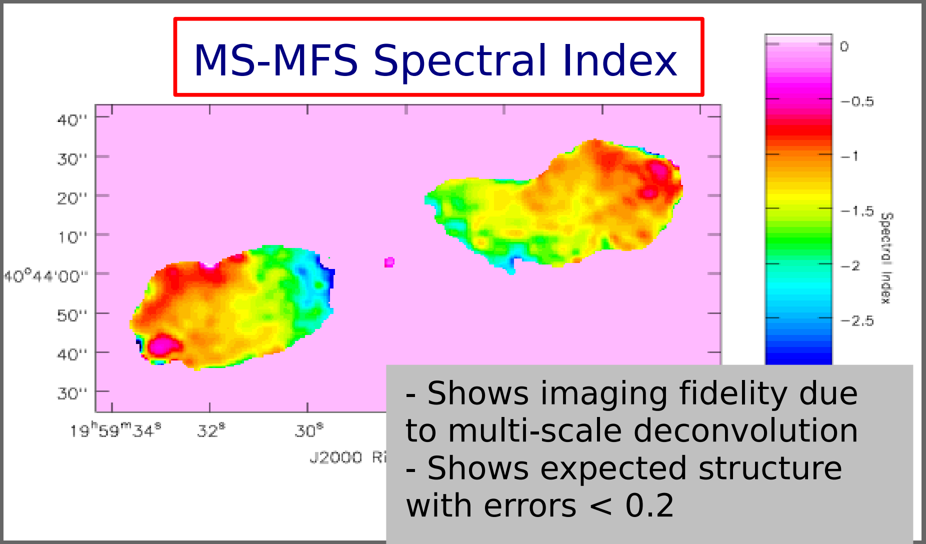
… various approximations to make the approximate Hessian-inverse tractable

(Rau, Phd Thesis, 2010, Rau & Cornwell, 2011)

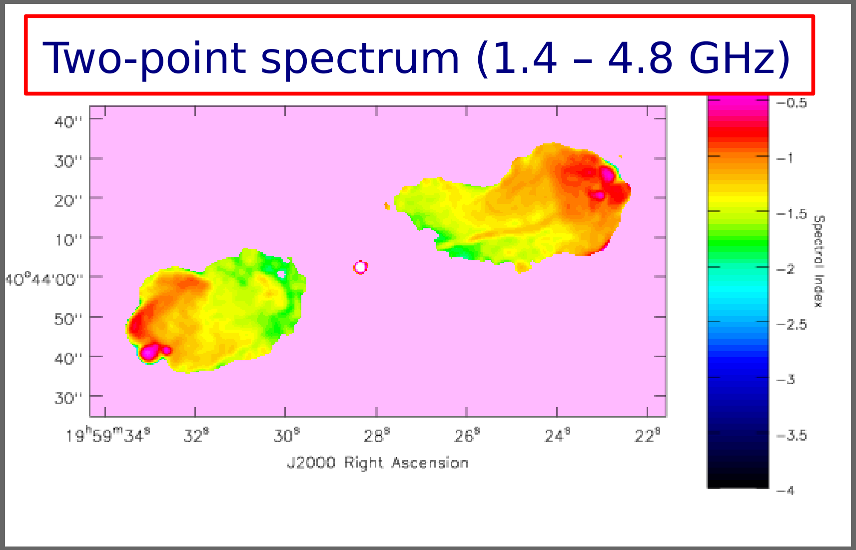
Example : Comparison of MS-MFS with multi-channel imaging

Data : 20 VLA snapshots at 9 frequencies across L-band + wide-band self-calibration

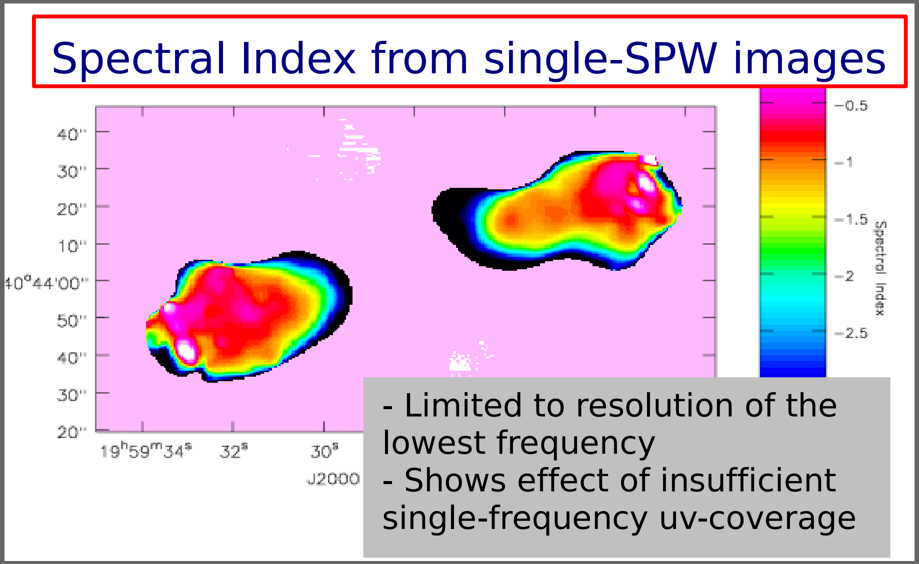




- Shows imaging fidelity due to multi-scale deconvolution
- Shows expected structure with errors < 0.2



C.Carilli et al, Ap.J. 1991.
(VLA A,B,C,D Array at L and C band)



- Limited to resolution of the lowest frequency
- Shows effect of insufficient single-frequency uv-coverage

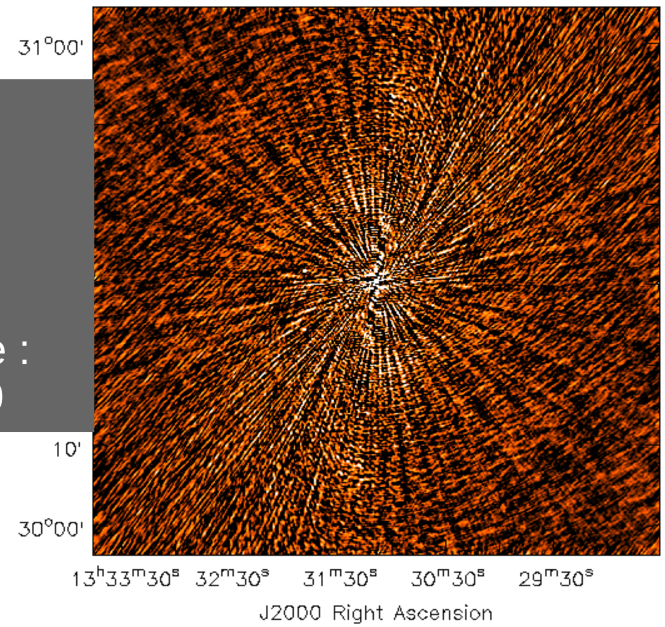
Example : Wide-band Dynamic Range (vs) Taylor-order

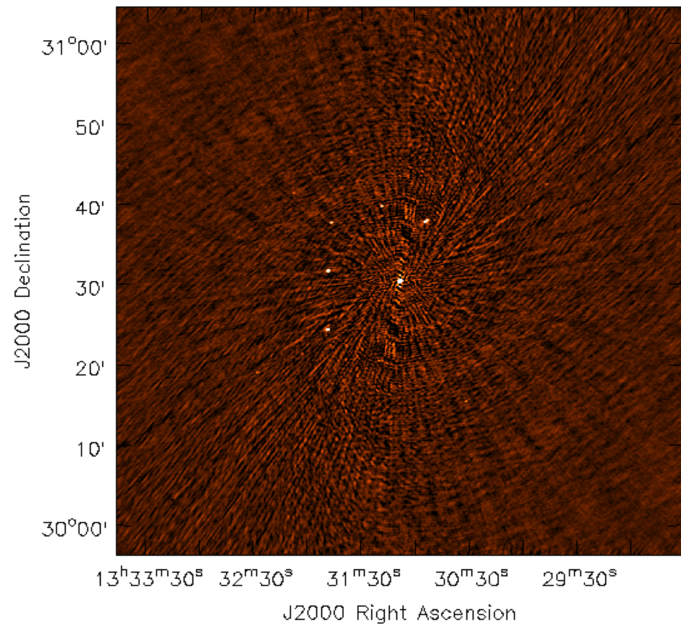
(I=14.4 Jy/bm, alpha = -0.47, BW=1.1GHz at Lband, 30 min)

NTERMS = 1

Rms :
9 mJy -- 1 mJy

Dynamic Range :
1600 -- 13000





NTERMS = 2

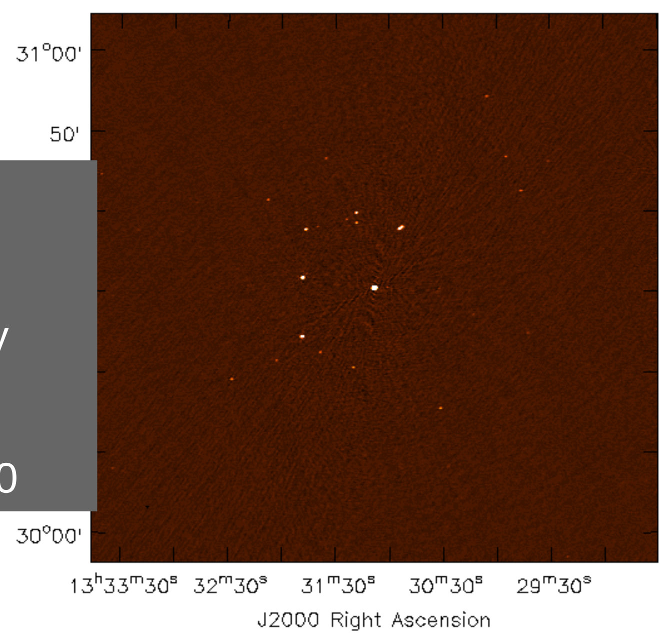
Rms :
1 mJy -- 0.2 mJy

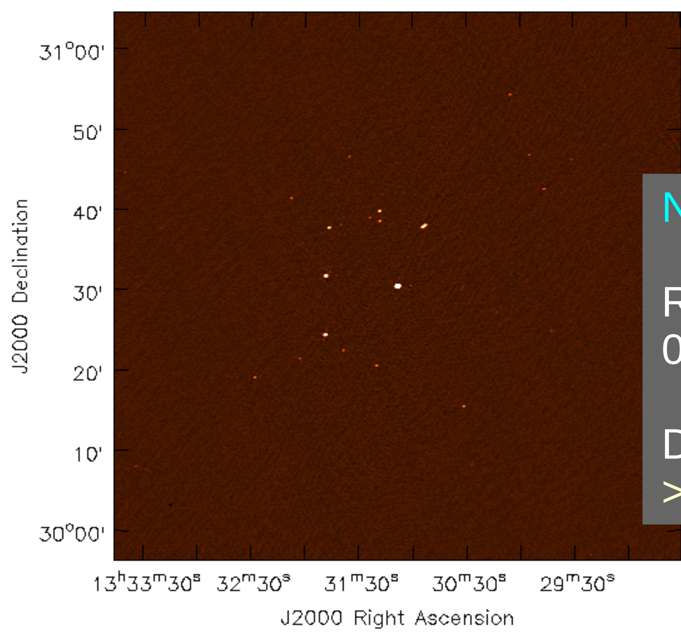
Dynamic Range :
10,000 -- 17,000

NTERMS = 3

Rms :
0.2 mJy -- 85 uJy

Dynamic Range :
65,000 -- 170,000



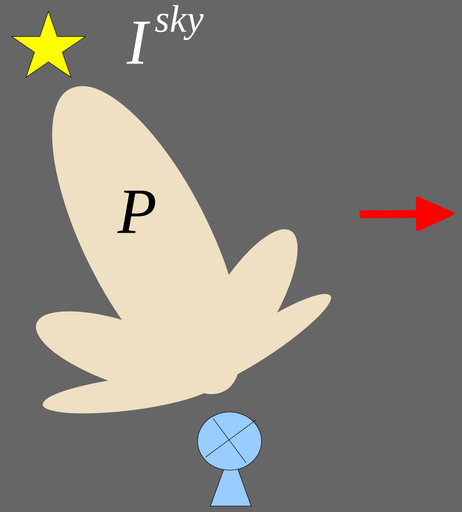


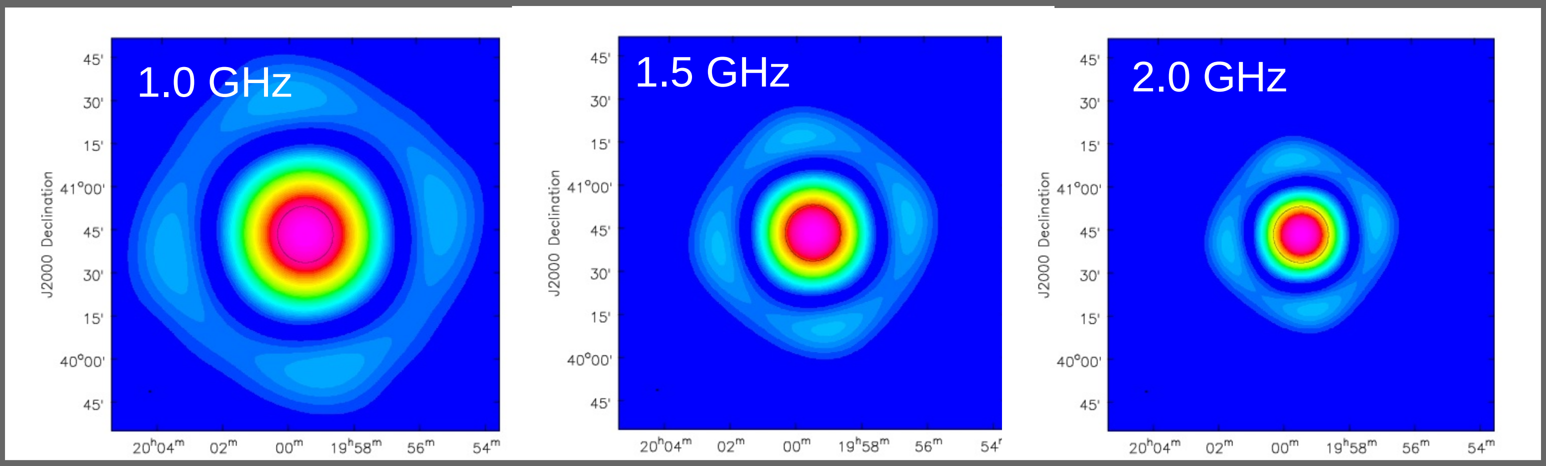
NTERMS = 4

Rms
0.14 mJy -- 80 uJy

Dynamic Range :
>110,000 -- 180,000

Imaging over a wide field-of-view (broad-band)





Frequency-dependent Primary-Beams

+

Multi-Frequency-Synthesis (MS-MFS)

=

Artificial spectral structure
(away from the center)

In practice, need to model and correct for P as a function of time, frequency, and antenna.

Measurement Equation :

$$V_\nu = [S_\nu][F](P_\nu \cdot I_\nu^{sky})$$

- V_ν: Data
- $[S_\nu]$: Spatial frequency coverage
- $[F]$: Fourier transform
- P_ν: Primary Beam (assume known)
- I_ν^{sky}: Sky Brightness

Wide-Band and Wide-Field Image Reconstruction

Multi-scale Wide-band model : 'Blobs' with amplitudes given by polynomials in ν

MS-MFS model : $I_\nu^{sky} = \sum_s \sum_t \left(\frac{\nu - \nu_0}{\nu_0} \right)^t \left[I_s^{shp} * I_{s,t}^m \right]$

MS-MFS
(Rau &Cornwell 2011, A&A)

Wide-field effects (per time/freq/antenna): Convolutions in the UV-domain

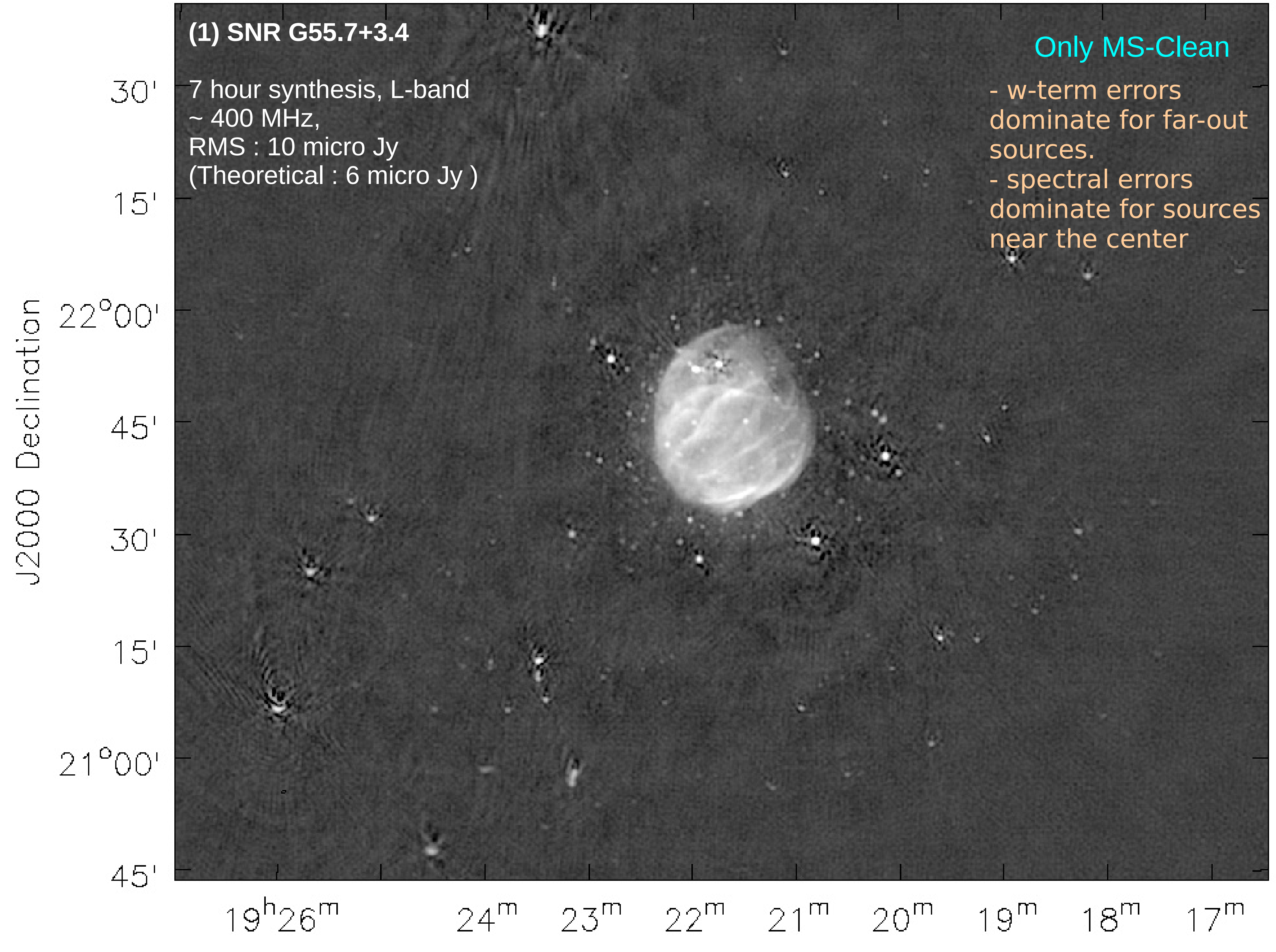
AWP-Projection model : $V_\nu = [S_\nu][G_\nu][F] I_\nu^{sky}$

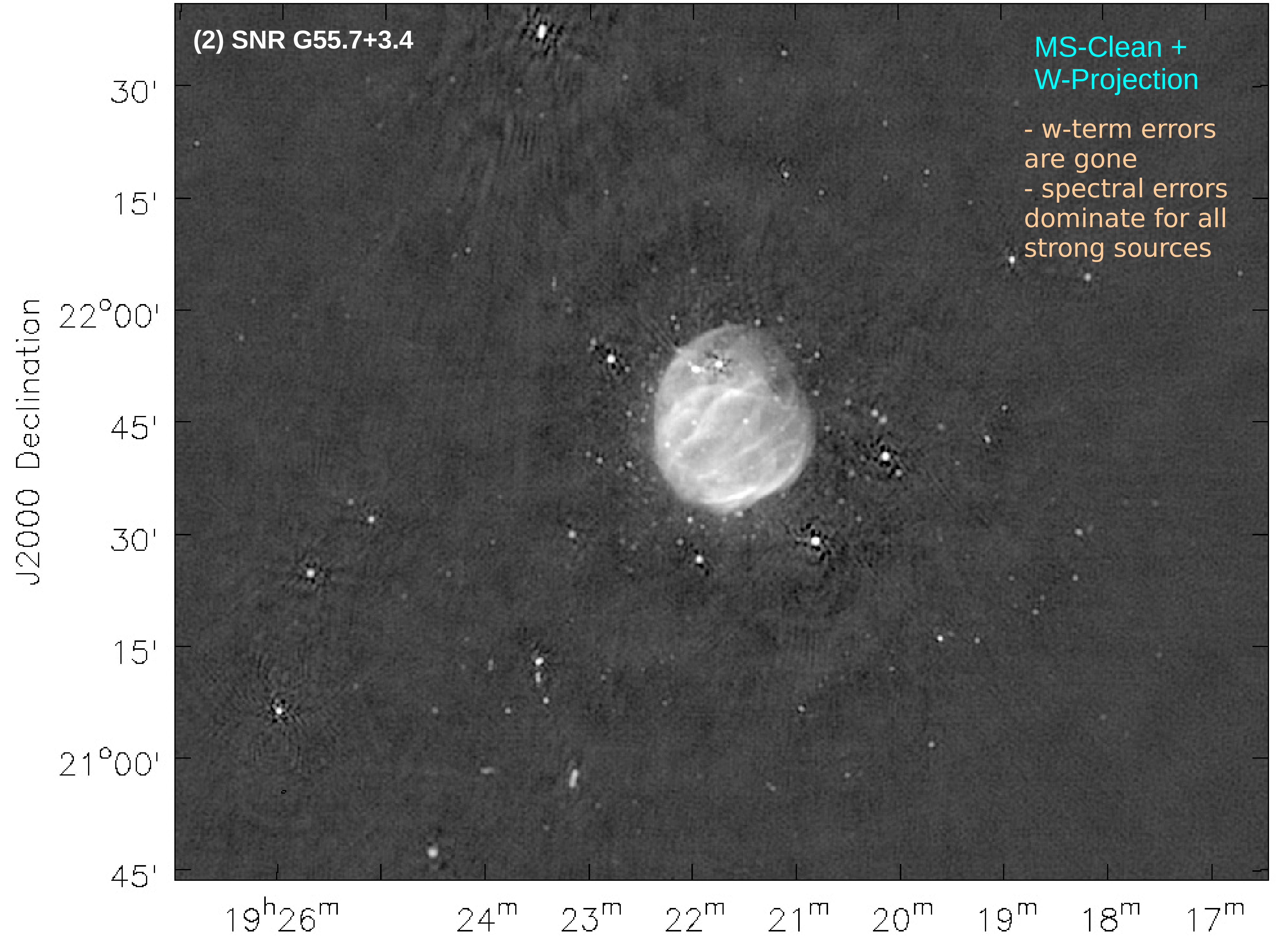
A/W-Projection
(Bhatnagar, Cornwell, Golap
2008,2009, A&A)

Image Reconstruction : Minimize L2 (iterate between approximate and exact steps)

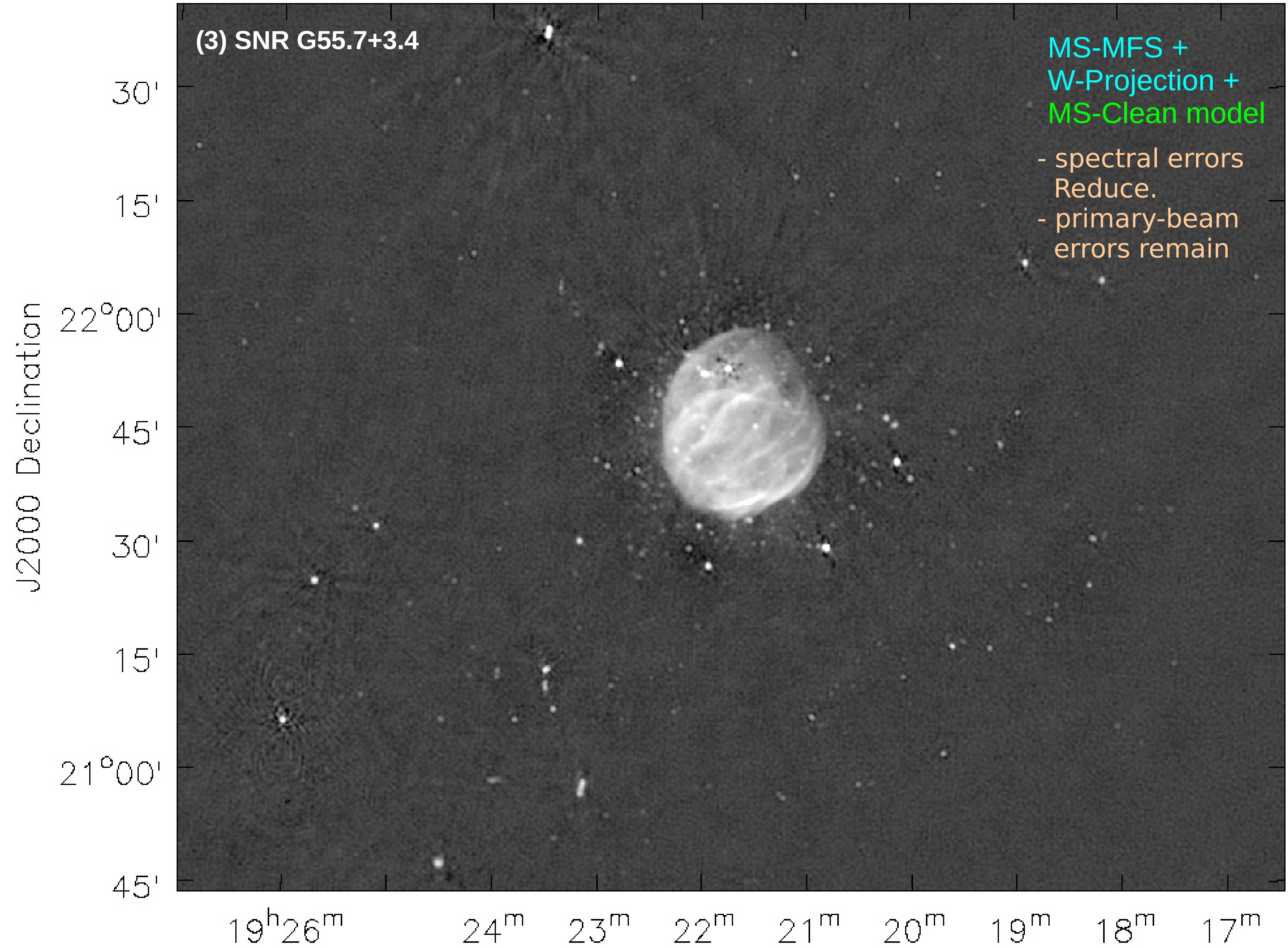
Cotton-Schwab algorithm : $I_{i+1}^m = I_i^m + [H^+][A^T W](V^{obs} - [A] I_i^m)$

(Linear-Algebra formulation that combines all of the above :
Rau, Bhatnagar, Cornwell, Voronkov, 2009, IEEE ; Rau, PhD Thesis, 2010)

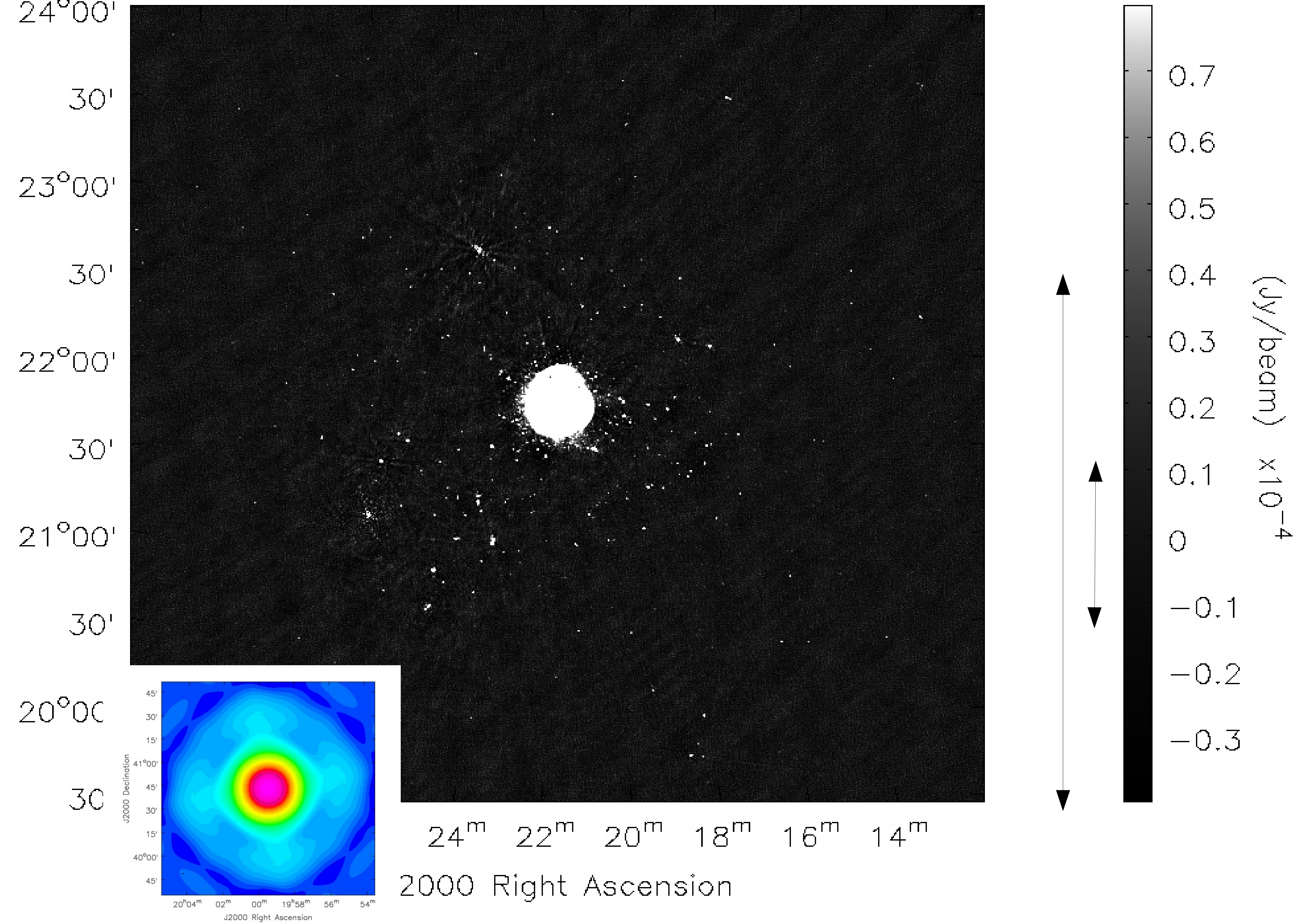

(1) SNR G55.7+3.4
7 hour synthesis, L-band
~ 400 MHz,
RMS : 10 micro Jy
(Theoretical : 6 micro Jy)
Only MS-Clean
- w-term errors dominate for far-out sources.
- spectral errors dominate for sources near the center
J2000 Declination
30'
15'
22°00'
45'
30'
15'
21°00'
45'
19h26m
24m
23m
22m
21m
20m
19m
18m
17m



(2) SNR G55.7+3.4
MS-Clean +
W-Projection
- w-term errors
are gone
- spectral errors
dominate for all
strong sources
J2000 Declination
30'
15'
22°00'
45'
30'
15'
21°00'
45'
19h26m
24m
23m
22m
21m
20m
19m
18m
17m


(3) SNR G55.7+3.4
MS-MFS +
W-Projection +
MS-Clean model
- spectral errors Reduce.
- primary-beam errors remain
J2000 Declination
30'
15'
22°00'
45'
30'
15'
21°00'
45'
19h26m
24m
23m
22m
21m
20m
19m
18m
17m

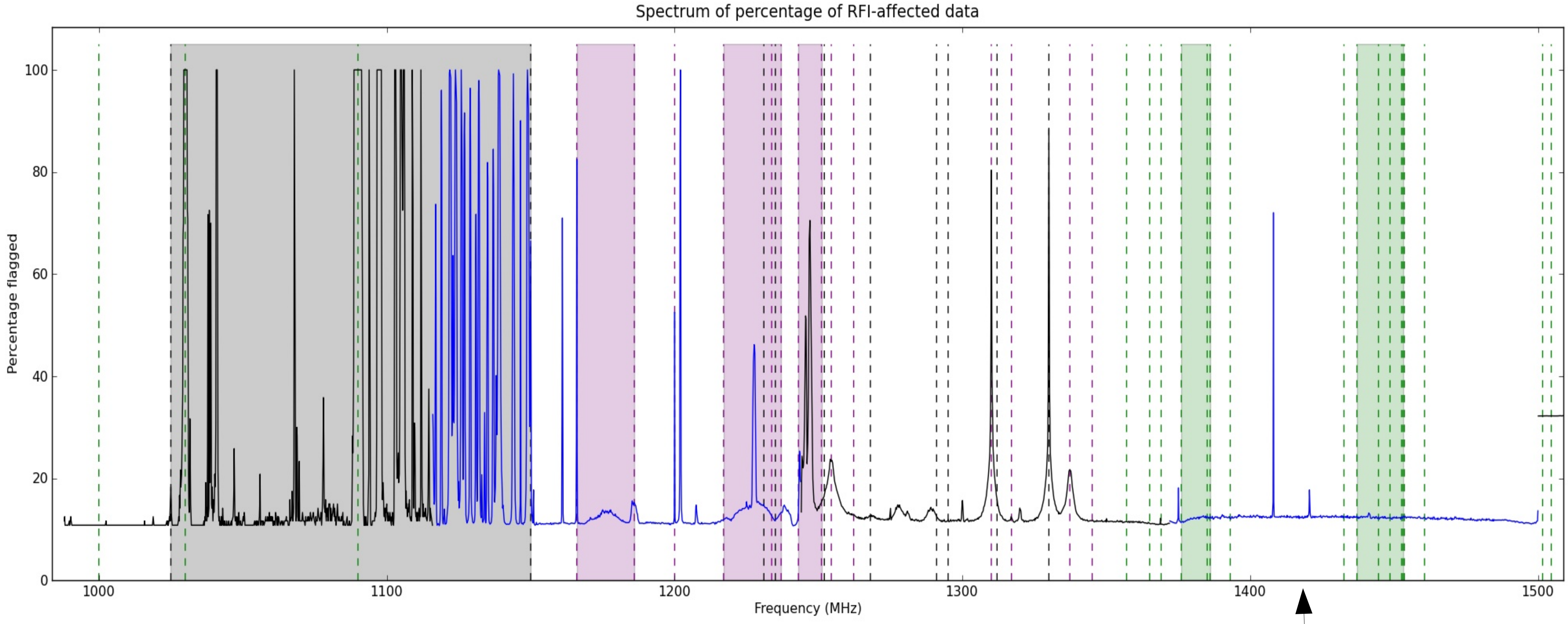
SNR G55.7+3.4 : 4 deg x 4 deg FOV : ONE pointing : EVLA L-Band (450 MHz bandwidth)



Radio Frequency Interference – Outlier detection and masking

1-2 GHz band : Usable = 500 MHz with rough flagging, 800 MHz if done carefully.

Tools for automatic 'flagging' exist in our software; people are beginning to use/trust them.



Plots of RFI at the EVLA between 1 GHz and 50 GHz :
http://www.aoc.nrao.edu/~mrupen/EVLA_RFI

Example summary-plot from CASA/TFCrop -
% of data flagged + known RFI (vs frequency)

Summary

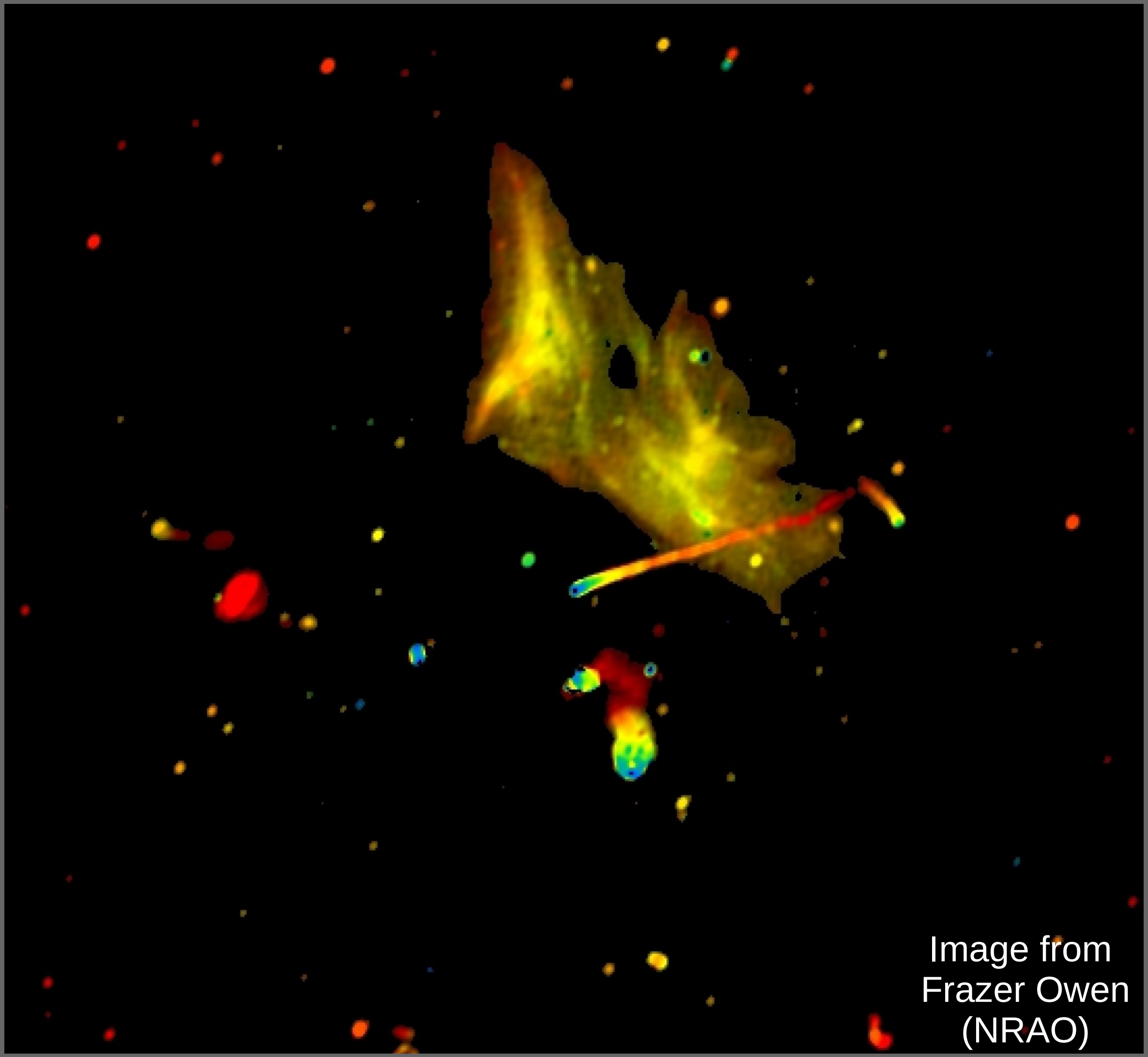
Broad-band receivers
=> better sensitivity

To achieve this sensitivity
=> Need spatial and spectral reconstructions along with corrections for wide-field instrumental effects.

We have a mathematical and software framework that describes and implements this (CASA : http://casa.nrao.edu)

The EVLA has been producing wide-band data from Fall 2010.

Astrophysical results are being obtained using these methods



Abell-2256 : intensity-weighted spectral-index

Ongoing work : HPC methods + more software integration + more efficient minor-cycle algorithms + uncertainty estimates, ... and much more.