Observing Gravitational Radiation with QSO Proper Motions and the SKA

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We discuss the ability of the SKA to observe QSO proper motions induced by long-wavelength gravitational radiation. We find that the SKA, configured for VLBI with multiple beams at high frequency (8 GHz), is sensitive to a dimensionless characteristic strain of roughly 10^{-13}, comparable to (and with very different errors than) other methods in the 1/yr frequency band such as pulsar timing.

1. Introduction

The history of astronomy has been that of our measurement of photons from astrophysical objects and, more recently, the early universe (the CMB). Analogously, the universe is thought to be suffused by gravitational radiation — a background from sources in the early universe (e.g., inflation) and "foregrounds" from the gravitational interaction of massive astrophysical objects, such as the massive black holes in the centers of galaxies. This gravitational radiation perturbs spacetime, altering path-lengths and lensing light rays, throughout the universe. The small amplitude of the perturbations (strains of $h \sim 10^{-15}$) makes these effects very difficult to observe. In this section, we examine the time-dependent "gravitational lensing" which results in apparent proper motions of distant galaxies, potentially observable by the SKA.

2. GW-induced Proper motions

The pattern of proper motions induced by a single gravity wave with "+" polarization, amplitude h, and frequency ω in the $\hat{z}$ direction is [1–3]

$$\mu(\omega, h, \hat{z}) = \frac{1}{2}\omega h \sin\omega t \sin\theta(\hat{\theta}\cos 2\phi - \hat{\phi}\sin 2\phi) . \tag{1}$$

The pattern from an arbitrary wave can be calculated by superposition of this pattern at different amplitudes and directions. Obviously, the particular pattern is not preserved by superposition, but the power spectrum of the pattern is [3]. We calculate the power spectrum by decomposing the pattern into **vector spherical harmonics**, $\mathbf{Y}^A_{\ell m}(\hat{x})$, $(A = E, B)$ the generalization of the usual scalar functions $Y_{\ell m}$ to vectors defined on the sphere. In fact, these are closely analogous to the tensor spherical harmonics used in the study of weak lensing and of the polarization of the CMB. Indeed the goal – to measure the power spectrum of the geometrical pattern – is much the same in both cases. We decompose the total pattern into

$$\begin{aligned} \mu(\omega) &= \sum_{\ell,m,A} a^A_{\ell m}\mathbf{Y}^A_{\ell m} \\ &= \omega h_c(\omega) \sum_{\ell,m,A} \hat{a}^A_{\ell m}\mathbf{Y}^A_{\ell m} \end{aligned} \tag{2}$$

where the coefficients $a^A_{\ell m}$ are functions of frequency and the $\hat{a}_{\ell m}$ separates out the spatial and frequency dependence. The quantity $h_c(\omega)$ is the characteristic strain at a given frequency [4]. We can finally define the power spectra

$$\begin{aligned} M^A_\ell(\omega) = \langle |a^A_{\ell m}|^2\rangle &= \omega^2 h_c^2(\omega)\langle|\hat{a}^A_{\ell m}|^2\rangle \\ &= \omega^2 h_c^2(\omega)\hat{M}^A_\ell \end{aligned} \tag{3}$$

However, we do not directly observe the proper motion, μ, itself, which is an angular velocity.

Rather, we measure the position of objects at two (or more) epochs separated by time δt – the integral of the angular velocity. The power spectrum of this position difference is then an integral over the μ power spectrum. In analogy to Eq. 2, we define $\theta_{\ell m}(\delta t)$ as the transform of the position difference pattern, with power spectrum

$$\langle\left|\theta_{\ell m}(\delta t)^A\right|^2\rangle = 2\int \frac{d\omega}{\omega^2}\, M_\ell^A(\omega)\left[1-\cos(\omega\delta t)\right] . \quad (4)$$

In fact, Gwinn et al [3] show that a full 5/6 of the power in gravitational radiation is at $\ell = 2$ (the quadrupole); in practice there is little need to measure the entire spectrum. However, the *time*-frequency (ω) spectrum depends on the frequency content of the gravitational radiation background:

$$M_\ell^A(\omega) = \omega h_c^2(\omega)\left|\hat{a}_{\ell m}\right|^2 \quad (5)$$

and

$$M_2^E + M_2^B = \frac{2\pi}{3} H_0^2 \Omega_{\rm GW}(\omega) \quad (6)$$

where $\Omega_{\rm GW}(\omega)$ gives the contribution to the cosmological critical density of the GW background per logarithmic integral of frequency.

3. Proper motion error analysis

¿From the study of the CMB, we recall that the error at a particular multipole is given by [5,6]

$$(\delta M_\ell)^2 \simeq \frac{2}{(2\ell+1)f_{\rm sky}}\left(M_\ell + w^{-1}\right)^2 \quad (7)$$

where w gives the weight (inverse variance) per solid angle and we have assumed observations uniformly spread around a fraction $f_{\rm sky}$ of the sky. This formula is just the usual one for the variance of the square of a quantity with known variance: $(2\ell+1)f_{\rm sky}$ is the number of modes at a given ℓ. The M_ℓ term then gives the sample variance, and the w^{-1} term the noise variance (i.e., resulting from interferometric phase errors).

It remains, then, only to insert the expected signal for M_ℓ and the noise characteristics of the SKA. In the band of frequencies probed by the SKA, the dominant contribution is likely to be massive black holes (MBHs) in close binaries at the centers of galaxies:

$$\begin{aligned} h_c(\omega)^2 \quad &\sim \quad \left(10^{-15}\right)^2 \\ &\times \left(\frac{\omega}{\rm yr^{-1}}\right)^{-4/3} \langle\left(\frac{\mathcal{M}}{10^7 M_\odot}\right)^{5/3}\rangle I_{h^2} \, . \end{aligned} \quad (8)$$

where $\mathcal{M}^{5/3} = M_1 M_2 (M_1+M_2)^{-1/3}$ gives the so-called “chirp mass” of the system, angle brackets denote the average over the massive black hole mass function, and the dimensionless factor I_{h^2} gives an integral over the merger rate of black holes [4].

Beneath this foreground potentially lies the cosmological background from an epoch of inflation [7]:

$$\begin{aligned} h_c^2(\omega) \quad &\sim \quad \left(10^{-17}\right)^2 \\ &\times \left(\frac{H_*}{10^{-4} M_{\rm Pl}}\right)^2 \left(\frac{\omega}{\rm yr^{-1}}\right)^{n_T-2} 10^{9n_T} \end{aligned} \quad (9)$$

where H_* is the energy scale of inflation, n_T is the Tensor spectral index from inflation (usually, $n_T \simeq 0$ and $n_T < 0$), and we have ignored the so-called running of the index; this formula gives a generous upper bound and thus any realistic background is well below any astrophysical foregrounds such as Eq. 8.

¿From Eqs. 3–4, we see that we need integrals of the form

$$\int d\omega h_c^2(\omega)\left[1-\cos(\omega\delta t)\right] \, . \quad (10)$$

For these power-law spectra, we of course just pick out the amplitude at $\omega \sim 1/\delta t$.

4. Discussion

For the proposed measurements with the SKA, we will likely only be placing limits on the gravitational radiation background and so the noise term will be dominant. The weight per solid angle is given by $w = N\sigma^{-2}/(4\pi f_{\rm sky})$ for N proper motion observations with error σ^2 on each observation. From Ref. [8], we can expect an rms position accuracy of $10\mu{\rm as} = 5\times10^{-11}$rad with the SKA used in VLBI mode at (say) 8 GHz. Without "multibeams," the SKA will be able to measure the positions of roughly 10^4 point-like QSOs in about one month; with 100 multi-beams this increases to the roughly 10^6 objects available on the sky [9]. This gives a total weight $w^{-1} = (10^{-13})^2$ for the multibeam configuration and $f_{\rm sky} = 1/2$. This indicates that our sensitivity to gravitational radiation will be at a level of roughly $h \sim 10^{-13}$, competitive with other methods such as pulsar timing [4]. In more detail, we see that this noise contribution will likely still be dominant for $\omega \sim \mathrm{yr}^{-1}$ for the signals in Eqs. 8–9. Without multi-beaming, we of course lose another factor of 10 in sensitivity to h, and the method is less competitive. However, this calculation assumes that the position errors are uncorrelated; in practice phase errors from the troposphere will correlate position errors, and this can mimic the low angular frequency pattern of the gravitational waves. We have also ignored the loss of some low-frequency information to the "geoedetic" information required for VLBI, as discussed in Ref. [3]. Nonetheless, these are very different systematic problems than those to which pulsar timing will be sensitive.

We thus propose to use the SKA configured for VLBI in multibeam mode at 8 GHz, observing a preselected population of 10^6 pointlike AGN in two epochs, separated by at least one year (more epochs and a longer time difference both increase the sensitivity).

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