Simulation Tests of Sub-Sample Delay Tracking in the Proposed WIDAR Correlator for the Expanded Very Large Array

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Brent Carlson, October 3, 2000

ABSTRACT

The proposed WIDAR (Wideband Interferometric Digital ARchitecture) correlator for the Expanded Very Large Array (EVLA) uses digital filters to split the wide-band into narrower sub-bands. These sub-bands are individually correlated and, using anti-aliasing techniques, seamlessly "stitched" together to yield the wide-band cross-power spectrum. This technique is employed so that baseline-based digital hardware in the correlator can operate efficiently without the need for time de-multiplexing techniques. A positive side-effect of this sub-banding is that it is possible to perform fully digital sub-sample delay tracking—eliminating the need to use other, less precise methods such as sample clock phase modification. Until now, the digital sub-sample delay tracking method has been explained theoretically, but has never been tested. This short memo presents a number of simulation results to demonstrate that this method does indeed perform as expected.

Background

In the WIDAR correlator [1][2] the wide-band signal is divided into narrower sub-bands with digital filters. The sub-bands are then correlated and the results are seamlessly stitched together to yield the wide-band cross-power spectrum. Digital delay tracking that occurs at the original wide-band sample rate contains a delay error that imposes a phase slope that varies (as the delay error changes) between +/- 90^{o} at the highest frequency and zero at DC. If this error is not corrected, it will seriously degrade the correlated signal. Since the WIDAR correlator correlates individual sub-bands, and since the correlator contains phase rotators, the phase error can be tracked as it changes with time. The residual phase error is reduced to the phase error within a sub-band, a fraction of the phase error across the wide band. This concept is shown in Figure 1.

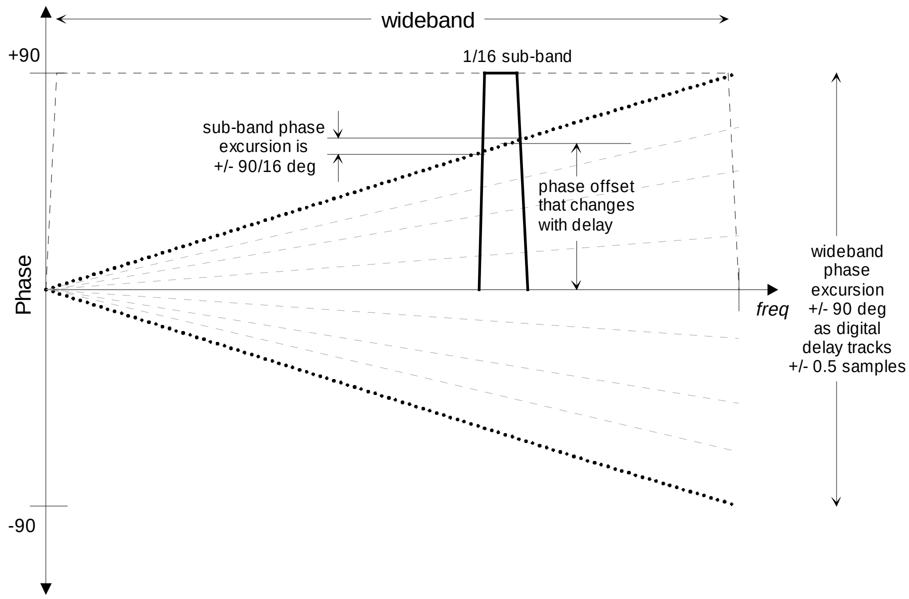


Figure 1 Diagram that shows how the digital delay error imposes a residual phase slope across the wide band that changes as the delay error changes. Since the sub-band is (in this example and in the proposed correlator for the EVLA) 1/16th the wide band, the phase excursion is 1/16th of the total phase excursion across the wideband if the phase offset is tracked as delay changes.

In the actual EVLA implementation, the wide-band signal will be "quasi-baseband" from 2 to 4 GHz, and sampled at 4 Gs/s. Thus, the lower edge of the sampled band[1] will see a +/-90^o phase excursion and the upper edge of the band will see a +/-180^o phase excursion. However, the phase slope will be no different than if the wide-band signal was at baseband (0 to 2 GHz) and so the following results and analysis are still applicable.

Simulation Methods

The *noise* simulator used in [1] was modified to include a station-based fractional sample delay (**FSD**) FIR filter after the frequency shift is imposed. This is the same filter [3], used in the same part of the signal path as in the simulator used to test the correlator described in [4] before it was constructed. The code was modified so that only the station-based FSD error was imposed. That is, station-based *integer* delay errors were simply discarded. Station-based integer delay error insertion and removal are well understood precise digital processes that have no effect on the FSD tracking described here.

In the *correlator* simulator, the WIDAR sub-band phase-offset tracker did not concern itself with how many bits were used to represent phase. Thus, for each sample, total phase (i.e. frequency offset phase + delay error tracking phase) was calculated with floating point precision, and then truncated to 4 bits before being used for fringe stopping. In the actual implementation of the correlator [1], these individual phases will be some large number of bits (probably 8) that will be added together before being truncated to 4 bits for use in the correlator chip. It is asserted that the difference between

[1] i.e. after being corrected for sideband flipping so as to maintain consistent frequency sense.

the simulation and the actual correlator implementation is not a significant source of error (i.e. because of the large number of bits that can be used in the actual correlator).

Simulation Results

Initially, a fixed FSD error was imposed in the X and Y station data such that the maximum *baseline* delay error would be present. This result is shown in Figure 2 for one sample of baseline delay error resulting in a phase excursion across the band of 180^o. Note that in this case there is no coherence loss because the delay error is fixed and not changing with time. This test helps to quantify the performance of the FSD filter and ensure that the correct phase-slope is indeed introduced. The band-edge roll-off and anomalous phase behaviour at the upper edge of the band is from the FSD filter [3].

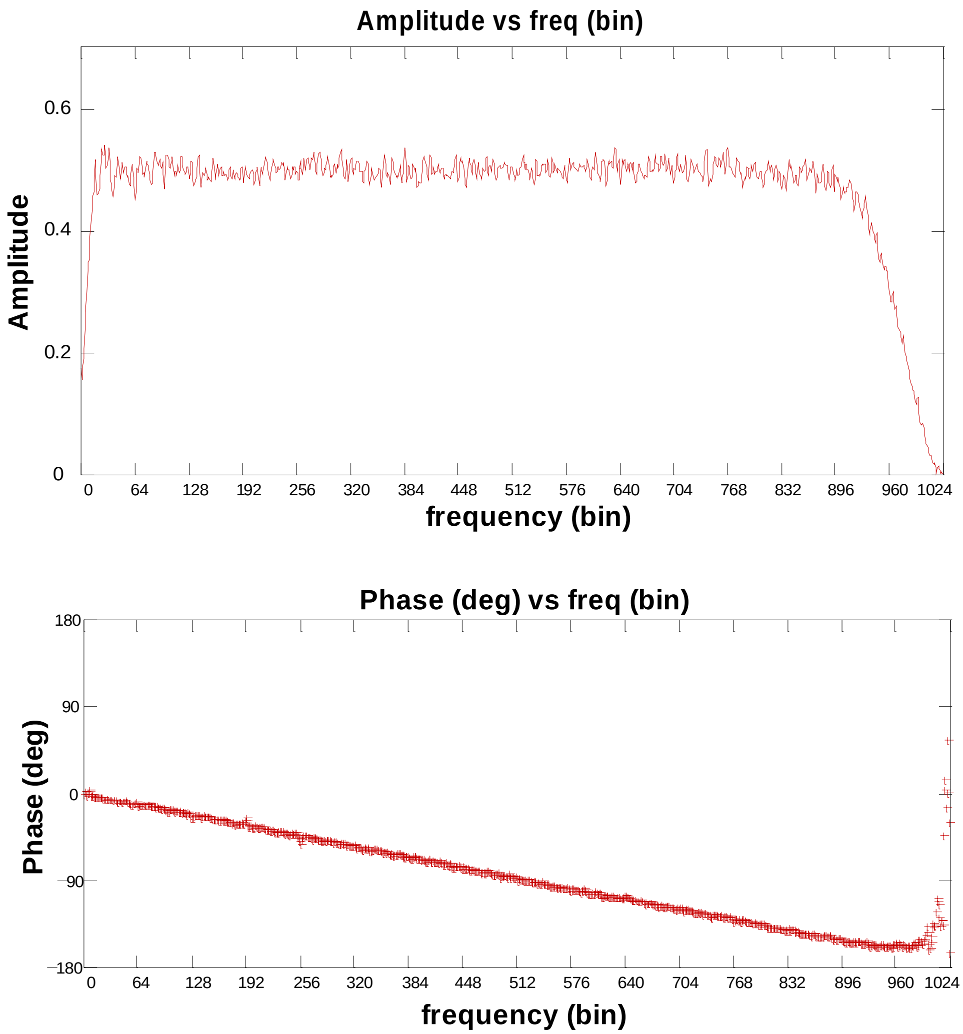


Figure 2 Amplitude and phase versus frequency of a 16 sub-band WIDAR correlation where there is one sample of *baseline* delay error. The phase slope versus frequency clearly shows the delay error that translates to a phase error of -180^o at the highest frequency. The roll-off at the edges of the band is due to non-ideal behaviour of the fractional sample delay filter (FSD) used to introduce the delay. The amplitude scale is linear, the expected correlation is 0.5, and 4×10^6 samples were correlated.

In Figure 3, the same fixed delay error is present, but each sub-band has introduced a compensating phase offset such that the phase offset at the center of the sub-band is zero. In the figure, the phase scale is somewhat magnified compared to Figure 2 and so the phase looks noisier. Clearly, the phase excursion in each sub-band is now only $180^o/16$, or 11.25^o as expected.

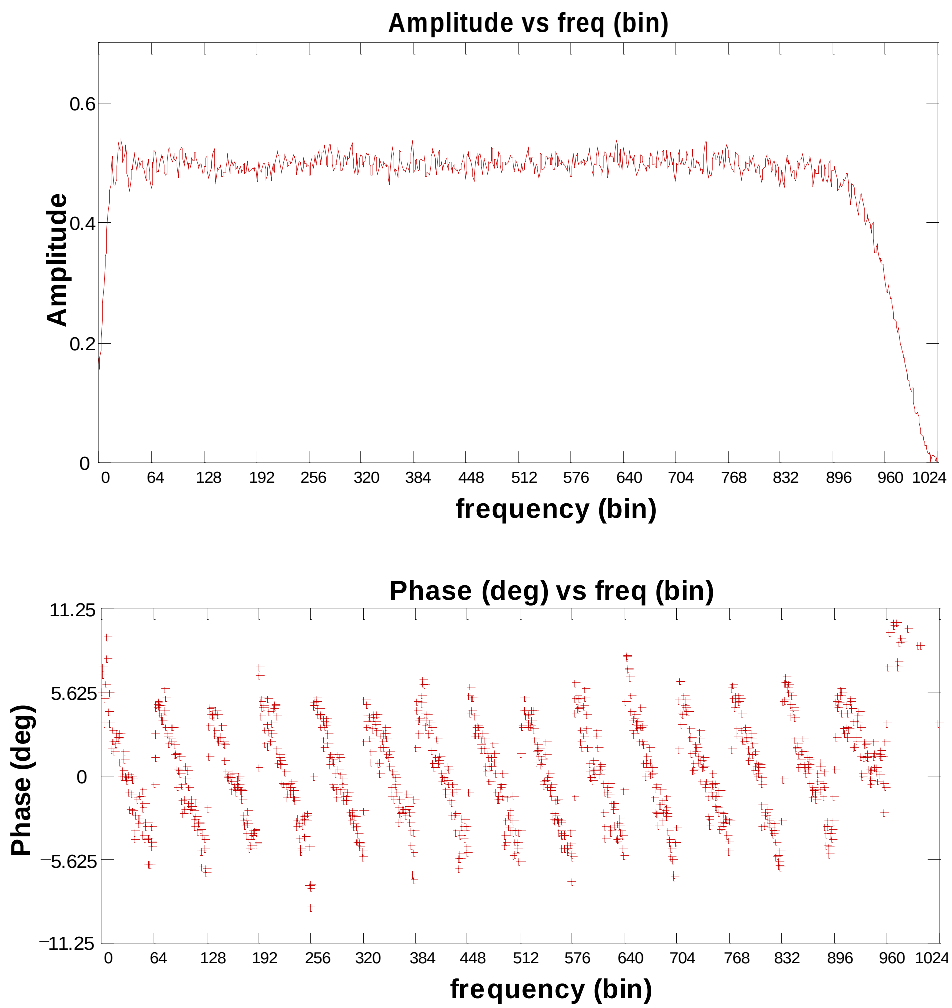


Figure 3 The WIDAR correlator has introduced a phase offset in each sub-band such that the phase error at the center of each sub-band is zero. The phase excursion is thus only $180^o/16$, or 11.25^o in each sub-band.

In Figure 4, the WIDAR phase offset is introduced but with the baseline delay error opposite in sign to that of Figure 3. Figure 3 and 4 indicate the phase error extremes that each sub-band will see. During geometrical delay tracking, the baseline delay error will smoothly oscillate between the two extremes, thus tracing out a uniform phase error path that is +/-5.625^o (or +/- $1/32^{nd}$ of a sample) at the sub-band edges[2]. This error introduces

[2] It is important to note that the apparent gain in baseline delay tracking precision is indeed real: the gain results from phase tracking set to the *center* of the sub-band rather than the edge of the sub-band.

a maximum coherence loss at the edge of the band [5] of 0.6 %. The cross-power amplitude and phase versus frequency when tracking (simulated) geometrical delay for the WIDAR correlator is shown in Figure 5. Within the integration, the baseline delay changed by 13.6 samples so that there were many sweeps of phase slope through the two extremes of +/-180^o across the wide band.

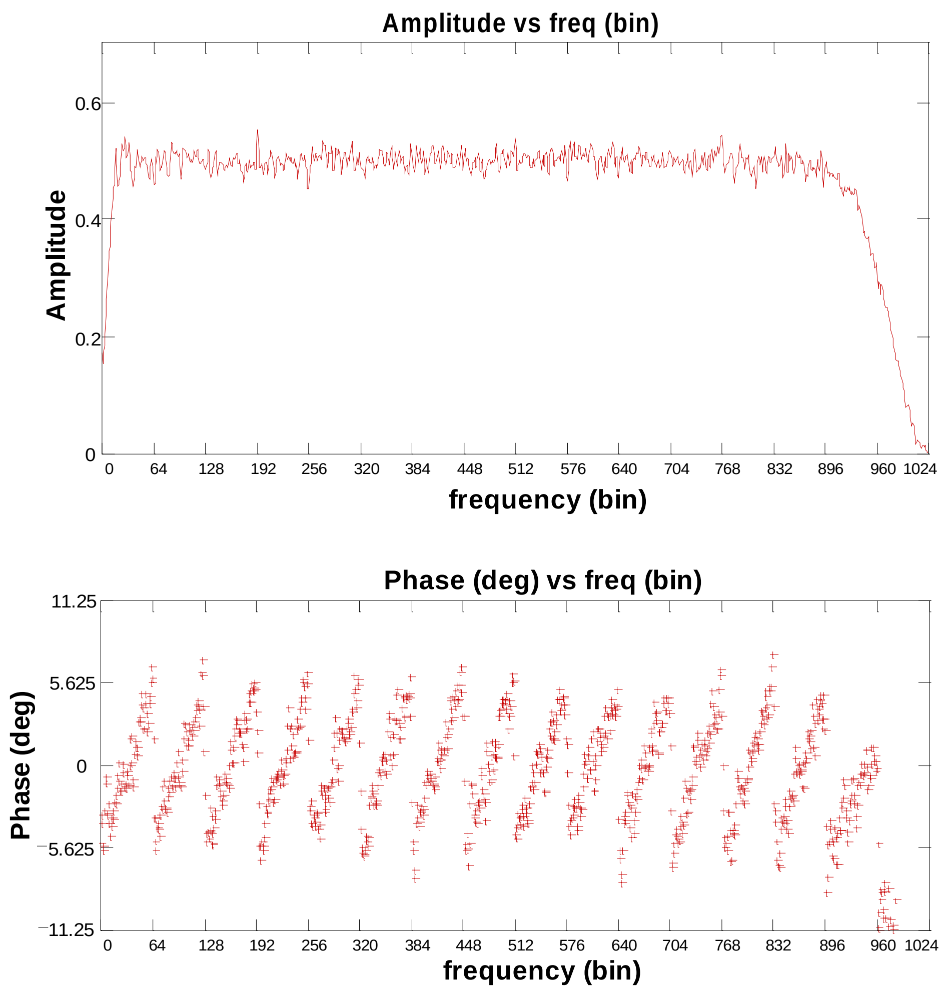


Figure 4 One sample of fixed baseline delay error with WIDAR phase-offset compensation. This is the same as Figure 3 but with an opposite delay error (phase slope).

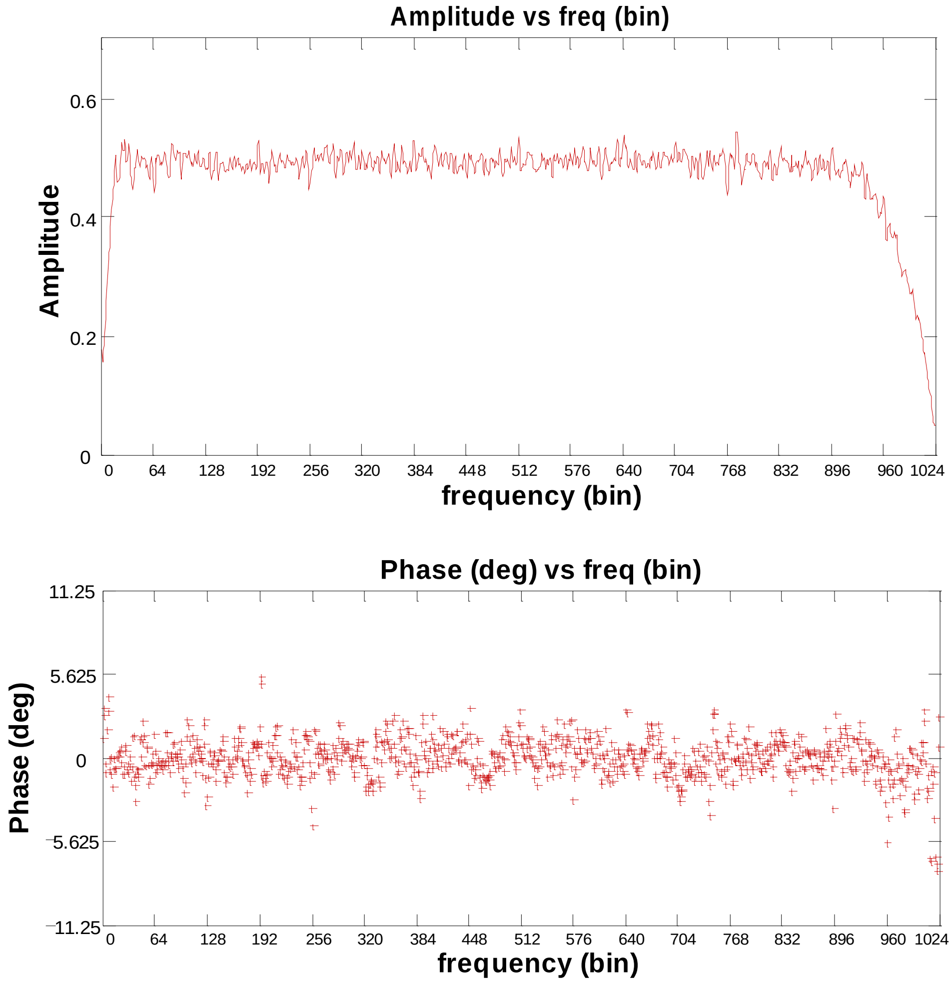
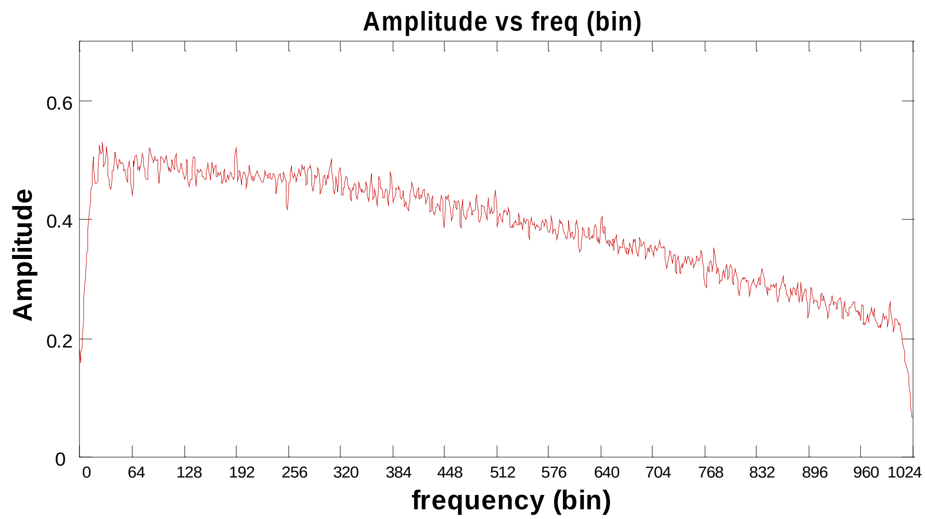


Figure 5 Amplitude and phase versus frequency where there is continuous (simulated) geometrical delay and delay tracking with a WIDAR correlator. In this example, there are 13.6 samples of baseline delay change within the integration time.

In all of the previous examples, Hanning windowing using "Windowing Method 1" [1] was employed. There does not appear to be any anomalous behaviour at the sub-band boundaries in the stitched together wideband spectrum.

Finally, for comparison purposes and to ensure that a continuous delay function is used, the WIDAR sub-band phase tracking is *turned off* with the same geometric delay model of Figure 5. This result is shown in Figure 6. Note the residual phase slope and the severe coherence loss as a function of frequency.



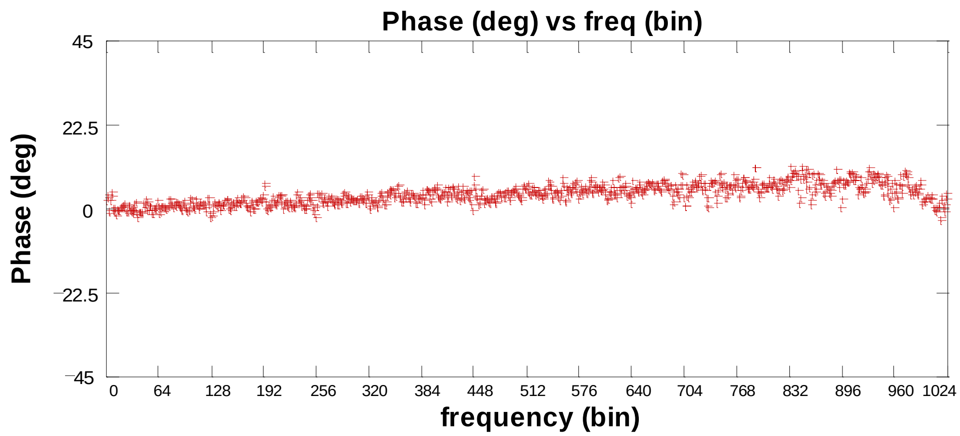


Figure 6 Continuous (simulated) geometric delay with the WIDAR sub-band delay tracking *turned off*. The increasing coherence loss with frequency and residual phase slope is obvious.

Conclusions

Simulation results in this memo have demonstrated that the WIDAR correlator sub-sample delay tracking operates as expected. With 16 sub-bands, it is possible to achieve +/-1/32nd of a sample of *baseline* delay tracking resulting in a negligible coherence loss of 0.6%.

References

[1] Carlson, B., A Proposed WIDAR Correlator for the Expansion Very Large Array Project: Discussion of Capabilities, Implementation, and Signal Processing, NRC-EVLA Memo# 001, May 18, 2000.

[2] Carlson, B.R., Dewdney, P.E., Efficient wideband digital correlation, *Electronics Letters*, IEE, Vol. 36 No. 11, p987, 25th May, 2000.

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