Two transporter axles broke in 1991. The remaining axles were inspected and were found to have cracks at the 1 inch radius on the shoulder fillet. These axles had been in service approximately 12 years. Subsequently 25 new axles were made by Penn Machine Company and installed on the transporter. The second set of axles were made out of a higher quality 4140 steel. This second set of axles started developing cracks in the same location after about 15 years of service. We are now at the point where this second set of axles needs to be replaced. These axles are very expensive to replace and broken axles can cause serious safety concerns. Therefore, it is imperative that we thoroughly understand these failures and design a more robust axle for replacement. This memo presents an analysis of these failures and a redesigned axle that should eliminate the crack problem.

The stress analysis on the original transporter axles is presented in E-Systems report No. 416-15406, Performance Analysis and Discussion for the VLA transporter vehicle dated October 10, 1974. With the help of hindsight, it is clear that some of the original assumptions were not adequately conservative. In particular, I believe that the wheel loads are higher and the fatigue endurance limit of the axles is lower than originally anticipated. The original analysis resulted in a fatigue factor of safety with respect to the endurance limit (which was assumed to be 30 ksi) of 1.38. The endurance limit is the minimum stress of a material where fatigue failure occurs. In theory this would have meant that the axles would never crack. In appendix A, I present a more thorough analysis using a 20% higher wheel load and the conservatively calculated endurance limit of 19.2 ksi. These assumptions yield a factor of safety of 0.95 resulting in a useful life of approximately 12 years.

The above analysis clearly shows that the cracks on the original axle are caused by fatigue due to cyclical loading on the shoulder fillet. The diameter change at the shoulder fillet is an area of stress concentration where the localized stress is significantly elevated. One way to reduce this stress concentration is to reduce the center section diameter. This is one of those cases where making the part smaller actually makes it stronger. The proposed new axle design reduces the center section diameter from 8.25” to 6.875”. This decreases the stress riser and also lowers the material cost by approximately $600 per axle. The new axle design also reduces the stress riser at the shoulder fillet by making the diameter transition much more gradual. The finite element analysis result shown in Figure 1 verifies that the new axle design reduces the stress at the shoulder fillet by approximately 15%.
In theory, the improved axle geometry would decrease the stress sufficiently that the original 4140 steel could be used with a fatigue factor of safety of 1.15. This does not allow very much leeway for unforeseen effects that could be present. Therefore, I suggest we use a harder 4340 HTSR steel for the new transporter axles. The endurance limit of this material is approximately 35 ksi. This is about 30% higher than the 4140 steel used on the second set of axles. The harder material combined with the improved geometry would increase our fatigue factor of safety to 1.4. This should be adequate that we have a high level of confidence that the newer axles will not fail in fatigue.

The improvements at the fillet shoulder are summarized in the following table.
<table>
<thead>
<tr>
<th>Property</th>
<th>Existing Axle 4140 steel</th>
<th>Improved Geometry with 4140 steel</th>
<th>Improved Geometry with 4340 steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress at shoulder fillet without stress concentration (ksi)</td>
<td>20.1</td>
<td>20.1</td>
<td>20.1</td>
</tr>
<tr>
<td>Endurance limit of steel bar (ksi)</td>
<td>27.1</td>
<td>27.1</td>
<td>35.2</td>
</tr>
<tr>
<td>Endurance limit reduced for stress concentration at shoulder fillet. (ksi)</td>
<td>19.2</td>
<td>22.7</td>
<td>28.9</td>
</tr>
<tr>
<td>Factor of Safety with respect to endurance limit.</td>
<td>0.95</td>
<td>1.13</td>
<td>1.43</td>
</tr>
<tr>
<td>Calculated life expectancy (years)</td>
<td>12.1</td>
<td>Stress is below endurance limit.</td>
<td>Stress is below endurance limit.</td>
</tr>
</tbody>
</table>

The mechanical drawing for the new axle is shown on the following page.
NOTE:
1. REMOVE ALL BURS AND SHARP EDGES.
2. THE DRAWING SUPERSEDES DRAWING NUMBERS 401-25860 AND 401-25816.
3. MATE S. & TOL. FOR LIM. MAXIMUM PLACES PASSING DEPTH. HARDENED SURFACE GROOVES MUST BE PROJECTED. MACHINED BETWEEN ANGLES IN BLUE SHADING.

**Table: Location Specifications**

<table>
<thead>
<tr>
<th>LOCATION</th>
<th>SPECIFICATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CENTERLINE</td>
<td>.000</td>
</tr>
<tr>
<td>VERTICAL</td>
<td>.000</td>
</tr>
<tr>
<td>HORIZONTAL</td>
<td>.000</td>
</tr>
<tr>
<td>END CUT</td>
<td>.000</td>
</tr>
</tbody>
</table>

**Angles**

- **Angle A:** 13°
- **Angle B:** 32°

**Table 1: Involute Spline Data (ASA B5.15-1960)**

<table>
<thead>
<tr>
<th>REFERENCE</th>
<th>MEASUREMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>MINOR DIAMETER</td>
<td>3.1293/3.1073</td>
</tr>
<tr>
<td>MAJOR DIAMETER</td>
<td>3.4627/3.4577</td>
</tr>
<tr>
<td>PITCH DIAMETER</td>
<td>3.3333</td>
</tr>
<tr>
<td>BASE DIAMETER</td>
<td>2.8868</td>
</tr>
<tr>
<td>PITCH</td>
<td>6/12</td>
</tr>
<tr>
<td>PRESSURE ANGLE</td>
<td>30°</td>
</tr>
</tbody>
</table>

**Material:**

- STEEL 4340 QTSR MINIMUM YIELD STRENGTH 120 KSI.
- HARDNESS BETWEEN 32 AND 39 RC (300-360 BNH).
- MATERIAL CERTIFICATIONS MUST BE PROVIDED.

**Date:**

- 1/20/2012

**Patent:**

- TRANSPORTER AXLE
- US 9690191

**Revision History:**

- INITIAL RELEASE: 3/14/2012
- REV. 2: 1/20/2012

**Notes:**

1. REMOVE ALL BURS AND SHARP EDGES.
2. THIS DRAWING SUPERSEDES DRAWING NUMBERS 401-25860 AND 401-25816.
3. MATE S. & TOL. FOR LIM. MAXIMUM PLACES PASSING DEPTH. HARDENED SURFACE GROOVES MUST BE PROJECTED. MACHINED BETWEEN ANGLES IN BLUE SHADING.
Appendix A
TRANSPORTER AXLE FAILURE ANALYSIS

Input Parameters:

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Load Values:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dia1: 3.4-in</td>
<td>Torque: 8000 ft-lbf</td>
</tr>
<tr>
<td>Rad1: .5-in</td>
<td>MaxWheelLoad: 43276 lbf</td>
</tr>
<tr>
<td>L1: .1875-in</td>
<td>FatigueWheelLoad: 26477 lbf</td>
</tr>
<tr>
<td>Dia2: 5.9-in</td>
<td>FatigueWheelLoad: 32000 lbf</td>
</tr>
<tr>
<td>Rad2: 1-in</td>
<td>(q = .82)</td>
</tr>
<tr>
<td>L2: 12.6875-in</td>
<td></td>
</tr>
<tr>
<td>Dia3: 8.25-in</td>
<td></td>
</tr>
</tbody>
</table>

Material Properties
4140 Steel:

\(F_y = 50000\) psi
\(F_u = 104\) ksi

\(q = \) Notch Sensitivity For 4140 steel From Figure 5-19, Shigley

The above Fatigue Load values were used in the original analysis. This assumed 317 ton Transporter + antenna evenly distributed on wheels. However the transporter is loaded more heavily on one side. The Fatigue wheel load below is a guess.
Maximum Load Condition

The maximum wheel load occurs from the maximum dynamic load. For the condition of vehicle speed 5 MPH and a wind speed of 25 MPH.

Torsional Shear Stress

\[ \tau_1 = \frac{16 \text{ Torque}}{\pi \cdot \text{Dia}^3} \]

\[ \tau_1 = 12439.548 \text{ psi} \]

If input torque is transmitted to the track at the first wheel then we can assume:

\[ \tau_2 = 0 \]
\[ \tau_3 = 0 \]
\[ \tau_4 = 0 \]

Shear Stress Between Wheel and Load

\[ S_{s1} = \frac{(4 \text{MaxWheelload})}{\pi \cdot \text{Dia}^2} \]

\[ S_{s1} = 4766.5 \text{ psi} \]

\[ S_{s2} = \frac{(4 \text{MaxWheelload})}{\pi \cdot \text{Dia}^2} \]

\[ S_{s2} = 1582.9 \text{ psi} \]

\[ S_{s4} = 0 \]

Total Shear Stress

\[ \tau_{xy1} = \tau_1 + S_{s1} \]

\[ \tau_{xy1} = 17206 \text{ psi} \]

\[ \tau_{xy2} = \tau_2 + S_{s2} \]

\[ \tau_{xy2} = 1582.9 \text{ psi} \]

\[ \tau_{xy4} = 0 \]
The above analysis shows that the axle should perform satisfactory for the maximum load condition under static load conditions.
Fatigue Calculations

Endurance Limit Modifying Factors

For most steels, the mean endurance limit of rotating beam specimens is approximately 1/2 of the ultimate strength of the material. This endurance limit corresponds to approximately 1 million cycles. The endurance limit is further reduced by the following Endurance Limit Modifying Factors.

<table>
<thead>
<tr>
<th>Surface Factor</th>
<th>Figure 5-17</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polished Specimen</td>
<td>1</td>
</tr>
<tr>
<td>Ground</td>
<td>.89</td>
</tr>
<tr>
<td>Machined</td>
<td>.65 - .8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Size Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kb = 1 d &lt; 0.3 in</td>
</tr>
<tr>
<td>Kb = .85 0.3 &lt; d &lt; 2 in</td>
</tr>
<tr>
<td>Kb = .75 d &gt; 2 in</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reliability factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reliability 0.5 Zr = 1</td>
</tr>
<tr>
<td>Reliability 0.9 Zr = 1.288</td>
</tr>
<tr>
<td>Reliability 0.95 Zr = 1.645</td>
</tr>
<tr>
<td>Reliability 0.99 Zr = 2.326</td>
</tr>
<tr>
<td>Reliability 0.999 Zr = 3.091</td>
</tr>
<tr>
<td>Reliability 0.9999 Zr = 3.719</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Temperature factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kd = 1 T &lt; 160 F</td>
</tr>
<tr>
<td>Kd = 1 T &gt; 160 F</td>
</tr>
</tbody>
</table>

\[ K = \frac{K_a K_b K_c K_d}{Z_r} \]

Endurance Limit

\[ S_e = 0.5 \times \text{Fut} \times K \]

\[ S_e = 27.1 \text{ ksi} \]
Fatigue at Point 1

At point 1 the axle is subjected to combined stress due to both shear and bending.

Shear Stress From Load

\[
S_{s1} = \frac{4(FatigueWheelLoad)}{\pi Dia^2}
\]

\[S_{s1} = 3524.5 \text{ psi}\]

Maximum Shear Stress (Torque + Shear)

\[
\tau_{xy1} = \tau_1 + S_{s1} \quad \tau_{xy1} = 15964.1 \text{ psi}
\]

\[
\tau_{xy1} = \tau_{y1} - S_{s1} \quad \tau_{xy1} = \tau_{y1} + S_{s1}
\]

\[
\tau_{xy1} = \frac{\tau_{xy1\max} + \tau_{xy1\min}}{2} \quad \tau_{xy1a} = \tau_{xy1\max} - \tau_{xy1\min} \quad \tau_{xy1\mean} = 12.4 \text{ ksi} \quad \tau_{xy1a} = 7 \text{ ksi}
\]

Bending Moment

\[
M_1 = FatigueWheelLoad \cdot L_1 \quad M_1 = 500 \text{ ft-lbf}
\]

Bending Stress

\[
\sigma_{x1} = \frac{32M_1}{\pi Dia^3} \quad \sigma_{x1} = 1554.9 \text{ psi}
\]

\[
\sigma_{x1\max} = \sigma_{x1\min} = -\sigma_{x1} \quad \sigma_{x1\mean} = 0 \text{ ksi} \quad \sigma_{x1a} = 1.6 \text{ ksi}
\]

\[
\sigma_{y1\mean} = 0 \quad \sigma_{y1a} = 0
\]
Von Mises Failure Criteria:

\[ \tau_{xy \text{mean}} = \left( \frac{\sigma_{x \text{mean}} - \sigma_{y \text{mean}}}{2} \right)^2 + \tau_{xy} \text{mean} \]

\[ \tau_{xy \text{a}} = \left( \frac{\sigma_{x \text{a}} - \sigma_{y \text{a}}}{2} \right)^2 + \tau_{xy \text{a}} \text{mean} \]

\[ \sigma_{1 \text{mean}} = \frac{\sigma_{x \text{mean}} + \sigma_{y \text{mean}}}{2} + \tau_{xy \text{mean}} \]

\[ \sigma_{2 \text{mean}} = \frac{\sigma_{x \text{mean}} - \sigma_{y \text{mean}}}{2} - \tau_{xy \text{mean}} \]

\[ \sigma_{1 \text{mean}}^2 = \left( \sigma_{1 \text{mean}}^2 - \sigma_{1 \text{mean}} \sigma_{2 \text{mean}} + \sigma_{2 \text{mean}}^2 \right)^{\frac{1}{2}} \]

\[ \sigma_{2 \text{mean}} = -12439.5 \text{ psi} \]

\[ \sigma_{1 \text{mean}} = 12439.5 \text{ psi} \]

\[ \sigma_{1 \text{a}} = \frac{\sigma_{x \text{a}} + \sigma_{y \text{a}}}{2} + \tau_{xy \text{a}} \]

\[ \sigma_{2 \text{a}} = \frac{\sigma_{x \text{a}} - \sigma_{y \text{a}}}{2} - \tau_{xy \text{a}} \]

\[ \sigma_{1 \text{a}}^2 = \left( \sigma_{1 \text{a}}^2 - \sigma_{1 \text{a}} \sigma_{2 \text{a}} + \sigma_{2 \text{a}}^2 \right)^{\frac{1}{2}} \]

\[ \sigma_{2 \text{a}} = -6314.4 \text{ psi} \]

Because the radius at point 1 is a stress riser we must further reduce the endurance limit of the steel. The Stress concentration Factor from Roark's Formulas for Stress and Strain (Table 37 Case 17). Valid for \( h_1 \) between 2 and 20.

\[ k_{11} = 1.225 + .831 h_1^3 - 0.1 h_1 \]

\[ k_{21} = -3.79 + .958 h_1^3 - 2.57 h_1 \]

\[ k_{31} = 7.374 - 4.834 h_1^5 + .862 h_1 \]

\[ k_{41} = -3.309 + 3.046 h_1^5 - .595 h_1 \]

\[ K_{11} = k_{11} + k_{21} \left( \frac{b_1}{\text{Dia}2} \right) + k_{31} \left( \frac{b_1}{\text{Dia}2} \right)^2 + k_{41} \left( \frac{b_1}{\text{Dia}2} \right)^3 \]

\[ h_1 := \frac{\text{Dia}2 - \text{Dia}1}{2} \]

\[ h_1 > \frac{h_1}{\text{Rad}1} \]

\[ h_1 = 2.5 \]

\[ K_{11} = 1.58 \]
In the present fluctuating stress, a modified Goodman Diagram can be used to relate the stress to the strength.

**Modified Goodman Line**

\[ y = \frac{\sigma_{vm}}{\sigma_{vm - mean}} \]

\[ \text{fatigue} = \frac{\sigma_{fatigue}}{\sigma_{vm}} \]

\[ \text{fatigue} = \frac{\sigma_{fatigue}}{\sigma_{vm}} \]

**Factor of Safety**

\[ n_{fatigue} = 1.14 \]
Fatigue at Point 2

At point 2 the axle is subjected to combined stress due to both shear and bending.

Shear Stress From Load

\[
\sigma_{\text{Sx2}} = \frac{(4\text{FatigueWheelLoad})}{\pi \cdot \text{Dia}^2} \\
\sigma_{\text{Sx2}} = 1170.5 \text{ psi}
\]

Maximum Shear Stress (Torque + Shear)

\[
\tau_{\text{xy2}} = \tau_2 + Ss2 \\
\tau_{\text{xy2}} = 1170.5 \text{ psi} \\
\tau_{\text{xy2min}} = \tau_2 - Ss2 \\
\tau_{\text{xy2max}} = \tau_2 + Ss2 \\
\tau_{\text{xy2mean}} = \frac{\tau_{\text{xy2max}} + \tau_{\text{xy2min}}}{2} \\
\tau_{\text{xy2a}} = \tau_{\text{xy2max}} - \tau_{\text{xy2min}} \\
\tau_{\text{xy2mean}} = 0 \text{ ksi} \\
\tau_{\text{xy2a}} = 2.3 \text{ ksi}
\]

Bending Moment

\[
M_{\text{xx2}} = \text{FatigueWheelLoad} \cdot L2 \\
M_{\text{xx2}} = 33833.3 \text{ ft lb}
\]

Bending Stress

\[
\sigma_{\text{sx2}} = \frac{32M_{\text{xx2}}}{\pi \cdot \text{Dia}^2} \\
\sigma_{\text{sx2}} = 20135.9 \text{ psi} \\
\sigma_{\text{sx2min}} = -\sigma_{\text{sx2}} \\
\sigma_{\text{sx2max}} = \sigma_{\text{sx2}} \\
\sigma_{\text{sx2mean}} = \frac{\sigma_{\text{sx2max}} + \sigma_{\text{sx2min}}}{2} \\
\sigma_{\text{sx2a}} = \sigma_{\text{sx2max}} - \sigma_{\text{sx2min}} \\
\sigma_{\text{sx2mean}} = 0 \text{ ksi} \\
\sigma_{\text{sx2a}} = 20.1 \text{ ksi}
\]

\[
\sigma_{\text{sy2mean}} = 0 \\
\sigma_{\text{sy2a}} = 0
\]
Von Mises Failure Criteria:

\[
\sigma_{12}^2 = \left(\frac{\sigma_{x2}^2 + \sigma_{y2}^2}{2} + \tau_{xy2}\right)^2
\]

\[
\sigma_{12a} = \left(\frac{\sigma_{x2a} + \sigma_{y2a}}{2} + \tau_{xy2a}\right)^2
\]

Since the Mean Von Misses Stress is zero then the factor of safety is

\[
\text{factor of safety} = \frac{S_e}{\sigma_{vm2a}} = 1.319
\]
Fatigue at Point 3

After the bearing the axle is subjected to cyclical bending stress. The Shear stress is zero at this point.

Bending Moment

\[ M_3 = \text{FatigueWheelLoad \cdot l \cdot d} \]

\[ M_3 = 33.833.3 \text{ ft \cdot lbf} \]

Stress due to bending at radius

\[ \sigma_3 = \frac{32M_3}{\pi Dia^2_3} \]

\[ \sigma_3 = 2013.9 \text{ psi} \]

Because the radius at point 3 is a stress riser we must further reduce the endurance limit of the steel. The Stress concentration Factor from Roark’s Formulas for Stress and Strain (Table 37 Case 17). Valid for \( h_3 \) between 0 and 2.

Endurance limit with stress concentration

\[ k_{13} = 0.927 + 1.149 \cdot h_3^5 - 0.066 \cdot h_3 \]

\[ k_{23} = 0.015 - 3.281 \cdot h_3^5 + 0.837 \cdot h_3 \]

\[ k_{33} = 0.47 + 1.76 \cdot h_3^5 - 0.506 \cdot h_3 \]

\[ k_{43} = -0.790 + 0.417 \cdot h_3^5 - 0.246 \cdot h_3 \]

\[ K_{13} = k_{13} + k_{23} \left( 2 \frac{h_3}{Dia_3} \right) + k_{33} \left( 2 \frac{h_3^3}{Dia_3} \right)^2 + k_{43} \left( 2 \frac{h_3^5}{Dia_3} \right)^3 \]

\[ K_{13} = 1.5 \]

\[ Ke_3 := \frac{1}{1 + q (K_{13} - 1)} \]

\[ Ke_3 = 0.709 \]

\[ Se_3 := Se \cdot Ke_3 \]

\[ Se_3 = 19.2 \text{ ksi} \]
The factor of safety is then

\[ F_s := \frac{Se3}{\sigma x3} \quad F_s = 0.954 \]

The expected part life can be estimated.

\[ m = \frac{\log \left( \frac{9 \cdot \text{Fut}}{Se3} \right)}{3} \quad m = 0.2 \]

\[ b = \log \frac{(0.9 - \text{Fut})^2}{Se3 \text{ psi}} \quad b = 5.7 \]

\[ N = 10 \left( \frac{b}{m} \right) \left( \frac{1}{m} \right) \left( \frac{\sigma x3}{\text{psi}} \right) \quad N = 816002.4 \]

cycles per year --- 4 configurations, 20 antennas per configuration, average distance moved 1.5 miles

Wheel diameter 3’

\[ \text{Cpy} := \frac{4 \times 20 \times 1.5 \times 5280}{\pi \times 3 \times \text{yr}} \quad \text{Cpy} = 67227 \frac{1}{\text{yr}} \]

\[ \text{Life} := \frac{N}{\text{Cpy}} \quad \text{Life} = 12.1 \text{ yr} \]
As can be seen from the chart below, the axle stress is slightly higher than the endurance limit of the material.

\[
\begin{align*}
S_0 &= \frac{F_{ut}}{ksi} \quad S_1 &= \frac{F_{ut}}{ksi} \quad S_2 &= \frac{S_{ct}}{ksi} \quad S_3 &= \frac{S_{ct}}{ksi} \\
N_0 &= 100 \quad N_1 &= 1000 \quad N_2 &= 1000000 \quad N_3 &= 10000000 \\
i &= 1 \ldots 4 \quad S_i &= \frac{S_{ct}^3}{ksi}
\end{align*}
\]

**S-N Diagram**

Fatigue Life (cycles)

4140 Steel De-rated for size, surface finish...etc.

Stress Level at point 3