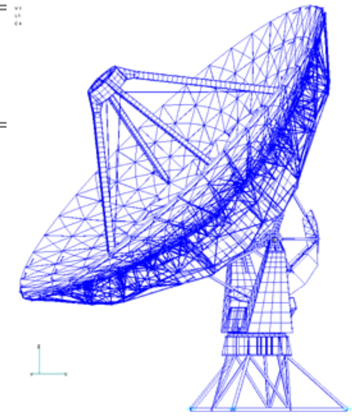
NATIONAL RADIO ASTRONOMY OBSERVATORY
SOCORRO, NEW MEXICO



EVLA MEMO #158

TRANSPORTER AXLE FAILURES

MARCH 27, 2012

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Two transporter axles broke in 1991. The remaining axles were inspected and were found to have cracks at the 1 inch radius on the shoulder fillet. These axles had been in service approximately 12 years. Subsequently 25 new axles were made by Penn Machine Company and installed on the transporter. The second set of axles were made out of a higher quality 4140 steel. This second set of axles started developing cracks in the same location after about 15 years of service. We are now at the point where this second set of axles needs to be replaced. These axles are very expensive to replace and broken axles can cause serious safety concerns. Therefore, it is imperative that we thoroughly understand these failures and design a more robust axle for replacement. This memo presents an analysis of these failures and a redesigned axle that should eliminate the crack problem.

The stress analysis on the original transporter axles is presented in E-Systems report No. 416-15406, Performance Analysis and Discussion for the VLA transporter vehicle dated October 10, 1974. With the help of hindsight, it is clear that some of the original assumptions were not adequately conservative. In particular, I believe that the wheel loads are higher and the fatigue endurance limit of the axles is lower than originally anticipated. The original analysis resulted in a fatigue factor of safety with respect to the endurance limit (which was assumed to be 30 ksi) of 1.38. The endurance limit is the minimum stress of a material where fatigue failure occurs. In theory this would have meant that the axles would never crack. In appendix A, I present a more thorough analysis using a 20% higher wheel load and the conservatively calculated endurance limit of 19.2 ksi. These assumptions yield a factor of safety of 0.95 resulting in a useful life of approximately 12 years.

The above analysis clearly shows that the cracks on the original axle are caused by fatigue due to cyclical loading on the shoulder fillet. The diameter change at the shoulder fillet is an area of stress concentration where the localized stress is significantly elevated. One way to reduce this stress concentration is to reduce the center section diameter. This is one of those cases where making the part smaller actually makes it stronger. The proposed new axle design reduces the center section diameter from 8.25” to 6.875”. This decreases the stress riser and also lowers the material cost by approximately $600 per axle. The new axle design also reduces the stress riser at the shoulder fillet by making the diameter transition much more gradual. The finite element analysis result shown in Figure 1 verifies that the new axle design reduces the stress at the shoulder fillet by approximately 15%.

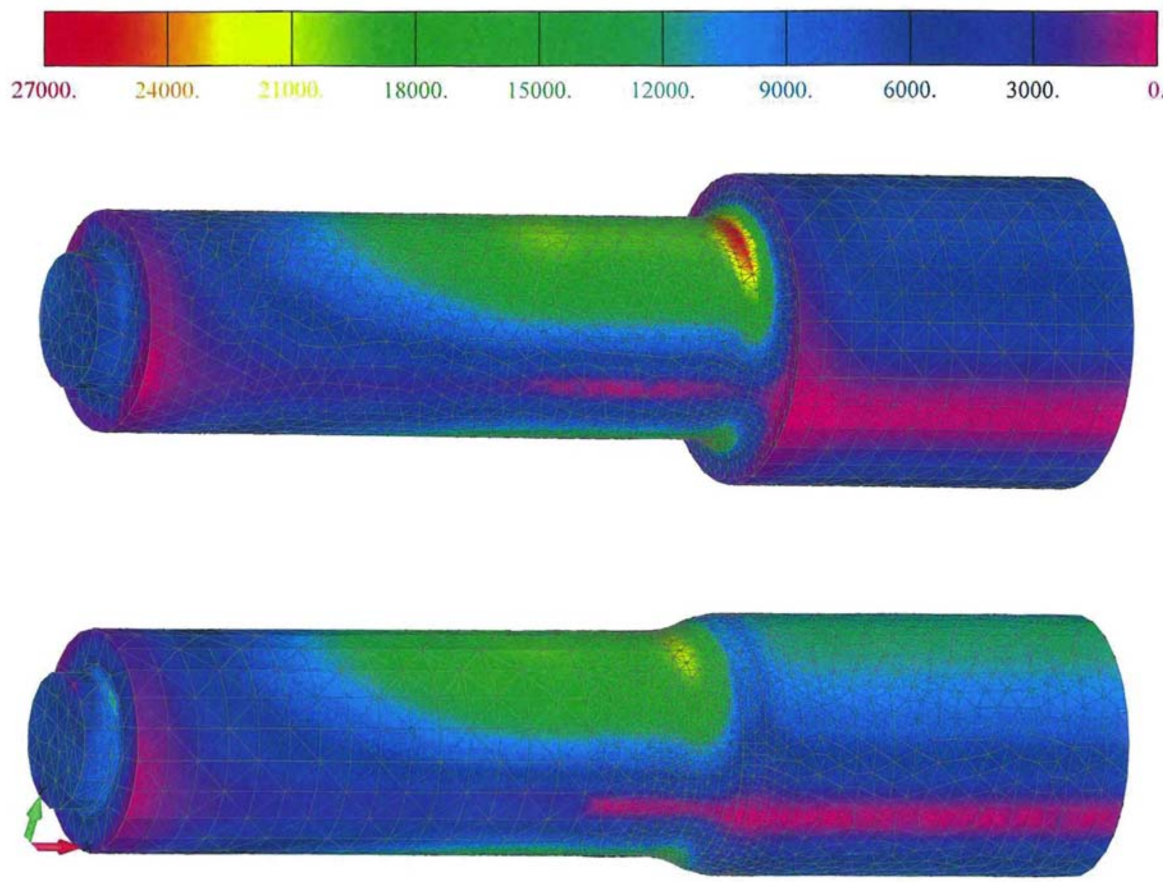


Figure 1. Comparison of the Von Mises Stress of the original axle geometry (top) to the optimized geometry (lower).

In theory, the improved axle geometry would decrease the stress sufficiently that the original 4140 steel could be used with a fatigue factor of safety of 1.15. This does not allow very much leeway for unforeseen effects that could be present. Therefore, I suggest we use a harder 4340 HTSR steel for the new transporter axles. The endurance limit of this material is approximately 35 ksi. This is about 30% higher than the 4140 steel used on the second set of axles. The harder material combined with the improved geometry would increase our fatigue factor of safety to 1.4. This should be adequate that we have a high level of confidence that the newer axles will not fail in fatigue.

The improvements at the fillet shoulder are summarized in the following table.

Property	Existing Axle 4140 steel	Improved Geometry with 4140 steel	Improved Geometry with 4340 steel
Stress at shoulder fillet without stress concentration (ksi)	20.1	20.1	20.1
Endurance limit of steel bar (ksi)	27.1	27.1	35.2
Endurance limit reduced for stress concentration at shoulder fillet. (ksi)	19.2	22.7	28.9
Factor of Safety with respect to endurance limit.	0.95	1.13	1.43
Calculated life expectancy (years)	12.1	Stress is below endurance limit	Stress is below endurance limit

The mechanical drawing for the new axle is shown on the following page.

NOTES
1. REMOVE ALL BURS AND SHARP EDGES.

2. THIS DRAWING SUPERCEDES E-SYSTEMS DRAWING NUMBERS 401-25860 AND 401-25816.

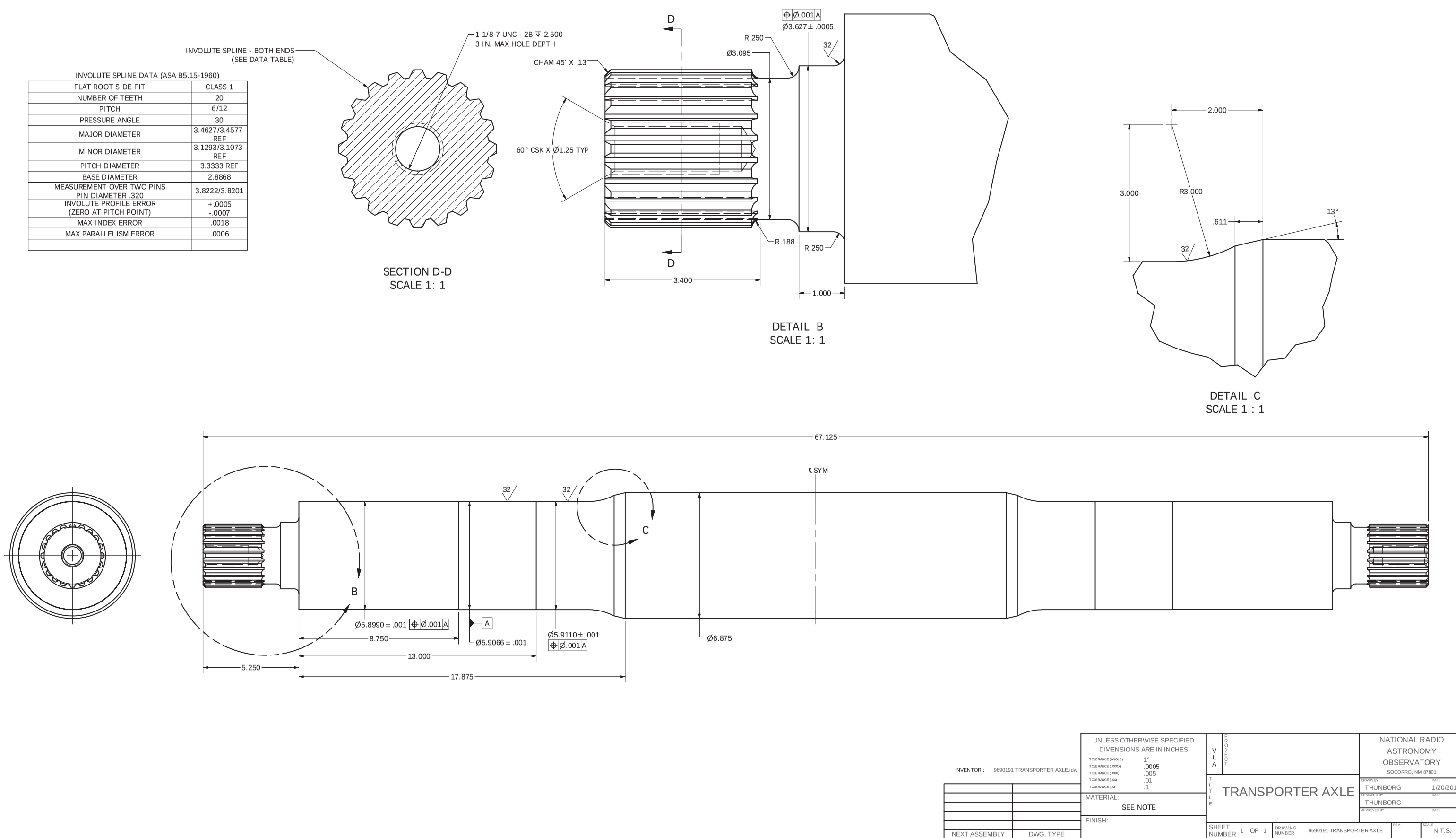
3. MATL: STEEL 4340 QTSR MINIMUM YIELD STRENGTH 120 KSI. MATERIAL CERTIFICATIONS MUST BE PROVIDED. HARDNESS BETWEEN 32 AND 39 RC (300-360 BNH).

REVISION HISTORY

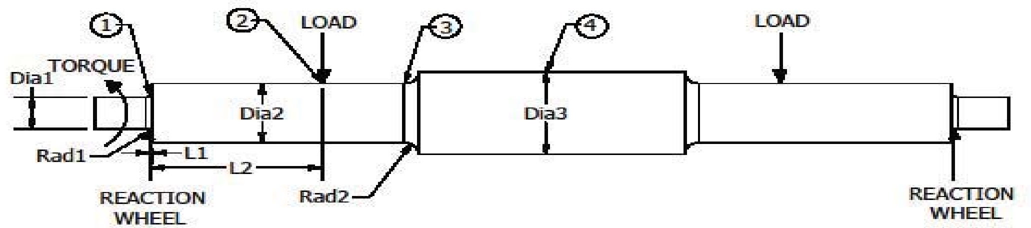
ZONE	REV	DESCRIPTION	DATE	APPROVED
	1	INITIAL RELEASE	3/14/2012	

INVOLUTE SPLINE DATA (ASA B5.15-1960)

FLAT ROOT SIDE FIT	CLASS 1
NUMBER OF TEETH	20
PITCH	6/12
PRESSURE ANGLE	30
MAJOR DIAMETER	3.4627/3.4577 REF
MINOR DIAMETER	3.1293/3.1073 REF
PITCH DIAMETER	3.3333 REF
BASE DIAMETER	2.8868
MEASUREMENT OVER TWO PINS PIN DIAMETER .320	3.8222/3.8201
INVOLUTE PROFILE ERROR (ZERO AT PITCH POINT)	+.0005 -.0007
MAX INDEX ERROR	.0018
MAX PARALLELISM ERROR	.0006



Appendix A
TRANSPORTER AXLE FAILURE ANALYSIS



Input Parameters:

Dimensions:

Dia1 := 3.4·in   Rad1 := .5·in   L1 := .1875·in

Dia2 := 5.9·in   Rad2 := 1·in   L2 := 12.6875·in

Dia3 := 8.25·in

Load Values:

Torque := 8000·ft·lbf

MaxWheelLoad := 43276·lbf

FatigueWheelLoad := 26477·lbf

The above Fatigue Load values were used in the original analysis. This assumed 317 ton Transporter + antenna evenly distributed on wheels.
However the transporter is loaded more heavily on one side. The Fatigue wheel load below is a guess.

FatigueWheelLoad := 32000·lbf

Material Properties 4140 Steel:

Fy := 50000·psi

Fut := 104·ksi

q = Notch Sensitivity For 4140 steel From Figure 5-19, Shigley   q := .82

Maximum Load Condition

The Maximun wheel load occurs from the maximum dynamic load. For the condition of vehicle speed 5 MPH and a wind speed of 25 MPH.

Torsional Shear Stress

$$\tau 1 := 16 \cdot \frac{Torque}{\pi \cdot Dia1^3} \qquad \tau 1 = 12439.548\ psi$$

If input torque is transmitted to the track at the first wheel then we can assume:

$$\tau 2 := 0$$
$$\tau 3 := 0$$
$$\tau 4 := 0$$

Shear Stress Between Wheel and Load:

$$Ss1 := \frac{(4MaxWheelLoad)}{\pi \cdot Dia1^2} \qquad Ss1 = 4766.5 \cdot psi$$

$$Ss2 := \frac{(4MaxWheelLoad)}{\pi \cdot Dia2^2} \qquad Ss2 = 1582.9 \cdot psi$$

$$Ss4 := 0$$

Total Shear Stress

$$\tau xy1 := \tau 1 + Ss1 \qquad \tau xy1 = 17206 \cdot psi$$

$$\tau xy2 := \tau 2 + Ss2 \qquad \tau xy2 = 1582.9 \cdot psi$$

$$\tau xy4 := 0$$

Bending Moment

$M1 := MaxWheelLoad \cdot L1 \qquad M1 = 676.2 \cdot ft \cdot lbf$

$M2 := MaxWheelLoad \cdot L2 \qquad M2 = 45755.4 \cdot ft \cdot lbf$

$M4 := MaxWheelLoad \cdot L2 \qquad M4 = 45755.4 \cdot ft \cdot lbf$

Bending Stress

$$\sigma x1 := 32 \cdot \frac{M1}{\pi \cdot Dia1^3} \qquad \sigma x1 = 2102.9 \cdot psi$$

$$\sigma x2 := 32 \cdot \frac{M2}{\pi \cdot Dia2^3} \qquad \sigma x2 = 27231.2 \cdot psi$$

$$\sigma x4 := 32 \cdot \frac{M4}{\pi \cdot Dia3^3} \qquad \sigma x4 = 9960 \cdot psi$$

$\sigma y1 := 0 \qquad \sigma y2 := 0 \qquad \sigma y4 := 0$

Von Mises Failure Criteria:

$$\tau max1 := \left[\left(\frac{\sigma x1 - \sigma y1}{2}\right)^2 + \tau xy1^2\right]^{.5} \qquad \tau max1 = 17238.1 \cdot psi$$

$$\tau max2 := \left[\left(\frac{\sigma x2 - \sigma y2}{2}\right)^2 + \tau xy2^2\right]^{.5} \qquad \tau max2 = 13707.3 \cdot psi$$

$$\tau max4 := \left[\left(\frac{\sigma x4 - \sigma y4}{2}\right)^2 + \tau xy4^2\right]^{.5} \qquad \tau max4 = 4980 \cdot psi$$

Factor of Safety:

$$\sigma 11 := \frac{\sigma x1 + \sigma y1}{2} + \tau max1 \qquad \sigma 21 := \frac{\sigma x1}{2} - \tau max1 \qquad \sigma 11 = 18289.6 \cdot psi \quad \sigma 21 = -16186.7 \cdot psi$$

$$\sigma vm1 := \left(\sigma 11^2 - \sigma 11 \cdot \sigma 21 + \sigma 21^2\right)^{.5} \qquad \sigma vm1 = 29875.8 \cdot psi \qquad Fvm1 := \frac{Fy}{\sigma vm1} \qquad Fvm1 = 1.7$$

$$\sigma 12 := \frac{\sigma x2 + \sigma y2}{2} + \tau max2 \qquad \sigma 22 := \frac{\sigma x2}{2} - \tau max2 \qquad \sigma 12 = 27322.9 \cdot psi \quad \sigma 22 = -91.7 \cdot psi$$

$$\sigma vm2 := \left(\sigma 12^2 - \sigma 12 \cdot \sigma 22 + \sigma 22^2\right)^{.5} \qquad \sigma vm2 = 27368.9 \cdot psi \qquad Fvm2 := \frac{Fy}{\sigma vm2} \qquad Fvm2 = 1.8$$

$$\sigma 14 := \frac{\sigma x4 + \sigma y4}{2} + \tau max4 \qquad \sigma 24 := \frac{\sigma x4}{2} - \tau max4 \qquad \sigma 14 = 9960 \cdot psi \quad \sigma 24 = 0 \cdot psi$$

$$\sigma vm4 := \left(\sigma 14^2 - \sigma 14 \cdot \sigma 24 + \sigma 24^2\right)^{.5} \qquad \sigma vm4 = 9960 \cdot psi \qquad Fvm4 := \frac{Fy}{\sigma vm4} \qquad Fvm4 = 5$$

The above analysis shows that the axle should perform satisfactory for the maximum load condition under static load conditions.

Endurance Limit Modifying Factors

For most steels, the mean endurance limit of rotating beam specimens is approximately 1/2 of the ultimate strength of the material. This endurance limit corresponds to approximately 1 million cycles. The endurance limit is further reduced by the following Endurance Limit Modifying Factors.

Surface Factor	Figure 5-17 Polished Specimen = 1 Ground = .89 Machined = .65 - .8	$Ka := .8$
Size Factor	Kb =1 d<0.3 in Kb=.85 0.3<d<2 in Kb=.75 d>2 in	$Kb := .75$
Reliability factor	Reliability 0.5 Zr=1 Reliability 0.9 Zr=1.288 Reliability 0.95 Zr=1.645 Reliability 0.99 Zr=2.326 Reliability 0.999 Zr=3.091 Reliability 0.9999 Zr=3.719	$Zr := 1.645$
	$Kc := 1 - 0.08 \cdot Zr$	$Kc = 0.868$
Temperature factor	Kd = 1 T<160 F T > 160 F	$Kd := 1$

Sum of all effects

$K := Ka \cdot Kb \cdot Kc \cdot Kd$ $\qquad K = 0.521$

Endurance Limit

$Se := 0.5 \cdot Fut \cdot K$ $\qquad Se = 27.1 \cdot ksi$

Fatigue at Point 1

At point 1 the axle is subjected to combined stress due to both shear and bending.

Shear Stress From Load

$$Ss1 := \frac{(4FatigueWheelLoad)}{\pi \cdot Dia1^2} \qquad Ss1 = 3524.5 \cdot psi$$

Maximum Shear Stress (Torque + Shear)

$$\tau xy1 := \tau 1 + Ss1 \qquad \tau xy1 = 15964.1 \cdot psi \qquad \tau xy1min := \tau 1 - Ss1 \qquad \tau xy1max := \tau 1 + Ss1$$

$$\tau xy1mean := \frac{\tau xy1max + \tau xy1min}{2} \qquad \tau xy1a := \tau xy1max - \tau xy1min \qquad \tau xy1mean = 12.4 \cdot ksi \qquad \tau xy1a = 7 \cdot ksi$$

Bending Moment

$$M1 := FatigueWheelLoad \cdot L1 \qquad M1 = 500 \cdot ft \cdot lbf$$

Bending Stress

$$\sigma x1 := 32 \cdot \frac{M1}{\pi \cdot Dia1^3} \qquad \sigma x1 = 1554.9 \cdot psi \qquad \sigma x1min := -\sigma x1 \qquad \sigma x1max := \sigma x1$$

$$\sigma x1mean := \frac{\sigma x1max + \sigma x1min}{2} \qquad \sigma x1a := \frac{\sigma x1max - \sigma x1min}{2} \qquad \sigma x1mean = 0 \cdot ksi \qquad \sigma x1a = 1.6 \cdot ksi$$

$$\sigma y1mean := 0 \qquad \sigma y1a := 0$$

Von Mises Failure Criteria:

$$\tau xy1mean := \left[\left(\frac{\sigma x1mean - \sigma y1mean}{2}\right)^2 + \tau xy1mean^2\right]^{.5} \qquad \tau xy1mean = 12439.5 \cdot psi$$

$$\tau xy1a := \left[\left(\frac{\sigma x1a - \sigma y1a}{2}\right)^2 + \tau xy1a^2\right]^{.5} \qquad \tau xy1a = 7091.8 \cdot psi$$

$$\sigma 11mean := \frac{\sigma x1mean + \sigma y1mean}{2} + \tau xy1mean \qquad \sigma 21mean := \frac{\sigma x1mean - \sigma y1mean}{2} - \tau xy1mean$$

$$\sigma 11mean = 12439.5 \cdot psi \qquad \sigma 21mean = -12439.5 \cdot psi$$

$$\sigma vm1mean := \left(\sigma 11mean^2 - \sigma 11mean \cdot \sigma 21mean + \sigma 21mean^2\right)^{.5} \qquad \sigma vm1mean = 21.5 \cdot ksi$$

$$\sigma 11a := \frac{\sigma x1a + \sigma y1a}{2} + \tau xy1a \qquad \sigma 21a := \frac{\sigma x1a}{2} - \tau xy1a$$

$$\sigma 11a = 7869.3 \cdot psi \qquad \sigma 21a = -6314.4 \cdot psi$$

$$\sigma vm1a := \left(\sigma 11a^2 - \sigma 11a \cdot \sigma 21a + \sigma 21a^2\right)^{.5} \qquad \sigma vm1a = 12.3 \cdot ksi$$

Because the radius at point 1 is a stress riser we must further reduce the endurance limit of the steel. The Stress concentration Factor from Roark's Formulas for Stress and Strain (Table 37 Case 17). Valid for hr1 between 2 and 20.

$$h1 := \frac{Dia2 - Dia1}{2} \qquad hr1 := \frac{h1}{Rad1} \qquad hr1 = 2.5$$

$$k11 := 1.225 + .831 \cdot hr1^{.5} - .01hr1 \qquad k11 = 2.5$$

$$k21 := -3.79 + .958hr1^{.5} - .257 \cdot hr1 \qquad k21 = -2.9$$

$$k31 := 7.374 - 4.834 \cdot hr1^{.5} + .862hr1 \qquad k31 = 1.9$$

$$k41 := -3.809 + 3.046hr1^{.5} - .595 \cdot hr1 \qquad k41 = -0.5$$

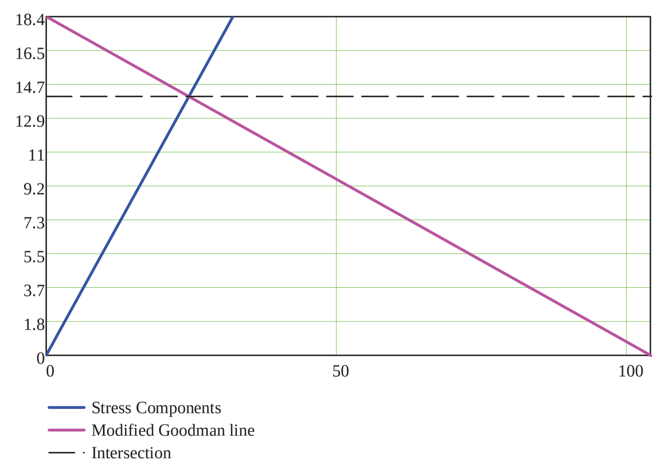
$$Kt1 := k11 + k21 \cdot \left(2 \cdot \frac{h1}{Dia2}\right) + k31 \cdot \left(2 \cdot \frac{h1}{Dia2}\right)^2 + k41 \cdot \left(2 \cdot \frac{h1}{Dia2}\right)^3 \qquad Kt1 = 1.58$$

$Ke1 := \frac{1}{[1 + q\cdot(Kt1 - 1)]}$ $Ke1 = 0.678$ $Se1 := Se\cdot Ke1$ $Se1 = 18.4\cdot ksi$

In the presents fluctuating stress A modified Goodman Diagram can be used to relate the stress to the strength.

Modified Goodman Line $i := 0 .. 150$ $x_i := i$ $y1_i := \frac{-Se1\cdot x_i}{Fut} + \frac{Se1}{ksi}$ $Sm1 := \frac{Se1}{\left(\frac{Se1}{Fut} + \frac{\sigma vm1a}{\sigma vm1mean}\right)}$ $Sa1 := \frac{\sigma vm1a\cdot Sm1}{\sigma vm1mean}$

$ymg1_i := \frac{\sigma vm1a\cdot x_i}{\sigma vm1mean}$ $nfatigue1 := \frac{Sa1}{\sigma vm1a}$ $Sa1_r := Sa1$



Factor of Safety

$nfatigue1 = 1.14$

Fatigue at Point 2

At point 2 the axle is subjected to combined stress due to both shear and bending.

Shear Stress From Load

$Ss2 := \frac{(4FatigueWheelLoad)}{\pi \cdot Dia2^2}$ $\quad Ss2 = 1170.5 \cdot psi$

Maximum Shear Stress (Torque + Shear)

$\tau xy2 := \tau 2 + Ss2$ $\quad \tau xy2 = 1170.5 \cdot psi$ $\quad \tau xy2min := \tau 2 - Ss2$ $\quad \tau xy2max := \tau 2 + Ss2$

$\tau xy2mean := \frac{\tau xy2max + \tau xy2min}{2}$ $\quad \tau xy2a := \tau xy2max - \tau xy2min$ $\quad \tau xy2mean = 0 \cdot ksi$ $\quad \tau xy2a = 2.3 \cdot ksi$

Bending Moment

$M2 := FatigueWheelLoad \cdot L2$ $\quad M2 = 33833.3 \cdot ft \cdot lbf$

Bending Stress

$\sigma x2 := 32 \cdot \frac{M2}{\pi \cdot Dia2^3}$ $\quad \sigma x2 = 20135.9 \cdot psi$ $\quad \sigma x2min := -\sigma x2$ $\quad \sigma x2max := \sigma x2$

$\sigma x2mean := \frac{\sigma x2max + \sigma x2min}{2}$ $\quad \sigma x2a := \frac{\sigma x2max - \sigma x2min}{2}$ $\quad \sigma x2mean = 0 \cdot ksi$ $\quad \sigma x2a = 20.1 \cdot ksi$

$\sigma y2mean := 0$ $\quad \sigma y2a := 0$

Von Mises Failure Criteria:

$$\tau xy2mean := \left[\left(\frac{\sigma x2mean - \sigma y2mean}{2}\right)^2 + \tau xy2mean^2\right]^{.5} \qquad \tau xy2mean = 0 \cdot psi$$

$$\tau xy2a := \left[\left(\frac{\sigma x2a - \sigma y2a}{2}\right)^2 + \tau xy2a^2\right]^{.5} \qquad \tau xy2a = 2340.9 \cdot psi$$

$$\sigma 12mean := \frac{\sigma x2mean + \sigma y2mean}{2} + \tau xy2mean \qquad \sigma 22mean := \frac{\sigma x2mean - \sigma y2mean}{2} - \tau xy2mean \qquad \sigma 12mean = 0 \cdot psi$$

$$\sigma 22mean = 0 \cdot psi$$

$$\sigma vm2mean := \left(\sigma 12mean^2 - \sigma 12mean \cdot \sigma 22mean + \sigma 22mean^2\right)^{.5} \qquad \sigma vm2mean = 0 \cdot ksi$$

$$\sigma 12a := \frac{\sigma x2a + \sigma y2a}{2} + \tau xy2a \qquad \sigma 22a := \frac{\sigma x2a}{2} - \tau xy2a \qquad \sigma 12a = 20404.4 \cdot psi$$

$$\sigma 22a = -268.6 \cdot psi$$

$$\sigma vm2a := \left(\sigma 12a^2 - \sigma 12a \cdot \sigma 22a + \sigma 22a^2\right)^{.5} \qquad \sigma vm2a = 20.5 \cdot ksi$$

Since The Mean Von Misses Stress is zero then the **factor of safety** is

$$nfatigue2 := \frac{Se}{\sigma vm2a} \qquad nfatigue2 = 1.319$$

Fatigue at Point 3

After the bearing the axle is subjected to cyclical bending stress. The Shear stress is zero at this point.

Bending Moment

$M3 := FatigueWheelLoad \cdot L2 \qquad M3 = 33833.3 \cdot ft \cdot lbf$

Stress due to bending at radius

$$\sigma x3 := 32 \cdot \frac{M3}{\pi \cdot Dia2^{3}} \qquad \sigma x3 = 20135.9 \cdot psi$$

Because the radius at point 3 is a stress riser we must further reduce the endurance limit of the steel. The Stress concentration Factor from Roark's Formulas for Stress and Strain (Table 37 Case 17). Valid for hr3 between 0 and 2.

Endurance limit with stress concentration

$$h3 := \frac{Dia3 - Dia2}{2} \qquad hr3 := \frac{h3}{Rad2} \qquad hr3 = 1.2$$

$k13 := .927 + 1.149 \cdot hr3^{.5} - .086 hr3 \qquad k13 = 2.1$

$k23 := .015 - 3.281 hr3^{.5} + .837 \cdot hr3 \qquad k23 = -2.6$

$k33 := .847 + 1.716 \cdot hr3^{.5} - .506 hr3 \qquad k33 = 2.1$

$k43 := -.790 + .417 hr3^{.5} - .246 \cdot hr3 \qquad k43 = -0.6$

$$Kt3 := k13 + k23 \cdot \left(2 \cdot \frac{h3}{Dia3}\right) + k33 \cdot \left(2 \cdot \frac{h3}{Dia3}\right)^{2} + k43 \cdot \left(2 \cdot \frac{h3}{Dia3}\right)^{3} \qquad Kt3 = 1.5$$

$$Ke3 := \frac{1}{1 + q \cdot (Kt3 - 1)} \qquad Ke3 = 0.709$$

$Se3 := Se \cdot Ke3 \qquad Se3 = 19.2 \cdot ksi$

The factor of safety is then

$$Fs := \frac{Se3}{\sigma x3} \qquad Fs = 0.954$$

The expected part life can be estimated.

$$m := \frac{\log\left(.9 \cdot \frac{Fut}{Se3}\right)}{3} \qquad m = 0.2 \qquad b := \log\left[\frac{(.9 \cdot Fut)^2}{Se3 \cdot psi}\right] \qquad b = 5.7$$

$$N := \frac{10^{\left(\frac{b}{m}\right)}}{\left(\frac{\sigma x3}{psi}\right)^{\left(\frac{1}{m}\right)}} \qquad N = 816002.4$$

cycles per year --- 4 configurations, 20 antennas per configuration, average distance moved 1.5 miles

Wheel diameter 3'

$$Cpy := \frac{4 \cdot 20 \cdot 1.5 \cdot 5280}{\pi \cdot 3 \cdot yr} \qquad Cpy = 67227 \cdot \frac{1}{yr}$$

$$Life := \frac{N}{Cpy} \qquad Life = 12.1 \cdot yr$$

As can be seen from the chart below, the axle stress is slightly higher than the endurance limit of the material.

$St_0 := \frac{Fut}{ksi}$ $St_1 := .9 \cdot \frac{Fut}{ksi}$ $St_2 := \frac{Se3}{ksi}$ $St_3 := \frac{Se3}{ksi}$

$N_0 := 100$ $N_1 := 1000$ $N_2 := 1000000$ $N_3 := 10000000$

$i := 1 .. 4$ $S3_i := \frac{\sigma x3}{ksi}$