# EVLA MEMO #10

# PREDICTED OPTICAL FIBER LENGTH VARIATIONS DUE TO TEMPERATURE CHANGES

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I. Abstract

This memo uses Duhamel's theorem to predict the change of path length due to temperature in the optical fiber reference distribution system. The predicted change will be shown to be quite significant. The significance of the change will require a fiber optic stabilization system for the Expanded VLA.

II. Introduction

The expanded VLA project because of the increased bandwidth requirements will require conversion from the waveguide system to a fiber optic system. The expanded VLA will digitize the astronomical data and modulate this digital data over the fiber link. Each antenna will require certain LO reference frequencies to maintain a coherent phase and frequency relationship. These LO reference frequencies are to be modulated on to the fiber optic system for distribution to each antenna. The fiber optic distribution of the signals requires that the path be stabilized in order to preserve coherence. The primary instability in the path is due to temperature variations of the fiber. I shall attempt to model the temperature variations of the path in order to better understand the requirements of the system.

III. System Model

The system will be modeled by breaking down the path into segments that are exposed to different environments. Table I shows the different segments with their corresponding maximum expected path length.

Table I: Fiber Segments

| Segment | Max expected path length |
|---|---|
| 1. Control building segment<br>( Antenna back end rack to underground segment) | 30 meters |
| 2. Underground segment<br>(Control building to Antenna pad) | 23000 meters |
| 3. Antenna Pad segment<br>(from underground to antenna entry point) | 8 meters |
| 4. Antenna segment<br>(antenna entry point to demodulation rack) | 20 meters |

Exposed segments are expected to be insulated. Antenna and control building temperature variations are expected to be less than $\pm 1^{\circ} C$ per hour from a nominal temperature. Analysis will be performed using both regular single mode fiber and temperature compensated single mode fiber.

## IV. Analysis

Analysis is being done to determine the over all length variation due to temperature. The length change will be shown as an average per second change.

### Segment 1

$\Delta L$ =7ppm/$^{\circ} C$ x .000030 Mm x 2$^{\circ} C$ /hr x 1hr/3600s =0 .12 µm/s For 7ppm fiber

$\Delta L$ =1ppm/$^{\circ} C$ x .000030 Mm x 2$^{\circ} C$ /hr x 1hr/3600s = 0.016 µm/s For 1ppm fiber

### Segment 2

For segment 2, we have to calculate the temperature variations for the depth of the fiber. In order to make these calculations, I am going to make the assumptions that outside temperature varies sinusoidally on a seasonal basis. From Duhamel's theorem we know that

$$T(x,t) = x / (2 * \sqrt{\pi * \alpha}) * \int_0^t T_s(\lambda) * e^{-\left[x^2/(4*k(t-\lambda))\right]} \Big/ (t-\lambda)^{\frac{3}{2}} \, d\lambda \qquad 1$$

This is a boundary value problem where $T_s(\lambda)$ is the surface temperature. Given by:

$$T_s(\lambda) = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t) + M \qquad 2$$

M is an offset term which can be ignored since we are only interested in the temperature change. The first term is the daily variation and the second term is the seasonal variation.

This problem has been solved by Carslaw and Jaeger (1). The solution is

$$T(x,t) = e^{-x*\sqrt{\omega/2\kappa}} * \cos\left(\omega t - x * \sqrt{\omega / 2\kappa}\right) \qquad 3$$

Note : That as $\omega$ gets large the function decreases much faster. This allows us to ignore the daily variations.

$\kappa$ is called the Diffusivity constant and is a measured quantity depending upon the type of soil. $\kappa$ for average soil is 0.0045 $cm^2/s$

$A_2$ is the amplitude of the average seasonal variation. Peak lows are down to about 10 $^\circ F$ or -12 $^\circ C$, peak highs are about 95 $^\circ F$ or 35 $^\circ C$, average lows are about 30 $^\circ F$ or -1 $^\circ C$, average highs are about 80 $^\circ F$ or 27 $^\circ C$. $A_2$ will then be 14 $^\circ C$. Temperature data is actual for the 1999 to 2000 year.

$\omega_2$ is a 1 year period 2*$\pi$ /365=0.017 cycles/day

Substituting eq 3 reduces to:

$$T(x,t) = e^{-x*\sqrt{.017/(2*(0.0045*3600*24/(100*100)))}}$$
$$* \cos\left(.017t - x*\sqrt{.017/(2*(0.0045*3600*24/(100*100)))}\right)$$

Or

$$T(x,t) = e^{-.47*x} * \cos(.017t - .47x) \text{ Meters/day}$$

Looking only at the amplitude $|T(x,t)| * A_2$

For a 1 meter depth temperature magnitude will be 8.7 $°C$

For a 1.5 meter depth (5 ft) Temperature magnitude will be 6.9 $°C$

This means that for a 1 meter depth the temperature will vary on average 17.4 $°C$ per year peak to peak. And for a 1.5 meter depth the temperature will vary on average 13.8 $°C$ per year peak to peak. Note the peaks are 180 days apart.

Calculating the average per second amount of length change of the fiber for segment 2

At 1 meter 7ppm/ $°C$ x 17.4 $°C$ /yr/24hr/3600s/180x .023Mm=0.18µm/s For 7ppm fiber

At 1.5 meters 7ppm/ $°C$ x 13.8 $°C$ /yr/24hr/3600s/180x .023Mm=0.14µm/s For 7ppm fiber

At 1 meter 1ppm/ $°C$ x 17.4 $°C$ /yr/24hr/3600s/180x .023Mm=0.025µm/s For 1ppm fiber

At 1.5 meters 1ppm/°C x 13.8 $°C$ /yr/24hr/3600s/180x .023Mm=0.02µm/s For 1ppm fiber

Segment 3

Calculating the length variations for an exposed length of fiber will be very difficult due to the high number of variables such as maximum temperature changes, wind effects, and solar effects. I will make what I believe is a worst case analysis where I assume a 10 $°C$ change in one hour. I, however;

recommend that the fiber for segment 3 be placed in a steel pipe wrapped with heat tape and insulated. This will protect it from animals as well as the weather.

$\Delta L$ =7ppm/ $^\circ C$ x .000008 Mm x 10 $^\circ C$ /hr x 1hr/3600s =0 .15µm/s For 7ppm fiber

$\Delta L$ =1ppm/ $^\circ C$ x .000008 Mm x 10 $^\circ C$ /hr x 1hr/3600s = 0.022m/s For 1ppm fiber

Segment 4

$\Delta L$ =7ppm/ $^\circ C$ x .000020 Mm x 2 $^\circ C$ /hr x 1hr/3600s =0 .077 µm/s For 7ppm fiber

$\Delta L$ =1ppm/ $^\circ C$ x .000020 Mm x 2 $^\circ C$ /hr x 1hr/3600s = 0.011 µm/s For 1ppm fiber

V. Analysis Results

Adding together all of the variations from each of the segments and assuming the burial depth is 1.5 meters we get:

.12µm/s+.14µm/s+.15µm/s+.077µm/s=.487µm/s for 7 ppm fiber

.016µm/s+.02µm/s+.022µm/s+.011µm/s=.069µm/s for 1 ppm fiber

We can now calculate the required amount of change needed for the stabilization system. If we choose to put 500 meters of 7ppm fiber onto a thermal electric plate then the temperature of the fiber on the plate needs to change at the following rate:

7ppm x $\Delta T$ x .0005Mm=.487µm/s for 7 ppm fiber

$\Delta T$ =.139 m $^\circ C$ /s

7ppm x ΔT x .0005Mm=.069µm/s for 1 ppm fiber

$\Delta T$ =.02 m $^\circ C$ /s

As a rule of thumb, we will need 10x for a stable system or

$\Delta T$ =1.39 m$^\circ C$/s For 7ppm fiber

$\Delta T$ =0.2 m$^\circ C$/s For 1ppm fiber

We will also need to know the amount of correction available in the system.

For segment 2 the total amount of fiber length change at a burial depth of 1.5 m over a one year period (which is actually 6 months peak to peak) will be:

7ppm x 13.8$^\circ C$ x .023 Mm=2.2 meters For 7 ppm fiber

1ppm x 13.8$^\circ C$ x .023 Mm=.32 meters For 1 ppm fiber

If we have a thermoelectric plate that can change ±50 $^\circ C$ with 500 meters of 7 ppm fiber then:

One half of the total change is 7ppm x .000500 Mm x 50 $^\circ C$ = .175 meters. This means for 7 ppm fiber we would have to reset or recalibrate the phase correction system at least once a month and for 1 ppm fiber about every 6 months.

## VI. Other Results

During my reference search for a good way to stabilize the fiber, I stumbled across a paper by Calhoun, etal. (2) In this paper, thermocouples were buried at various depths at NASA’s Goldstone Tracking Station. The results for actual temperature measurements at this site agree very closely with my predicted results.

## VII. Conclusion

Seasonal temperature variations, contrary to popular beliefs, are high even at depths of 1.5 meters. Stabilization of the fiber becomes more difficult and more important. With such a high rate of temperature change I recommend we choose 1 ppm temperature compensated fiber and bury it at a depth of 1.5 meters. This has also been recommend to me by JPL (3). JPL points out that plows are available which economically allow burial at 1.5 meters.

## References

1. Carslaw, H. S. and Jaeger, J. C., *Conduction of Heat in Solids*, Oxford at the Clarendon Press, 1959. pp 67 and 497.

2. Calhoun, Malcolm; Kuhnle, Paul; and Law, Julius; *Fiber Optic Reference Frequency Distribution to Remote Beam Waveguide Antennas*, Jet Propulsion Laboratory and California Institute of Technology, Pasadena, California.

3. Private communication with George Lutes of JPL.