EVLA Memo 82
Headroom Requirements for the EVLA

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Abstract

A simple analysis is performed to indicate the required headroom for EVLA amplifiers operating in a high-RFI environment. The approach taken is to keep the power in 3rd-order harmonics below the total system noise power when the originating RFI power saturates the digitizer. This analysis is based on total system noise power, and is extended for the spectral line case. It is concluded that for an 8-bit digitizer, 35 dB of headroom between the standard operating point and the 1% compression point will be sufficient for all but the most extreme cases of RFI.

1 Introduction

The EVLA, by utilizing very wide bandwidths, will be observing far more than astronomical signals. Local sources of interference will unavoidably be visible to the signal processing chain. As these undesired signals will commonly be much stronger than natural emissions, a very high linearity of the signal processing chain, beginning with the analog amplifiers, and ending with the digital correlator will be necessary to preserve the astronomical information.

The memo considers the linearity characteristics for a multi-level digitizing system intended for high-RFI environments, and calculates the third-order intercept characteristics for an analog amplifier needed to match the linearity of the quantizer.

2 Interference and Noise Power with a 2n-level sampler

We imagine power arriving at a sampler with b bits, so there are $n = 2^b/2 = 2^{b-1}$ levels on each side of zero. The input power is arranged so that the rms noise voltage (in the absence of any RFI) corresponds to the kth level (k need not be an integer). As an example, if $b = 8$, then the SNR loss due to quantization is about 4% when k is about 2. The remaining $2^{b-1} - k$ levels are required to handle strong RFI signals.

Now presume there are N quasi-sinusoid input RFI signals, each of peak amplitude m levels. The peak amplitude of all of these, presuming an unlikely simultaneous coherent summation, is Nm levels. This voltage should not exceed the number of levels assigned to cover the RFI, to prevent saturation of the digitizer. Hence, the peak acceptable voltage, per interferor, in sampler units, is

$$m = \frac{2^{b-1} - k}{N}.$$

The power in each RFI tone is (presuming a unit resistance) $P_r = m^2/2$. Thus the total power in all N tones will be $P_I = NP_r$. The ratio of this total to the noise power ($P_n = k^2$) is then:

$$\frac{P_I}{P_n} = \frac{(2^{b-1} - k)^2}{2Nk^2} \tag{1}$$

For an 8-bit sampler ($b = 8$), with $N = 1$ and $k = 2$, the ratio is 33 dB. For $N = 5$, it is 26 dB. These values appear sufficient to handle even the very strong DME signals from aircraft as near as a few tens of kilometers without satuation of the digitizer.

If N, the number of interferors, is large, it can be argued that the probability of observing all of them in phase is low, so the consequence of this event can be ignored. The above analysis can be repeated assuming the signals are incoherent, with a gaussian-like distribution of the voltage sum. We would then set the topmost bit to α times the rms voltage of the incoherent RFI sum, giving (including an offset of k bits to handle the noise)

$$m = \frac{2^{b-1} - k}{\alpha\sqrt{N}} \tag{2}$$

as the number of bits needed per interferor to prevent saturation. We might expect $\alpha \sim 3$ to keep the fraction of saturated samples low.

With this approach, the ratio of interference to noise power is independent of the number of interferors, and is found to be:

$$\frac{P_I}{P_n} = \left(\frac{2^{b-1} - k}{\alpha k}\right)^2. \tag{3}$$

With $\alpha = 3$, $k = 2$, and $b = 8$, the ratio is 26 dB.

Thus, we conclude that setting the 2nd level to approximately the rms noise voltage will provide at least 26 dB of unsaturated response to interfering signals in an 8-bit sampler.

3 Third Order Harmonics

Pairs of strong signals input to a non-linear device create intermodulation products which can appear in the desired passband. The most serious of these are from the 3rd-order, with power $P_3 = P^3/P_0^2$, where P is the RFI signal power of each of the two interfering signals, and P_0 is the so-called 3rd-order intercept power. We desire to keep these 3rd order harmonic powers at or below the system noise power, and seek an expression for P_0.

Presume there are N sources of RFI power, each of power P_r. With the N sources we have $N(N-1)/2$ intermodulation products, each with power $P_3 = P_r^3/P_0^2$. Thus, the total power in all third-order products is, presuming all of them are in the desired pass-band,

$$P_3 = \frac{N(N-1)}{2}\frac{P_r^3}{P_0^2}. \tag{4}$$

Our condition is to keep this harmonic power below the total system noise power, P_n, hence $P_3 < P_n$. This sets a minimum condition on the third-order intercept, P_0:

$$P_0 > \sqrt{\frac{N(N-1)P_r^3}{2P_n}}. \tag{5}$$

The headroom is defined as the ratio of the third-order intercept to the system noise power. The required headroom to meet the preceding condition is then:

$$\frac{P_0}{P_n} > \sqrt{\frac{N(N-1)}{2}\left(\frac{P_r}{P_n}\right)^3} = \sqrt{\frac{N-1}{2N^2}\left(\frac{P_I}{P_n}\right)^3}, \tag{6}$$

where we have used $P_r = P_I/N$.

We can now incorporate the results of the last section, which gave the maximum interference-to-noise power ratio for a given number of sampler bits for a linear response. In utilizing these results, we are in essence setting the linearity of the analog system (through the intercept point P_0) to match that of the digital system.

For the case where we set the maximum sampler level to match the maximum possible coherent sum of N signals, we use equation 1 to find

$$\frac{P_0}{P_n} = \frac{\sqrt{N(N-1)}}{4N^3k^3}(2^{b-1}-k)^3 \tag{7}$$

For $b = 8$, $N = 2$, and $k = 2$, the necessary headroom is 39 dB. For $N = 5$, it is 33 dB. (It is less with more interferors because the strength of the interferors is based on their voltage sum not exceeding the sampler's highest bit). Typically, the third-order intercept point is 10 dB above the 1 dB compression point, and 22 dB above the 1% compression point. So for the worst case of two strong interferors of equal strength, the required headroom from the standard operating point (determined by system noise power) to the 1 dB compression point should be approximately 29 dB, and 17 dB to the 1% compression point.

For the case of a large number of interferors, we can reasonably argue their harmonic intermods will be incoherent, and use equation (2) to find

$$\frac{P_0}{P_n} = \sqrt{\frac{N-1}{2N^2}}\left(\frac{2^{b-1}-k}{\alpha k}\right)^3. \tag{8}$$

For $N = 5$, $\alpha = 3$ and $k = 2$, the headroom from the standard operating point to the third-order intercept is 34 dB, or 12 dB above the 1% compression point. The reason this expression appears more restrictive than the 'coherent sum' is because the coherent sum scenario saturates the quantizer at much lower levels of total input RFI power, hence lowering the associated third-order compression requirements on the analog system.

4 Extension to Spectral Line Observations

The preceding analysis provides the required headroom in the analog amplifier chain necessary to ensure that its limiting response for third-order harmonics matches the saturation level of the digital system. The argument is based on limiting the harmonic power to equal the system noise seen across the full input bandwidth. This is not appropriate for a spectral line system, as the harmonic response is (nearly) a pure tone, so its power will be 'concentrated' within very few channels. The limiting condition must be based on the noise power within the spectral resolution – normally very much less than that of the full bandpass. We thus desire a more linear analogue system than that indicated by the preceding analysis.

The simplest approach is simply to increase the requirements for the third-order intercept by the square root of the number of channels – as the SNR of the interfering signal will rise with the square root of the channelwidth, providing that the RFI is not spectrally resolved. For the EVLA's WIDAR correlator, the maximum number of spectral channels in the non-recirculating modes is 16384. If all of these are assigned to a single polarization, then the third-order intercept required to keep harmonic responses below the noise in a single channel increases by 21 dB. Thus we would require 38 dB headroom from the standard operating point to the 1% compression level to handle this worst-case scenario. A more common scenario occurs when the channels are distributed amongst all four polarization combinations. In this case, the headroom requirement is 32 dB from the standard operating point to the 1% compression point.

Alternatively, the maximum velocity resolution to be used in normal astrophysical work is about 1 km/sec, which corresponds to a channelwidth of 5×10^{-6} of the bandwidth for a 2:1 BWR receiver. This condition requires an extra 27 dB of headroom, or 44 dB from the standard operating point to the 1% compression point. This is most certainly a worst-case approach, as it assumes two full-time input signals sufficient to saturate the digitizer, and does not account for the rotating differential phase of the saturating signals. Taking into consideration the impulsive nature of very strong RFI, (so that powerful impulsive events can be easily edited out) and the reduction of the harmonic response in the image plane due to fringe-winding, we suggest that 35 dB headroom from the standard operating point to the 1% compression point is a sufficient goal for the EVLA analog electronics.