MHD THEORY OF PULSAR WINDS AND PLERIONS

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Abstract

Young, rapidly rotating pulsars are surrounded by compact synchrotron nebulae, the Crab nebula being the best studied and most known example. These nebulae are continuously pumped by electron-positron plasma and magnetic field emanating from the pulsar in the form of relativistic, magnetized wind. High-resolution images in the X-ray and optical bands revealed a peculiar "jet-torus" structure in the inner parts of these nebulae. This discovery clearly indicates significant anisotropy of the pulsar wind, which has been ignored so far in simplified theoretical models of the nebula. The recent radio and IR observations of the Crab suggest, contrary to what has been assumed in the most popular models, that the radio emitting electrons are accelerated now in the same region as the ones responsible for the optical to X-ray emission. All these findings provide important clues about the general physical processes such as self-collimation and energy dissipation in relativistic, magnetized outflows. I discuss physics of pulsar winds and surrounding nebulae focusing attention on attempts to incorporate the new observational results into a self-consistent picture.

1 Introduction

Andrew Melatos has already reviewed variety of phenomena observed in pulsar wind nebulae (PWNe) and outlined basic theoretical problems (see these proceedings). Now I can focus upon the basic physical processes in relativistic, magnetized outflows. The central question the theory should answer is how rotational energy of the pulsar is transferred eventually to relativistic particles in the nebula. It is widely accepted that the pulsar loses its rotational energy predominantly on generation of a relativistic wind dominated by electromagnetic fields rather than plasma (the Poynting-to-matter energy flux ratio $\sigma \gg 1$). As the wind is confined within a non-relativistic surrounding, it must terminate at a strong reverse shock. The shocked plasma inflates a bubble within the surrounding medium and produces the observed non-thermal electromagnetic emission from the radio to the γ-ray band. The most serious problem with this picture is that a strong shock and a non-relativistic post-shock flow arise only if the energy at the shock front is carried mainly by the particles (Rees & Gunn 1974). So the question arises of what mechanism transforms so efficiently the electromagnetic energy into the plasma energy (the σ problem).

I am going to discuss energy transformation processes in Poynting-dominated outflows and attempt to develop a self-consistent picture of the pulsar wind and surrounding nebula. The observational results about the Crab nebula will serve predominantly as a guide in our analysis of PWNe. The Crab is the prototype of plerions, PWNe confined within supernova remnants, and development of the theory was promoted predominantly by new findings about this nebula. Of course this does not mean that all PWNe are similar to the Crab. However the Crab is studied much better than other sources and therefore the theory should be first of all confronted with the comprehensive observational data about this source.

2 Pulsar wind

According to conventional wisdom, pulsars emit an electron-positron plasma (maybe with some admixture of ions), which carries away energy in the form of an ultra-relativistic magnetized wind together with large-amplitude waves. The energy of the ejected plasma is small compared with the energy of generated electromagnetic fields. Therefore the total power lost by a rotating, magnetized neutron star may be estimated using the formula for a magnetic dipole rotating in vacuum (e.g., Michel 1991). However, the presence of plasma changes the physical picture significantly; for example, even an axisymmetric rotator surrounded by plasma loses energy by driving a plasma wind, in contrast with an aligned magnetic dipole in vacuum. In the case of an oblique rotator, the energy lost in the wind can be regarded as shared between a steady, axisymmetric component of the Poynting flux and one oscillating with the rotational period of the star.

It is sometimes instructive to discuss the system pulsar-pulsar wind-PWN in terms of electrical engineering. The pulsar may be considered as a rotating magnet generating an electromagnetic field like in man-made electric power generators. This electromagnetic field excites currents in the plasma of the wind; the energy is transferred by these currents (including the displacement current) and is eventually released in the nebula so that the basic question of the theory is: what is the nature of the load where the electromagnetic energy is released? In order to answer this question, one should consider electromagnetic structure of the pulsar wind.

Let us first consider the steady state magneto-hydrodynamic (MHD) wind. Magnetic field lines in the wind are wrapped backward by rotation of the star. Far beyond the light cylinder, the field becomes predominantly azimuthal whereas the flow becomes predominantly poloidal. The hoop stress of the azimuthal magnetic field is very large as compared with the inertia forces of the flow. Therefore it seems at first glance that the flow should be compressed toward the axis. However careful considerations show that such a collimation does not occur in ultra-relativistic flows because electric force nearly compensates hoop stresses (Tomimatsu 1994; Beskin, Kuznetsova & Rafikov 1998; Chiueh, Li & Begelman 1998; Bogovalov & Tsinganos 1999; Lyubarsky & Eichler 2001). Moreover, it is shown in the cited works that the flow remains practically ballistic in this case, i.e., it is not only nearly radial but its acceleration is negligibly small. The last statement is of special importance because it means that the electromagnetic energy is not transferred to the plasma in the case under consideration.

I should stress that it does not mean that σ remains large in any steady Poynting-dominated flow. Relativistic MHD equations do not forbid in principle acceleration of the flow until a significant fraction of the Poynting flux is converted into the kinetic energy. Moreover there are self-similar solutions demonstrating decreasing of the Poynting flux to the equipartition level or even below it (Li, Chiueh & Begelman 1992; Vlahakis 2004). However such an energy transformation occurs only if the flow lines are not radial. In the above-mentioned solutions, this is achieved by a special choice of the poloidal flux distribution such that the poloidal magnetic field diverges at the rotation axis, the total poloidal flux is infinite and the magnetic surfaces do not converge to origin (note that although the poloidal field is small beyond the light cylinder, the stress of this field may be not negligible in some cases because the hoop stress is nearly compensated by the electric force). Such structures may be formed above a disk but hardly ever may be matched to a non-singular source subtending a finite magnetic flux.

There is only one exact solution of the relativistic MHD equations describing a magnetosphere of a rotating star, namely that of the split monopole (Michel 1973). In this solution, the plasma flows radially from the origin to infinity. Of course the flow lines in the realistic (dipole) magnetosphere can not be radial everywhere so this solution is degenerate. However Ingraham (1973) and Michel (1974) showed that independently of the field structure at the origin, the flow lines beyond the light cylinder become radial. Their solution may be considered only as an intermediate asymptotic because it was obtained in the so called force-free approximation, which is violated far enough from the light cylinder. However the asymptotic analysis of the full set of MHD equations show that deviations from the radial flow grow only logarithmically so that noticeable collimation of the flow, both and transformation of a significant fraction of the Poynting flux into the kinetic energy of the flow, occur only at unreasonably large distances (Tomimatsu 1994; Beskin, Kuznetsova & Rafikov 1998; Chiueh, Li & Begelman 1998; Bogovalov & Tsinganos 1999; Lyubarsky & Eichler 2001; Heyvaerts & Norman 2003). Therefore one can conclude that a steady wind remains Poynting dominated unless highly contrived boundary conditions are satisfied.

However the real pulsar wind is by no means steady. Variable electromagnetic fields, which an obliquely rotating pulsar magnetosphere excites near the light cylinder, propagate outward in the form of waves. The waves decay relatively easily, because dissipation processes can come into play on the short scale-length associated with a wavelength (Usov 1975; Michel 1982; 1994; Coroniti 1990; Melatos & Melrose 1996;

Lyubarsky & Kirk 2001; Kirk & Skjæraasen 2003; Lyubarsky 2003a,b). Therefore dissipation of waves is an important mechanism of energy transformation in the pulsar wind.

There is a variety of electromagnetic waves in rarefied magnetized plasmas. However it seems reasonable to assume that only MHD waves (those satisfying the condition $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$) may be generated by the rotating magnetosphere. The reason is that according to the convenient view, the plasma density in the pulsar wind is sufficiently large to screen the electric field in the proper plasma frame so that low-frequency electromagnetic waves are heavily damped (e.g., Asseo et al. 1978; Melatos & Melrose 1996).

In the equatorial belt of the flow, the magnetic field line at a given radius alternates in direction with the frequency of rotation, being connected to a different magnetic pole every half-period. Michel (1971) pointed out that the flow in this zone should evolve into regions of cold magnetically dominated plasma separated by very narrow, hot current sheets. According to the standard classification of MHD waves, such a structure should be considered as an entropy wave. The formation of this pattern may be imagined as follows: in the axisymmetric case, a current sheet separates the two hemispheres with with opposite polarity beyond the light cylinder; as the obliquity increases, this sheet begins to oscillate about the equatorial plane. Such a picture is observed in the solar wind. In a quasi-radial flow the amplitude of these oscillations grows linearly with the radius, and at large distances one can imagine locally quasi-spherical current sheets following each other and separating the stripes of magnetized plasma with opposed polarity (the striped wind). Bogovalov (1999) has found an exact solution for the oblique split monopole case, which has precisely this structure.

At high latitudes, where the magnetic field does not change sign, the magnetic oscillations are transferred away in the form of fast magneto-sonic and Alfven waves; generation of such waves by the rotating, slightly non-axisymmetric magnetosphere was considered by Bogovalov (2001a).

The X-ray image of the inner Crab nebula (Weisskopf 2000) clearly suggests that most of the energy is transported in the equatorial belt of pulsar wind. In this region, energy is transferred predominantly by alternating fields (the mean field at the equator of the wind from an obliquely rotating magnetosphere is zero). Therefore the fate of the striped wind is of special importance. The alternating fields in the striped wind decay because of the current starvation in current sheets separating strips with the opposite magnetic field (Usov 1975; Michel 1982, 1994). The current required to sustain them goes as r^{-1} whereas the available number of charge carriers falls off faster, as r^{-2}. Eventually the particle number density becomes insufficient to maintain the necessary current and the alternating fields annihilate. The possibility of observing this decay as high energy pulsar radiation was examined by Kirk, Skjæraasen & Gallant (2002). Lyubarsky & Kirk (2001) showed that the flow significantly accelerates in the course of the field dissipation and this dilates the timescale over which the wave decays. Therefore the dissipation radius exceeds the radius of the termination shock unless the dissipation rate is so high that the field annihilates with the sound velocity (Kirk & Skjæraasen 2003). Thus one should conclude that the Poynting flux in the striped wind need not dissipate until the wind enters the termination shock. In this case all the energy should release within the termination shock where the flow is sharply compressed (see the next section).

3 The pulsar wind termination shock and spectra of plerions

According to MHD models (Rees & Gunn 1974; Kennel & Coroniti 1984a,b; Emmering & Chevalier 1987), the pulsar wind terminates in a standing shock at a radius defined by the condition that the confining pressure balances the momentum flux of the wind. At the shock front the wind energy is released into the relativistic particles responsible for the observed radiation. The observed brightness and the spectral index distributions in the Crab are generally consistent with the assumption that the relativistic particles are accelerated at the termination shock and then fill in the nebula, spending the acquired energy on synchrotron emission and pdV work (Kennel & Coroniti 1984b; Amato et al. 2000).

Plerions emit synchrotron radiation from the radio to the γ-ray band. In the radio band the spectrum is flat; the spectral flux may be presented as a power law function of the frequency, $\mathcal{F}_\nu \propto \nu^{-\alpha}$, with the spectral index α = 0–0.3. At high frequencies the spectrum steepens and the typical spectral slope in the X-ray band is $\alpha > 1$. The overall spectrum of the Crab may be described as a broken power law with the spectral breaks around 10^{13} Hz, few$\times 10^{15}$ Hz and around 100 keV. The energy spectrum of the emitting electrons (and positrons – below by electrons I mean both electrons and positrons) extends from $\leq$ 100 MeV to $\sim 10^9$ MeV; the plasma energy density is dominated by electrons with $E \sim 10^6$ MeV radiating in the ultraviolet band.

The conventional wisdom is that all but (1–3)$\times 10^{-3}$ fraction of the Poynting flux has already been converted into the plasma kinetic energy by the time the flow arrives the termination shock (Kennel & Coroniti 1984a,b; Emmering & Chevalier 1987, Begelman & Li 1992). Kennel & Coroniti (1984b) postulated that the wind from the Crab pulsar is accelerated to the Lorentz factor $\Gamma_w \approx 3\times 10^6$ and at the termination shock, a power-law particle spectrum is formed at the energies exceeding $E \sim m_e c^2 \Gamma_w \sim 1$ TeV. These electrons emit from the ultraviolet to the γ-ray bands. Optical electrons appear in the nebula as a result of synchrotron cooling of TeV electrons.

The formation of the power-law spectrum at $E > m_e c^2 \Gamma_w$ was considered by Hoshino et al. (1992). They suggested that the pulsar wind is loaded by ions; then ion cyclotron waves are collectively emitted at the shock front and pairs are accelerated by resonant absorption of these waves. Their simulations of the relativistic shock in electron-positron-ion plasma confirmed that a power-law spectrum is formed by such a resonant absorption. This model not only accounts for the optical-to-X-ray spectrum of the Crab but offers an explanation of the synchrotron emission enhancements observed in the vicinity of the termination shock as the wisps (Scargle 1969; Hester et al. 2002). Gallant & Arons (1994) and Spitkovsky & Arons (2004) argue that the wisps arise in the regions where reflection of the ions in the self-consistent magnetic field causes compressions of the electron-positron plasma.

A stumbling block for this model is the radio-to-infrared emission, which is generated by electrons with energies 10^2–10^5 MeV. According to the shock acceleration theory, the particle spectrum remains Maxwellian below $m_e c^2 \Gamma_w \sim 1$ TeV. Therefore the observed power-law radio-to-infrared spectrum cannot be reproduced. Kennel & Coroniti (1984b) and Atoyan (1999) got around this problem assuming that the low energy electrons were injected at a very early stage of the plerion evolution. The synchrotron lifetime of the radio emitting electrons significantly exceeds the plerion age. Therefore one can not exclude a priori that the overall spectrum depends on history of the nebula. Arons (1998) proposed that that the radio electrons may be accelerated now but not in the same place as high-energy electrons; the X-ray electrons may be injected in the equatorial belt whereas the radio ones in the polar region. However the apparent continuity of the overall spectrum of the nebula from the radio to the γ-ray band favors for a single population of emitting electrons. Moreover the spectral break at 10^{13} Hz may be simply accounted for the synchrotron burn off effect assuming that particles emitting from the radio to the optical bands are injected more or less homogeneously in time with the single power law energy distribution. This view is supported by Gallant & Tuffs (1999, 2002) who found that the infra-red spectral index in the central parts of the Crab is close to that in the radio, and gradually steepens outward. Recent observations of the wisps in the radio band (Bietenholtz & Kronberg 1992; Bietenholtz, Frail & Hester 2001) also suggest that the radio emitting electrons are accelerated now in the same region as the ones responsible for the optical to X-ray emission.

If the radio emitting electrons are injected into the Crab nebula till the present time, the injection rate of electrons should be about $\dot{N} = 10^{40}$–10^{41} s^{-1}. This is roughly compatible with the estimates of the particle loss rate from the pulsar magnetosphere, based on the observed optical pulses (Shklovsky 1970). The spindown power divided by this pair output yields an average Lorenz-factor $\Gamma_w = 6\times 10^4/\dot{N}_{40}$, incompatible with the generally accepted value of few$\times 10^6$. Analysis of the radio spectrum yields even more severe constraints on the pulsar wind parameters and possible mechanisms of the particle acceleration at the termination shock.

The observed spectral slope in the radio band, $\alpha = 0.26$, implies the energy spectrum of the injected electrons of the form $N(E) \propto E^{-1.5}$. In this case, most of the particles find themselves at the low energy end of the distribution whereas particles at the upper end of the distribution dominate the energy density of the plasma. Taking into account that no sign of a low frequency cut-off is observed in the Crab spectrum down to about 10 MHz, one concludes that the above spectrum extends down to $E_{min} \leq 100$ MeV. The kinetic energy of the upstream flow needs to be very low if most of the electrons end up at about 100 MeV. If the pulsar spin-down power, L_{sd}, is converted into the kinetic energy of the wind, the above upper limit on the low-frequency break in the Crab spectrum suggests that the wind Lorentz-factor, Γ_w, does not exceed a few hundred. On the other hand, the electron energy spectrum $E^{-1.5}$ implies that the total energy per electron in the flow is much larger than 100 MeV so there seems to be an energy reservoir in the flow that eventually distributes itself mostly to a small fraction of the particles. Gallant et al. (2002), modifying the original idea of Hoshino et al. (1992), suggested that the wind is loaded by such amount of ions that their kinetic energy dominates the wind energy flux. At the shock front, the ions collectively emit about one half of their energy as cyclotron waves and then the radio emitting pairs are accelerated by resonant absorption of these waves. The necessary ion injection rate, $\sim L_{sd}/(m_p c^2 \Gamma_w) \sim 10^{39}$ s^{-1}, vastly exceeds the fiducial Goldreich-Julian elementary charge loss rate, $\sim 3\times10^{34}$ s^{-1}. Although one can not exclude by observations that pulsars emit the required amount of ions, the available pulsar models do not assume an ion outflow with the rate exceeding the Goldreich-Julian charge loss rate (Cheng & Ruderman 1980; Arons 1983).

Thus the conventional two-step model (Poynting flux $\rightarrow$ kinetic energy of the wind $\rightarrow$ accelerated particles) faces severe difficulties in both steps: the Poynting flux can scarcely be totally transformed into kinetic energy flux in the wind (see Sect. 2), neither can the kinetic energy dominated upstream flow produce the observed energy distribution of particles in the nebula. For this reason, a one-step process (Poynting flux $\rightarrow$ accelerated particles) looks preferable (Lyubarsky 2003b). If most of the spin-down energy is still stored in the striped magnetic field when the flow enters the termination shock, the pre-shock Lorentz factor of the flow is relatively small, which implies a low cut-off in the energy distribution of the accelerated particles. When the flow enters the shock, the plasma is compressed, so that the alternating magnetic fields easily annihilate, transferring the energy to the particles.

It follows from both analytical and numerical studies that particles are readily accelerated in the course of the magnetic field reconnection to form an energy distribution with a power-law index about unity (Romanova & Lovelace 1992; Zenitani & Hoshino 2001; Larrabee, Lovelace & Romanova 2002). Electrons with such a distribution emit synchrotron radiation with a flat spectrum, which led Romanova & Lovelace (1992) and Birk, Crusius-Watzel & Lesch (2001) to suggest that reconnection plays a crucial role in flat-spectrum extragalactic radio sources. It seems natural to assume that the radio-to-optical emission of plerions is also generated by electrons accelerated in the course of reconnection of the alternating magnetic field at the pulsar wind termination shock.

The high-energy spectrum may be attributed to the Fermi acceleration of particles pre-accelerated in the reconnection process. It has been argued that the first-order Fermi mechanism cannot operate in a perpendicular shock (such that the magnetic field is perpendicular to the flow velocity), as the particles are prevented from diffusing back upstream by the fact that their guiding centers must follow the field lines, which run parallel to the shock front. However field annihilation at the shock may leave just downstream a highly turbulent small-scale structure of the field with the chaotic component significantly exceeding the regular field. Particle scattering off these strong inhomogeneities may result in spatial diffusion necessary for the Fermi mechanism to operate. It is interesting that Fermi acceleration at an ultra-relativistic shock yields an energy distribution $E^{-2.2}$ (Bednarz & Ostrowski 1998; Gallant & Achterberg 1999; Kirk et al. 2000; Achterberg et al. 2001), exactly what is necessary to explain the X-ray spectrum of the Crab.

As the first step in the study of the physics of the shock in the striped wind, Lyubarsky (in progress) performed 1.5-dimensional particle-in-cell simulations of such a shock (1.5-dimensional means that the code is one-dimensional in the coordinate space but two-dimensional in the momentum space). The results of simulations show that the alternating fields are efficiently dissipated within the shock structure and the post-shock MHD parameters (plasma density, temperature and the mean magnetic field) are the same as if the energy of the alternating field has already been converted into the plasma kinetic energy upstream of the shock. Therefore the MHD flow downstream of the shock is independent of whether the alternating field dissipated in the upstream flow or just at the shock. Expectations to find a non-thermal tail in the post-shock particle spectrum were not realized, the Maxwellian distribution was found downstream of the shock. This may be attributed to a highly idealized one-dimensional field structure in the simulations. Reconnection typically occurs in cross-points and therefore two- or three-dimensional simulations are necessary in order to reproduce correctly particle dynamics within the shock structure.

4 MHD flow within the nebula

The Crab and similar nebulae may be considered as bubbles inflated in the surrounding medium by the pulsar wind. It is the wind plasma heated to relativistic temperatures at the termination shock that fills the nebula and produces the observed non-thermal electromagnetic emission from the radio to the γ-ray band. Early spherically symmetrical MHD models of the nebula (Rees & Gunn 1974, Kennel & Coroniti 1984a,b) seemed to describe its main properties perfectly well. In particular, an apparent central hole in the nebula brightness distribution (Scargle 1969) was identified with the termination shock in the theoretical models of the nebula. The radius of the hole, about 0.1 pc, is compatible with the radius of the termination shock estimated from the balance between the momentum flux in the wind and the confining pressure. The observed elongation of the nebula was attributed to the hoop stress of the toroidal magnetic field (Begelman & Li 1992; van der Swaluw 2003).

The recent *Chandra* and *HST* images show in unprecedented detail the structure of the high energy plasma in the Crab nebula (Weisskopf et al. 2000; Hester et al. 2002). A remarkable jet-torus structure, already revealed in earlier observations (Brinkmann, Aschenbach & Langmeier 1985; Hester et al. 1995), is now clearly resolved. As similar structures have been found in other PWNe (Gaensler, Pivovaroff & Garmire 2001; Helfand, Gotthelf & Halpern 2001; Pavlov et al. 2001; Gaensler et al. 2002; Lu et al. 2002), this is likely to be a generic phenomenon. The inner boundary of the X-ray torus coincides with the estimated position of the standing shock. Rapidly moving wisps and variable knots were also found in this region (Scargle 1969; Hester et al. 1995, 2002). The intriguing "jet-torus" dichotomy prompted calls for re-examining of the theory. The most radical proposals involve abolishing of the MHD approximation altogether and developing of purely electromagnetic models (Blandford 2002; Michel 2002). However, such a dramatic turn seems to be a bit premature as the condition of spherical symmetry utilized in the classical MHD models may simply be too restrictive and the effects of relaxing of this constraint still remain to be fully investigated.

The observed structure suggests that the pulsar wind is highly anisotropic, most of the energy being transported in the equatorial belt. Although the angular structure of the relativistic wind from the obliquely rotating pulsar magnetosphere still can not be derived from the first principles, the available idealized models do show that the energy flux have a maximum at the equator. In the wind from the split monopole, the Poynting flux is proportional to $\sin^2\theta$ (Michel 1973), where θ is the polar angle, so that the energy flows predominantly along the equatorial plane. Such a distribution seems to be the generic feature of axisymmetric MHD flows because the toroidal field generally decreases toward the axis; this conclusion is supported by the asymptotic solution for the axisymmetric Poynting-dominated winds in the far zone (Ingraham 1973; Michel 1974). The situation is less clear for the wind from oblique rotators however Bogovalov (1999) demonstrated that the angular distribution of the Poynting flux from an obliquely rotating split monopole is the same as from the aligned one so in this case also most of the energy flows along the equator. Taking this into account it seems reasonable to assume that the real pulsar wind exhibit the same feature. An important point is that the termination shock in such a wind is non-spherical, being significantly closer to the pulsar at the axis than at the equator (Lyubarsky 2002; Bogovalov & Khangoulyan 2002).

Crab's jet as well as jets of other pulsars appears to originate from the pulsar and propagate along the rotational axis. This seems to indicate that they are formed within the pulsar wind and the collimation by magnetic hoop stress suggests itself. However, such a collimation is found to be extremely ineffective in ultra-relativistic flows (see Sect. 2). Moreover, jets of such an origin would have to be ultra-relativistic whereas the observed jets are certainly not. Indeed, the mere fact that both the jet and the counter-jet are seen in the images of the Crab and the Vela nebulae alone rules out ultra-relativistic speeds. Moreover, the direct observations of the proper motion in Crab's and Vela's jets indicate rather moderate velocities of $\approx (0.3\text{–}0.7)c$ only (Hester et al. 2002; Pavlov et al. 2003). Lyubarsky (2002) pointed out that although magnetic collimation is futile in the pulsar wind itself, it becomes much more effective downstream of the termination shock where the flow is no longer relativistic and therefore Crab's jet could be formed via magnetic collimation in this region. Annihilation of the alternating field (see the previous sections) ensures that in the equatorial belt, where most of the energy is transferred, the residual magnetic field is low. However the magnetization of the high latitude flow may be significant. This naturally results in separation of the post-shock flow into an equatorial disk and a magnetically collimated polar jet. In this model, the observed jet-like feature arises naturally because of the axial compression of the synchrotron emitting plasma. The jet appears to originate from the pulsar simply because along the symmetry axis the termination shock is much closer to the wind origin.

Komissarov & Lyubarsky (2003a,b) have simulated numerically the flow produced by the anisotropic pulsar wind within a plerionic nebula. They assumed the same angular distribution of the total energy flux in the wind, f_{tot}, as in the split monopole solution (Michel 1973; Bogovalov 1999), namely

$$f_{tot} = \frac{f_0}{r^2}(\sin^2\theta + 1/\sigma_0) \qquad (1)$$

where r and θ are the usual spherical coordinates, f_0 and $\sigma_0 \gg 1$ constants. The first term in the brackets represents the Poynting flux whereas the second one accounts for the small initial contribution of particles. At larger distances, a significant part of the Poynting flux may be converted into the kinetic energy but the total energy per particle is conserved along the streamlines and, thus, the angular distribution (1) remains unchanged.

As it was discussed in the previous sections, the wave decay is the main mechanism of the energy transformation in the pulsar wind. The post-shock MHD parameters are independent of where exactly the waves decayed therefore one can specify only the total energy flux, f_{tot}, and the mean field, B, in the wind assuming for simplicity that all the wave energy have already been converted into the flow kinetic energy, $\rho\gamma v = f_{tot} - cB^2/4\pi$. For the ultra-relativistic flow, $v \to c$, the post-shock plasma is relativistically hot and the dynamics of the downstream flow depends only on the product $\rho\gamma$. Therefore, the angular distributions of density and velocity in the wind may be chosen quite arbitrarily provided the product $\rho\gamma$ is fixed.

The distribution of the mean field in the pulsar wind is

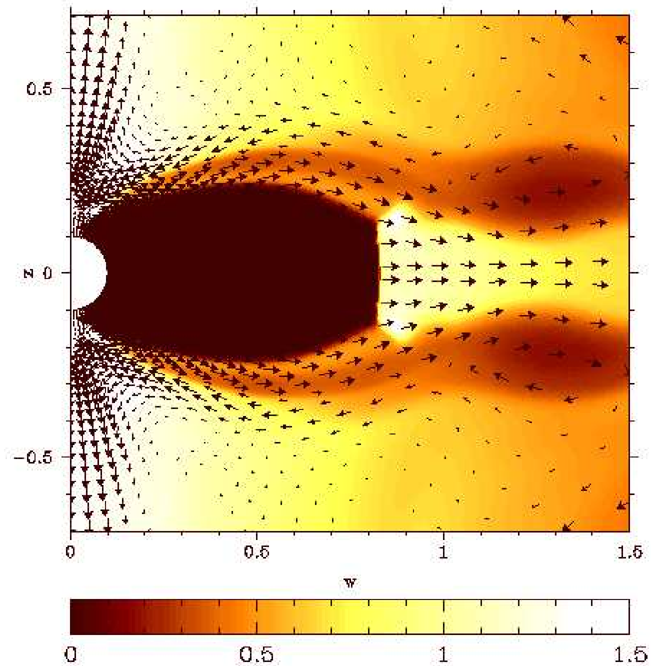

Figure 1: The flow around the termination shock (Komissarov & Lyubarsky 2003a). The flow velocity is shown by arrows, the gas pressure by color.

not known but in any case the mean field goes to zero at the equator of the flow from the obliquely rotating magnetosphere. Moreover the mean field should also vanish at the axis of the flow because a singular current cannot be presented in a realistic flow. Taking into account that the field is frozen in a radial flow, a simple model distribution was chosen for the mean field:

$$B = \sqrt{\frac{4\pi f_0}{c}}\frac{\xi}{r}\sin\theta\left(1 - \frac{2\theta}{\pi}\right) \qquad (2)$$

where the free parameter $\xi \leq 1$ controls the magnetization of the wind. The ratio of the energy transported by the mean electromagnetic fields to the energy transported by the particles is $\sigma_m = 0.1\xi^2$.

The simulated structure of he flow in the vicinity of the termination shock is shown in Fig. 1. One can see that the termination shock is highly non-spherical being significantly closer to the pulsar in the polar zone than in the equatorial plane. Most of the downstream flow is initially confined to the equatorial plane. The magnetic hoop stresses stop the outflow in the surface layers of this equatorial disk and redirect it into magnetically confined polar jets. So the jets are formed beyond the termination shock. Velocities both in the disk and in the jet were found to be about $0.5c$, close

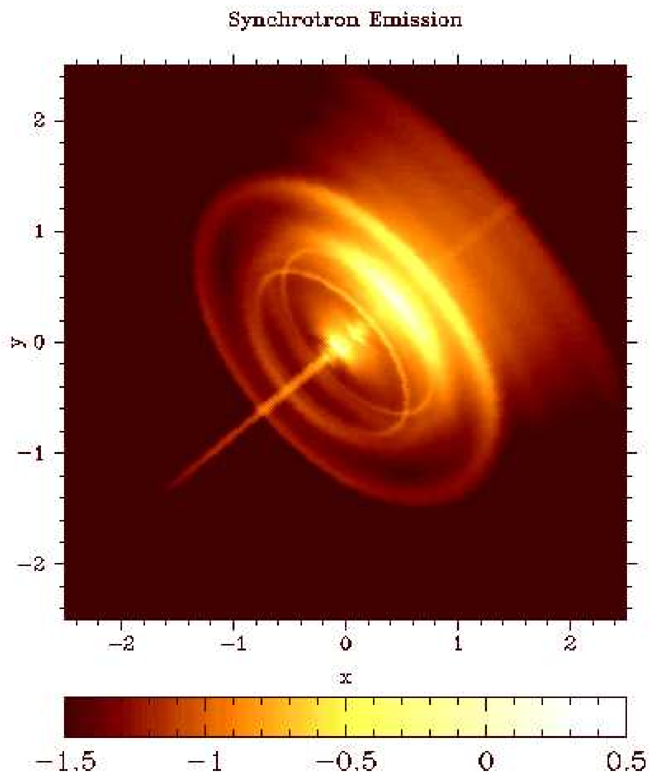

Figure 2: The simulated synchrotron image of the nebula (Komissarov & Lyubarsky 2003a).

to those inferred from the observations (Hester et al. 2002; Pavlov et al. 2003).

The synchrotron images of the nebula were also simulated, taking into account the relativistic beaming effect and particle energy losses (Fig. 2). These images are strikingly similar to the images of the Crab and other pulsar wind nebulae obtained by *Chandra* and *HST*. They exhibit both a system of rings, giving the impression of an equatorial disk-like or even toroidal structure, and well-collimated polar jets, which visually appear to originate directly from the pulsar but in fact don't. The jet is magnetically collimated; it appears only if the mean magnetization parameter is not too small, $\sigma_m \geq 4\times10^{-3}$, which is only slightly higher than the previously found estimates for the Crab wind, 1–3×10^{-3} (Kennel & Coroniti 1984a,b; Emmering & Chevalier 1987; Begelman & Li 1992; Bucciantini et al. 2003). These estimates were obtained under the assumption that the plasma flow in the nebula is axisymmetric such that the concentric magnetic tubes expand and the magnetic energy grows. The extremely low magnetization of the pulsar wind then follows from the requirement for the magnetic energy density to remain less than the plasma energy density everywhere in the nebula. Begelman (1998) suggested that these limits may be abandoned if one takes into account that the kink instability destroys the axisymmetry and makes possible the flow expansion without the magnetic field growth; in this case the magnetization of the flow in the post-shock region may be not so small.

The simulated maps reveal a bright central source which has nothing to do with emission from the pulsar itself. This is the relativistically beamed emission from the high velocity plasma passed the shock at high latitudes. The plasma enters the shock there at small angle and therefore the post-shock velocity remains high, 0.8–0.9*c*. This region is projected very close to the pulsar and may be identified with the very bright knot discovered by Hester et al. (1995) 150 AU to the southeast of the Crab pulsar. All these results show that the MHD model captures many properties of the Crab nebula quite well and in spite of some quantitative differences with the observational data, is basically correct.

5 Conclusion

There is still no a solid answer on the question of how the rotational energy of the pulsar is transferred to radiating particles in the PWN. The picture outlined here may be considered only as hypothetical. The important assumption is that the energy flux in the pulsar wind has a maximum at the equator of the flow; this is supported both by the X-ray image of the Crab and by available highly idealized models for relativistic, magnetized winds. In the equatorial belt, the energy is transferred predominantly by alternating magnetic fields therefore dissipation of these fields should be considered as the main energy conversion mechanism in the pulsar wind. The structure of the flow in the nebula is essentially independent of where the waves decayed; the only important point is that the residual magnetization in the equatorial belt is rather low, well in line with conventional wisdom, whereas in the polar region the magnetization may be significant enough for the collimating hoop stress to become dynamically important downstream of the termination shock. The observed jet-torus structure naturally builds up in such a flow.

More detailed information about the structure of the pulsar wind and the physical processes within it may be obtained from the study of the termination shock. The self-consistent theory should explain how the observed very wide energy spectrum of radiating electrons is formed. The evidence has been accumulating that the electrons radiating from the radio to the γ-ray band are accelerating together. This places severe limits on the pulsar wind parameters and particle acceleration mechanisms. Variable small-scale structures observed in the vicinity of the termination shock also provide important clues about the physics of the pulsar wind.

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