EVLA Memo #93

Optimization of the LWA antenna station configuration minimizing side lobes.

Leonid Kogan

National Radio Astronomy Observatory, Socorro, New Mexico, USA

Aaron Cohen

Naval Research Laboratory, Washington, DC USA

May 4, 2005

Abstract

This memo is a duplicate of LWA memo #21 and is posted here as well because of its relevance to the optimization of the EVLA phase II arrays. The algorithm for optimization of an array configuration to minimize side lobes, designed by Leonid Kogan ([1]), has been applied to optimize the configuration of a dipole station of the Long Wavelength Array (LWA). The results of optimization are given for different areas of optimization on the sky including full sky semi-sphere; for different minimum spacing between the station antennas. For an array phased to zenith, the optimization is done to minimize sidelobes all the way to the horizons by optimizing in a circle defined by the radius $|\sin(z)| \leq 1$, where z is the angle from zenith. *For an array that will potentially be phased to any location above the horizon, the optimization radius should be twice as large to optimize the beam pattern over the entire sky.* Thus for the whole semisphere optimization with a range of array pointings covering the whole semisphere as well, optimization at zenith pointing must be done for $|\sin(z)| \leq 2$!

1 Introduction

The Long Wavelength Array (LWA) elements (antenna stations) will consist of 256 identical dipoles working in the wide range of the frequencies from approximately 20 MHz to 80 MHz. We'll consider that all 256 dipoles are located in a plane. We use the algorithm of optimization of an array configuration minimizing side lobes designed by Leonid Kogan ([1]). The algorithm minimizes the biggest side lobe in the given area of optimization on the sky and as a result makes all side lobes identical. The algorithm is coded into the AIPS task CONFI. The most important parameters of CONFI are the area optimization on the sky, maximum size of the array and minimum spacing between the antennas.

The beam pattern of the antenna station phased at the direction of unit vector $\vec{e_0}$ is described by the following equation:

$$B(\vec{e}) = \left| \frac{1}{N} \sum_{k=1}^{N} \exp(-j2\pi \frac{\vec{r}_k}{\lambda} \cdot (\vec{e} - \vec{e_0})) \right|^2 \qquad (1)$$

where $\vec{e}$ is the unit vector directed to the point at the sky;
$\vec{e_0}$ is the unit vector where the antenna station phased;
$\vec{r_k}$ is the position of antenna k;
λ is the wavelength;
N is number of antennas in the array;
$\cdot$ in the exponent stands for the scalar product of the vectors.

Let's choose the following natural coordinate system for vectors $\vec{r_k}$ and $\vec{e}$, $\vec{e_0}$:
$\vec{u}$, $\vec{v}$ are located at the station plane and are perpendicular to each other;
$\vec{w}$ is perpendicular to the station plane (Figure (1)).

If all antennas are located in the $u - v$ plane then the w coordinate of the antenna position is zero for all antennas. If the array is phased to the perpendicular direction (relatively the array plane), then we have $\vec{e_0} = \{0, 0, 1\}$, and therefore the scalar product of the vector $\vec{e_0}$ with any antenna position vector is equal to 0 and can be removed from Equation (1)

Vector $\vec{e}$ at the chosen coordinate system is determined as:
$\vec{e} = \{\sin(z) * \cos(az),\ \sin(z) * \sin(az),\ \cos(z)\}$

where z is the angle between $\vec{e}$ and zenith to the station plane
az is azimuth of $\vec{e}$ at the station plane

The w component of $\vec{e}$ ($\cos(z)$) does not contribute to the scalar product in the case of the plane array (our case) because all antenna position vectors have zero w components.

Typically another coordinate system is used for the sky position description: the w-axis is directed to the center of the considered sky area. In our case the w-axis can be directed towards the vector $\vec{e_0}$. Such a coordinate system is useful for a small field of view (high frequency VLA) but has some disadvantages for such a wide field of view array as an LWA station, which is virtually the entire sky. The chosen coordinate system has the following advantages:
1. It needs only one parameter 'z' to describe a big physically motivated area on the sky. For example the full semisphere is described as $|z| \leq 90°, |sin(z)| \leq 1$.
2.The chosen coordinate system allows one to forget about the w component of the vector $\vec{e}$ for any direction of pointing ($\vec{e_0}$).
3. The area of optimization carried out for phasing to zenith ($\vec{e_0} = \{0,\ 0,\ 1\}$) can simply be shifted linearly to the desired pointing if we consider $\sin(z)$ instead of z at the plane reproduction of the sky. This fact will be demonstrated in a following section of this memo.

2 Some results for the station phased to zenith

We started optimization with an initial configuration of a hexagon configuration (Figure 2). If we set the antenna station size as a circle with diameter 100 meters then the spacing is $\sim 5.5m$. We carried out the optimization with minimum allowing spacing 2m, and at the highest LWA frequency of 80 MHz ($\lambda = 3.75m$), considering that the effective circle of optimization will be even greater at longer wavelengths. (Figure 3 shows an example of the station configuration after optimization.) We carried out two sets of optimization:
1. radius optimization $60°, |sin(z)| = 0.87$.
2. radius optimization $90°, |sin(z)| = 1.00$. The full semisphere.

In the first case the maximum side lobe level in the optimization region is only 0.0025 (26dB). In the second case the maximum side lobe level is 0.007 (21.5dB). The question is what is better: 21.5dB for the whole sky semisphere or 26dB but only optimized within 60° from zenith? Figure 4 shows the flat behavior of the side lobes inside of the optimization area, as expected within this optimized region.

We carried out the optimization starting with an expanded hexagon configuration where the density of antennas decreases towards the edge. The array size was selected as circle of 110m diameter with spacing at the center 3m allowing the minimum spacing at the optimization process 2m. The optimization was carried out for the whole semisphere $|z| < 90°, |sin(z)| < 1$ The resulting beam pattern is shown in Figure 4.

3 The station phased for an arbitrary direction

The configuration found at the optimization at zenith (see Figures 3, 4) was used to calculate the beam pattern phased towards $|z0| = 60°, |az0| = 45°$ as an example. The calculation of the beam pattern has been done using Equation (1). Equation (1) describes the move from zenith to the given direction of phasing as a linear shift if we use sin(z) instead of z. Figure 5 confirms this fact. Looking at Figure 5, we can see that having optimized the side lobes inside the whole semisphere (for the array phased to zenith) we have a big non optimized area at the low-right area when the array is phased towards the up-left corner of the semisphere. Therefore *we need to make the optimization radius twice as large when optimizing the beam pattern of an array phased to zenith if we want to have low side lobes everywhere for the array phased up to edge of the field of view.* In particular for the whole semisphere we have to carry out optimization at the following range of $\sin(z)$: $|\sin(z)| < 2$

4 Conclusion

The ability to optimize an LWA dipole station configuration minimizing side lobes throughout the entire sky field for any pointing in the sky has been demonstrated. However, there is a trade-off between the achieved level of the side lobes and other factors such as sky area of optimization, size of the array, minimum spacing between antennas and other factors. The optimization can be done with the array phased to zenith. *We need to double the radius of the optimization region of the beam pattern of the array phased to zenith in order to have low sidelobes for the whole sky for the array phased up to any pointing in the sky.* In particular for the whole semisphere we have to carry out optimization at the following range of $\sin(z)$: $|\sin(z)| < 2$

References

[1] L. Kogan, Optimizing a Large Array Configuration to Minimize the Side lobes, IEEE Transactions on Antennas and Propagation, vol 48, NO 7, July 2000, p 1075

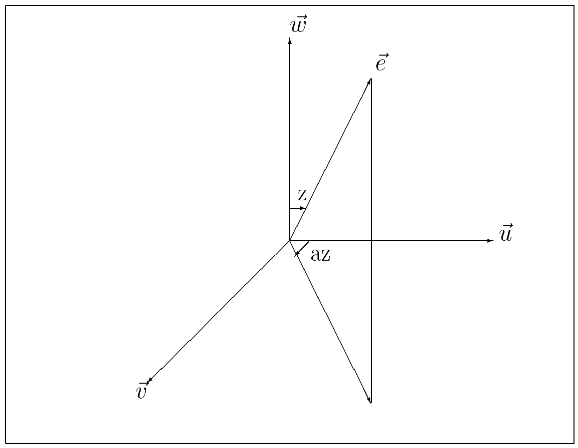

Figure 1: Coordinate system of the antenna station. The array antennas are located in the $u - v$ plane. The w-axis is perpendicular to the array plane. The angle z is the angle between the w-axis and the given direction on the sky. The angle az is the azimuth of the given direction on the sky at the array plane.

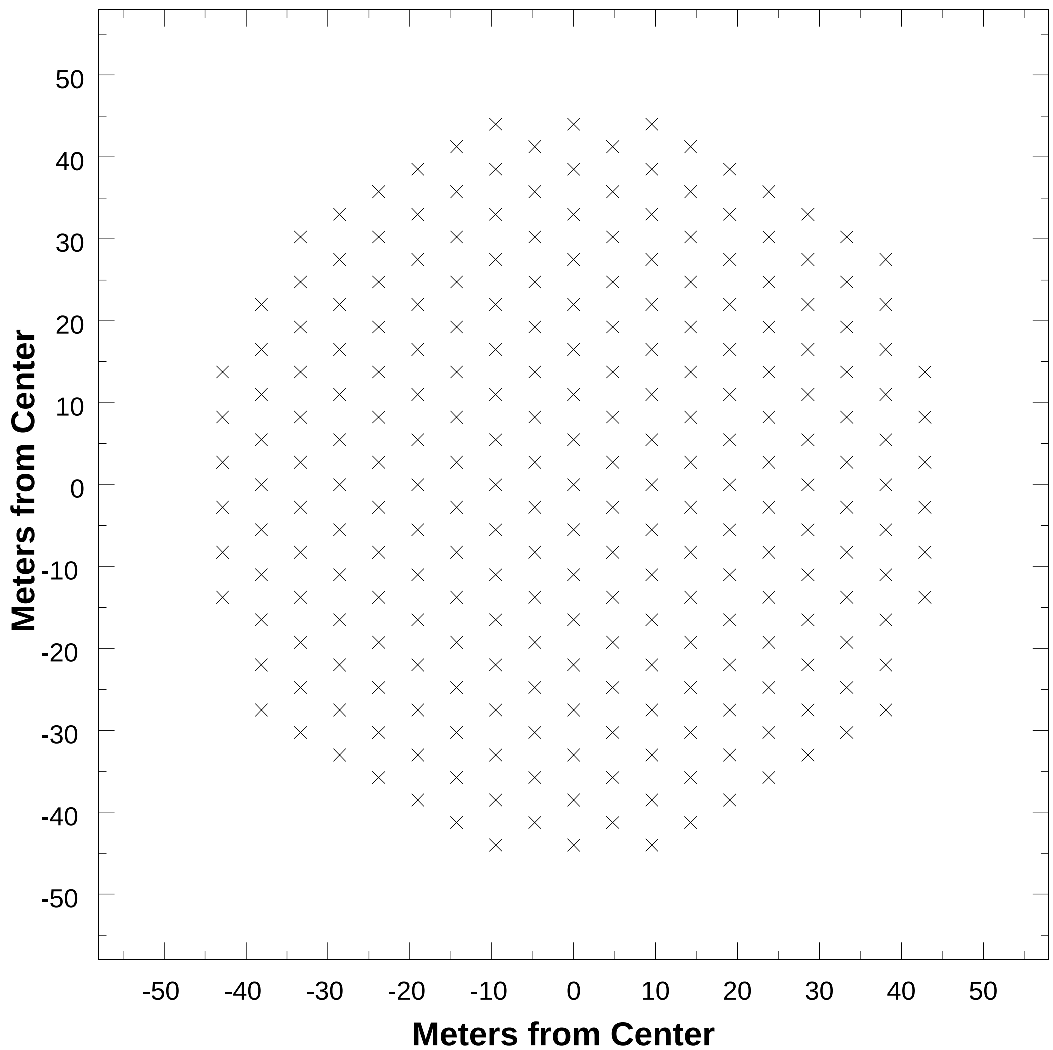

Figure 2: Hexagonal station configuration used as a starting point for the optimization.

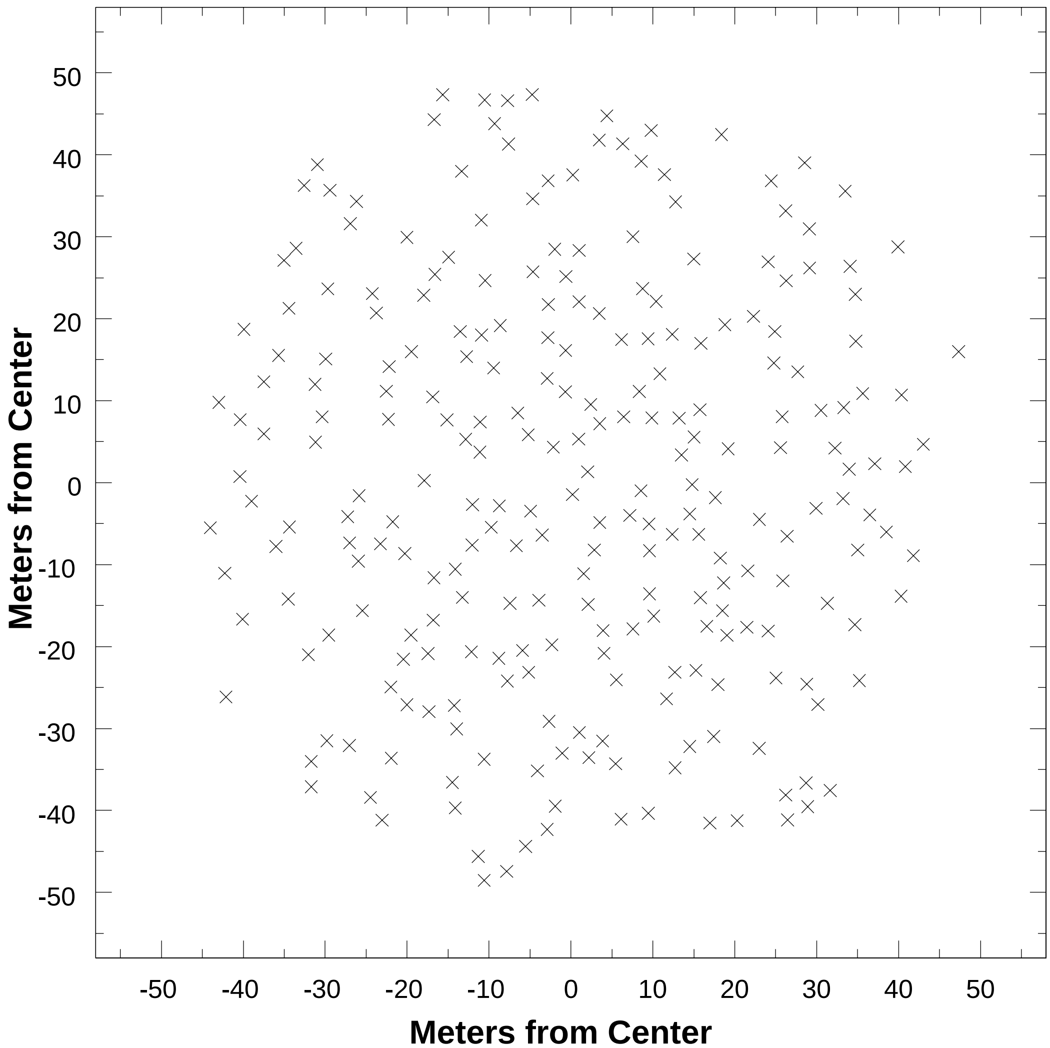

Figure 3: Station configuration after being optimized for sidelobe suppression with the constraint of a 2.0 meter minimum spacing between dipoles.

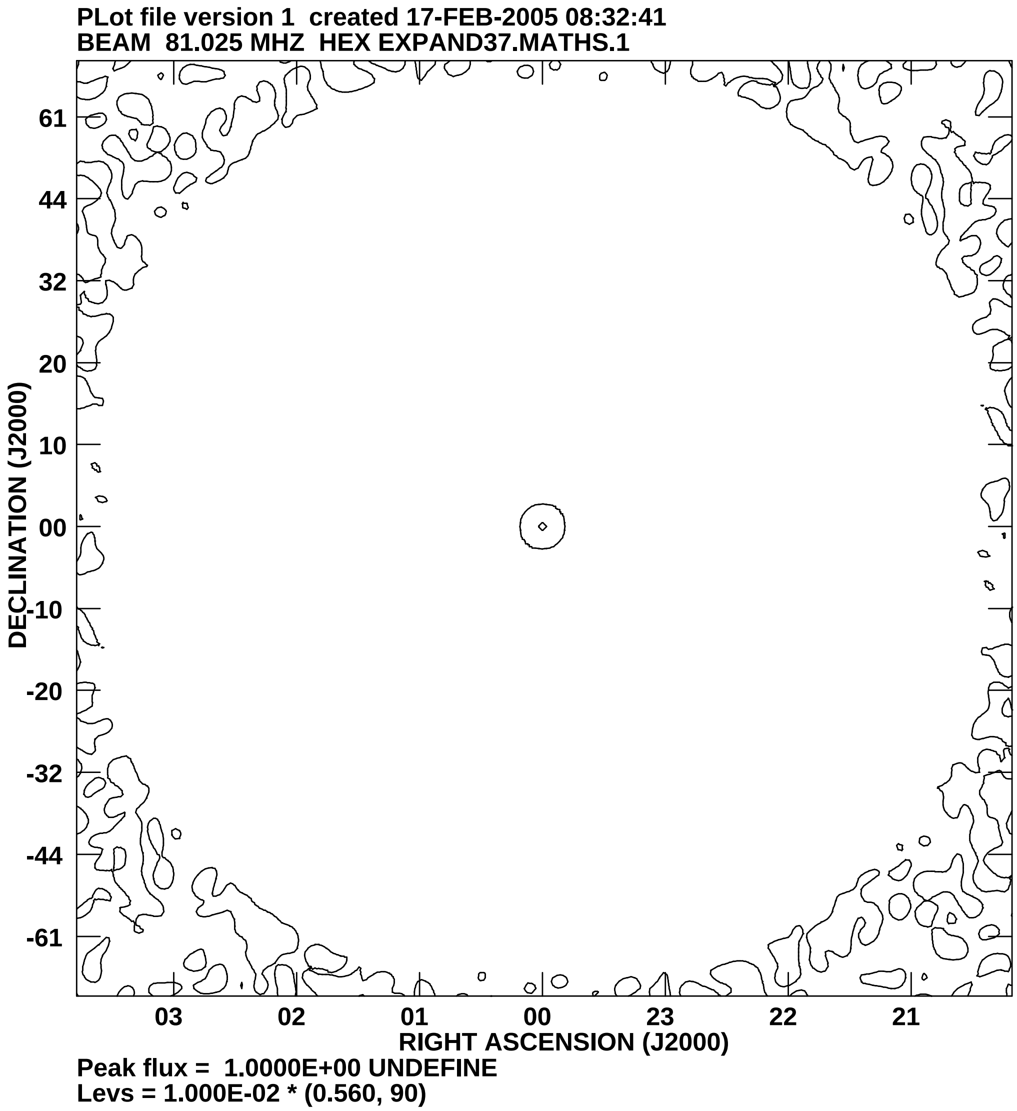

Figure 4: The two dimensional beam pattern created as a result of the optimization of the array with 256 antennas located inside of the circle 110 m diameter with minimum spacing 2 meters. The sky area of optimization is the whole semisphere $|z| \leq 90°, |sin(z)| < 1$. The polar coordinate at the plane corresponds to the radius equaled $|sin(z)|$. The side lobes are less than 0.0056 (22.5dB) inside of the circle $|z| < 90°, |sin(z)| < 1$

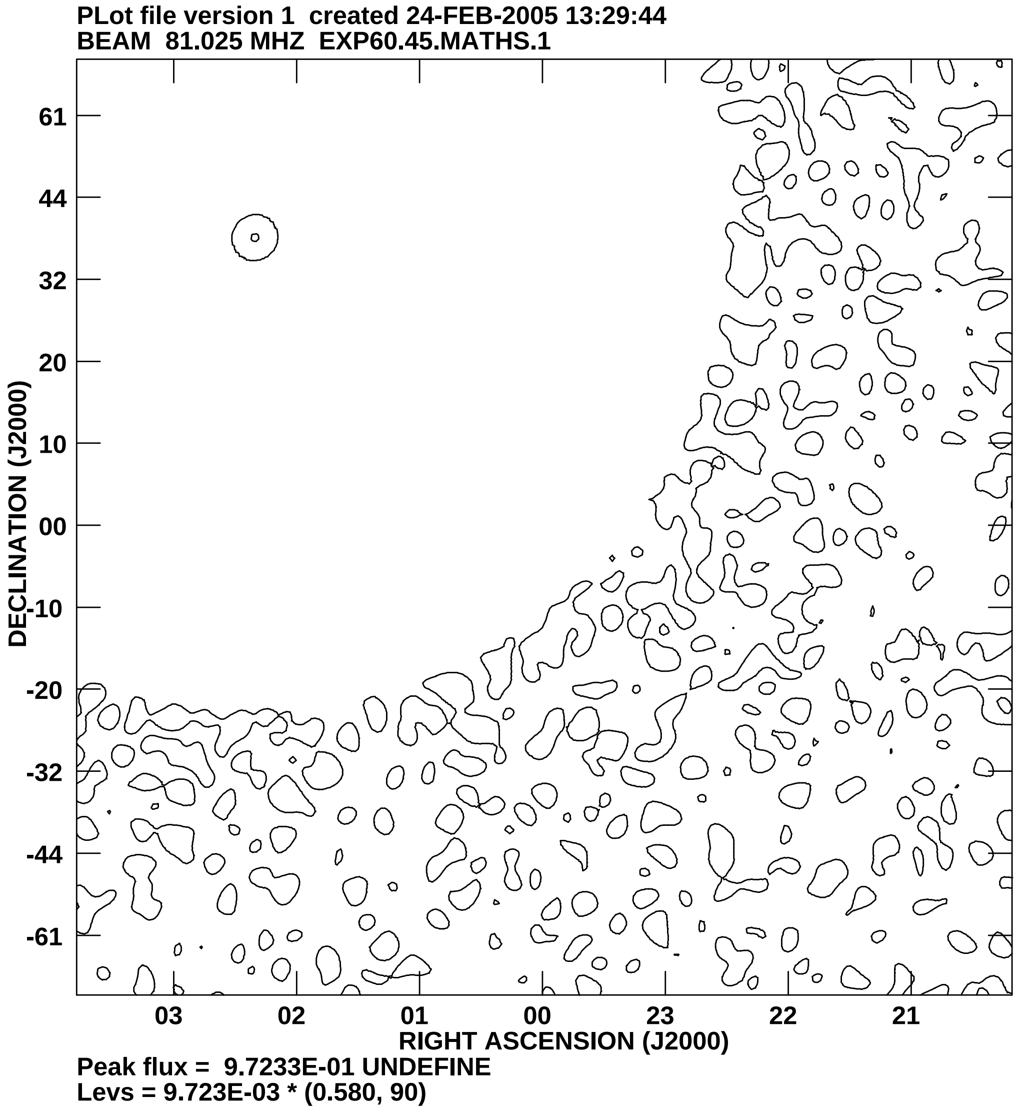

Figure 5: The two dimensional beam pattern phased towards $|z0| = 60°, |az0| = 45°$. The same configuration used here as for the plot at the figure 4. Equation (1) is used to calculate the beam pattern. It is seen that this plot coincides with the beam pattern of the figure 4 but shifted from the center (z=0) to $|z0| = 60°, |az0| = 45°$.